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EXISTENCE AND UNIQUENESS OF THE PRODUCERS' OPTIMAL ADJUSTMENT TRAJECTORY IN A DEBREU-TYPE ECONOMY

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Abstract. The aim of this paper is the analysis of adjustment processes in a Debreu-type economy. The reasons taken into account, e.g. incentives, cooperation of economic agents under full access to information, the way of sending messages described formally, are the basis for defining adjustment trajectories.

Some reasons, such as introducing new legal requirements or implementing new profitable technologies formulated in mathematical language, can contribute to the transformation of the production sector and induce an appropriate way of adjusting the producers' plans of action.

This survey relies on an examination of the relationships between quantities of goods and quantities of the productive factors used to produce them. As a result, the optimal producers' trajectories, due to the criterion of cost minimization, are defined. The paper also contains some remarks on the uniqueness of the trajectories under study.

Keywords: private ownership economy, adjustment process, adjustment trajectory.

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1. Motivation

The studies on the adjustment processes in this framework have their origin in [Lipieta 2010] and [Lipieta 2013]. At the beginning, the economy in which producers want or have to change their productive activity because of certain reasons such as introducing new legal requirements (e.g. the reduction of CO_2 emissions into the atmosphere), implementing new profitable technologies (innovations), new trends and fashions for some commodities, is considered. The above reasons, formulated in mathematical language, can contribute to making decisions on the transformation of the production sector and induce an appropriate way of adjusting the producers' plans of action.

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This paper is aimed at modeling some adjustment processes with respect to cost minimization in situations when producers have to remove a harmful commodity from their plans of action, or to limit its amount proportionally to the amount of this output, in the production of which a harmful commodity is used.

On the basis of the previous considerations, the following corollary can be formulated: under some assumptions, there are infinitely many continuous and linear trajectories of changes of the production sector of a Debreu-type economy not disturbing, at given prices, the equilibrium in the economy, as well as not making agents worse off. This conclusion leads us to the research hypothesis: in given initial conditions, there is exactly one best (optimal) with respect to the given criterion, continuous and linear trajectory of changes not disturbing the equilibrium in the economy under study.

Initially, the Debreu private ownership economy is considered. The origin definition of the above structure was formulated by [Debreu 1959] and, since then, it has been studied and modified (see among others [Radner 1972; Magill, Quinzii 2002; Malawski 1999; Ciałowicz, Malawski 2011; Mas-Colell et al. 1995; Lipieta 2013]. A modification of the economy defined by Debreu is called a Debreu-type economy.

The paper is organized as follows: in the second part the adjustment processes in a Debreu economy are defined, in the third part the basic model is presented, the fourth part is devoted to the analysis of some kinds of producers' adjustment trajectories in a Debreu-type economy.

2. Adjustment processes

The sequence of activities of economic agents in points of time $t = 0, 1, \ldots, \tau$, where $\tau \in \{1, 2, \ldots\}$, resulting in offered goods and services is called the economic process. Point $t = 0$ means the beginning of the process, point $t = \tau$ its end. To every economic process are assigned the effects of agents' activities in time $t = \tau$, done in the framework of the given process. They are called the results of the economic process, in short the results or outcomes. The set of outcomes will be denoted by Z.

Let $K = \{k_1, k_2, \ldots, k_{\underline{k}}\}, \underline{k} \in \mathbb{N}$, be the set of economic agents active on the market. All characteristics determining an individual as the k-th agent in the given economic process form the so called environment of that agent. This will be denoted by e^k. The set of all feasible environments of agent k is marked by $E^k (e^k \in E^k)$. On the basis of the above, the set

$$E \overset{def}{=} E^{k_1} \times E^{k_2} \times \ldots \times E^{\mathscr{k}}$$

is called the set of environments (see [Arrow, Intriligator 1987]). If the environments of agent k are changed in time, then they will be denoted by $e^k(t)$ for $t \in \{0, 1, \ldots, \tau\}$. Notice that for every t and k, $e^k(t) \in E^k$.

Let M^k denote the set of the messages (information) to be used on the market by agent k. The elements of set M^k (messages) will be denoted by m^k. As above, $m^k(t)$ stands for the message of agent $k \in K$ at time $t \in \{0, 1, \ldots, \tau\}$. The vector $m = (m^{k_1}, m^{k_2}, \ldots, m^{k_{\mathscr{k}}})$ is called the $\mathscr{k}$-tuple message if $m^k \in M^k$ for every $k \in K$. The process of exchanging messages may be represented by a system of difference equation of the form

$$m^k(t+1) = f^k\left(m^{k_1}(t), m^{k_2}(t), \ldots, m^{k_{\mathscr{k}}}(t), e(t)\right),\ t = 0,1,\ldots,\tau-1\ ; k \in K. \quad (1)$$

Then, for every $k \in K$ and $M \subset M^{k_1} \times \ldots \times M^{k_{\mathscr{k}}}$, the function $f^k : M \times E \to M^k$ is called the agent k's response function (see also: [Arrow, Intriligator 1987]).

Definition 1. (see: [Arrow, Intriligator 1987]) *A $\mathscr{k}$-tuple message*

$$\overline{m} = \left(\overline{m}^{k_1}, \overline{m}^{k_2}, \ldots, \overline{m}^{k_{\mathscr{k}}}\right) \in M$$

is stationary if, for every $k \in K$, it satisfies the equation

$$\overline{m}^k = f^k\left(\overline{m}^{k_1}, \overline{m}^{k_2}, \ldots, \overline{m}^{k_{\mathscr{k}}}, \overline{e}\right). \quad (2)$$

Definition 2. *If $h : M \to Z$ is the outcome function, then the structure*

$$\left(M^{k_1}, \ldots, M^{k_{\mathscr{k}}}, \ldots, f^{k_1}, \ldots, f^{k_{\mathscr{k}}}, h\right), \quad (3)$$

is called the adjustment process.

If the components of the given environment $e(0) = (e^{k_1}(0), e^{k_2}(0), \ldots, e^{k_{\mathscr{k}}}(0)) \in E$ form a Debreu-type economy, then the adjustment process (3) is called the adjustment process in a Debreu-type economy.

3. Model

Consider the set of agents $K = A \cup B$, where

- $A = \{a_1, a_2, \ldots, a_m\}$ is the set of consumers, $m \in \mathbb{N}$,
- $B = \{b_1, b_2, \ldots, b_n\}$ is the set of producers, $n \in \mathbb{N}$.

It is assumed that $A \cap B = \varnothing$. This means that an agent $k \in K$ can be indexed by b as well as by a, if he/she is both the producer and the consumer. Hence $\kappa = m + n$ and the environment of agent k depends on his/her role in the given process.

To every producer $b \in B$ is assigned a nonempty production set $Y^b \subset \mathbb{R}^\ell$ of his/her feasible production plans. Every consumer a is represented by a nonempty consumption set X^a of his/her feasible consumption plans, an initial endowment $\omega^a \in \mathbb{R}^\ell$ and a preference relation $\preceq^a \subset X^a \times X^a$. Hence, the environment $e^k(0)$ of every agent $k \in K = A \cup B$ is of the form

$$e^k(0) = \left(y(k), \chi(k), e(k), \varepsilon(k), \theta(k,\cdot)\right),$$

where:

$$\begin{array}{llll} y(k) = Y^k & \text{for } k \in B, & y(k) = \{0\} & \text{for } k \notin B \\ \chi(k) = X^k & \text{for } k \in A, & \chi(k) = \{0\} & \text{for } k \notin A \\ e(k) = \omega^k & \text{for } k \in A, & e(k) = 0 & \text{for } k \notin A \\ \varepsilon(k) = \preceq^k & \text{for } k \in A, & \varepsilon(k) = \{\varnothing\} & \text{for } k \notin A \end{array}$$

the mapping $\theta : K \times K \to [0,1]$ satisfies:

$$\theta(k,\cdot) \equiv 0 \text{ for } k \notin A, \ \theta(\cdot,k) \equiv 0 \text{ for } k \notin B$$

$$\forall b \in B \sum_{a \in A} \theta(a,b) = 1.$$

By the above, we get that the set of environments E^k of every agent $k \in K$ is of the form

$$E^k = \mathbb{R}^\ell \times P(\mathbb{R}^\ell) \times P(\mathbb{R}^\ell) \times \mathbb{R}^\ell \times P(\mathbb{R}^{2\ell}) \times \mathcal{F}\left(K,[0,1]\right),$$

with $\mathcal{F}\left(K,[0,1]\right) \overset{def}{=} \left\{ f \mid f : K \to [0,1] \right\}$, while the set of environment is given by

$$E \overset{def}{=} E^{k_1} \times E^{k_2} \times \ldots \times E^{k_\kappa}.$$

Remark 1. It is easy to see (compare to [Lipieta 2013]) that the components of the environment $e(0)$ form the private ownership economy $\varepsilon_q = \left(\mathbb{R}^\ell, P_q, C_q, \theta, \omega\right)$, where

$P_q = (B, \mathbb{R}^\ell; y, p)$ is the quasi-production system,

$C_q = (A, \mathbb{R}^\ell, \Xi; \chi, e, \varepsilon, p)$ is the quasi-consumption system,

$\omega = \sum_{a \in A} \omega^a$.

Mark Ξ stands for the set of all preference relations on $\mathbb{R}^\ell \times \mathbb{R}^\ell$. Moreover, if at the given price vector $p \in \mathbb{R}^\ell$:

$$\forall b \in B \ \ \eta^b\left(p\right) \stackrel{def}{=} \{y^{b*} \in y(b) : p \circ y^{b*} = \max\{p \circ y^b : y^b \in y(b)\}\} \neq \varnothing,$$

then

$$\eta : B \ni b \to \eta^b\left(p\right) \subset \mathbb{R}^\ell$$

is called the correspondence of supply at price system p,

$$\pi : B \ni b \to \pi\left(b\right) = p \circ y^{b*} \in \mathbb{R}$$

is called the maximal profit function at price system p.

Similarly, if at the given price vector $p \in \mathbb{R}^\ell$, for every $a \in A$,

$$w^a = p \circ \omega^a + \sum_{a \in A} \theta\left(a,b\right) \cdot \pi^b(p),$$

$$\beta^a\left(p\right) = \{x \in \chi\left(a\right) : p \circ x \le p \circ \omega^a\} \neq \varnothing$$

and

$$\varphi^a\left(p\right) = \left\{x^{a*} \in \beta^a\left(p\right) : \forall x^a \in \beta^a\left(p\right) x^a \preceq^a x^{a*}\right\} \neq \varnothing,$$

then

- $\beta : A \ni a \to \beta^a\left(p\right) \subset \mathbb{R}^\ell$ is the correspondence of budget sets at price system p, which to every consumer $a \in A$ assigns his/her set of budget constraints $\beta^a\left(p\right) \subset \chi\left(a\right)$ at price system p and initial endowment ω^a,
- $\varphi : A \ni a \to \varphi^a(p) \subset \mathbb{R}^\ell$ is the demand correspondence at price system p, which to every consumer $a \in A$ assigns the consumption plans maximizing his/her preference on the budget set $\beta^a\left(p\right)$.

Additionally, the sequence $(x^{a_1 *}, \ldots, x^{a_m *}, y^{b_1 *}, \ldots, y^{b_n *}, p)$, for which

$$\forall a \in A x^{a*} \in \varphi^a\left(p\right),$$

$$\forall b \in B,\ y^{b*} \in \eta^b\left(p\right),$$

$$\sum_{a \in A} x^{a*} - \sum_{b \in B} y^{b*} = \omega,$$

is called the state of equilibrium in economy ε_q. If there exists a state of equilibrium in economy ε_q, then we say that ε_q is in equilibrium. Then price p is called the equilibrium price vector. The private ownership economy ε_q in which a state of equilibrium exists is called a Debreu economy. The set of all states of equilibrium at the given price system p will be denoted by $S_e(p)$.

The aim of this paper is to model the adjustment processes with respect to cost minimization in a situation when the producers have to eliminate a harmful commodity from their plans of action or to limit its amount proportionally to the amount of this output, in the production of which the harmful commodity is used. The second case makes sense if the producers do not have a technology which lets them get rid of the harmful commodity from their plans of action.

Hence, the production plans after modification are contained in the subspace $V \overset{def}{=} \ker \tilde{g}$, where

$$\tilde{g} : \mathbb{R}^\ell \ni (x_1, \ldots, x_\ell) \to x_{l_0} \in \mathbb{R}, \tag{4}$$

if the producers have eliminated the commodity $l_0 \in \{1, \ldots, \ell\}$ from their plans of action or

$$\tilde{g} : \mathbb{R}^\ell \ni (x_1, \ldots, x_\ell) \to x_{l_0} - c \cdot x_l \in \mathbb{R}, \tag{5}$$

under the assumption that $c > 0$, if the producers have to limit the amount of the commodity l_0 proportionally to the amount of the commodity l, $l_0 \neq l$. So, in both cases, the commodity l_0 is “not wanted” by producers as well as by consumers. Hence,

$$\forall a \in A \ X^a \subset V, \tag{6}$$

for functional $\tilde{g}$ of the form (4) or (5). Let us notice that the mapping $Q : \mathbb{R}^\ell \to V$

$$Q(x) = x - \tilde{g}(x) \cdot q, \tag{7}$$

is the projection on subspace *V*, determined by vector $q \in \mathbb{R}^\ell$ (see [Lipieta 1999]) satisfying

$$\tilde{g}(q) = 1 \tag{8}$$

(see [Lipieta 2010]). Let mapping $\tilde{Q} : \mathbb{R}^\ell \times [0, \tau] \to \mathbb{R}^\ell$ be of the form $\tilde{Q}(x, t) = x - \frac{t}{\tau} \tilde{g}(x) \cdot q$.

The plans realized by economic agents can be regarded as messages sent by them to other economic agents. Then

$$M^k = \bigcup_{t \in [0,\tau]} \tilde{Q}\left(\eta^k(p), t\right) \text{ for } k \in B \quad \text{and} \quad M^k = \varphi^k(p) \text{ for } k \in A.$$

Let

$$M = \{(x^1,\dots,x^m,y^1,\dots,y^n) : \exists\,(x^{a_1},\dots,x^{a_m},y^{b_1},\dots,y^{b_n},p) \in S_e(p)\,|$$
$$[\left(\forall i \in \{1,\dots,m\}\, x^i = x^{a_i}\right) \wedge \qquad (9)$$
$$(\exists t \in [0,\tau]\ \forall j \in \{1,\dots,n\}\, y^j = y^{b_j} - \frac{t}{\tau}\cdot \tilde{g}(y^{b_j})\cdot q)]\}.$$

It is easy to see that $M \subset M^{k_1} \times \ldots \times M^{k_{\ell}}$.

Theorem 1. *If $p \in \mathbb{R}^{\ell}$ is the equilibrium price vector in a Debreu economy ε_q and*

$$\forall a \in A\, \omega^a \in V\,, \qquad (10)$$

then the structure

$$(M^{k_1},\dots,M^{k_{\ell}}, f^{k_1},\dots,f^{k_{\ell}}, h)\,, \qquad (11)$$

where

- M *is of the form* (9)
- $f^k : M \times E \to M^k$, *for every* $k \in K$, *is of the form*

$$f^k\left(x^{k_1},x^{k_2},\dots,x^{k_{\ell}};e\right) = x^k - \frac{1}{\tau}\cdot \tilde{g}(x^k(0))\cdot q\,, \qquad (12)$$

for $\tilde{g}$ satisfying (4) *or* (5) *as well as $q \in \mathbb{R}^{\ell}$ obtained by* (8),

- $h : M \to Z$, $h\left(x^1,\dots,x^m,y^1,\dots,y^n\right) \overset{def}{=} \left(x^1,\dots,x^m,Q\left(y^1\right),\dots,Q\left(y^n\right)\right)$

is the outcome function with the set of outcomes

$$Z \overset{def}{=} \{(x^{a_1*},\dots,x^{a_m*},y^{b_1*},\dots,y^{b_n*}) \in V^{m+n} : \sum_{a\in A} x^{a*} - \sum_{b\in B} y^{b*} = \omega\,,$$
$$\forall a \in A\, x^{a*} \in \varphi^a(p), \forall b \in B,\ y^{b*} \in Q(\eta^k(p))\}$$

is the adjustment process in a Debreu economy ε_q.

Proof. Firstly we show that set Z by the thesis of the theorem is not empty. The reasoning is similar to the proof of theorem 4.2 in [Lipieta 2010]. Namely, if $p \notin V^T$, then there exists vector $q \in \mathbb{R}^{\ell}$ satisfying

$$\begin{cases} \tilde{g}(q) = 1 \\ p \circ q = 0 \end{cases}. \qquad (13)$$

Then any projection determined by vector q satisfies

$$p \circ Q(x) = p \circ x \quad \text{for every} \quad x \in \mathbb{R}^{\ell}.$$

If $y^{b*} \in \eta^b(p)$, then for every $b \in B$, vector $Q(y^{b*})$ maximizes the profit of producer b at price p on the set $Q(Y^b)$. Moreover, by (10)

$$\sum_{a\in A} x^{a*} - \sum_{b\in B} y^{b*} = \sum_{a\in A} \omega^a \quad \Rightarrow \quad \sum_{a\in A} x^{a*} - \sum_{b\in B} Q(y^{b*}) = \sum_{a\in A} \omega^a . \tag{14}$$

If $p \in V^T$, then for every $x \in \mathbb{R}^\ell$

$$p \circ Q(x) = 0 .$$

Hence, if $y^{b*} \in \eta^b(p)$, then for every $b \in B$, vector $Q(y^{b*})$ also maximizes the profit of producer b on the set $Q(Y^b)$. In this case, condition (14) is also satisfied.

By (2) and (12), we get that

$$x^k(t) = x^k(t-1) - \frac{1}{\tau} \cdot \tilde{g}\left(x^k(0)\right) \cdot q \text{ for } t \in \{1,...,\tau\}.$$

Consequently

$$x^k(1) = x^k(0) - \frac{1}{\tau} \cdot \tilde{g}\left(x^k(0)\right) \cdot q ,$$

$$x^k(2) = x^k(1) - \frac{1}{\tau} \cdot \tilde{g}\left(x^k(0)\right) \cdot q = x^k(0) - \frac{2}{\tau} \cdot \tilde{g}\left(x^k(0)\right)$$

and so on. After t steps, for every $t \in \{0,1,\ldots,\tau\}$,

$$x^k(t) = x^k(0) - \frac{t}{\tau} \cdot \tilde{g}\left(x^k(0)\right) \cdot q .$$

By the above and by the definition of subspace V,

$$\left(x^1(\tau),\ldots,x^m(\tau),y^1(\tau),\ldots,y^n(\tau)\right) \in Z ,$$

which ends the proof.

□

To every adjustment process of the form (11) is assigned the adequate mapping of the form (7). The mapping Q of the form (7) will be called the producers' adjustment trajectory. Defining the optimal producers' adjustment trajectory under the given criterion, we get the optimal adjustment process of the form (11) due to the same criterion.

As we can see, the mapping Q and consequently the adjustment process (11) is determined by a vector $q \in \mathbb{R}^\ell$ satisfying (8), for given subspace $V = \ker \tilde{g}$, where $\tilde{g}$ is of the form (4) or (5). Additionally, if $p \notin V^T$, then

$q \in \mathbb{R}^{\ell}$ satisfies the system of equalities (13) with the given equilibrium price vector $p \in \mathbb{R}^{\ell}$. The equality (8) has infinitely many solutions for $\ell \geq 2$. The system of equalities (13), for given $p \notin V^T$ has infinitely many solutions for $\ell > 2$. Hence if $\ell > 2$, then there are infinitely many adjustment processes of the form (11) in a Debreu economy ε_q.

4. The main results

Let $\varepsilon_q = \left(\mathbb{R}^{\ell}, P_q, C_q, \theta, \omega\right)$ be a Debreu economy in which at given price vector $p \in \mathbb{R}^{\ell}$, for every $b \in B$, vector y^{b*} maximizes, at price $p \in \mathbb{R}^{\ell}$, the profit on the set Y^b and, for every $a \in A$, vector x^{a*} maximizes the preference relation $\preceq^a$ on the set $\beta^a(p, w^a)$, where $w^a = p \circ \omega^a + \sum_{a \in A} \theta(a,b) \cdot \pi^b(p)$.

We assume that the producers adjusting their plans of action also want to minimize costs of transformation. This results in keeping the smallest difference between every production plan and its modification. Hence, we determine, for every $x = (x_1, \ldots, x_\ell) \in \mathbb{R}^{\ell}$, the norm

$$\|x\| = max\{|x_l| : l \in \{1, 2, \ldots, \ell\}\}. \tag{15}$$

Suppose that the changes in production that have to be done are described by functional $\tilde{g}$ of the form (4) or (5). Let vector $q \in \mathbb{R}^{\ell}$ satisfy condition (8) and additionally (13), if $p \notin V^T$. Let Q be the mapping of the form (7) determined by vector q. For every $x \in \mathbb{R}^{\ell}$

$$dist(x,V) \leq \left\|\left(Id - Q\right)\left(x\right)\right\| \leq \left\|Id - Q\right\| dist(x,V), \tag{16}$$

(see for example [Cheney 1966]), where

$$\left\|Id - Q\right\| = \sup\left\{\left\|\left(Id - Q\right)\left(x\right)\right\| : x \in \mathbb{R}^{\ell} \wedge \|x\| \leq 1\right\}. \tag{17}$$

It is easily seen, by (16) and (17), that $\|Id - Q\| \geq 1$. If the norm $\|Id - Q\|$ is not large, then the production plans and their modifications are close, in the meaning of distance. This is the reason for which the mapping Q determined by vector q with possibly the smallest number $\|Id - Q\|$ is the optimal producers' adjustment trajectory under the criterion of distance minimization.

Let $\tilde{g}$ be the mapping of the form (4) or (5) and $V = \ker \tilde{g}$. Then
$\mathcal{P}\left(\mathbb{R}^{\ell},V\right) = \{Q : \mathbb{R}^{\ell} \to V : \exists q \in \mathbb{R}^{\ell}\ \tilde{g}\left(q\right) = 1 \wedge \forall x \in \mathbb{R}^{\ell}\ Q\left(x\right) = x - \tilde{g}(x) \cdot q\}$.

Under the above assumptions. the following is true:

Theorem 2. *Let functional* $\tilde{g}$ *be of the form* (4) *or* (5). *If* $p \in V^T$, *then there exists* $q_0 \in \mathbb{R}^{\ell}$ *such that*

$$\left\|Id - Q_0\right\| = inf\left\{\left\|Id - Q\right\| : Q \in \mathcal{P}\left(\mathbb{R}^{\ell},V\right)\right\} \tag{18}$$

where

$$Q_0\left(x\right) = x - \tilde{g}\left(x\right) \cdot q_0 .$$

The projection Q_0 *satisfying condition* (18) *is the unique one.*

Proof. The proof is the consequence of theorem 1 by [Lipieta 1999] as well as theorem 3 by [Lipieta 2010]. Vector q_0 determining projection Q_0 has only one coordinate, namely l_0, different from zero. The l_0 coordinate of vector q_0 has to be equal to 1. Moreover $\left\|Id - Q_0\right\| = 1$. There are no more projections from set $\mathcal{P}\left(\mathbb{R}^{\ell},V\right)$ satisfying (18).

□

Assume that $p \notin V^T$. Then

$$\mathcal{P}\left(\mathbb{R}^{\ell},V;p\right) \overset{def}{=} \{Q \in \mathcal{P}\left(\mathbb{R}^{\ell},V\right) :\ p \circ q = 0\} .$$

We assume additionally that the commodity l_0 is the most expensive. Now we have

Theorem 3. *Let functional* $\tilde{g}$ *be of the form* (4). *If* $p \notin V^T$ *and*

$$\left|p_{l_0}\right| \geq \sum_{s=1,\ s \neq l_0}^{\ell} \left|p_s\right| > 0 , \tag{19}$$

then there exists $q_0 \in \mathbb{R}^{\ell}$ *such that*

$$\left\|Id - Q_0\right\| = inf\left\{\left\|Id - Q\right\| : Q \in \mathcal{P}\left(\mathbb{R}^{\ell},V;p\right)\right\},$$

where

$$Q_0\left(x\right) = x - \tilde{g}\left(x\right) \cdot q_0 .$$

Proof. Consider functional $\tilde{g}$ of the form (4). If $Q \in \mathcal{P}\left(\mathbb{R}^{\ell}, V\right)$ is determined by vector $q \in \mathbb{R}^{\ell}$, then by (17)

$$\|Id - Q\| = \max\{|q_l| : l \in \{1, 2, \ldots, \ell\}\}.$$

By (13), $q_{l_0} = 1$ and $\max\{|q_l| : l \in \{1, 2, \ldots, \ell\}, l \neq l_0\} \geq \frac{|p_{l_0}|}{\sum_{l=1, l\neq l_0}^{\ell} |p_l|}$.

Consequently

$$\|Id - Q_0\| \geq \max\left\{1, \frac{|p_{l_0}|}{\sum_{l=1, l\neq l_0}^{\ell} |p_l|}\right\}.$$

Vector $q \in \mathbb{R}^{\ell}$ whose coordinates are of the form

$$q_{l_0} = 1 \quad \text{and} \quad q_l = \frac{-sgn\ (p_l)}{\sum_{l=1, l\neq l_0}^{\ell} |p_l|} \cdot p_{l_0} \quad \text{for} \quad l \in \{1, 2, \ldots, \ell\} \setminus \{l_0\}$$

determines the projection Q_0 for which

$$\|Id - Q_0\| = \max\left\{1; \frac{|p_{l_0}|}{\sum_{l=1, l\neq l_0}^{\ell} |p_l|}\right\} = \frac{|p_{l_0}|}{\sum_{l=1, l\neq l_0}^{\ell} |p_l|}.$$

Hence Q_0 satisfies the thesis of the theorem.

□

In the further part of the paper we assume that:

$$|p_{l_0}| \geq \sum_{s=1,\ s\neq l_0}^{\ell} |p_s| \quad \text{or} \quad |p_l| \geq \sum_{s=1,\ s\neq l}^{\ell} |p_s|, \tag{20}$$

which means that the commodity l_0 or the commodity l by (5) is the most expensive. Now, the following is true:

Theorem 4. *Let functional* $\tilde{g}$ *be of the form* (5). *If* $p \notin V^T$ *satisfies* (20) *as well as* $p_{l_0}, p_l > 0$, *then there exists* $q_0 \in \mathbb{R}^{\ell}$ *such that*

$$\|Id - Q_0\| = inf\left\{\|Id - Q\| : Q \in \mathcal{P}\left(\mathbb{R}^{\ell}, V; p\right)\right\},$$

where

$$Q_0(x) = x - \tilde{g}(x) \cdot q_0.$$

Proof. By (17)

$$\|Id-Q\|=(1+c)\cdot\max\{|q_l|:l\in\{1,2,\dots,\ell\}\}. \tag{21}$$

Combining conditions (5) and (13), we get that

$$q_{l_0}=1+cq_l$$

as well as

$$p_{l_0}(1+cq_l)+\sum_{s=1,\ s\neq l_0}^{\ell}p_sq_s=0,$$

where $c>0$. Hence

$$p_{l_0}=-cp_{l_0}q_l-\sum_{s=1,\ s\neq l_0}^{\ell}p_sq_s.$$

Consequently,

$$\max\{|q_s|:s\in\{1,2,\dots,\ell\}\setminus\{l_0\}\}\geq\frac{|p_{l_0}|}{c|p_{l_0}|+\sum_{s=1,\ s\neq l_0}^{\ell}|p_s|}.$$

By the above, for every $Q\in\mathcal{P}(\mathbb{R}^\ell,V;p)$

$$\|Id-Q\|\geq(1+c)\cdot\frac{|p_{l_0}|}{c|p_{l_0}|+\sum_{s=1,\ s\neq l_0}^{\ell}|p_s|}. \tag{22}$$

Define

$$q_s^0=\frac{-sgn\ (p_s)\cdot p_{l_0}}{c|p_{l_0}|+\sum_{s=1,\ s\neq l_0}^{\ell}|p_s|}\quad\text{for}\quad s\in\{1,2,\dots,\ell\}\setminus\{l_0\} \tag{23}$$

and

$$q_{l_0}^0=\frac{\sum_{s=1,\ s\neq l_0}^{\ell}|p_s|}{c|p_{l_0}|+\sum_{s=1,\ s\neq l_0}^{\ell}|p_s|}. \tag{24}$$

Then q^0 satisfies (13). Moreover, if $|p_{l_0}|\geq\sum_{s=1,\ s\neq l_0}^{\ell}|p_s|$, then

$$|q_{l_0}^0|\leq\max\{|q_s^0|:s\in\{1,2,\dots,\ell\}\setminus\{l_0\}\}. \tag{25}$$

By (25), vector q^0 defined in (23) and (24), determines mapping Q_0 satisfying (18), where

$$\left\| Id - Q_0 \right\| = (1+c) \cdot \frac{\left| p_{l_0} \right|}{c\left| p_{l_0} \right| + \sum_{s=1,\ s \neq l_0}^{\ell} \left| p_s \right|} .$$

If $\left| p_l \right| \geq \sum_{s=1,\ s \neq l}^{\ell} \left| p_s \right|$, then we prove, in the same way as above, that there exists vector $q^0 \in \mathbb{R}^{\ell}$, precisely,

$$q_s^0 = \frac{-sgn\ (p_s) \cdot \left| p_l \right|}{\left| p_l \right| + c\sum_{s=1,\ s \neq l}^{\ell} \left| p_s \right|} \quad \text{for} \quad s \in \{1, 2, \ldots, \ell\} \setminus \{l\} \tag{26}$$

and

$$q_l^0 = \frac{\sum_{s=1,\ s \neq l}^{\ell} \left| p_s \right|}{\left| p_l \right| + c\sum_{s=1,\ s \neq l}^{\ell} \left| p_s \right|} . \tag{27}$$

determining mapping Q_0 satisfying (18), where

$$\left\| Id - Q_0 \right\| = (1+c) \cdot \frac{\left| p_l \right|}{\left| p_l \right| + c\sum_{s=1,\ s \neq l}^{\ell} \left| p_s \right|} .$$

In both cases, only the projection Q_0 satisfies condition (19).

□

If all producers adjust their plans of action with respect to the criterion of distance minimization, then they change their production as the mapping Q_0 indicates. Hence at point $t = \tau$ there will be equilibrium in the modified form of economy ε_q.

The trajectories defined in theorems 2-4 minimize the distance between the initial and final production plans in the given initial conditions. Therefore they are the optimal producers' adjustment trajectories. Moreover, the maximal profits and the consumers' optimal plans are not changed during modification of the production sphere, which means that after transformation the economic agents are not worse off than at the beginning.

5. Conclusion

If, in the given initial conditions, there is exactly one optimal producers' adjustment trajectory in a Debreu economy, then the producers, who aim at cost minimization, will change their plans of action due to this trajectory. If all producers modify their activities under the above criterion, then there will be equilibrium in the economy after modification.

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