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Existence and Stability of Periodic Solution for Cohen-Grossberg Neural Networks with Time-Varying and Distributed Delays[1]

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Abstract

In this paper, a generalized model of Cohen-Grossberg neural networks with periodic coefficients and both time-varying and distributed delays is investigated. By employing Mawhin's continuation theorem, analytic methods, inequality technique and M-matrix theory, some sufficient conditions ensuring the existence, uniqueness and global exponential stability of the periodic oscillatory solution for Cohen-Grossberg neural networks with both time-varying and distributed delays are obtained. Two examples are given to show the effectiveness of the obtained results.

Keywords: Cohen-Grossberg neural networks; delays; global exponential stability; periodic oscillatory solution

1 Introduction

In recent years, the dynamical characteristic such as stability and periodicity of Hopfield network, cellular neural network and bidirectional associative memory neural network play an important rule in the pattern recognition, associative memory, and combinatorial optimization (see, e.g., [1]-[17], and the references cited therein). Among models of neural networks, the Cohen-Grossberg neural

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network [18] is an important one, which can be described by the following ordinary differential equations:

$$\frac{d_i(x_i(t))}{dt} = -a_i(x_i(t))\Big[a_i(x_i(t)) - \sum_{j=1}^{n} c_{ij} f_j(x_j(t))\Big], \quad i = 1, 2, \cdots, n, \quad (1.1)$$

where, $n \geq 2$ is the number of neurons in the network; $x_i(t)$ denotes the state variable of the ith neuron at time t ; $f_j(x_j(t))$ denotes the activation function of the j th neuron at time t; the feedback matrix $C = (c_{ij})_{n\times n}$ indicates the strength of the neuron interconnections within the network; $a_i(x_i(t))$ represents an amplification function; $b_i(x_i(t))$ is an appropriately behaved function such that the solutions of model (1.1) remain bounded.

Due to their promising potential applications in areas such as pattern recognition and optimization. The network (1.1) have attracted increasing interest in scientific community (see, e.g., [18]-[31], and references cited therein).

In reality, time delays inevitably exist in biological and artificial neural networks due to the finite switching speed of neurons and amplifiers. It is also important to incorporate time delay in various neural networks. In recent years, there exist some results on global asymptotical stability, global exponential stability and periodic solutions for the neural networks with constant delays or time-varying delays (see, e.g., [1]-[6],[8],[10]-[17],[19]-[24],[29]-[31]). Although the use of finite delays in models with delayed feedback provides a good approximation to simple circuits consisting of a small number of neurons, neural networks usually should have a spatial extent due to the presence of a multitude of parallel pathways with a variety of axon sizes and lengths. Thus, there will be a distributed of propagation delays in finite or/and infinite time (see, e.g., [7],[9], [25]-[28]). In the case, the signal propagation is no longer instantaneous and cannot be modeled with finite delays or infinite delays. A more appropriate and ideal way is to incorporate finite delays and infinite delays (see, e.g., [27, 28]). However, as we well know, besides delay effect, the nonautonomous phenomenon often occurs in many realistic systems. From the view point of reality, it should also be taken into account evolutionary processes of some practical systems as well as disturbances of external influence such as varying environment of biological systems and so on. Particularly, when we consider a long-time dynamical behavior of a system, the parameters of the system usually will arise change along with time. In addition, in many applications, the property of periodic oscillatory solutions of a neural networks also is great interest. Therefore, the research on the nonautonomous neural networks with delays is very important in like manner.

Motivated by the above discussions, in this paper, we consider a class of periodic Cohen-Grossberg neural networks with both variable and distributed delays described by the following system of integro-differential equations:

$$\begin{aligned}\frac{dx_i(t)}{dt} &= -\alpha_i(x_i(t))\Big[\beta_i(x_i(t)) - \sum_{j=1}^{n} a_{ij}(t) f_j(x_j(t)) \\ &\quad - \sum_{j=1}^{n} b_{ij}(t) f_j(x_j(t-\tau_{ij}(t))) \\ &\quad - \sum_{j=1}^{n} c_{ij}(t) \int_{-\infty}^{t} K_{ij}(t-s) f_j(x_j(s))ds + I_i(t)\Big] \qquad (1.2)\end{aligned}$$

for $i = 1, 2 \cdots, n$. Where $n \geq 2$ is the number of neurons in the network, $x_i(t)$ is the state of the ith neuron at time t; f_i denotes the activation function; $\alpha_i(x_i(t))$ presents an amplification function; $\beta_i(x_i(t))$ is an appropriately behaved function; $I_i(t)$ denotes external input to the $i$the neuron; $a_{ij}(t), b_{ij}(t), c_{ij}(t)$ denote the connection strengths of the jth neuron on the ith neuron, respectively; $\tau_{ij}(t)$ corresponds to the transmission delay and satisfies $0 \leq \tau_{ij}(t) \leq \tau$ (τ is a constant); K_{ij} is the delay kernel.

To the best of our knowledge, few authors have considered dynamical behavior of the periodic Cohen-Grossberg neural networks with both variable and distributed delays. This paper studies the existence, uniqueness and global exponential stability of the periodic oscillatory solution for the periodic Cohen-Grossberg neural networks with both variable and distributed delays. Several sufficient conditions ensuring the existence, uniqueness and global exponential stability of the periodic oscillatory solution will be established for the system (1.2).

The rest of this paper is organized as follows. In section 2, we introduce some notations and preliminaries. We shall use Mawhin's continuation theorem [34] to establish the existence of periodic solutions of model (1.2) in section 3. In section 4, we give stability analysis of the periodic oscillatory solution. Remarks and examples are given to illustrate our theory in section 5. Finally, in section 6 we give the conclusion.

2 Preliminaries

Throughout this paper we assume that:

(**H1**) Each function $\alpha_i(u)$ is bounded, positive and globally Lipschitz continuous, i.e. there exist constants $\underline{\alpha}_i$, $\bar{\alpha}_i$ *and* L_i such that

$$0 < \underline{\alpha}_i \leq \alpha_i(u) \leq \bar{\alpha}_i < +\infty, \ for\ u \in R, \ i = 1, 2, \cdots, n,$$

$$|\alpha_i(u) - \alpha_i(v)| \leq L_i|u - v|, \ for\ u, v \in R, \ i = 1, 2, \cdots, n.$$

(**H2**) For each function $\beta_i(u) \in C^1(R, R)$, the inverse function $\beta^{-1}(\cdot)$ is locally Lipschitz continuous, and there exists a positive constant β_i such that $\dot{\beta}_i(u) \geq \beta_i > 0$. For each $i \in \{1, 2, \cdots, n\}$, $u = 0$ supposed to a zero point of $\beta_i(u)$, moreover, $\dot{\beta}_i(u)$ locally exists at $u = 0$ (Due to the monotonicity of $\beta_i(u)$, its zero point is unique).

(**H3**) $a_{ij}(t), b_{ij}(t), c_{ij}(t), \tau_{ij}(t), I_i(t)$ are continuously periodic functions defined on $t \in [0, +\infty)$ with common period $\omega > 0$, $i, j = 1, 2, \cdots, n.$

(**H4**) Each function $f_i(u)$ is globally Lipschitz continuous, i.e. there exists a constant $F_i > 0$ such that

$$|f_i(u) - f_i(v)| \leq F_i|u - v|, \quad i = 1, 2, \cdots, n.$$

(**H5**) The delay kernel $K_{ij} : [0, +\infty) \to [0, +\infty)$ is piecewise continuous function and satisfies:

(i) $\int_0^\infty K_{ij}(s)ds = 1, \qquad i, j = 1, 2, \cdots, n.$

(ii) $\int_0^\infty sK_{ij}(s)ds < \infty, \qquad i, j = 1, 2, \cdots, n.$

(iii) There exists a positive number μ such that

$$\int_0^\infty se^{\mu s}K_{ij}(s)ds < \infty, \qquad i, j = 1, 2, \cdots, n.$$

(**H6**) $E - \Gamma$ is a nonsingular M-matrix, where E is an identical matrix and $\Gamma = (\Gamma_{ij})$ is in the form of

$$\Gamma_{ij} = \frac{1}{\beta_i}\Big(|\overline{a}_{ij}| + |\overline{b}_{ij}| + |\overline{c}_{ij}|\Big)F_j, \quad i, j = 1, 2, \cdots, n.$$

Let $PC = C((-\infty, 0], R^n)$ be the linear space of bounded and continuous functions which map $(-\infty, 0]$ into R^n. The initial conditions associated with model (1.2) are of the form

$$x_i(t) = \varphi_i(t), \qquad -\infty < t \leq 0 \tag{2.3}$$

in which $\varphi_i(\cdot)$ is bounded continuous $(i = 1, 2, \cdots, n)$. For $\varphi \in PC$, $\|\varphi\|$ is defined as

$$\|\varphi\| = \sup_{-\infty<s\leq 0} \Big(\sum_{i=1}^{n} |\varphi_i(s)|^r \Big)^{\frac{1}{r}},$$

then PC is a Banach space of continuous functions which map $(-\infty, 0]$ into R^n with the topology of uniform convergence.

To begin with, we introduce some notations and recall some basic definitions.

For an $n \times n$ matrix A, $|A|$ denotes the absolute value matrix given by $|A| = (|a_{ij}|)_{n\times n}$. Let $h(t)$ be a continuous periodic ω-function, we denote

$$|\underline{h}| = \min_{t\in[0,\omega]} |h(t)|, \qquad |\overline{h}| = \max_{t\in[0,\omega]} |h(t)|.$$

Definition 2.1 *A function $x : (-\infty, +\infty) \to R^n$ is said to be the special solution of system* (1.2) *with initial condition* (2.1) *if x is a continuous function and satisfies model* (1.2) *for $t \geq 0$, and $x(s) = \varphi(s)$ for $s \in (-\infty, 0]$.*

Henceforth, we let $x(t, \varphi)$ denote the special solution of (1.2) with initial condition $\varphi \in PC$.

Definition 2.2 *The periodic solution $x(t, \varphi)$ of system* (1.2) *is said to be globally exponentially stable, if there exist positive constants ε and κ such that every solution $x(t, \phi)$ of* (1.2) *satisfies*

$$\|x(t, \phi) - x(t, \varphi)\| \leq \kappa\|\phi - \varphi\|e^{-\varepsilon t} \qquad for\ all\ t \geq 0.$$

Definition 2.3 [32] *A real matrix $D = (d_{ij})_{n\times n}$ is said to be a nonsingular M-matrix if $d_{ij} \leq 0$, $i, j = 1, 2, \cdots, n$, $i \neq j$, and all successive principal minors of D are positive.*

To the nonsingular M-matrix, we have

Lemma 2.1 [32] *Each of the following conditions is equivalent:*

(i) *D is a nonsingular M-matrix.*

(ii) *D^{-1} exists and D^{-1} is a nonnegative matrix.*

(iii) *The diagonal elements of D are all positive and there exists a positive vector d such that $Dd > 0$ or $D^T d > 0$.*

Lemma 2.2 [33] *Let $a, b \geq 0, p > 1$, then*

$$a^{p-1}b \leq \frac{p-1}{p}a^p + \frac{1}{p}b^p.$$

3 Existence of periodic oscillatory solution

In this section, based on the Mawhin's continuation theorem, we study the existence of at least one periodic solution of (1.2). First, we shall make some preparations.

Let X, Y be normed vector spaces, $L : \mathrm{Dom}\, L \subset X \to Y$ be a linear mapping, and $N : X \to Y$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if dim Ker L=codim Im $L < +\infty$ and Im L is closed in Y. If L is a Fredholm mapping of index zero and there exist continuous projectors $P : X \to X$ and $Q : Y \to Y$ such that Im P=Ker L, Ker Q=Im L=Im $(I-Q)$, it follows that mapping $L|_{\mathrm{Dom}L\cap\mathrm{Ker}P} : (I-P)X \to$Im L is invertible. We denote the inverse of that mapping by K_P. If Ω is an open bounded subset of X, the mapping N will be called L-compact on $\overline{\Omega}$ if $QN(\overline{\Omega})$ is bounded and $K_P(I-Q)N : \overline{\Omega} \to X$ is compact. Since Im Q is isomorphic to Ker L, there exists an isomorphism $J : \mathrm{Im}\, Q \to$ Ker L.

Now, we introduce Mawhin's continuation theorem [34, p.40] as follows.

Lemma 3.1 *Let $\Omega \subset X$ be an open bounded set and let $N : X \to Y$ be a continuous operator which is L-compact on $\overline{\Omega}$. Assume*

(a) *For each $\lambda \in (0, 1), x \in \partial\Omega \cap \mathrm{Dom}\, L, Lx \neq \lambda Nx$*

(b) *For each $x \in \partial\Omega \cap \mathrm{Ker}\ L, QNx \neq 0$*

(c) $\deg(JNQ, \Omega \cap \mathrm{Ker}\ L, 0) \neq 0$.

Then $Lx = Nx$ has at least one solution in $\overline{\Omega} \cap \mathrm{Dom}\, L$.

Theorem 3.1 *Assume that* **(H1)**-**(H6)** *hold, then the system* (1.2) *has at least one ω-periodic solution.*

Proof. To apply the continuation theorem of coincidence degree theory and establish the existence of an ω-periodic solution of (1.2), we take

$$X = Y = \{x \in C(R, R^n) : x(t + \omega) = x(t), t \in R\}$$

and denote

$$\|x\| = \sup_{t\in[0,\omega]} \{|x_i(t)|, i = 1, 2, \dots, n\},$$

then X is a Banach space. Set

$$L : \mathrm{Dom}\ \ L \cap X, \quad Lx = \dot{x}(t), \quad x \in X$$

where $\mathrm{Dom}\, L = \{x \in C^1(R, R^n)\}$, and $N : X \to X$ such that

$$\begin{aligned} Nx_i &= -\alpha_i(x_i(t))\Big[\beta_i(x_i(t)) - \sum_{j=1}^n a_{ij}(t) f_j(x_j(t)) \\ &\quad - \sum_{j=1}^n b_{ij}(t) f_j(x_j(t-\tau_{ij}(t))) \\ &\quad - \sum_{j=1}^n c_{ij}(t) \int_{-\infty}^t K_{ij}(t-s) f_j(x_j(s)) ds + I_i(t)\Big] \\ &= -\alpha_i(x_i(t)) g_i(t, x(t)), \quad i = 1, 2, \ldots, n, \end{aligned}$$

where

$$\begin{aligned} g_i(t, x(t)) &= \beta_i(x_i(t)) - \sum_{j=1}^n a_{ij}(t) f_j(x_j(t)) - \sum_{j=1}^n b_{ij}(t) f_j(x_j(t-\tau_{ij}(t))) \\ &\quad - \sum_{j=1}^n c_{ij}(t) \int_{-\infty}^t K_{ij}(t-s) f_j(x_j(s)) ds + I_i(t). \end{aligned}$$

Define two projectors P and Q as

$$Qx = Px = \frac{1}{\omega} \int_0^\omega x(s) ds, \quad x \in X.$$

Clearly, Ker $L = R^n$,

$$\mathrm{Im}\, L = \{(x_1, x_2, \ldots, x_n)^T \in X : \int_0^\omega x_i(t) dt = 0,\ i = 1, 2, \ldots, n\}$$

is closed in X. Moreover, P and Q are continuous projectors such that

$$\mathrm{Im}\ P = R^n = \mathrm{Ker}\ L, \quad \mathrm{Ker}\ Q = \mathrm{Im}\ L = \mathrm{Im}\ (I - Q)$$

and

$$\dim \mathrm{Ker}\ L = \mathrm{codim}\, \mathrm{Im}\, L = n.$$

Hence, L is a Fredholm mapping of index 0. On the other hand, it is not hard to obtain the inverse $K_P :\ \mathrm{Im}L \to \mathrm{Ker}\ P \cap \mathrm{Dom}\ L$ of $L|_{\mathrm{Dom}L \cap \mathrm{Ker}P}$ as follows:

$$(K_P x)_i(t) = \int_0^t x_i(s) ds \in \mathrm{Ker}\ P \cap \mathrm{Dom}\ L, \quad i = 1, 2, \cdots, n,$$

it follows that

$$\begin{aligned} (K_P(I-Q)Nx)_i(t) &= (K_P Nx)_i(t) - (K_P QNx)_i(t) \\ &= -\int_0^t \alpha_i(x_i(s)) g_i(s, x(s)) ds \\ &\quad + \frac{t}{\omega} \int_0^\omega \alpha_i(x_i(s)) g_i(s, x(s)) ds \end{aligned}$$

for $i = 1, 2, \cdots, n$.

Hence, $QN : X \to R^n$ and $K_p(I-Q)N : X \to X$ are both continuous, by generalizing the famous Arzela-Ascoli theorem, $QN(\overline{\Omega})$ and $K_P(I-Q)N(\overline{\Omega})$ are relatively compact for any open bounded set $\Omega \subset X$. Therefore, N is L-compact on $\overline{\Omega}$ for any open bounded set $\Omega \subset X$.

Corresponding to the operator equation $Lx = \lambda Nx,\ \lambda \in (0,1)$, we have

$$\begin{aligned}\frac{dx_i(t)}{dt} &= -\lambda\alpha_i(x_i(t))\Big[\beta_i(x_i(t)) - \sum_{j=1}^{n} a_{ij}(t)f_j(x_j(t)) \\ &\quad - \sum_{j=1}^{n} b_{ij}(t)f_j(x_j(t-\tau_{ij}(t))) \\ &\quad - \sum_{j=1}^{n} c_{ij}(t)\int_{-\infty}^{t} K_{ij}(t-s)f_j(x_j(s))ds + I_i(t)\Big] \qquad (3.1)\end{aligned}$$

for $i = 1, 2, \cdots, n$. Since for each $i \in \{1, 2, \cdots, n\}$, $x_i(t)$, as the component of $x(t)$, is continuously differentiable and $x_i(0) = x_i(\omega)$, so that, there exists $t_i \in [0, \omega]$ such that $|x_i(t_i)| = \max_{t\in[0,\omega]} |x_i(t)|$ and $\dot{x}_i(t_i) = 0$, so we have

$$0 = -\lambda\alpha_i(x_i(t_i))g_i(t_i, x(t_i)).$$

From (**H1**) and $\lambda \in (0,1)$, we get

$$\begin{aligned}\beta_i(x_i(t_i)) &= \sum_{j=1}^{n} a_{ij}(t_i)f_j(x_j(t_i)) + \sum_{j=1}^{n} b_{ij}(t_i)f_j(x_j(t_i-\tau_{ij}(t_i))) \\ &\quad + \sum_{j=1}^{n} c_{ij}(t_i)\int_{-\infty}^{t_i} K_{ij}(t_i-s)f_j(x_j(s))ds - I_i(t_i),\end{aligned}$$

it follows that

$$\begin{aligned}x_i(t_i) &= \beta_i^{-1}\Big(\sum_{j=1}^{n} a_{ij}(t_i)f_j(x_j(t_i)) + \sum_{j=1}^{n} b_{ij}(t_i)f_j(x_j(t_i-\tau_{ij}(t_i))) \\ &\quad + \sum_{j=1}^{n} c_{ij}(t_i)\int_{-\infty}^{t_i} K_{ij}(t_i-s)f_j(x_j(s))ds - I_i(t_i)\Big) \\ &\quad -\beta_i^{-1}(0) + \beta_i^{-1}(0).\end{aligned}$$

Due to (**H2**), we know that $(\beta_i^{-1}(u))' \leq \frac{1}{\beta_i}$, $\beta_i^{-1}(u)$ is locally Lipschitz, and $\beta_i^{-1}(0) = 0$, so that

$$|x_i(t_i)| \leq \frac{1}{\beta_i}\Big|\sum_{j=1}^{n} a_{ij}(t_i)f_j(x_j(t_i)) + \sum_{j=1}^{n} b_{ij}(t_i)f_j(x_j(t_i-\tau_{ij}(t_i)))$$

$$
\begin{aligned}
&\quad + \sum_{j=1}^{n} c_{ij}(t_i) \int_{-\infty}^{t_i} K_{ij}(t_i - s) f_j(x_j(s)) ds - I_i(t_i) \Big| \\
&\leq \frac{1}{\beta_i} \Big[\sum_{j=1}^{n} |\overline{a}_{ij}| F_j |x_j(t_j)| + \sum_{j=1}^{n} |\overline{b}_{ij}| F_j |x_j(t_j)| \\
&\quad + \sum_{j=1}^{n} |\overline{c}_{ij}| F_j \int_{0}^{+\infty} K_{ij}(s) |x_j(t_j)| ds + |\overline{I}_i| \\
&\quad + \sum_{j=1}^{n} |\overline{a}_{ij}| |f_j(0)| + \sum_{j=1}^{n} |\overline{b}_{ij}| |f_j(0)| + \sum_{j=1}^{n} |\overline{c}_{ij}| |f_j(0)| \Big] \\
&= \frac{1}{\beta_i} \sum_{j=1}^{n} \Big[|\overline{a}_{ij}| + |\overline{b}_{ij}| + |\overline{c}_{ij}| \Big] F_j |x_j(t_j)| + p_i, \qquad (3.2)
\end{aligned}
$$

where $p_i = \frac{1}{\beta_i} \Big[|\overline{I}_i| + \sum\limits_{j=1}^{n} \Big(|\overline{a}_{ij}| + |\overline{b}_{ij}| + |\overline{c}_{ij}| \Big) |f_j(0)| \Big]$.

Then, by defining the vector $p = (p_1, p_2, \cdots, p_n)^T$ and through the calculation of vector inequalities, from inequality (3.2), we have

$$
(E - \Gamma)(|x_1(t_1)|, |x_2(t_2)|, \cdots, |x_n(t_n)|)^T \leq p,
$$

where $E - \Gamma$ is the nonsingular M-matrix defined in (**H6**). Let

$$
m = (m_1, m_2, \cdots, m_n)^T = (E - \Gamma)^{-1} p \geq 0,
$$

which implies that $(|x_1(t_1)|, |x_2(t_2)|, \cdots, |x_n(t_n)|)^T \leq m$, that is, $|x_i(t_i)| \leq m_i$ for $i = 1, 2, \cdots, n$. On the other hand, due to $E - \Gamma$ is a nonsingular M-matrix, so there exists a positive vector $l = (l_1, l_2, \cdots, l_n)^T > 0$ such that $(E - \Gamma) l > 0$, let $\mu = kl = (\mu_1, \mu_2, \cdots, \mu_n)^T$ be a positive vector such that $k(E - \Gamma) l = (E - \Gamma)\mu > p$.

We take

$$
\Omega = \{ x(t) \in X \ : \ |x_i(t)| < \mu_i, \forall\ t \in R,\ i = 1, 2, \cdots, n \}, \qquad (3.3)
$$

which satisfies condition (a) of Lemma 3.1. If $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T \in \partial\Omega \cap \text{Ker } L$, then $x(t)$ is a constant vector in R^n, and there exists some $i \in \{1, 2, \cdots, n\}$ such that $|x_i| = \mu_i$. It follows that

$$
\begin{aligned}
(QNx)_i &= -\frac{1}{\omega} \int_{0}^{\omega} \alpha_i(x_i) \Big[\beta_i(x_i) - \sum_{j=1}^{n} a_{ij}(t) f_j(x_j) \\
&\quad - \sum_{j=1}^{n} b_{ij}(t) f_j(x_j) - \sum_{j=1}^{n} c_{ij}(t) f_j(x_j) + I_i(t) \Big] dt.
\end{aligned}
$$

We claim that $|(QNx)_i| > 0$.

By way of contradiction, suppose that $|(QNx)_i| = 0$, i.e.,

$$-\frac{1}{\omega}\int_0^{\omega}\alpha_i(x_i)\Big[\beta_i(x_i) - \sum_{j=1}^{n} a_{ij}(t)f_j(x_j) - \sum_{j=1}^{n} b_{ij}(t)f_j(x_j) - \sum_{j=1}^{n} c_{ij}(t)f_j(x_j) + I_i(t)\Big]dt = 0.$$

Then there exists some $t^* \in [0,\omega]$ such that

$$\alpha_i(x_i)\Big[\beta_i(x_i) - \sum_{j=1}^{n} a_{ij}(t^*)f_j(x_j) - \sum_{j=1}^{n} b_{ij}(t^*)f_j(x_j) - \sum_{j=1}^{n} c_{ij}(t^*)f_j(x_j) + I_i(t^*)\Big] = 0,$$

which implies that

$$\begin{aligned}|x_i| &= \beta_i^{-1}\Big(\sum_{j=1}^{n} a_{ij}(t^*)f_j(x_j) + \sum_{j=1}^{n} b_{ij}(t^*)f_j(x_j) + \sum_{j=1}^{n} c_{ij}(t^*)f_j(x_j) - I_i(t^*)\Big)\\ &\le \frac{1}{\beta_i}\Big|\sum_{j=1}^{n}\Big[|\overline{a}_{ij}| + |\overline{b}_{ij}| + |\overline{c}_{ij}|\Big]|f_j(x_j)| + |\overline{I}_i|\Big|\\ &\le \frac{1}{\beta_i}\sum_{j=1}^{n}\Big[|\overline{a}_{ij}| + |\overline{b}_{ij}| + |\overline{c}_{ij}|\Big]F_j|x_j| + p_i\\ &= \sum_{j=1}^{n}\Gamma_{ij}|x_j| + p_i.\end{aligned}$$

Thereby, we have

$$\mu_i = |x_i| \le \sum_{j=1}^{n}\Gamma_{ij}|x_j| + p_i \le \sum_{j=1}^{n}\Gamma_{ij}\mu_j + p_i,$$

this implies that $((E-\Gamma)\mu)_i \le p_i$, which contradicts $(E-\Gamma)\mu > p$. Therefore condition (b) of Lemma 3.1 is satisfied.

Next, we intend to show that the topological degree is nonzero. To do this, defined a homotopical map $H(x,\lambda)$ ($\lambda \in [0,1]$) by

$$H(x,\lambda) = -\lambda(\alpha_1(x_1)\beta_1(x_1), \alpha_2(x_2)\beta_2(x_2)\cdots, \alpha_n(x_n)\beta_n(x_n))^T + (1-\lambda)QNx,$$

where $x = (x_1, x_2, \cdots, x_n)^T \in \overline{\Omega} \cap \text{Ker } L = \overline{\Omega} \cap R^n$.

Arbitrarily taking $x \in \text{Ker } L \cap \partial\Omega$ and $\lambda \in [0, 1]$, we have, for all $i = 1, 2, \cdots, n,\ |x_i| = \mu_i$ and

$$\begin{aligned} H(x,\lambda)_i &= -\lambda\alpha_i(x_i)\beta_i(x_i) - (1-\lambda)\alpha_i(x_i)\frac{1}{\omega}\int_0^\omega \Big[\beta_i(x_i) - \sum_{j=1}^n a_{ij}(t)f_j(x_j) \\ &\quad - \sum_{j=1}^n b_{ij}(t)f_j(x_j) - \sum_{j=1}^n c_{ij}(t)f_j(x_j) + I_i(t)\Big]dt \\ &= -\alpha_i(x_i)\beta_i(x_i) + (1-\lambda)\frac{\alpha_i(x_i)}{\omega}\Big[\sum_{j=1}^n f_j(x_j)\int_0^\omega \Big(a_{ij}(t) + b_{ij}(t) \\ &\quad + c_{ij}(t)\Big)dt - \int_0^\omega I_i(t)dt\Big]. \end{aligned}$$

Actually, we claim that $H(x, \lambda)_i \neq 0$ for all i. If this is not true, then there exists a $k \in \{1, 2, \cdots, n\}$ such that, for $t^* \in [0, \omega]$ and $\lambda \in [0, 1]$,

$$\beta_k(x_k) = (1-\lambda)\Big[\sum_{j=1}^n f_j(x_j)\Big(a_{kj}(t^*) + b_{kj}(t^*) + c_{kj}(t^*)\Big) - I_k(t^*)\Big],$$

and then

$$\begin{aligned} |x_k| &= \Big|\beta_k^{-1}\Big((1-\lambda)\Big[\sum_{j=1}^n f_j(x_j)\Big(a_{kj}(t^*) + b_{kj}(t^*) + c_{kj}(t^*)\Big) - I_k(t^*)\Big]\Big| \\ &\le \frac{1-\lambda}{\beta_k}\Big|\sum_{j=1}^n f_j(x_j)\Big(a_{kj}(t^*) + b_{kj}(t^*) + c_{kj}(t^*)\Big) - I_k(t^*)\Big| \\ &\le \frac{1}{\beta_k}\Big|\sum_{j=1}^n f_j(x_j)\Big(a_{kj}(t^*) + b_{kj}(t^*) + c_{kj}(t^*)\Big) - I_k(t^*)\Big| \\ &\le \frac{1}{\beta_k}\sum_{j=1}^n \Big[|\overline{a}_{kj}| + |\overline{b}_{kj}| + |\overline{c}_{kj}|\Big]F_j|x_j| + p_k. \end{aligned}$$

Similarly, we obtain $|x_k| \le m_k < \mu_k$, which contradicts that $|x_k| = \mu_k$, because of $x \in \text{Ker } L \cap \partial\Omega$. It is therefore concluded that $H(x, \lambda) \neq 0$ for every $x \in \text{Ker } L \cap \partial\Omega$.

Hence, using the homotopy invariance theorem, we obtain

$$\deg(JQN, \Omega \cap \text{Ker } L, 0) \neq 0.$$

To summarize, we have proved that Ω satisfies all the conditions of Lemma 3.1. This completes the proof.

4 Stability analysis for the periodic solution

In this section, sufficient conditions on the global exponential stability are deduced for the ω-periodic solutions of the CGNN with both variable and distributed delays. Let $x_i(s) = \varphi_i(s),\ i = 1, 2, \cdots, n,\ s \in (-\infty, 0]$, be the initial conditions of the neural network, where $\varphi_i : (-\infty, 0] \to R$ is a continuous function.

Suppose that $\overline{x}(t)$ is a periodic solution of system (1.2). Set $y(t) = x(t) - \overline{x}(t)$ with any solution $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T$ of System (1.2). Hence, System (1.2) can be transformed into the form as follows:

$$\begin{aligned} \frac{dy_i(t)}{dt} = & -\alpha_i(x_i(t))\Big\{\beta_i(x_i(t)) - \beta_i(\overline{x}_i(t)) - \sum_{j=1}^{n} a_{ij}(t)\Big[f_j(x_j(t)) - f_j(\overline{x}_j(t))\Big] \\ & - \sum_{j=1}^{n} b_{ij}(t)\Big[f_j(x_j(t-\tau_{ij}(t))) - f_j(\overline{x}_j(t-\tau_{ij}(t)))\Big] \\ & - \sum_{j=1}^{n} c_{ij}(t)\int_{-\infty}^{t} K_{ij}(t-s)\Big[f_j(x_j(s)) - f_j(\overline{x}_j(s))\Big]ds\Big\} \\ & - [\alpha_i(x_i(t)) - \alpha_i(\overline{x}_i(t))]g_i(t, \overline{x}(t)) \end{aligned} \tag{4.1}$$

for $i = 1, 2, \cdots, n$, and correspondingly, the initial condition becomes $y_i(s) = \psi_i(s) = \phi_i(s) - \varphi_i(s),\ s \in (-\infty, 0]$, where $\phi_i(s)$ and $\varphi_i(s)$ are, respectively, the initial condition of solutions $x_i(t)$ and $\overline{x}_i(t)$. Furthermore, from the arguments in the last section, it is not hard to obtain the estimation for the periodic solution $\overline{x}(t)$: $|g_i(t, \overline{x}(t))| \le M_i$ for all i, where $M_i = \beta_i(m_i) + \beta_i m_i$.

Theorem 4.1 *Under hypothesis* (**H1**)-(**H6**), *there exists exactly one ω-periodic solution of model* (1.2) *and all other solutions of model* (1.2) *converge exponentially to it as $t \to +\infty$, if there exist real constants $\alpha_{ij}, \beta_{ij}, \gamma_{ij}, \sigma_{ij}$ $(i, j = 1, 2, \cdots, n)$ and $r > 1$ such that $\Lambda = D - P - Q$ is a nonsingular M-matrix, where*

$D = \mathrm{diag}(d_1, d_2, \cdots, d_n)$ *with* $d_i = \underline{\alpha}_i\beta_i - L_iM_i$,

$P = \mathrm{diag}(p_1, p_2, \cdots, p_n)$ *with* $p_i = \frac{r-1}{r}\overline{\alpha}_i\sum\limits_{j=1}^{n} F_j^{\frac{r-\sigma_{ij}}{r-1}}\Big(|\overline{a}_{ij}|^{\frac{r-\alpha_{ij}}{r-1}} + |\overline{b}_{ij}|^{\frac{r-\beta_{ij}}{r-1}} + |\overline{c}_{ij}|^{\frac{r-\gamma_{ij}}{r-1}}\Big)$, $Q = (q_{ij})_{n\times n}$ *with* $q_{ij} = \frac{1}{r}\overline{\alpha}_i F_j^{\sigma_{ij}}\Big(|\overline{a}_{ij}|^{\alpha_{ij}} + |\overline{b}_{ij}|^{\beta_{ij}} + |\overline{c}_{ij}|^{\gamma_{ij}}\Big)$.

proof. Since Λ is a nonsingular M-matrix, from Lemma 2.4, we know that there exists a vector $l = (l_1, l_2, \cdots, l_n)^T > 0$ such that $\Lambda l > 0$, that is

$$l_i(d_i - p_i) - \frac{1}{r}\overline{\alpha}_i\sum_{j=1}^{n} l_j F_j^{\sigma_{ij}}\Big(|\overline{a}_{ij}|^{\alpha_{ij}} + |\overline{b}_{ij}|^{\beta_{ij}} + |\overline{c}_{ij}|^{\gamma_{ij}}\Big) > 0 \tag{4.2}$$

for $i = 1, 2, \cdots, n$. Let us define function

$$\begin{aligned} h_i(\theta) &= l_i(\frac{\theta}{r} - d_i + p_i) + \frac{1}{r}\overline{\alpha}_i \sum_{j=1}^{n} l_j F_j^{\sigma_{ij}} \Big(|\overline{a}_{ij}|^{\alpha_{ij}} \\ &\quad + |\overline{b}_{ij}|^{\beta_{ij}} e^{\tau\theta} + |\overline{c}_{ij}|^{\gamma_{ij}} \int_0^{+\infty} e^{\theta s} K_{ij}(s) ds \Big) \end{aligned} \tag{4.3}$$

for $i = 1, 2, \cdots, n$, where $\tau = \max_{1 \le i,j \le n} \{\tau_{ij}\}$. Obviously, $h_i(\theta)$ is continuous on $[0, +\infty)$, for $i = 1, 2, \cdots, n$. From (4.2) and assumption (**H5**), we know that $h_i(0) < 0,\ i = 1, 2, \cdots, n$. From the continuity of $h_i(\theta)$, we know that there exists a constant $\theta_i \in [0, +\infty)$ such that

$$\begin{aligned} h_i(\theta_i) &= l_i(\frac{\theta_i}{r} - d_i + p_i) + \frac{1}{r}\overline{\alpha}_i \sum_{j=1}^{n} l_j F_j^{\sigma_{ij}} \Big(|\overline{a}_{ij}|^{\alpha_{ij}} \\ &\quad + |\overline{b}_{ij}|^{\beta_{ij}} e^{\tau\theta_i} + |\overline{c}_{ij}|^{\gamma_{ij}} \int_0^{+\infty} e^{\theta_i s} K_{ij}(s) ds \Big) \le 0 \end{aligned} \tag{4.4}$$

for $i = 1, 2, \cdots, n$. Choose ε such that $0 < \varepsilon < \min\{\theta_1, \theta_2, \cdots, \theta_n\}$, then

$$\begin{aligned} h_i(\varepsilon) &= l_i(\frac{\varepsilon}{r} - d_i + p_i) + \frac{1}{r}\overline{\alpha}_i \sum_{j=1}^{n} l_j F_j^{\sigma_{ij}} \Big(|\overline{a}_{ij}|^{\alpha_{ij}} \\ &\quad + |\overline{b}_{ij}|^{\beta_{ij}} e^{\tau\varepsilon} + |\overline{c}_{ij}|^{\gamma_{ij}} \int_0^{+\infty} e^{\varepsilon s} K_{ij}(s) ds \Big) < 0 \end{aligned} \tag{4.5}$$

for $i = 1, 2, \cdots, n$.

Next, calculating $D^+|y_i(t)|$ along with (4.1), we have

$$\begin{aligned} \frac{d^+|y_i(t)|}{dt} &= \mathrm{sgn}(y_i(t))(y_i'(t)) \\ &= -\mathrm{sgn}(x_i(t) - \overline{x}_i(t))\alpha_i(x_i(t)) \Big\{ \beta_i(x_i(t)) - \beta_i(\overline{x}_i(t) \\ &\quad - \sum_{j=1}^{n} a_{ij}(t)[f_j(x_j(t)) - f_j(\overline{x}_j(t))] \\ &\quad - \sum_{j=1}^{n} b_{ij}(t)[f_j(x_j(t - \tau_{ij}(t))) - f_j(\overline{x}_j(t - \tau_{ij}(t)))] \\ &\quad - \sum_{j=1}^{n} c_{ij}(t) \int_{-\infty}^{t} K_{ij}(t - s)[f_j(x_j(s)) - f_j(\overline{x}_j(s))] ds \Big\} \\ &\quad - \mathrm{sgn}(x_i(t) - \overline{x}_i(t))[\alpha_i(x_i(t)) - \alpha_i(\overline{x}_i(t))] g_i(t, \overline{x}(t)) \\ &\le -\mathrm{sgn}(x_i(t) - \overline{x}_i(t))\alpha_i(x_i(t))\dot{\beta}_i(\xi_i)(x_i(t) - \overline{x}_i(t)) \end{aligned}$$

$$
\begin{aligned}
&+\Big|\alpha_i(x_i(t)\Big\{\sum_{j=1}^{n}a_{ij}(t)[f_j(x_j(t))-f_j(\overline{x}_j(t))]\\
&+\sum_{j=1}^{n}b_{ij}(t)[f_j(x_j(t-\tau_{ij}(t)))-f_j(\overline{x}_j(t-\tau_{ij}(t)))]\\
&+\sum_{j=1}^{n}c_{ij}(t)\int_{-\infty}^{t}K_{ij}(t-s)[f_j(x_j(s))-f_j(\overline{x}_j(s))]ds\Big\}\Big|\\
&+\Big|\alpha_i(x_i(t))-\alpha_i(\overline{x}_i(t))\Big|\Big|g_i(t,\overline{x}(t))\Big|\\
\le\;& -\underline{\alpha}_i\beta_i|y_i(t)|+\bar{\alpha}_i\Big\{\sum_{j=1}^{n}|a_{ij}(t)|\Big|f_j(x_j(t))-f_j(\overline{x}_j(t))\Big|\\
&+\sum_{j=1}^{n}|b_{ij}(t)|\Big|f_j(x_j(t-\tau_{ij}(t)))-f_j(\overline{x}_j(t-\tau_{ij}(t)))\Big|\\
&+\sum_{j=1}^{n}|c_{ij}(t)|\int_{-\infty}^{t}K_{ij}(t-s)\Big|f_j(x_j(s))-f_j(\overline{x}_j(s))\Big|ds\Big\}\\
&+L_iM_i|y_i(t)|\\
\le\;& -\underline{\alpha}_i\beta_i|y_i(t)|+\bar{\alpha}_i\Big\{\sum_{j=1}^{n}|\overline{a}_{ij}|F_j|x_j(t)-\overline{x}_j(t)|\\
&+\sum_{j=1}^{n}|\overline{b}_{ij}|F_j|x_j(t-\tau_{ij}(t))-\overline{x}_j(t-\tau_{ij}(t))|\\
&+\sum_{j=1}^{n}|\overline{c}_{ij}|F_j\int_{-\infty}^{t}K_{ij}(t-s)|x_j(s)-\overline{x}_j(s)|ds\Big\}\\
&+L_iM_i|y_i(t)|\\
=\;& -d_i|y_i(t)|+\overline{\alpha}_i\Big[\sum_{j=1}^{n}|\overline{a}_{ij}|F_j|y_j(t)|+\sum_{j=1}^{n}|\overline{b}_{ij}|F_j|y_j(t-\tau_{ij}(t))|\\
&+\sum_{j=1}^{n}|\overline{c}_{ij}|F_j\int_{-\infty}^{t}K_{ij}(t-s)|y_j(s)|ds\Big],
\end{aligned}
\tag{4.6}
$$

Furthermore, let $Y_i(t)=e^{\varepsilon t}|y_i(t)|^r$, and calculate the upper right Dini derivative $D^+Y_i(t)$ of $Y_i(t)$ along the solution of (1.2), from (4.5), (4.6), assumption (**H5**) and Lemma 2.5, we get

$$
\begin{aligned}
D^+Y_i(t)\;&=\;\varepsilon Y_i(t)+re^{\varepsilon t}|y_i(t)|^{r-1}\frac{d^+|y_i(t)|}{dt}\\
&\le\;\varepsilon Y_i(t)+re^{\varepsilon t}|y_i(t)|^{r-1}\Big\{-d_i|y_i(t)|+\overline{\alpha}_i\Big[\sum_{j=1}^{n}|\overline{a}_{ij}|F_j|y_j(t)|
\end{aligned}
$$

$$
\begin{aligned}
&\quad + \sum_{j=1}^{n} |\overline{b}_{ij}| F_j |y_j(t-\tau_{ij}(t))| \\
&\quad + \sum_{j=1}^{n} |\overline{c}_{ij}| F_j \int_{-\infty}^{t} K_{ij}(t-s)|y_j(s)|ds\Big]\Big\} \\
= &\quad (\varepsilon - rd_i)Y_i(t) + e^{\varepsilon t}\overline{\alpha}_i \Big[\sum_{j=1}^{n} r|\overline{a}_{ij}|F_j|y_i(t)|^{r-1}|y_j(t)| \\
&\quad + \sum_{j=1}^{n} r|\overline{b}_{ij}|F_j|y_i(t)|^{r-1}|y_j(t-\tau_{ij}(t))| \\
&\quad + \sum_{j=1}^{n} \int_{-\infty}^{t} K_{ij}(t-s) r|\overline{c}_{ij}|F_j|y_i(t)|^{r-1}|y_j(s)|ds\Big] \\
= &\quad (\varepsilon - rd_i)Y_i(t) \\
&\quad + e^{\varepsilon t}\overline{\alpha}_i \Big[\sum_{j=1}^{n} r\Big(|\overline{a}_{ij}|^{\frac{r-\alpha_{ij}}{r(r-1)}} F_j^{\frac{r-\sigma_{ij}}{r(r-1)}}|y_i(t)|\Big)^{r-1}\Big(|\overline{a}_{ij}|^{\frac{\alpha_{ij}}{r}} F_j^{\frac{\sigma_{ij}}{r}}|y_j(t)|\Big) \\
&\quad + \sum_{j=1}^{n} r\Big(|\overline{b}_{ij}|^{\frac{r-\beta_{ij}}{r(r-1)}} F_j^{\frac{r-\sigma_{ij}}{r(r-1)}}|y_i(t)|\Big)^{r-1}\Big(|\overline{b}_{ij}|^{\frac{\beta_{ij}}{r}} F_j^{\frac{\sigma_{ij}}{r}}|y_j(t-\tau_{ij}(t))|\Big) \\
&\quad + \sum_{j=1}^{n} \int_{-\infty}^{t} K_{ij}(t-s) r\Big(|\overline{c}_{ij}|^{\frac{r-\gamma_{ij}}{r(r-1)}} F_j^{\frac{r-\sigma_{ij}}{r(r-1)}}|y_i(t)|\Big)^{r-1} \\
&\quad \times\Big(|\overline{c}_{ij}|^{\frac{\gamma_{ij}}{r}} F_j^{\frac{\sigma_{ij}}{r}}|y_j(s)|\Big)ds\Big] \\
\leq &\quad (\varepsilon - rd_i)Y_i(t) \\
&\quad + e^{\varepsilon t}\overline{\alpha}_i \Big[\sum_{j=1}^{n} (r-1)|\overline{a}_{ij}|^{\frac{r-\alpha_{ij}}{r-1}} F_j^{\frac{r-\sigma_{ij}}{r-1}}|y_i(t)|^r + \sum_{j=1}^{n} |\overline{a}_{ij}|^{\alpha_{ij}} F_j^{\sigma_{ij}}|y_j(t)|^r \\
&\quad + \sum_{j=1}^{n} (r-1)|\overline{b}_{ij}|^{\frac{r-\beta_{ij}}{r-1}} F_j^{\frac{r-\sigma_{ij}}{r-1}}|y_i(t)|^r + \sum_{j=1}^{n} |\overline{b}_{ij}|^{\beta_{ij}} F_j^{\sigma_{ij}}|y_j(t-\tau_{ij}(t))|^r \\
&\quad + \sum_{j=1}^{n} \int_{-\infty}^{t} K_{ij}(t-s)(r-1)|\overline{c}_{ij}|^{\frac{r-\gamma_{ij}}{r-1}} F_j^{\frac{r-\sigma_{ij}}{r-1}}|y_i(t)|^r \\
&\quad + \sum_{j=1}^{n} \int_{-\infty}^{t} K_{ij}(t-s)|\overline{c}_{ij}|^{\gamma_{ij}} F_j^{\sigma_{ij}}|y_j(s)|^r ds\Big] \\
= &\quad Y_i(t)\Big[(\varepsilon - rd_i) + (r-1)\overline{\alpha}_i \sum_{j=1}^{n} F_j^{\frac{r-\sigma_{ij}}{r-1}} \\
&\quad \times\Big(|\overline{a}_{ij}|^{\frac{r-\alpha_{ij}}{r-1}} + |\overline{b}_{ij}|^{\frac{r-\beta_{ij}}{r-1}} + |\overline{c}_{ij}|^{\frac{r-\gamma_{ij}}{r-1}}\Big)\Big] \\
&\quad + \overline{\alpha}_i \sum_{j=1}^{n} F_j^{\sigma_{ij}}\Big(|\overline{a}_{ij}|^{\alpha_{ij}} e^{\varepsilon t}|y_j(t)|^r + |\overline{b}_{ij}|^{\beta_{ij}} e^{\varepsilon\tau_{ij}} e^{\varepsilon(t-\tau_{ij})}|y_j(t-\tau_{ij}(t))|^r
\end{aligned}
$$

$$
\begin{aligned}
& +|\overline{c}_{ij}|^{\gamma_{ij}}\int_{-\infty}^{t} e^{\varepsilon(t-s)}K_{ij}(t-s)e^{\varepsilon s}|y_j(s)|^r ds\Big) \\
\leq\ & Y_i(t)\Big[(\varepsilon - rd_i) + (r-1)\overline{\alpha}_i\sum_{j=1}^{n} F_j^{\frac{r-\sigma_{ij}}{r-1}} \\
& \times\Big(|\overline{a}_{ij}|^{\frac{r-\alpha_{ij}}{r-1}} + |\overline{b}_{ij}|^{\frac{r-\beta_{ij}}{r-1}} + |\overline{c}_{ij}|^{\frac{r-\gamma_{ij}}{r-1}}\Big)\Big] \\
& +\overline{\alpha}_i\sum_{j=1}^{n} F_j^{\sigma_{ij}}\Big(|\overline{a}_{ij}|^{\alpha_{ij}}Y_j(t) + |\overline{b}_{ij}|^{\beta_{ij}}e^{\varepsilon\tau}Y_j(t-\tau_{ij}(t)) \\
& +|\overline{c}_{ij}|^{\gamma_{ij}}\int_{-\infty}^{t} e^{\varepsilon(t-s)}K_{ij}(t-s)Y_i(s)ds\Big) \\
=\ & r\Big\{Y_i(t)\Big[(\frac{\varepsilon}{r} - d_i) + \frac{r-1}{r}\overline{\alpha}_i\sum_{j=1}^{n} F_j^{\frac{r-\sigma_{ij}}{r-1}} \\
& \times\Big(|\overline{a}_{ij}|^{\frac{r-\alpha_{ij}}{r-1}} + |\overline{b}_{ij}|^{\frac{r-\beta_{ij}}{r-1}} + |\overline{c}_{ij}|^{\frac{r-\gamma_{ij}}{r-1}}\Big)\Big] \\
& +\frac{1}{r}\overline{\alpha}_i\sum_{j=1}^{n} F_j^{\sigma_{ij}}\Big(|\overline{a}_{ij}|^{\alpha_{ij}}Y_j(t) + |\overline{b}_{ij}|^{\beta_{ij}}e^{\varepsilon\tau}Y_j(t-\tau_{ij}(t)) \\
& +|\overline{c}_{ij}|^{\gamma_{ij}}\int_{-\infty}^{t} e^{\varepsilon(t-s)}K_{ij}(t-s)Y_i(s)ds\Big)\Big\} \\
=\ & r\Big[Y_i(t)\Big(\frac{\varepsilon}{r} - d_i + p_i\Big) + \frac{1}{r}\overline{\alpha}_i\sum_{j=1}^{n} F_j^{\sigma_{ij}}\Big(|\overline{a}_{ij}|^{\alpha_{ij}}Y_j(t) \\
& +|\overline{b}_{ij}|^{\beta_{ij}}e^{\varepsilon\tau}Y_j(t-\tau_{ij}(t)) + |\overline{c}_{ij}|^{\gamma_{ij}}\int_{-\infty}^{t} e^{\varepsilon(t-s)}K_{ij}(t-s)Y_i(s)ds\Big)\Big] \quad (4.7)
\end{aligned}
$$

for $i = 1, 2, \cdots, n$.

Defining the curve $\Gamma = \Big\{z(k) = (kl_1, kl_2, \cdots, kl_n) \ : \ k > 0\Big\}$ and the set $\Theta(z) = \Big\{u \ : \ 0 \leq u \leq z, \ z \in \Gamma\Big\}$. It is obvious that $\Theta(z(k)) \supset \Theta(z(k'))$ as $k > k'$. Let $k_0 = \frac{(1+\delta)\|\phi-\varphi\|^r}{\min\limits_{1\leq i\leq n}\{l_i\}}$ (δ is a positive constant), then

$$
Y_i(s) = e^{\varepsilon s}|x_i(s) - \overline{x}_i(s)|^r \leq \|\phi - \varphi\|^r < l_i k_0, \quad -\infty < s \leq 0, \tag{4.8}
$$

for $i = 1, 2, \cdots, n$.

In the following, we will prove that

$$
Y_i(t) < l_i k_0, \quad i = 1, 2, \cdots, n \tag{4.9}
$$

for $t > 0$. No loss of generality, we assume that there exist some i_0 and $t^* > 0$ such that

$$
Y_{i_0}(t^*) = l_{i_0}k_0, \quad D^+Y_{i_0}(t^*) \geq 0,
$$

$$
Y_i(t) \leq l_i k_0, \quad -\infty < t \leq t^*, \quad i = 1, 2, \cdots, n.
$$

Then, from (4.7) and (4.5), we get

$$
\begin{aligned}
D^+Y_{i_0}(t^*) \leq\ & r\Big[Y_{i_0}(t^*)\Big(\frac{\varepsilon}{r}-d_{i_0}+p_{i_0}\Big)+\frac{1}{r}\overline{\alpha}_i\sum_{j=1}^{n}F_j^{\sigma_{ij}}\Big(|\overline{a}_{ij}|^{\alpha_{ij}}Y_j(t^*)\\
&+|\overline{b}_{ij}|^{\beta_{ij}}e^{\varepsilon\tau}Y_j(t^*-\tau_{ij}(t^*))\\
&+|\overline{c}_{ij}|^{\gamma_{ij}}\int_{-\infty}^{t^*}e^{\varepsilon(t^*-s)}K_{ij}(t^*-s)Y_i(s)ds\Big)\Big]\\
\leq\ & r\Big[\Big(\frac{\varepsilon}{r}-d_{i_0}+p_{i_0}\Big)l_{i_0}k_0+\frac{1}{r}\overline{\alpha}_i\sum_{j=1}^{n}F_j^{\sigma_{ij}}\Big(|\overline{a}_{ij}|^{\alpha_{ij}}l_jk_0\\
&+|\overline{b}_{ij}|^{\beta_{ij}}e^{\varepsilon\tau}l_jk_0+|\overline{c}_{ij}|^{\gamma_{ij}}\int_{-\infty}^{t^*}e^{\varepsilon(t^*-s)}K_{ij}(t^*-s)l_jk_0ds\Big)\Big]\\
&=rk_0\Big[l_{i_0}\Big(\frac{\varepsilon}{r}-d_{i_0}+p_{i_0}\Big)+\frac{1}{r}\overline{\alpha}_i\sum_{j=1}^{n}l_jF_j^{\sigma_{ij}}\Big(|\overline{a}_{ij}|^{\alpha_{ij}}\\
&+|\overline{b}_{ij}|^{\beta_{ij}}e^{\varepsilon\tau}+|\overline{c}_{ij}|^{\gamma_{ij}}\int_{0}^{+\infty}e^{\varepsilon s}K_{ij}(s)ds\Big)\Big]<0,
\end{aligned}
$$

this is a contradiction, so (4.9) holds. Let $\kappa=\left(\frac{(1+\delta)\sum_{i=1}^{n}l_i}{\min_{1\leq i\leq n}\{l_i\}}\right)^{\frac{1}{r}}$, from (4.9) we get

$$
\begin{aligned}
\|x(t)-\overline{x}(t)\| &= \Big(\sum_{i=1}^{n}|x_i(t)-\overline{x}_i(t)|^r\Big)^{\frac{1}{r}}\\
&\leq \Big(\sum_{i=1}^{n}k_0l_ie^{-\varepsilon t}\Big)^{\frac{1}{r}}\\
&= \left(\frac{(1+\delta)\sum_{i=1}^{n}l_i}{\min_{1\leq i\leq n}\{l_i\}}\right)^{\frac{1}{r}}\|\phi-\varphi\|e^{-\frac{\varepsilon}{r}t}\\
&= \kappa\|\phi-\varphi\|e^{-\frac{\varepsilon}{r}t},
\end{aligned}
$$

that is

$$\|x(t)-\overline{x}(t)\|\leq\kappa\|\phi-\varphi\|e^{-\frac{\varepsilon}{r}t} \tag{4.10}$$

for $t\geq 0$. The proof is completed.

Corollary 4.1 *Under hypothesis* (**H1**)-(**H6**)*, there exists exactly one ω-periodic solution of model* (1.2) *and all other solutions of model* (1.2) *converge exponentially to it as $t\to+\infty$, if $\Lambda_1=D-\overline{\alpha}(|\bar{A}|+|\bar{B}|+|\bar{C}|)F$ is a nonsingular M-matrix, where $D=\mathrm{diag}(d_1,d_2,\cdots,d_n)$ with $d_i=\underline{\alpha}_i\beta_i-L_iM_i$, $|\bar{A}|=(|\overline{a}_{ij}|)_{n\times n}$, $|\bar{B}|=(|\overline{b}_{ij}|)_{n\times n}$, $|\bar{C}|=(|\overline{c}_{ij}|)_{n\times n}$, $F=\mathrm{diag}(F_1,F_2,\cdots,F_n)$, $\overline{\alpha}=\mathrm{diag}(\overline{\alpha}_1,\overline{\alpha}_2,\cdots,\overline{\alpha}_n)$.*

proof. Take $\alpha_{ij} = \beta_{ij} = \gamma_{ij} = \sigma_{ij} = 1$, and let $r \to 1^+$, then Λ in Theorem 4.1 turns to Λ_1. The proof is completed.

As $\alpha_i(x_i(t)) = \alpha_i,\ \beta_i(x_i(t)) = d_i(t)x_i(t)$, model (1.2) may reduce to the following model:

$$\begin{aligned}\frac{dx_i(t)}{dt} &= -d_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t)f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(t)f_j(x_j(t-\tau_{ij}(t))) \\ &\quad + \sum_{j=1}^{n} c_{ij}(t)\int_{-\infty}^{t} K_{ij}(t-s)f_j(x_j(s))ds + I_i(t), \qquad (4.11)\end{aligned}$$

for $i = 1, 2, \cdots, n$, where $d_i(t) > 0$ is continuously periodic functions defined on $t \in [0, +\infty)$ with common period $\omega > 0,\ i = 1, 2, \cdots, n$. For model (4.11), by applying Theorem 3.2 and Theorem 4.1, we can easily obtain the following results.

Theorem 4.2 *Under hypothesis* (**H3**)-(**H5**)*, there exists exactly one ω-periodic solution of model* (4.11) *and all other solutions of model* (4.11) *converge exponentially to it as $t \to +\infty$, if there exist real constants $\alpha_{ij}, \beta_{ij}, \gamma_{ij}, \sigma_{ij}$ $(i, j = 1, 2, \cdots, n)$ and $r > 1$ such that $\Lambda = D - P - Q$ is a nonsingular M-matrix, where*

$D = \text{diag}(\underline{d}_1, \underline{d}_2, \cdots, \underline{d}_n)$,

$P = \text{diag}(p_1, p_2, \cdots, p_n)$ *with*

$p_i = \frac{r-1}{r}\sum_{j=1}^{n} F_j^{\frac{r-\sigma_{ij}}{r-1}}\left(|\overline{a}_{ij}|^{\frac{r-\alpha_{ij}}{r-1}} + |\overline{b}_{ij}|^{\frac{r-\beta_{ij}}{r-1}} + |\overline{c}_{ij}|^{\frac{r-\gamma_{ij}}{r-1}}\right)$,

$Q = (q_{ij})_{n\times n}$ *with* $q_{ij} = \frac{1}{r}F_j^{\sigma_{ij}}\left(|\overline{a}_{ij}|^{\alpha_{ij}} + |\overline{b}_{ij}|^{\beta_{ij}} + |\overline{c}_{ij}|^{\gamma_{ij}}\right)$.

Corollary 4.2 *Under hypothesis* (**H3**)-(**H5**)*, there exists exactly one ω-periodic solution of model* (4.11) *and all other solutions of model* (4.11) *converge exponentially to it as $t \to +\infty$, if $D - (|\bar{A}| + |\bar{B}| + |\bar{C}|)F$ is a nonsingular M-matrix, where*

$$D = \text{diag}(\underline{d}_1, \underline{d}_2, \cdots, \underline{d}_n), \qquad |\bar{A}| = (|\overline{a}_{ij}|)_{n\times n}, \qquad |\bar{B}| = (|\overline{b}_{ij}|)_{n\times n},$$

$$|\bar{C}| = (|\overline{c}_{ij}|)_{n\times n}, \qquad F = \text{diag}(F_1, F_2, \cdots, F_n).$$

5 Remarks and examples

Remark 5.1 Some famous neural network models become a special case of model (1.2). For example, Refs. [19, 20, 23, 25, 26], and as model (1.2) becomes neural networks model (4.11), it contains those models studied by many authors, see, for example, Refs. [1]-[17]. Thus the results of this paper can be applied to the recurrent neural networks with and/or without delays. Moreover, our results need only the activation function f_i satisfies the assumption (**H4**), not requiring the activation function f_j to be bounded and monotone nondecreasing. In addition, we do not demand that variable delay function $\tau_{ij}(t)$ is differentiable. Therefore, we improve some previous results.

Remark 5.2 In [26], the authors considered a special case model (1.2) as $(a_{ij}(t))_{n\times n} = (b_{ij}(t))_{n\times n} = 0,\ (c_{ij}(t))_{n\times n} = (c_{ij})_{n\times n}$, the sufficient conditions given in Theorem 4.3-Theorem 4.5 not only require f_j to be bounded, but also rely on the estimation of constant $\bar{C}_i$, but $\bar{b}_i$ in the definition of $\bar{C}_i$ depends on the value of solution $x(t)$ at $t = 0$ (Remark 4.2 in [26, p.11]). Hence the estimation of constant $\bar{C}_i$ is difficult when $\alpha_i(x)$ is not a constant. The estimation of constant M_i in this paper is independent of the solution of model (1.2).

In order to illustrate the feasibility of our above-established criteria in the preceding sections, we provide concrete examples. Although the selection of the coefficients and functions in the examples is somewhat artificial, the possible application of our theoretical theory is clearly expressed.

Example 5.1 Consider the following model

$$\begin{aligned}\frac{dx_i(t)}{dt} &= -\alpha_i(x_i(t))\Big[\beta_i(x_i(t)) - \sum_{j=1}^{n} a_{ij}(t)f_j(x_j(t)) \\ &\quad - \sum_{j=1}^{n} b_{ij}(t)f_j(x_j(t-\tau_{ij}(t))) \\ &\quad - \sum_{j=1}^{n} c_{ij}(t)\int_{-\infty}^{t} K_{ij}(t-s)f_j(x_j(s))ds + I_i(t)\Big],\ i = 1,2, \qquad (5.1)\end{aligned}$$

where the coefficients and functions are taken as

$$\alpha_1(x) = \alpha_2(x) = 2 + \frac{1}{10\pi}\arctan x, \quad \underline{\alpha}_1 = \underline{\alpha}_2 = 1, \quad \overline{\alpha}_1 = \overline{\alpha}_2 = 3,$$

$$L_1 = L_2 = \frac{1}{30}, \quad \beta_1(x) = \beta_2(x) = x, \quad \beta_1 = \beta_2 = 1,$$

$$f_1(x) = f_2(x) = \frac{1}{2}(|x+1| + |x-1|), \quad F_1 = F_2 = 1,$$

$$(a_{ij}(t)) = \begin{pmatrix} -\frac{1}{24}\sin t & -\frac{1}{24}\cos t \\ -\frac{1}{24}\sin 2t & -\frac{1}{24}\cos 4t \end{pmatrix}, (b_{ij}(t)) = \begin{pmatrix} -\frac{1}{24}\sin t & -\frac{1}{24}\cos t \\ -\frac{1}{24}\sin 2t & -\frac{1}{24}\cos 4t \end{pmatrix},$$

$$(c_{ij}(t)) = \begin{pmatrix} -\frac{1}{24}\sin t & -\frac{1}{24}\cos t \\ -\frac{1}{24}\sin 2t & -\frac{1}{24}\cos 4t \end{pmatrix}, \quad (\tau_{ij}(t)) = \begin{pmatrix} \cos^2 t & 2\cos^2 t \\ 3\sin^2 t & 4\sin^2 t \end{pmatrix},$$

$$(K_{ij}(s)) = \begin{pmatrix} e^{-2s} & e^{-s} \\ e^{-s} & e^{-2s} \end{pmatrix}, \quad I(t) = \begin{pmatrix} I_1(t) \\ I_2(t \end{pmatrix} = \begin{pmatrix} \frac{5}{12}\sin^2 t \\ \frac{5}{12}\cos t \end{pmatrix}.$$

It is not hard to verify the validity of *(***H1***)-(***H5***)*, and it is easy to calculate that

$$(|\overline{a}_{ij}|) = (|\overline{b}_{ij}|) = (|\overline{b}_{ij}|) = \begin{pmatrix} \frac{1}{24} & \frac{1}{24} \\ \frac{1}{24} & \frac{1}{24} \end{pmatrix}, \quad |\overline{I}| = \begin{pmatrix} \frac{5}{12} \\ \frac{5}{12} \end{pmatrix}.$$

It follows that

$$E-\Gamma=\begin{pmatrix}\frac{7}{8} & -\frac{1}{8}\\ -\frac{1}{8} & \frac{7}{8}\end{pmatrix},\quad (E-\Gamma)^{-1}=\frac{4}{3}\begin{pmatrix}\frac{7}{8} & \frac{1}{8}\\ \frac{1}{8} & \frac{7}{8}\end{pmatrix},\quad p=\begin{pmatrix}\frac{2}{3}\\ \frac{2}{3}\end{pmatrix},$$

Obviously, $E-\Gamma$ is a nonsingular M-matrix, that is, (**H6**) holds, and

$$m=(E-\Gamma)^{-1}p=(\frac{8}{9},\ \frac{8}{9})^T,$$

hence

$$M=(M_1,M_2)^T=(\frac{16}{9},\ \frac{16}{9})^T.$$

It follows that

$$\begin{aligned}\Lambda_1 &= D-\overline{\alpha}(|\bar{A}|+|\bar{B}|+|\bar{C}|)F\\ &= \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}-\begin{pmatrix}\frac{1}{30} & 0\\ 0 & \frac{1}{30}\end{pmatrix}\begin{pmatrix}\frac{16}{9} & 0\\ 0 & \frac{16}{9}\end{pmatrix}-\begin{pmatrix}3 & 0\\ 0 & 3\end{pmatrix}\begin{pmatrix}\frac{1}{8} & \frac{1}{8}\\ \frac{1}{8} & \frac{1}{8}\end{pmatrix}\\ &= \begin{pmatrix}\frac{571}{1080} & -\frac{3}{8}\\ -\frac{3}{8} & \frac{571}{1080}\end{pmatrix},\end{aligned}$$

Therefor Λ_1 is a nonsingular M-matrix, from Theorem 3.2 and Corollary 4.2, we know that system (5.1) has exactly one 2π-periodic solution, and the 2π-periodic solution of system (5.1) is globally exponentially stable.

Remark 5.3 When $(a_{ij}(t))=0,\ (b_{ij}(t))=0,\ (c_{ij}(t))=\begin{pmatrix}\frac{1}{8} & \frac{1}{8}\\ \frac{1}{8} & \frac{1}{8}\end{pmatrix}$ in Example 5.3, System (5.1) become to Cohen-Crossberg neural networks with distributed delays. Here, it is not hard to know that it has exactly one 2π-periodic solution which is globally exponentially stable. However, those results given in Theorem 4.3-Theorem 4.5 in [26] cannot be applied to here.

Example 5.2 Consider the following model:

$$\begin{cases}\frac{dx_1(t)}{dt}=-d_1(t)x_1(t)+b_{11}(t)f_1(x_1(t-\tau_{11}(t)))+c_{11}(t)\int\limits_{-\infty}^{t}e^{-(t-s)}f_1(x_1(s))ds\\ \qquad +c_{12}(t)\int\limits_{-\infty}^{t}e^{-2(t-s)}f_2(x_2(s))ds-2\cos t,\\ \frac{dx_2(t)}{dt}=-d_2(t)x_2(t)+b_{21}(t)f_1(x_1(t-\tau_{21}(t)))+b_{22}(t)f_2(x_2(t-\tau_{22}(t)))\\ \qquad +c_{21}(t)\int\limits_{-\infty}^{t}e^{-2(t-s)}f_1(x_1(s))ds\\ \qquad +c_{22}(t)\int\limits_{-\infty}^{t}e^{-(t-s)}f_2(x_2(s))ds+3\sin t\end{cases}\tag{5.2}$$

where $d_1(t) = 5 + \sin t$, $d_2(t) = 5 - 0.5\cos t$, $b_{11}(t) = 1 - 0.5\sin t$, $b_{12}(t) = 0$, $b_{21}(t) = \sin t$, $b_{22}(t) = \cos t$, $\tau_{11}(t) = 0.2 + 3|\cos\frac{t}{2}|$, $\tau_{21}(t) = 0.3 + |\sin\frac{t}{2}|$, $\tau_{22}(t) = 1 - \sin t$, $c_{11}(t) = \cos t$, $c_{12}(t) = 0.5 + 0.5\sin t$, $c_{21} = 0.5 + 0.5\sin t$, $c_{22} = 1 - 0.5\sin t$, $f_i(x) = \frac{1}{2}(|x+1| + |x-1|)$, $i = 1, 2$.

It is easy to check that assumptions (**H3**)-(**H5**) hold, and $F_1 = F_2 = 1$, $\underline{d}_1 = 4$, $\underline{d}_2 = 4.5$, $\overline{b}_{11} = 2$, $\overline{b}_{12} = 0$, $\overline{b}_{21} = 1$, $\overline{b}_{22} = 1$, $\overline{c}_{11} = 1$, $\overline{c}_{12} = 1$, $c_{21} = 1$, $\overline{c}_{22} = 1.5$, $0.2 \le \tau_{11}(t) \le 3.2$, $0.3 \le \tau_{21}(t) \le 2.3$, $0 \le \tau_{22}(t) \le 2$. Thus

$$\begin{aligned} D - (|\overline{B}| + |\overline{C}|)F &= \begin{pmatrix} 3.5 & 0 \\ 0 & 4.5 \end{pmatrix} - \left(\begin{pmatrix} 4 & 0 \\ 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 1 & 1.5 \end{pmatrix} \right) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1.5 & -1 \\ -2 & 2 \end{pmatrix}. \end{aligned}$$

Obviously, $D - (|\overline{B}| + |\overline{C}|)F$ is a nonsingular M-matrix, from Corollary 4.4, model (5.2) has exactly one 2π-periodic solution and all other solutions of model (5.2) converge exponentially to it as $t \to +\infty$.

6 Conclusions

In this paper, a class of periodic Cohen-Grossberg neural networks with both variable and distributed delays have been studied. Some sufficient conditions for the existence and exponential stability of the periodic solutions have been established. These obtained results are new and they complement previously known results. Moreover, Two examples are given to illustrate the effectiveness of the new results.

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