Exogenous Information, Endogenous Information and Optimal Monetary Policy*

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Abstract

Most of the analysis of optimal monetary policy is conducted with the Calvo model. This paper studies optimal monetary policy when the slow adjustment of the price level is due to imperfect information by decision-makers in firms. We consider two models: a model with exogenous dispersed information and a rational inattention model. In the model with exogenous dispersed information, complete stabilization of the price level is optimal after aggregate productivity shocks but not after markup shocks. By contrast, in the rational inattention model, complete stabilization of the price level is optimal both after aggregate productivity shocks and after markup shocks. Moreover, in the model with exogenous dispersed information, there is no value from commitment to a future monetary policy. By contrast, in the rational inattention model, there is value from commitment to a future monetary policy because then the private sector can trust the central bank that not paying attention to certain variables is optimal.

JEL: E3, E5, D8.

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1 Introduction

This paper studies optimal macroeconomic policy when decision-makers in firms decide how much attention they devote to aggregate conditions. It seems like a good description of reality that decision-makers in firms have a limited amount of attention and decide how much attention they devote to aggregate conditions. What are the implications for the optimal conduct of monetary policy? To the best of our knowledge, this paper is the first paper that studies this question.

We model an economy with many firms, a representative household, and a government. Firms supply differentiated goods. These goods are produced with labor. The representative household consumes the different goods, supplies labor, and holds nominal government bonds and money. Money demand is derived from a cash-in-advance constraint. The economy is hit by aggregate productivity shocks and markup shocks (i.e., shocks that change the desired markup by firms). In each period, all firms set prices for their goods. Prices respond slowly to shocks either due to exogenous dispersed information by firms or due to rational inattention by decision-makers in firms. In the rational inattention model, decision-makers in firms choose the precision of their signals about aggregate productivity and the desired markup, subject to a cost of information flow. The central bank sets the money supply in response to shocks. The central bank aims to maximize expected utility of the representative household. We derive optimal monetary policy under commitment.

The main results are the following. First, in the model with exogenous dispersed information and in the rational inattention model, the optimal policy response to aggregate productivity shocks is to fully stabilize the price level in response to aggregate productivity shocks. The reason is the following. By offsetting fully the effect of aggregate productivity shocks on the profit-maximizing price (i.e., by increasing the money supply in response to a positive productivity shock so that the profit-maximizing price does not respond to an aggregate productivity shock), the central bank can replicate the response of the economy to aggregate productivity shocks under perfect information. Furthermore, the response of the economy to aggregate productivity shocks under perfect information is efficient. Hence, this policy is the optimal monetary policy response to aggregate productivity shocks. One feature of this policy is that prices do not respond to aggregate productivity shocks. Second, in the model with exogenous dispersed information, it is not optimal to fully stabilize the price level in response to markup shocks. By offsetting fully the effect of markup shocks on the profit-maximizing price, the central bank can in principle replicate the response of the economy to markup shocks under perfect information. However, the response of the economy to markup shocks under perfect information is inefficient. In particular, there is inefficient consumption variance. By offsetting only partially the effect of markup shocks on the profit-maximizing price, the central bank increases inefficient price dispersion but reduces inefficient consumption variance (relative to the perfect-information solution). Accepting some inefficient price dispersion in exchange for reduced consumption variance turns out to be the optimal monetary policy. At the optimal monetary policy, the profit-maximizing price, actual prices, and the price level respond to markup shocks. By contrast, in the rational inattention model, it is optimal to fully stabilize the price level in response to markup shocks. By counteracting the effect of markup shocks on the profit-maximizing price (i.e., by reducing the money supply in response to a positive markup shock), the central bank reduces the variance of the profit-maximizing price due to markup shocks, which now reduces both inefficient price dispersion *and* the attention that decision-makers in firms devote to markup shocks. The latter reduces the response of the price level to markup shocks and thereby reduces consumption variance due to markup shocks. Hence, the trade-off between price dispersion and consumption variance due to markup shocks disappears. Reducing the money supply more in response to a positive markup shock now reduces both price dispersion and consumption variance. The optimal monetary policy is to counteract the effect of markup shocks on the profit-maximizing price until the variance of the profit-maximizing price due to the markup shock is sufficiently small so that decision-makers in firms pay no attention to markup shocks. Thus, at the optimal monetary policy, prices do not respond to markup shocks. In summary, in the rational inattention model, the trade-off between price dispersion and consumption variance due to markup shocks disappears and therefore complete price level stability is optimal. This is important because the trade-off between price dispersion and consumption variance due to markup shocks has been emphasized a lot in the literature on optimal monetary policy in the New Keynesian model. Third, in the model with exogenous dispersed information, there is no value from commitment to a future monetary policy. By contrast, in the rational inattention model, there is value from commitment to a future monetary policy because then the private sector can trust the central bank that not paying attention to certain variables is optimal.

This paper is related to four papers that also study optimal monetary policy in models in which price setting firms have imperfect information. First, Ball, Mankiw and Reis (2005) study optimal monetary policy in the sticky information model of Mankiw and Reis (2002). The main difference between their paper and our paper is that in their paper the information structure is exogenous. In particular, in their paper the probability with which firms update their information sets is independent of monetary policy. Second, Adam (2007) studies optimal monetary policy in a model in which firms pay limited attention to aggregate variables. In his model the amount of attention that firms devote to aggregate variables is exogenous; whereas in the rational inattention model presented below the amount of attention that firms devote to aggregate variables is endogenous (and depends on monetary policy). We show that this changes optimal monetary policy in a fundamental way. Third, Lorenzoni (2010) and Angeletos and La'O (2008) study optimal monetary policy in models with dispersed information. In Lorenzoni (2010), price setting firms observe the history of the economy up to the previous period, the sum of aggregate and idiosyncratic productivity, and a noisy public signal about aggregate productivity. There are several differences between his paper and our paper: (i) in his paper the "noise" in the private signal concerning aggregate productivity is idiosyncratic productivity, while in our paper the noise arises from limited attention, (ii) in his paper the information structure is exogenous, while in our paper the information structure is endogenous, and (iii) in his paper the central bank has imperfect information, while we assume that the central bank has perfect information about the state of the economy. We make this assumption to derive the optimal monetary policy response to changes in fundamentals. Afterwards, we study whether the central bank can also implement this optimal monetary policy response with less information. Like in Lorenzoni (2010), agents in Angeletos and La'O (2008) observe the history of the economy up to the previous period, the sum of aggregate and idiosyncratic productivity, and a noisy public signal about aggregate productivity. In addition, in Angeletos and La'O (2008) agents observe noisy signals concerning endogenous variables with exogenous variance of noise. This creates an informational externality because a stronger response of agents to their private signals makes the signals concerning endogenous variables more informative. Angeletos and La'O (2008) study how this informational externality affects optimal fiscal and monetary policy. In summary, this paper is the first paper that studies optimal monetary policy in a model in which agents choose the attention that they allocate to aggregate variables.

This paper is also related to the literature on the social value of public information, for example, Morris and Shin (2002), Hellwig (2005), and Angeletos and Pavan (2007). In this literature, the main monetary policy question is whether the central bank should provide information about economic fundamentals. We instead ask how the central bank should set the money supply or a nominal interest rate in response to fundamentals. In addition, in the literature on the social value of public information the information structure (i.e., what agents observe) is typically exogenous.

The rest of the paper is organized as follows. Section 2 presents the model setup. Section 3 specifies the objective of the central bank. Section 4 states the optimal monetary policy problem under commitment in the model with an exogenous information structure and in the model with an endogenous information structure. Section 5 shows that there is a monetary policy that replicates the allocation under perfect information. Section 6 derives the optimal monetary policy response to aggregate productivity shocks. Section 7 derives the optimal monetary policy response to markup shocks. Section 8 contains additional results. Section 9 concludes.

2 Model setup

The economy is populated by a representative household, firms, and a government.

Household: The household's preferences in period zero over sequences of consumption and labor supply $\{C_t, L_t\}_{t=0}^{\infty}$ are given by

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\gamma}-1}{1-\gamma}-\frac{L_t^{1+\psi}}{1+\psi}\right)\right], \tag{1}$$

where C_t is composite consumption and L_t is labor supply in period t. Here E_0 denotes the expectation operator conditioned on information of the household in period zero. The parameter $\beta \in (0,1)$ is the discount factor. The parameter $\gamma > 0$ is the inverse of the intertemporal elasticity of substitution and the parameter $\psi \geq 0$ is the inverse of the Frisch elasticity of labor supply. Composite consumption in period t is given by a Dixit-Stiglitz aggregator

$$C_t = \left(\frac{1}{I}\sum_{i=1}^{I} C_{i,t}^{\frac{1}{1+\Lambda_t}}\right)^{1+\Lambda_t}, \tag{2}$$

where $C_{i,t}$ is consumption of good i in period t. There are I different consumption goods and the elasticity of substitution between those different consumption goods equals $(1 + 1/\Lambda_t)$ in period t. The variable Λ_t will equal the desired markup by firms in period t. Therefore, we call Λ_t the desired markup. We assume that the log of the desired markup follows a stationary Gaussian first-order autoregressive process

$$\ln(\Lambda_t) = (1-\rho_\lambda)\ln(\Lambda) + \rho_\lambda \ln(\Lambda_{t-1}) + \nu_t, \tag{3}$$

where the parameter $\Lambda > 0$, the parameter $\rho_\lambda \in [0,1)$, and the innovation ν_t is $i.i.d.N\left(0,\sigma_\nu^2\right)$. We call the innovation ν_t a markup shock. We introduce the markup shock in the model as an example of a shock that has the following property: the response of the economy to the shock under perfect information is inefficient.[1] We call this property of a shock the inefficiency property. In Section 6 we derive the optimal monetary policy response to a markup shock. In Section 8 we show that our results concerning the optimal monetary policy response to a markup shock extend to other shocks that have the inefficiency property.

The flow budget constraint of the representative household in period t reads

$$M_t + B_t = R_{t-1}B_{t-1} + W_t L_t + D_t - T_t + \left(M_{t-1} - \sum_{i=1}^{I} P_{i,t-1}C_{i,t-1}\right). \tag{4}$$

The right-hand side of the flow budget constraint is pre-consumption wealth in period t. Here B_{t-1} are the household's holdings of nominal government bonds between periods $t-1$ and t, R_{t-1} is the nominal gross interest rate on those bond holdings, W_t is the nominal wage rate in period t, D_t are nominal aggregate profits in period t, T_t are nominal lump sum taxes in period t, and the term in brackets are unspent nominal money balances carried over from period $t-1$ to period t. The representative household can transform his pre-consumption wealth in period t into money balances, M_t, and bond holdings, B_t. The purpose of holding money is to purchase goods. We assume that the representative household faces the following cash-in-advance constraint

$$\sum_{i=1}^{I} P_{i,t}C_{i,t} = M_t. \tag{5}$$

The representative household also faces a no-Ponzi-scheme condition.

We introduce the cash-in-advance constraint because it allows us to explain the intuition behind our results concerning optimal monetary policy in a very simple way. In Section 8 we show that our results concerning optimal monetary policy extend to a cashless economy à la Woodford (2003). Furthermore, recall that there are different formulations of the cash-in-advance constraint and note that in the formulation of the cash-in-advance constraint that we use there are no monetary frictions because wage income can be immediately transformed into cash and cash can be immediately spent

[1] We define efficiency formally in Section 3.

on goods. We decided to abstract from monetary frictions in our benchmark economy for two reasons: (i) abstracting from monetary frictions is common in the New Keynesian literature on optimal monetary policy and therefore abstracting from monetary frictions facilitates comparison of optimal monetary policy in the two models that we consider to optimal monetary policy in the standard New Keynesian model, and (ii) we think it is useful to study in isolation the implications of different frictions for optimal monetary policy. In Section 8 we consider an extension with monetary frictions and there we study how monetary frictions affect optimal monetary policy in the two models that we consider.

In every period, the representative household chooses a consumption vector, labor supply, nominal money balances, and nominal bond holdings. The representative household takes as given the nominal interest rate, the nominal wage rate, nominal aggregate profits, nominal lump sum taxes, and the prices of all consumption goods.

Firms: There are I firms. Firm i supplies good i. The technology of firm i is given by

$$Y_{i,t} = A_t L_{i,t}^{\alpha}, \tag{6}$$

where $Y_{i,t}$ is output and $L_{i,t}$ is labor input of firm i in period t. A_t is aggregate productivity in period t. The parameter $\alpha \in (0,1]$ is the elasticity of output with respect to labor input. The log of aggregate productivity follows a stationary Gaussian first-order autoregressive process

$$\ln(A_t) = \rho_a \ln(A_{t-1}) + \varepsilon_t, \tag{7}$$

where the parameter $\rho_a \in [0,1)$ and the innovation ε_t is $i.i.d.N\left(0, \sigma_\varepsilon^2\right)$. The processes $\{A_t\}$ and $\{\Lambda_t\}$ are assumed to be independent. We call the innovation ε_t an aggregate productivity shock. We introduce the aggregate productivity shock in the model as an example of a shock that has the following property: the response of the economy to the shock under perfect information is efficient. We call this property of a shock the efficiency property. In Section 6 we derive the optimal monetary policy response to an aggregate productivity shock. In Section 8 we show that the results concerning the optimal monetary policy response to an aggregate productivity shock extend to other shocks that have the efficiency property.

Nominal profits of firm i in period t equal

$$(1+\tau_p) P_{i,t} Y_{i,t} - W_t L_{i,t}, \tag{8}$$

where τ_p is a production subsidy paid by the government.

In every period, each firm sets a price and commits to supply any quantity at that price. Each firm takes as given composite consumption by the representative household, the nominal wage rate, and the following price index[2]

$$P_t = \left(\frac{1}{I} \sum_{i=1}^{I} P_{i,t}^{-\frac{1}{\Lambda_t}} \right)^{-\Lambda_t} I. \tag{9}$$

Government: There is a monetary authority and a fiscal authority. The monetary authority commits to set the money supply according to the following rule

$$\ln (M_t^s) = F_t (L) \varepsilon_t + G_t (L) \nu_t, \tag{10}$$

where M_t^s denotes the money supply in period t. $F_t (L)$ and $G_t (L)$ are infinite-order lag polynomials which can depend on t. The last equation simply says that the log of the money supply in period t can be any linear function of the sequence of shocks up to and including period t. We will ask the question which linear function is optimal.

Two remarks may be useful. First, the reader may wonder how the money market clears at a given money supply. In equilibrium, the endogenous variables (e.g., the price level, the nominal interest rate, and consumption) adjust such that the demand for money balances by the representative household equals the supply of money balances by the monetary authority (i.e., $M_t = M_t^s$). Second, in equation (10) we assume that the central bank can commit to a money supply rule. In Section 8 we show that the set of attainable allocations is identical when the central bank can commit to an interest rate rule of the form

$$\ln (R_t) = F_t (L) \varepsilon_t + G_t (L) \nu_t. \tag{11}$$

The drawback of an interest rate rule is that multiplicity of equilibria at a given monetary policy arises more easily. Therefore, we assume in the benchmark economy that the central bank can commit to a money supply rule and we postpone the discussion of unique implementation in the case of an interest rate rule to Section 8.

[2] Dixit and Stiglitz (1977), in their seminal article on monopolistic competition, also assume that there is a finite number of physical goods and that firms take the price index as given. Moreover, it seems to be a good description of the U.S. economy that there is a finite number of physical consumption goods and that firms take the consumer price index as given.

Next, consider fiscal policy. The government budget constraint in period t reads

$$T_t + B_t = R_{t-1}B_{t-1} + \tau_p \left(\sum_{i=1}^{I} P_{i,t}Y_{i,t} \right). \tag{12}$$

The government has to finance maturing nominal government bonds and the production subsidy. The government can collect lump sum taxes or issue new one-period nominal government bonds. We assume that the fiscal authority pursues a Ricardian fiscal policy. For ease of exposition, we assume that the fiscal authority fixes nominal government bonds at some non-negative level

$$B_t = B \geq 0. \tag{13}$$

Furthermore, we assume that the fiscal authority sets the production subsidy so as to correct, in the non-stochastic steady state, the distortion arising from monopolistic competition. Formally,

$$\tau_p = \Lambda. \tag{14}$$

Alternatively, one could assume that the fiscal authority sets the production subsidy so as to correct perfectly at each point in time the distortion arising from monopolistic competition. Formally,

$$\tau_{p,t} = \Lambda_t. \tag{15}$$

However, since in the United States fiscal policy has to be approved by Congress while monetary policy decisions are implemented directly by the Federal Reserve, we find it more realistic to assume that the fiscal authority *cannot* adjust the production subsidy quickly while the monetary authority *can* adjust the money supply quickly.

Information: We consider two models, one with an exogenous information structure and one with an endogenous information structure. In both models, the information set in period t of the decision-maker who is responsible for setting the price of good i is

$$\mathcal{I}_{i,t} = \mathcal{I}_{i,-1} \cup \{s_{i,0}, s_{i,1}, \ldots, s_{i,t}\}, \tag{16}$$

where $\mathcal{I}_{i,-1}$ contains any initial information that the price setter of firm i has in period minus one and $s_{i,t}$ is the signal that the price setter of firm i receives in period t. We assume that the structure of the economy is common knowledge in period minus one. Furthermore, in the model with an exogenous information structure, we assume that the price setter of firm i receives in every period

$t \geq 0$ a two-dimensional signal consisting of a noisy signal concerning aggregate productivity and a noisy signal concerning the desired markup:

$$s_{i,t} = \begin{pmatrix} \ln(A_t) + \eta_{i,t} \\ \ln(\Lambda_t/\Lambda) + \zeta_{i,t} \end{pmatrix}, \tag{17}$$

where the noise terms have the following properties: (i) the stochastic processes $\{\eta_{i,t}\}$ and $\{\zeta_{i,t}\}$ are independent of the stochastic processes $\{A_t\}$ and $\{\Lambda_t\}$, (ii) the stochastic processes $\{\eta_{i,t}\}$ and $\{\zeta_{i,t}\}$ are independent across firms and independent of each other, and (iii) the noise term $\eta_{i,t}$ is $i.i.d.N\left(0,\sigma_\eta^2\right)$ and the noise term $\zeta_{i,t}$ is $i.i.d.N\left(0,\sigma_\zeta^2\right)$. In the model with an exogenous information structure, the variances of noise σ_η^2 and σ_ζ^2 are structural parameters.

By contrast, in the model with an endogenous information structure, the variances of noise σ_η^2 and σ_ζ^2 are endogenous. Following the literature on rational inattention, we assume that the decision-maker who is responsible for setting the price of good i chooses the variances of noise subject to an information flow constraint. Formally, the price setter of firm i solves the following decision problem in period minus one:

$$\max_{\left(1/\sigma_\eta^2, 1/\sigma_\zeta^2\right) \in \mathbb{R}_+^2} \left\{ E_{i,-1}\left[\sum_{t=0}^{\infty} \beta^t \pi\left(P_{i,t}, P_t, C_t, W_t, A_t, \Lambda_t\right)\right] - \frac{\mu}{1-\beta}\kappa \right\}, \tag{18}$$

subject to equations (16)-(17) and in every period $t \geq 0$

$$P_{i,t} = \arg\max_{x \in \mathbb{R}_{++}} E_{i,t}[\pi\left(x, P_t, C_t, W_t, A_t, \Lambda_t\right)], \tag{19}$$

and

$$\frac{1}{2}\log_2\left(\frac{\sigma^2_{a|s_i^{t-1}}}{\sigma^2_{a|s_i^t}}\right) + \frac{1}{2}\log_2\left(\frac{\sigma^2_{\lambda|s_i^{t-1}}}{\sigma^2_{\lambda|s_i^t}}\right) = \kappa. \tag{20}$$

Here $E_{i,t}$ denotes the expectation operator conditioned on the information of the price setter of firm i in period t, π denotes the real profit function defined as the nominal profit function times the marginal utility of consumption of the representative household divided by P_t, $\sigma^2_{a|s_i^t}$ denotes the conditional variance of $a_t \equiv \ln(A_t)$ given information of the price setter of firm i in period t, and $\sigma^2_{\lambda|s_i^t}$ denotes the conditional variance of $\lambda_t \equiv \ln(\Lambda_t/\Lambda)$ given information of the price setter of firm i in period t. The variable κ is the per-period information flow concerning aggregate conditions and the parameter $\mu > 0$ is the per-period marginal cost of information flow. The decision-maker chooses the precision of the two signals in period minus one so as to maximize the expected discounted sum of profits net of the cost of information flow. The decision-maker takes into account how his/her choice of signal precision affects future price setting behavior. Furthermore, equation (20) measures the information flow concerning aggregate conditions and objective (18) states that information flow concerning aggregate conditions is costly. We interpret the cost μ as the price setter's opportunity cost of devoting attention to aggregate conditions.

Three remarks are in place before we proceed. First, we interpret the noise in the signal (17) as arising from the limited attention of the price setter of firm i. Therefore, we find it reasonable to assume that the noise is idiosyncratic. Second, in equation (17) we assume that the price setter of firm i receives independent signals concerning aggregate productivity and the desired markup. In Section 8 we show that this assumption has no effect on optimal monetary policy in the model with an endogenous information structure. We show that optimal monetary policy in the model with an endogenous information structure is exactly the same when the decision-maker can choose to receive signals concerning any linear combination of a_t and λ_t. Third, in the decision problem given in the previous paragraph the price setter of firm i chooses signal precision once and for all. In Section 8 we show that our propositions concerning optimal monetary policy in the model with an endogenous information structure also hold when the decision-maker chooses signal precision period by period.

We assume that the monetary authority has perfect information (i.e., in every period $t \geq 0$, the monetary authority knows the entire history of the economy up to and including period t). We make this assumption because we are interested in the optimal conduct of monetary policy. We think this is an interesting benchmark. In Section 8 we also consider an extension where the monetary authority only knows the price level and composite consumption. In addition, we assume that the representative household has perfect information. We make this assumption (i) to isolate the implications of information frictions on the side of price setters for optimal monetary policy, and (ii) to facilitate the comparison to optimal monetary policy in the simplest New Keynesian model, where the only friction apart from monopolistic competition is price stickiness.

Aggregation: When we compute the price index terms will appear that are linear in $\frac{1}{I}\sum_{i=1}^{I}\eta_{i,t}$ and $\frac{1}{I}\sum_{i=1}^{I}\zeta_{i,t}$. These averages are random variables with mean zero and variance $\frac{1}{I}\sigma_{\eta}^{2}$ and $\frac{1}{I}\sigma_{\zeta}^{2}$, respectively. We will neglect these terms because these terms have mean zero and a variance that can be made arbitrarily small by choosing a sufficiently high number of firms I. For example, one could set $I = 10^{100}$. We work with a finite number of firms rather than a continuum of firms because we find that it makes the derivation of the central bank's objective in the next section more transparent.

3 Objective of the central bank

We assume that the central bank's aim is to maximize expected utility of the representative household, given by equations (1)-(2).

We now derive a simple expression for expected utility by using the fact that one can express period utility at a feasible allocation as a function only of the consumption vector at time t, aggregate productivity at time t, and the desired markup at time t. First, at any feasible allocation the representative household has to supply the labor that is needed to produce the consumption vector

$$L_t = \sum_{i=1}^{I} \left(\frac{C_{i,t}}{A_t} \right)^{\frac{1}{\alpha}}. \tag{21}$$

Furthermore, equation (2) for the consumption aggregator can be written as

$$1 = \frac{1}{I} \sum_{i=1}^{I} \hat{C}_{i,t}^{\frac{1}{1+\Lambda_t}},$$

where $\hat{C}_{i,t} \equiv (C_{i,t}/C_t)$ denotes relative consumption of good i in period t. Rearranging yields

$$\hat{C}_{I,t} = \left(I - \sum_{i=1}^{I-1} \hat{C}_{i,t}^{\frac{1}{1+\Lambda_t}} \right)^{1+\Lambda_t}. \tag{22}$$

Substituting equations (21) and (22) into the period utility function in (1) yields the following expression for period utility at a feasible allocation

$$\begin{aligned} U\left(C_t, \hat{C}_{1,t}, \ldots, \hat{C}_{I-1,t}, A_t, \Lambda_t\right) &= \frac{C_t^{1-\gamma} - 1}{1-\gamma} \\ &\quad - \frac{1}{1+\psi} \left(\frac{C_t}{A_t} \right)^{\frac{1}{\alpha}(1+\psi)} \left[\sum_{i=1}^{I-1} \hat{C}_{i,t}^{\frac{1}{\alpha}} + \left(I - \sum_{i=1}^{I-1} \hat{C}_{i,t}^{\frac{1}{1+\Lambda_t}} \right)^{\frac{1}{\alpha}(1+\Lambda_t)} \right]^{1+\psi} \end{aligned} \tag{23}$$

Hence, expected utility at a feasible allocation equals

$$E\left[\sum_{t=0}^{\infty} \beta^t U\left(C_t, \hat{C}_{1,t}, \ldots, \hat{C}_{I-1,t}, A_t, \Lambda_t\right) \right]. \tag{24}$$

In summary, by substituting the technology and the consumption aggregator into the period utility function one can express period utility at time t as a function only of composite consumption, the consumption mix, aggregate productivity, and the desired markup at time t.

Next, we study the efficient allocation. The efficient allocation in period t, defined as the feasible allocation in period t that maximizes utility of the representative household, is

$$C_t^* = \left(\frac{\alpha}{I^{1+\psi}}\right)^{\frac{1}{\gamma-1+\frac{1}{\alpha}(1+\psi)}} A_t^{\frac{\frac{1}{\alpha}(1+\psi)}{\gamma-1+\frac{1}{\alpha}(1+\psi)}}, \tag{25}$$

and, for all $i = 1, \ldots, I-1$,

$$\hat{C}_{i,t}^* = 1. \tag{26}$$

Efficient composite consumption in period t is strictly increasing in aggregate productivity at time t. The efficient consumption mix in period t is to consume an equal amount of each good. Note that the efficient consumption vector is independent of the desired markup.

In the following sections, we work with a log-quadratic approximation of expected utility (24) around the non-stochastic steady state. In the rest of the paper, variables without time subscript denote values in the non-stochastic steady state and small variables denote log-deviations from the non-stochastic steady state (e.g., $c_t = \ln(C_t/C)$ and $\hat{c}_{i,t} = \ln\left(\hat{C}_{i,t}/\hat{C}_i\right)$). Due to the production subsidy (14), the non-stochastic steady state is efficient. Expressing the function U given by equation (23) in terms of log-deviations from the non-stochastic steady state and using $C = C^*$ and $\hat{C}_i = \hat{C}_i^*$ yields the following expression for period utility at a feasible allocation

$$\begin{aligned} & u\left(c_t, \hat{c}_{1,t}, \ldots, \hat{c}_{I-1,t}, a_t, \lambda_t\right) \\ = \; & \frac{C^{1-\gamma} e^{(1-\gamma)c_t} - 1}{1-\gamma} \\ & -\frac{C^{1-\gamma} e^{\frac{1}{\alpha}(1+\psi)(c_t - a_t)}}{\frac{1}{\alpha}(1+\psi)} \left[\frac{1}{I}\sum_{i=1}^{I-1} e^{\frac{1}{\alpha}\hat{c}_{i,t}} + \frac{1}{I}\left(I - \sum_{i=1}^{I-1} e^{\hat{c}_{i,t}\frac{1}{1+\Lambda e^{\lambda_t}}}\right)^{\frac{1}{\alpha}\left(1+\Lambda e^{\lambda_t}\right)}\right]^{1+\psi}. \end{aligned} \tag{27}$$

Proposition 1 *(Objective of the central bank) Let $\tilde{u}$ denote the second-order Taylor approximation to the period utility function u at the origin. Let E denote the unconditional expectation operator.*

Let x_t, z_t, and ω_t denote the following vectors

$$x_t = \begin{pmatrix} c_t & \hat{c}_{1,t} & \cdots & \hat{c}_{I-1,t} \end{pmatrix}', \tag{28}$$

$$z_t = \begin{pmatrix} a_t & \lambda_t \end{pmatrix}', \tag{29}$$

$$\omega_t = \begin{pmatrix} x_t' & z_t' & 1 \end{pmatrix}'. \tag{30}$$

Let $\omega_{n,t}$ denote the nth element of ω_t. Suppose that there exist two constants $\delta < (1/\beta)$ and $\phi \in \mathbb{R}$ such that, for each period $t \geq 0$ and for all n and k,

$$E\left|\omega_{n,t}\omega_{k,t}\right| < \delta^t \phi. \tag{31}$$

Then

$$E\left[\sum_{t=0}^{\infty} \beta^t \tilde{u}\left(x_t, z_t\right)\right] = E\left[\sum_{t=0}^{\infty} \beta^t \tilde{u}\left(x_t^*, z_t\right)\right] + \sum_{t=0}^{\infty} \beta^t E\left[\frac{1}{2}\left(x_t - x_t^*\right)' H \left(x_t - x_t^*\right)\right], \tag{32}$$

where the matrix H is given by

$$H = -C^{1-\gamma} \begin{bmatrix} \gamma - 1 + \frac{1}{\alpha}(1+\psi) & 0 & \cdots & \cdots & 0 \\ 0 & 2\frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} & \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} & \cdots & \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} \\ \vdots & \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} \\ 0 & \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} & \cdots & \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} & 2\frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} \end{bmatrix}, \tag{33}$$

and the vector x_t^* is given by

$$c_t^* = \frac{\frac{1}{\alpha}(1+\psi)}{\gamma - 1 + \frac{1}{\alpha}(1+\psi)} a_t, \tag{34}$$

and

$$\hat{c}_{i,t}^* = 0. \tag{35}$$

Proof. See Appendix A. ■

After the log-quadratic approximation of the period utility function (23), expected utility of the representative household is given by equation (32). The efficient consumption vector in period t is given by equations (34)-(35) and the utility loss in the case of a deviation from the efficient consumption vector in period t is given by the quadratic form in square brackets on the right-hand side of equation (32). The upper-left element of the matrix H determines the utility loss in the case of inefficient composite consumption, while the lower-right block of the matrix H determines the utility loss in the case of an inefficient consumption mix. Finally, condition (31) ensures that in the expression on the left-hand side of equation (32) one can change the order of integration and summation and the infinite sum converges. In the models that we consider, condition (31) is always satisfied.

4 The Ramsey problem

In this section, we state the problem of the central bank that aims to commit to the money supply rule that maximizes expected utility of the representative household.

In the model with an exogenous information structure, the problem of the central bank is

$$\max_{\{F_t(L),G_t(L)\}_{t=0}^{\infty}} E\left[\sum_{t=0}^{\infty}\beta^t U\left(C_t,\hat{C}_{1,t},\ldots,\hat{C}_{I-1,t},A_t,\Lambda_t\right)\right], \tag{36}$$

subject to

$$P_tC_t = M_t, \tag{37}$$

$$C_{i,t} = \left(\frac{P_{i,t}}{\frac{1}{I}P_t}\right)^{-\left(1+\frac{1}{\Lambda_t}\right)} C_t, \tag{38}$$

$$\frac{W_t}{P_t} = L_t^{\psi}C_t^{\gamma}, \tag{39}$$

$$P_t = \left(\frac{1}{I}\sum_{i=1}^{I}P_{i,t}^{-\frac{1}{\Lambda_t}}\right)^{-\Lambda_t} I, \tag{40}$$

$$E\left[\frac{\partial\pi\left(P_{i,t},P_t,C_t,W_t,A_t,\Lambda_t\right)}{\partial P_{i,t}}|\mathcal{I}_{i,t}\right] = 0, \tag{41}$$

$$\mathcal{I}_{i,t} = \mathcal{I}_{i,-1}\cup\left\{s_{i,0},s_{i,1},\ldots,s_{i,t}\right\}, \tag{42}$$

$$s_{i,t} = \begin{pmatrix}\ln\left(A_t\right)+\eta_{i,t}\\ \ln\left(\Lambda_t/\Lambda\right)+\zeta_{i,t}\end{pmatrix}, \tag{43}$$

$$L_t = \sum_{i=1}^{I}\left(\frac{C_{i,t}}{A_t}\right)^{\frac{1}{\alpha}}, \tag{44}$$

$$\ln\left(A_t\right) = \rho_a\ln\left(A_{t-1}\right)+\varepsilon_t, \tag{45}$$

$$\ln\left(\Lambda_t/\Lambda\right) = \rho_\lambda\ln\left(\Lambda_{t-1}/\Lambda\right)+\nu_t, \tag{46}$$

and

$$\ln\left(M_t\right) = F_t\left(L\right)\varepsilon_t + G_t\left(L\right)\nu_t. \tag{47}$$

Equations (37)-(40) are the household's optimality conditions.[3] Equation (41) is the firms' optimality condition and equations (42)-(43) specify the information set of the price setter of firm i in period t. Equation (44) is the labor market clearing condition, equations (45)-(46) specify the laws of motion of the exogenous variables, and equation (47) is the equation for the money supply.[4] The function U defined by equation (23) gives period utility at a feasible allocation, $F_t\left(L\right)$ and $G_t\left(L\right)$ are infinite-order lag polynomials which can depend on t, and the innovations ε_t, ν_t, $\eta_{i,t}$, and $\zeta_{i,t}$ have the properties specified in Section 2. In the model with an exogenous information structure, the variances of noise σ_η^2 and σ_ζ^2 are structural parameters. They do not depend on monetary policy.

By contrast, in the model with an endogenous information structure, the variances of noise σ_η^2 and σ_ζ^2 are given by the solution to the problem (18)-(20) and the central bank understands that the choice of the money supply rule affects the firms' choice of signal precision.

Next, in the model with an exogenous information structure, a log-quadratic approximation of the central bank's objective (36) around the non-stochastic steady state and a log-linear approximation of the equilibrium conditions (37)-(41) and (44) around the non-stochastic steady state yields the following linear quadratic Ramsey problem

$$\min_{\{F_t(L),G_t(L)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t E\left[\left(c_t - c_t^*\right)^2 + \delta \frac{1}{I}\sum_{i=1}^{I}\left(p_{i,t} - p_t\right)^2\right], \tag{48}$$

subject to

$$c_t^* = \frac{\phi_a}{\phi_c} a_t, \tag{49}$$

$$c_t = m_t - p_t, \tag{50}$$

$$p_t = \frac{1}{I}\sum_{i=1}^{I} p_{i,t}, \tag{51}$$

$$p_{i,t} = E\left[p_{i,t}^* | \mathcal{I}_{i,t}\right], \tag{52}$$

[3] We do not state the consumption Euler equation because here the consumption Euler equation is only a pricing equation that determines the equilibrium nominal interest rate.

[4] The requirement that each firm produces the quantity demanded is embedded in the profit function π and the money market clearing condition $M_t = M_t^s$ is embedded in equation (47).

$$p_{i,t}^{*} = p_t + \phi_c c_t - \phi_a a_t + \phi_\lambda \lambda_t, \tag{53}$$

$$\mathcal{I}_{i,t} = \mathcal{I}_{i,-1} \cup \{s_{i,0}, s_{i,1}, \ldots, s_{i,t}\}, \tag{54}$$

$$s_{i,t} = \begin{pmatrix} a_t + \eta_{i,t} \\ \lambda_t + \zeta_{i,t} \end{pmatrix}, \tag{55}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_t, \tag{56}$$

$$\lambda_t = \rho_\lambda \lambda_{t-1} + \nu_t, \tag{57}$$

and

$$m_t = F_t(L)\,\varepsilon_t + G_t(L)\,\nu_t, \tag{58}$$

where

$$\phi_c = \frac{\frac{\psi}{\alpha} + \gamma + \frac{1-\alpha}{\alpha}}{1 + \frac{1-\alpha}{\alpha}\frac{1+\Lambda}{\Lambda}} > 0, \tag{59}$$

$$\phi_a = \frac{\frac{\psi}{\alpha} + \frac{1}{\alpha}}{1 + \frac{1-\alpha}{\alpha}\frac{1+\Lambda}{\Lambda}} > 0, \tag{60}$$

$$\phi_\lambda = \frac{\frac{\Lambda}{1+\Lambda}}{1 + \frac{1-\alpha}{\alpha}\frac{1+\Lambda}{\Lambda}} > 0, \tag{61}$$

$$\delta = \frac{\frac{1+\Lambda-\alpha}{(1+\Lambda)\alpha}\left(1 + \frac{1}{\Lambda}\right)^2}{\gamma - 1 + \frac{1}{\alpha}(1+\psi)} > 0. \tag{62}$$

Here c_t^* is efficient composite consumption in period t, $p_{i,t}^*$ is the profit-maximizing price of firm i in period t, ϕ_c, ϕ_a and ϕ_λ are the coefficients in the equation for the profit-maximizing price, and δ is the relative weight on price dispersion in the central bank's objective.

In the model with an exogenous information structure, the variances of noise are exogenous. By contrast, in the model with an endogenous information structure, the variances of noise are given by the solution to problem (18)-(20) and the central bank understands that the money supply rule affects firms' allocation of attention. After a log-quadratic approximation of the profit function π the attention problem (18)-(20) reads

$$\min_{(1/\sigma_\eta^2, 1/\sigma_\zeta^2) \in \mathbb{R}_+^2} \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\omega}{2} E_{i,-1}\left[\left(p_{i,t} - p_{i,t}^*\right)^2\right] + \frac{\mu}{1-\beta}\kappa \right\}, \tag{63}$$

subject to

$$p_{i,t} = E\left[p_{i,t}^* | \mathcal{I}_{i,t}\right], \tag{64}$$

and

$$\frac{1}{2}\log_2\left(\frac{\sigma^2_{a|s_i^{t-1}}}{\sigma^2_{a|s_i^t}}\right)+\frac{1}{2}\log_2\left(\frac{\sigma^2_{\lambda|s_i^{t-1}}}{\sigma^2_{\lambda|s_i^t}}\right)=\kappa, \tag{65}$$

where

$$\omega=C^{-\gamma}\frac{WL_i}{P}\frac{\frac{1+\Lambda}{\Lambda}}{\alpha}\left(1+\frac{1-\alpha}{\alpha}\frac{1+\Lambda}{\Lambda}\right). \tag{66}$$

Here $p_{i,t}^*$ is the profit-maximizing price of firm i in period t given by equation (53), $\mathcal{I}_{i,t}$ is the information set of the price setter of firm i in period t given by equations (54)-(55), and the coefficient ω determines the loss in profit in the case of a suboptimal price.

5 Perfect information solution and the central bank's ability to replicate it

In this section, we derive the solution of the model under perfect information, and we show that the central bank can always replicate this solution under imperfect information. This intermediate result will be useful when we derive the optimal monetary policy response to aggregate productivity shocks in the next section.

Suppose that decision-makers who set prices have perfect information. Then, each firm charges the profit-maximizing price, and equations (50)-(53) imply that

$$c_t=\frac{\phi_a}{\phi_c}a_t-\frac{\phi_\lambda}{\phi_c}\lambda_t, \tag{67}$$

$$p_{i,t}-p_t=0, \tag{68}$$

and

$$p_t=m_t-c_t. \tag{69}$$

The economy's response to aggregate productivity shocks under perfect information is efficient, while the economy's response to markup shocks under perfect information is inefficient. To see this, note that price dispersion equals zero under perfect information, and compare equilibrium composite consumption given by equation (67) to efficient composite consumption given by equation (49). The reason for the inefficient response to markup shocks is that the efficient allocation is independent of the desired markup but under perfect information firms' actual markup varies with the desired markup. Finally, note that monetary policy has no effect on the equilibrium allocation under perfect information. Monetary policy only affects nominal variables. For example, the central bank can completely stabilize the price level by setting $m_t = \frac{\phi_a}{\phi_c} a_t - \frac{\phi_\lambda}{\phi_c} \lambda_t$ but this has no effect on welfare.

The central bank can replicate the perfect information solution under imperfect information with a particular monetary policy rule. More precisely, when the central bank sets $m_t = \frac{\phi_a}{\phi_c} a_t - \frac{\phi_\lambda}{\phi_c} \lambda_t$, then the equilibrium allocation under perfect information is also the unique equilibrium allocation under imperfect information. The proof is as follows. First, substituting equation (50) into equation (53) yields the following equation for the profit-maximizing price of firm i in period t

$$p_{i,t}^* = (1 - \phi_c)\, p_t + \phi_c \left(m_t - \frac{\phi_a}{\phi_c} a_t + \frac{\phi_\lambda}{\phi_c} \lambda_t \right). \tag{70}$$

Second, when $m_t = \frac{\phi_a}{\phi_c} a_t - \frac{\phi_\lambda}{\phi_c} \lambda_t$ then equations (51), (52), and (70) imply

$$p_t = (1 - \phi_c) \frac{1}{I} \sum_{i=1}^{I} E\left[p_t | \mathcal{I}_{i,t} \right]. \tag{71}$$

The unique solution to the last equation is $p_t = 0$. Thus, when $m_t = \frac{\phi_a}{\phi_c} a_t - \frac{\phi_\lambda}{\phi_c} \lambda_t$ then $p_t = 0$ is the unique equilibrium price level. Finally, substituting $m_t = \frac{\phi_a}{\phi_c} a_t - \frac{\phi_\lambda}{\phi_c} \lambda_t$ and $p_t = 0$ into equation (50) yields

$$c_t = \frac{\phi_a}{\phi_c} a_t - \frac{\phi_\lambda}{\phi_c} \lambda_t, \tag{72}$$

and substituting $m_t = \frac{\phi_a}{\phi_c} a_t - \frac{\phi_\lambda}{\phi_c} \lambda_t$ and $p_t = 0$ into equations (52) and (70) yields

$$p_{i,t} = 0, \tag{73}$$

implying

$$p_{i,t} - p_t = 0. \tag{74}$$

Hence, when the central bank sets $m_t = \frac{\phi_a}{\phi_c} a_t - \frac{\phi_\lambda}{\phi_c} \lambda_t$, then the equilibrium allocation under perfect information is also the unique equilibrium allocation under imperfect information. Moreover, the same arguments also apply shock by shock. By setting $F_t(L)\, \varepsilon_t = \frac{\phi_a}{\phi_c} a_t$ the central bank can replicate the response of the economy to aggregate productivity shocks under perfect information. By setting $G_t(L)\, \nu_t = -\frac{\phi_\lambda}{\phi_c} \lambda_t$ the central bank can replicate the response of the economy to markup shocks under perfect information.

We have derived the solution of the model under perfect information, and we have shown that the central bank can replicate this solution under imperfect information with a particular monetary policy rule. Hence, if the central bank conducts optimal monetary policy, welfare under imperfect information has to be weakly larger than welfare under perfect information.

6 Optimal monetary policy response to aggregate productivity shocks

In this section, we derive the optimal monetary policy response to aggregate productivity shocks. We begin with the model with an exogenous information structure and we then consider the model with an endogenous information structure. Equipped with the results of the previous section, the proofs are straightforward.

Proposition 2 *(Exogenous information structure) Consider the Ramsey problem (48)-(62), where the variances of noise σ_η^2 and σ_ζ^2 are structural parameters. If $\sigma_\eta^2 > 0$, the unique optimal monetary policy response to aggregate productivity shocks is*

$$F_t(L)\,\varepsilon_t = \frac{\phi_a}{\phi_c} a_t. \tag{75}$$

At this policy, the unique equilibrium response of the economy to aggregate productivity shocks equals the efficient response to aggregate productivity shocks, and the price level does not respond to aggregate productivity shocks.

Proof. First, when $F_t(L)\,\varepsilon_t = \frac{\phi_a}{\phi_c} a_t$, the unique equilibrium response of composite consumption to aggregate productivity shocks equals the perfect information response of composite consumption to aggregate productivity shocks, and the perfect information response of composite consumption to aggregate productivity shocks is efficient. See the previous section. Second, when $F_t(L)\,\varepsilon_t = \frac{\phi_a}{\phi_c} a_t$, there is no price dispersion caused by the noise in the signal concerning aggregate productivity. See again the previous section. Third, if $\sigma_\eta^2 > 0$, any monetary policy rule with $F_t(L)\,\varepsilon_t \neq \frac{\phi_a}{\phi_c} a_t$ yields an inefficient response of composite consumption to aggregate productivity shocks or price dispersion caused by the noise in the signal concerning aggregate productivity: if price setters put weight on the signal concerning aggregate productivity then there is price dispersion caused by the noise in the signal concerning aggregate productivity, and if price setters put no weight on the signal concerning aggregate productivity then the response of composite consumption to aggregate productivity shocks is inefficient. Finally, the choice of $F_t(L)$ affects neither the equilibrium response of composite consumption to markup shocks nor the extent to which there is price dispersion caused by the noise in the signal concerning the desired markup. ■

Next, we consider the model with an endogenous information structure, that is, the model in which price setters choose how much attention they devote to aggregate conditions. Compared to the model with an exogenous information structure, the result is the same and the proof is similar.

Proposition 3 *(Endogenous information structure) Consider the Ramsey problem (48)-(66), where the variances of noise σ_η^2 and σ_ζ^2 are given by the solution to problem (63)-(66). If $\mu > 0$, the unique optimal monetary policy response to aggregate productivity shocks is*

$$F_t(L)\,\varepsilon_t = \frac{\phi_a}{\phi_c} a_t. \tag{76}$$

At this policy, the unique equilibrium response of the economy to aggregate productivity shocks equals the efficient response to aggregate productivity shocks, decision-makers in firms who set prices devote no attention to aggregate productivity, and the price level does not respond to aggregate productivity shocks.

Proof. First, when the central bank sets $F_t(L)\,\varepsilon_t = \frac{\phi_a}{\phi_c} a_t$, then for any signal precision $\left(1/\sigma_\eta^2\right) \geq 0$ the unique equilibrium response of composite consumption to aggregate productivity shocks equals the efficient response of composite consumption to aggregate productivity shocks and there is no price dispersion caused by the noise in the signal concerning aggregate productivity. See the previous section. Second, when $F_t(L)\,\varepsilon_t = \frac{\phi_a}{\phi_c} a_t$, the price level and the profit-maximizing price do not respond to aggregate productivity shocks. See again the previous section. If $\mu > 0$, this implies that decision-makers in firms who set prices devote no attention to aggregate productivity. Third, if $\mu > 0$, any policy $F_t(L)\,\varepsilon_t \neq \frac{\phi_a}{\phi_c} a_t$ yields an inefficient response of composite consumption to aggregate productivity shocks or price dispersion caused by the noise in the signal concerning aggregate productivity: if price setters pay no attention to aggregate productivity then the response of composite consumption to aggregate productivity shocks is inefficient, and if price setters devote attention to aggregate productivity then there is price dispersion caused by the noise in the signal concerning aggregate productivity. Finally, the choice of $F_t(L)$ affects neither the equilibrium response of composite consumption to markup shocks nor the extent to which there is price dispersion caused by the noise in the signal concerning the desired markup. ■

7 Optimal monetary policy response to markup shocks

In this section, we derive the optimal monetary policy response to markup shocks. The main result is that complete price stabilization in response to markup shocks is never optimal in the model with an exogenous information structure, whereas complete price stabilization in response to markup shocks is always optimal in the model with an endogenous information structure. In Section 8 we show that this result for markup shocks extends to other shocks that have the inefficiency property (i.e., the property that the response of the economy to the shock under perfect information is inefficient). Hence, whether the information structure is exogenous or endogenous has a major implication for the optimal conduct of monetary policy.

For ease of exposition, we assume in the rest of this section that there are no aggregate productivity shocks. This assumption simplifies the notation in Propositions 4 and 5 and has no impact on the optimal monetary policy response to markup shocks.

7.1 Exogenous information structure

The next proposition specifies the optimal monetary policy response to markup shocks in the model with an exogenous information structure when the desired markup follows a white noise process.

Proposition 4 *(Exogenous information structure) Consider the Ramsey problem (48)-(62), where the variances of noise* σ_η^2 *and* σ_ζ^2 *are structural parameters. Suppose that* $\rho_\lambda = 0$, $\sigma_\nu^2 > 0$ *and* $a_{-1} = \sigma_\varepsilon^2 = 0$. *Consider policies of the form* $G_t(L)\,\nu_t = g_0\nu_t$ *and equilibria of the form* $p_t = \theta\lambda_t$. *The unique equilibrium at any monetary policy* $g_0 \in \mathbb{R}$ *is*

$$p_t = \frac{\phi_c g_0 + \phi_\lambda}{\phi_c + \frac{\sigma_\zeta^2}{\sigma_\lambda^2}}\lambda_t, \tag{77}$$

$$c_t = \frac{\frac{\sigma_\zeta^2}{\sigma_\lambda^2} g_0 - \phi_\lambda}{\phi_c + \frac{\sigma_\zeta^2}{\sigma_\lambda^2}}\lambda_t, \tag{78}$$

$$p_{i,t} - p_t = \frac{\phi_c g_0 + \phi_\lambda}{\phi_c + \frac{\sigma_\zeta^2}{\sigma_\lambda^2}}\zeta_{i,t}. \tag{79}$$

Furthermore, if $\sigma_\zeta^2 > 0$, *the unique optimal monetary policy* $g_0 \in \mathbb{R}$ *is*

$$g_0^* = \frac{(1 - \delta\phi_c)\,\phi_\lambda}{\frac{\sigma_\zeta^2}{\sigma_\lambda^2} + \delta\phi_c^2}. \tag{80}$$

At this policy, the price level strictly increases in response to a positive markup shock, composite consumption strictly falls in response to a positive markup shock, and there is inefficient price dispersion.

Proof. See Appendix B. ■

The most important result for the rest of this section is that, in the model with an exogenous information structure, complete price stabilization is not optimal. The intuition is the following. Suppose for a moment that the central bank does not change the money supply in response to markup shocks. Then the profit-maximizing price increases after a positive markup shock, implying that decision-makers who are responsible for setting prices put a positive weight on their signals concerning the desired markup. Thus, the price level increases in response to a positive markup shock and there is price dispersion caused by the noise in the signal concerning the desired markup. Furthermore, composite consumption falls in response to a positive markup shock because the money supply is constant and the price level increases after a positive markup shock. To reduce price dispersion, the central bank would have to lower the money supply in response to a positive markup shock. On the other hand, to reduce consumption variance, the central bank would have to increase the money supply in response to a positive markup shock. Hence, there is a trade-off between price dispersion and consumption variance. It depends on the value of $\delta\phi_c$ whether it is optimal for the central bank to increase, decrease or not change the money supply in response to a positive markup shock. However, it is never optimal to drive price dispersion to zero (by completely stabilizing the profit-maximizing price and thereby prices) because as price dispersion goes to zero the benefit of further reducing price dispersion goes to zero while the cost of further reducing price dispersion increases. For this reason, complete price stabilization is never optimal. By contrast, in the model with an endogenous information structure, we will find that complete price stabilization is always optimal.

Next, we consider the case of an autocorrelated desired markup to know whether or not complete price stabilization is optimal in the model with an exogenous information structure when $\rho_\lambda > 0$. When $\rho_\lambda > 0$, we solve the Ramsey problem (48)-(62) numerically. We turn this infinite-dimensional problem into a finite-dimensional problem by restricting $G_t(L)$ to be the same in each period and by restricting $G_t(L)$ to be the lag-polynomial of an ARMA(2,2) process. Following the procedure in Woodford (2002), one can then compute an exact linear rational expectations equilibrium of the model (49)-(62) *for a given monetary policy* by solving a Riccati equation. We then run a numerical optimization routine to obtain the optimal monetary policy.[5]

Figure 1 shows the optimal monetary policy response to a markup shock in the model with an exogenous information structure for the following parameter values: $\beta = 0.99$, $\gamma = 1$, $\psi = 0$, $\alpha = (2/3)$, $\Lambda = (1/3)$, $\sigma_\nu = 0.2$, and $\sigma_\zeta = 0.4$. The upper panel of Figure 1 shows optimal monetary policy in the case of $\rho_\lambda = 0$. The lower panel of Figure 1 shows optimal monetary policy in the case of $\rho_\lambda = 0.9$. For comparison, Figure 1 also shows optimal monetary policy in the Calvo model with perfect information and an average price duration of 2.5 quarters. The parameter value $\Lambda = (1/3)$ implies a steady-state price elasticity of demand of four, which is within the range of estimates of the price elasticity of demand in the Industrial Organization literature. The parameter values $\rho_\lambda = 0.9$ and $\sigma_\nu = 0.2$ are within the range of estimates of markup shocks in the New Keynesian literature. The Calvo parameter is taken from Nakamura and Steinsson (2008). The standard deviation of noise is set such that the model with imperfect information and the Calvo model yield the same response of the price level to a markup shock when the component of the profit-maximizing price driven by markup shocks is a random walk. The idea is that we aim to compare the model with imperfect information and the Calvo model for parameter values that imply the same degree of stickiness of the price level. All impulse responses are to a positive one standard deviation markup shock. A response equal to one means a one percent deviation from the non-stochastic steady state. Time is measured in quarters along the horizontal axis.

We obtain the following results. First of all, in the model with an exogenous information structure, complete price stabilization is still suboptimal when $\rho_\lambda > 0$. At the optimal monetary policy, the price level strictly increases on impact of a positive markup shock and composite consumption strictly falls on impact of a positive markup shock. Figure 1 shows this result for our benchmark parameter values. We solved the Ramsey problem (48)-(62) for many sets of parameter values with $\rho_\lambda > 0$ and we always obtained this result. Second, whether the optimal monetary policy response to markup shocks is similar in the model with exogenous imperfect information and in the Calvo model depends on ρ_λ. When $\rho_\lambda = 0.9$, we find that optimal monetary policy is roughly the same in the two models. When $\rho_\lambda = 0$, optimal monetary policy is quite different in the two models.

[5] We choose an ARMA(2,2) parameterization because it is well known from time series econometrics that an ARMA(p,q) parameterization is a very flexible and parsimonious parameterization.

Specifically, when $\rho_\lambda = 0$, the optimal policy in the model with exogenous imperfect information is to respond to the markup shock only in the period of the shock, while the optimal policy in the Calvo model is to respond to the markup shock also in the periods after the shock. The reason is the following. In the model with exogenous imperfect information, firms set prices period by period implying: (i) future monetary policy has no effect on today's price setting, and (ii) when $\rho_\lambda = 0$ today's markup shock creates no inefficiencies in future periods so long as the central bank does not respond to today's markup shock in future periods. Hence, when $\rho_\lambda = 0$, the optimal monetary policy in the model with exogenous imperfect information is to respond to the markup shock only in the period of the shock. In the Calvo model, price setting is forward looking and firms adjusting prices today but not tomorrow carry the markup shock forward. For these reasons, even when $\rho_\lambda = 0$, the optimal monetary policy in the Calvo model is to respond to the markup shock also in the periods after the shock. Despite these differences between the model with exogenous imperfect information and the Calvo model, the optimal monetary policy is roughly the same in the two models when $\rho_\lambda = 0.9$. We think this is an interesting result, but this is not the main result of this paper. We now turn to the main result of this paper.

7.2 Endogenous information structure

In this subsection, we specify the optimal monetary policy response to markup shocks in the model with an endogenous information structure. We begin with the case of an i.i.d. desired markup. In this case, we have a closed-form solution for the optimal monetary policy. It turns out that complete price stabilization in response to markup shocks is optimal. In fact, we find that in the model with an endogenous information structure complete price stabilization in response to markup shocks is optimal for all parameter values.

Proposition 5 *(Endogenous information structure) Consider the Ramsey problem (48)-(66), where the variances of noise σ_η^2 and σ_ζ^2 are given by the solution to problem (63)-(66). Suppose that $\rho_\lambda = 0$, $\sigma_\nu^2 > 0$, and $a_{-1} = \sigma_\varepsilon^2 = 0$. Consider policies of the form $G_t(L)\nu_t = g_0\nu_t$ and equilibria of the form $p_t = \theta\lambda_t$. Assume that $\mu > 0$ and define*

$$b \equiv \sqrt{\frac{\omega\left(\phi_c g_0 + \phi_\lambda\right)^2 \sigma_\lambda^2 \ln(2)}{\mu}}. \tag{81}$$

First, we characterize the set of equilibria at a given monetary policy $g_0 \in \mathbb{R}$. Here κ^* is the equilibrium attention devoted to the desired markup. If and only if $b \leq 1$, there exists an equilibrium with

$$\kappa^* = 0. \tag{82}$$

If and only if either $\phi_c \in \left(0, \frac{1}{2}\right]$ and $b \geq \sqrt{4\phi_c(1-\phi_c)}$ or $\phi_c > \frac{1}{2}$ and $b \geq 1$, there exists an equilibrium with

$$\kappa^* = \log_2\left(\frac{b + \sqrt{b^2 - 4\phi_c(1-\phi_c)}}{2\phi_c}\right). \tag{83}$$

If and only if $\phi_c \in \left(0, \frac{1}{2}\right]$ and $b \in \left[\sqrt{4\phi_c(1-\phi_c)}, 1\right]$, there exists an equilibrium with

$$\kappa^* = \log_2\left(\frac{b - \sqrt{b^2 - 4\phi_c(1-\phi_c)}}{2\phi_c}\right). \tag{84}$$

The equilibrium price level, composite consumption, and price dispersion are given by

$$p_t = \frac{\left(\phi_c g_0 + \phi_\lambda\right)\left(1 - 2^{-2\kappa^*}\right)}{1 - (1-\phi_c)\left(1 - 2^{-2\kappa^*}\right)}\lambda_t, \tag{85}$$

$$c_t = \left[g_0 - \frac{\left(\phi_c g_0 + \phi_\lambda\right)\left(1 - 2^{-2\kappa^*}\right)}{1 - (1-\phi_c)\left(1 - 2^{-2\kappa^*}\right)}\right]\lambda_t, \tag{86}$$

and

$$E\left[\left(p_{i,t} - p_t\right)^2\right] = \frac{\frac{\mu}{\omega}}{\ln(2)}\left(1 - 2^{-2\kappa^*}\right). \tag{87}$$

Second, we characterize optimal monetary policy. If $\phi_c \in \left[\frac{1}{2}, \infty\right)$, there exists a unique equilibrium for any monetary policy $g_0 \in \mathbb{R}$ and the unique optimal monetary policy is

$$g_0^* = \begin{cases} 0 & \text{if } \frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} \leq 1 \\ -\frac{\phi_\lambda}{\phi_c} + \frac{1}{\phi_c}\sqrt{\frac{\mu}{\omega\sigma_\lambda^2\ln(2)}} & \text{if } \frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} > 1 \end{cases}. \tag{88}$$

At this policy, decision-makers in firms who set prices devote no attention to the desired markup, and the price level does not respond to markup shocks.

Proof. See Appendix C. ■

The main result in Proposition 5 is that in the model with an endogenous information structure complete price stabilization in response to markup shocks is optimal when $\rho_\lambda = 0$, $\mu > 0$, and $\phi_c \in \left[\frac{1}{2}, \infty\right)$. The condition $\rho_\lambda = 0$ means that the desired markup follows a white noise process. The condition $\mu > 0$ means that there is some cost of devoting attention to the desired markup (this cost can be arbitrarily small or arbitrarily large). The condition $\phi_c \in \left[\frac{1}{2}, \infty\right)$ means that strategic complementarity in price setting is not so large such that there are multiple equilibria. Below we show analytically that in the case of an i.i.d. desired markup complete price stabilization in response to markup shocks is also optimal when $\mu = 0$ or $\phi_c \in \left(0, \frac{1}{2}\right)$. Hence, in the case of an i.i.d. desired markup complete price stabilization in response to markup shocks is *always* optimal in the model with an *endogenous information structure*. This result is in the starkest possible contrast to Proposition 4 stating that in the case of an i.i.d. desired markup complete price stabilization in response to markup shocks is *never* optimal in the model with an *exogenous information structure*.

To understand Proposition 5, let us first understand the optimal allocation of attention by decision-makers in firms because this is the new feature in the model with an endogenous information structure. The profit-maximizing price of good i in period t equals

$$\begin{aligned} p_{i,t}^{*} &= (1-\phi_c)\, p_t + \phi_c m_t - \phi_a a_t + \phi_\lambda \lambda_t \\ &= \left[(1-\phi_c)\,\theta + \phi_c g_0 + \phi_\lambda\right] \lambda_t. \end{aligned} \tag{89}$$

To find out about the current value of the profit-maximizing price, the decision-maker in firm i can pay attention to the desired markup. We let decision-makers in firms choose themselves how much attention they want to devote to the desired markup. The optimal attention devoted to the desired markup equals

$$\kappa^* = \begin{cases} \frac{1}{2}\log_2\left(\frac{\omega[(1-\phi_c)\theta+\phi_c g_0+\phi_\lambda]^2\sigma_\lambda^2\ln(2)}{\mu}\right) & \text{if } \frac{\omega[(1-\phi_c)\theta+\phi_c g_0+\phi_\lambda]^2\sigma_\lambda^2\ln(2)}{\mu} \geq 1 \\ 0 & \text{otherwise} \end{cases}. \tag{90}$$

The ratio $\frac{\omega[(1-\phi_c)\theta+\phi_c g_0+\phi_\lambda]^2\sigma_\lambda^2\ln(2)}{\mu}$ is the marginal benefit of paying attention to the desired markup at no attention devoted to the desired markup ($\kappa = 0$) divided by the marginal cost of paying attention to the desired markup. If this ratio exceeds one, the decision-maker pays some attention to the desired markup. If this ratio increases, the decision-maker pays more attention to the desired markup, which raises the signal-to-noise ratio in the signal concerning the desired markup. The benefit of paying attention to the desired markup depends on the variance of the profit-maximizing price due to the desired markup: $\left[(1-\phi_c)\,\theta + \phi_c g_0 + \phi_\lambda\right]^2 \sigma_\lambda^2$. This variance in turn depends on the behavior of other firms through θ and on monetary policy through g_0. In particular, as pointed out by Maćkowiak and Wiederholt (2009) and Hellwig and Veldkamp (2009), strategic complementarity in price setting leads to strategic complementarity in the allocation of attention.

When other firms are paying more attention to the desired markup, the price level responds more to the desired markup, which in the case of $(1-\phi_c)>0$ raises the incentive for an individual firm to pay attention to the desired markup. For this reason, multiple equilibria can in principle arise. However, when $\phi_c \in \left[\frac{1}{2},\infty\right)$, strategic complementarity in price setting is not strong enough for multiple equilibria to arise. In this case, if the compound parameter b defined by equation (81) is below one, the unique equilibrium attention is given by equation (82); while if the compound parameter b is above one, the unique equilibrium attention is given by equation (83). To illustrate this result, the upper panel of Figure 2 shows equilibrium attention as a function of the compound parameter b for $\phi_c = (1/2)$. By contrast, when $\phi_c \in \left(0,\frac{1}{2}\right)$, strategic complementarity in price setting is strong enough for multiple equilibria to arise at some values of the compound parameter b. To illustrate this result, the lower panel of Figure 2 shows equilibrium attention as a function of the compound parameter b for $\phi_c = (1/4)$.

Let us now turn to optimal monetary policy. Proposition 5 specifies optimal monetary policy when $\phi_c \in \left[\frac{1}{2},\infty\right)$ and thus there exists a unique equilibrium for any monetary policy $g_0 \in \mathbb{R}$. First, consider the case of $\frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} \le 1$. In this case, if the central bank does not respond to markup shocks (i.e., $g_0 = 0$), decision-makers in firms pay no attention to markup shocks because the marginal benefit of paying attention to random variation in the desired markup is smaller than the marginal cost of paying attention to random variation in the desired markup. Moreover, when neither the central bank nor firms respond to markup shocks, markup shocks create no inefficiencies. Hence, in the case of $\frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} \le 1$, a monetary policy of no response to markup shocks implements the efficient allocation and is therefore the optimal monetary policy. Second, consider the case of $\frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} > 1$. In this case, if the central bank does not respond to markup shocks (i.e., $g_0 = 0$), decision-makers in firms do pay attention to markup shocks because the marginal benefit of paying attention to random variation in the desired markup exceeds the marginal cost of paying attention to random variation in the desired markup. Hence, in the case of $\frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} > 1$, if the central bank does not respond to markup shocks, the price level increases after a positive markup shock, composite consumption falls after a positive markup shock, and there is price dispersion caused by the noise in the signal concerning the desired markup. Suppose instead that the central bank lowers the money supply after a positive markup shock (i.e., $g_0 < 0$). Recall first what happens in the model with an exogenous information structure: when the central bank lowers the money supply after a positive markup shock, price dispersion falls (because the profit-maximizing price increases by less after a positive markup shock implying that firms put less weight on their noisy signals) and consumption variance increases (because the reduction in the money supply after a positive markup shock amplifies the fall in consumption after a positive markup shock). Hence, there is a trade-off between price dispersion and consumption variance. Next return to the model with an endogenous information structure. In this model, there is one additional effect. When the central bank lowers the money supply after a positive markup shock, the profit-maximizing price increases by less after a positive markup shock, implying that the variance of the profit-maximizing price due to markup shocks falls. Decision-makers in firms pay less attention to markup shocks. Due to this one additional effect, the trade-off between price dispersion and consumption variance disappears. Think about consumption variance. On the one hand, the reduction in the money supply after a positive markup shock by itself amplifies the fall in consumption after a positive markup shock. On the other hand, the fact that decision-makers in firms are now paying less attention to markup shocks implies that the price level increases less after a positive markup shock, which by itself reduces the fall in consumption after a positive markup shock. It turns out that the second effect (less attention devoted to markup shocks) dominates for all parameter values. A monetary policy of reducing the money supply after a positive markup shock therefore *reduces* consumption variance. It discourages decision-makers in firms from paying attention to random variation in the desired markup. For this reason, the trade-off between price dispersion and consumption variance (which is a classic result in monetary economics) disappears. It is now straightforward to derive optimal monetary policy. So long as decision-makers in firms are paying some attention to markup shocks, the central bank can lower both price dispersion and consumption variance by counteracting the markup shock more strongly and thereby reducing the variance of the profit-maximizing price due to markup shocks. Once decision-makers in firms are paying no attention to markup shocks, price dispersion equals zero and reducing the money supply even further after a positive markup shock would only increase consumption variance. Hence, the optimal monetary policy is the one that makes decision-makers in firms just pay no attention to markup shocks. The lower part of equation (88) specifies this policy. At this policy, decision-makers in firms are paying no attention to markup shocks and thus prices do not respond to markup shocks. Complete price stabilization in response to markup shocks is optimal.

Figure 3 illustrates Proposition 5 for the following parameter values: $\gamma = 1$, $\psi = 0$, $\alpha = (2/3)$, $\Lambda = (1/3)$, $\rho_\lambda = 0$, and $\sigma_\lambda^2 = \left[(0.2)^2 / \left(1 - (0.9)^2\right)\right]$. Figure 3 depicts price setters' equilibrium attention devoted to markup shocks (κ^*) at the optimal monetary policy, the optimal monetary policy (g_0^*), and the loss in welfare due to markup shocks at the optimal monetary policy for different values of (μ/ω). Recall that $\mu > 0$ is the per-period marginal cost of attention of the decision-maker in a firm and $\omega > 0$ is the constant in the price setters' objective (63).

Proposition 5 specifies optimal monetary policy when $\phi_c \in \left[\frac{1}{2}, \infty\right)$, that is, when strategic complementarity in price setting is not large enough for multiple equilibria to arise. Proposition 6 specifies optimal monetary policy when $\phi_c \in \left(0, \frac{1}{2}\right)$, that is, when strategic complementarity in price setting is large enough for multiple equilibria to arise at some monetary policies $g_0 \in \mathbb{R}$. Before one can make a statement about optimal monetary policy in this case, one has to make an assumption about the central bank's attitude towards multiple equilibria. The most common assumption in the literature seems to be that central banks are very adverse to multiple equilibria. Therefore, we assume that the central bank aims to implement the best policy among all those monetary policies $g_0 \in \mathbb{R}$ that yield a unique equilibrium.

Proposition 6 *(Endogenous information structure) Consider the Ramsey problem (48)-(66), where the variances of noise σ_η^2 and σ_ζ^2 are given by the solution to problem (63)-(66). Suppose that $\rho_\lambda = 0$, $\sigma_\nu^2 > 0$, and $a_{-1} = \sigma_\varepsilon^2 = 0$. Consider policies of the form $G_t(L)\,\nu_t = g_0 \nu_t$ and equilibria of the form $p_t = \theta \lambda_t$. Assume that $\mu > 0$. If $\phi_c \in \left(0, \frac{1}{2}\right)$, there exist multiple equilibria for all $g_0 \in [\hat{g}_0, \bar{g}_0]$ where*

$$\hat{g}_0 = -\frac{\phi_\lambda}{\phi_c} + \frac{\sqrt{4\phi_c(1-\phi_c)}}{\phi_c}\sqrt{\frac{\mu}{\omega\sigma_\lambda^2 \ln(2)}}, \tag{91}$$

and

$$\bar{g}_0 = -\frac{\phi_\lambda}{\phi_c} + \frac{1}{\phi_c}\sqrt{\frac{\mu}{\omega\sigma_\lambda^2 \ln(2)}}. \tag{92}$$

If $\frac{\omega\phi_\lambda^2\sigma_\lambda^2 \ln(2)}{\mu} < 4\phi_c(1-\phi_c)$, the best policy among all $g_0 \in \mathbb{R}$ that yield a unique equilibrium is $g_0^* = 0$. If $\frac{\omega\phi_\lambda^2\sigma_\lambda^2 \ln(2)}{\mu} \geq 4\phi_c(1-\phi_c)$, the best policy among all $g_0 \in \mathbb{R}$ that yield a unique equilibrium is a g_0 marginally below $\hat{g}_0$. At this policy, decision-makers in firms who set prices devote no attention to the desired markup, and the price level does not respond to markup shocks.

Proof. See Appendix C. ■

The main result in Proposition 6 is that in the model with an endogenous information structure complete price stabilization in response to markup shocks is also optimal when $\phi_c \in \left(0, \frac{1}{2}\right)$. To understand this result, note the following. When $g_0 \in [\hat{g}_0, \bar{g}_0]$, the compound parameter b governing the benefit to the cost of paying attention to markup shocks lies in the interval $\left[\sqrt{4\phi_c (1-\phi_c)}, 1\right]$. Furthermore, the first half of Proposition 5 states that, if $\phi_c \in \left(0, \frac{1}{2}\right)$ and $b \in \left[\sqrt{4\phi_c (1-\phi_c)}, 1\right]$, then multiple equilibria arise. See Figure 2 for an illustration. Hence, if $\phi_c \in \left(0, \frac{1}{2}\right)$, the central bank has to choose $g_0 \notin [\hat{g}_0, \bar{g}_0]$ to avoid multiple equilibria. Next, think about optimal monetary policy. When $\frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} < 4\phi_c (1-\phi_c)$ we have $\hat{g}_0 > 0$. Thus, at the policy $g_0 = 0$, $\kappa^* = 0$ is the unique equilibrium. When the central bank and firms do not respond to markup shocks, those shocks create no inefficiencies. Hence, in the case of $\frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} < 4\phi_c (1-\phi_c)$, a monetary policy of no response to markup shocks is the optimal monetary policy. By contrast, when $\frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} \geq 4\phi_c (1-\phi_c)$ we have $\hat{g}_0 \leq 0$. Thus, at the policy $g_0 = 0$, $\kappa^* = 0$ is *not* the unique equilibrium. To understand optimal monetary policy in this case, consider the lower panel of Figure 2. When $g_0 > \bar{g}_0$ and thus $b > 1$, both price dispersion and consumption variance fall when the central bank *reduces* g_0 for the same reasons given below Proposition 5. When $g_0 < \hat{g}_0$ and thus $b < \sqrt{4\phi_c (1-\phi_c)}$, consumption variance falls when the central bank *increases* g_0. Finally, the no attention equilibrium at $g_0 = \hat{g}_0$ strictly dominates the high positive attention equilibrium at $g_0 = \bar{g}_0$. Hence, the best policy among all policies that yield a unique equilibrium is a g_0 marginally below $\hat{g}_0$. At this policy, decision-makers in firms who set prices pay no attention to random variation in the desired markup, and prices do not respond to markup shocks. Complete price stabilization in response to markup shocks is optimal.

Finally, we consider the case of an autocorrelated desired markup. When $\rho_\lambda > 0$, we solve the Ramsey problem (48)-(66) numerically. Figure 4 shows the optimal monetary policy response to a markup shock in the model with an endogenous information structure for the following parameter values: $\beta = 0.99$, $\gamma = 1$, $\psi = 0$, $\alpha = (2/3)$, $\Lambda = (1/3)$, $\rho_\lambda = 0.9$, and $\sigma_\nu = 0.2$. At the optimal monetary policy, decision-makers in firms who set prices devote no attention to random variation in the desired markup, and the price level does not respond to markup shocks. Complete price stabilization in response to markup shocks is optimal. We solved the Ramsey problem (48)-(66) for many sets of parameter values with $\rho_\lambda > 0$ and we always obtained this result.

8 Additional results and robustness of the main results

In this section, we present two additional results for the model with an endogenous information structure (Sections 8.1-8.2). Furthermore, we show that our main conclusions are robust to several modifications of the model with an endogenous information structure (Sections 8.3-8.6). Here we want to highlight two results. First, optimal monetary policy remains the same when decision-makers in firms can decide to receive signals concerning any linear combination of a_t and λ_t. Second, the optimality of complete price stabilization extends from markup shocks to a much larger class of shocks.

8.1 Welfare at the optimal monetary policy

We now study how welfare at the optimal monetary policy varies with the parameters governing the degree of information friction. These parameters are: the marginal cost of attention in the model with an endogenous information structure, μ, and the variances of noise in the model with an exogenous information structure, σ_η^2 and σ_ζ^2.

In the model with an endogenous information structure and $\rho_\lambda = 0$, the value of the central bank's objective (48) at the optimal policy specified in Propositions 3, 5, and 6 equals

$$\sum_{t=0}^{\infty} \beta^t E\left[(c_t - c_t^*)^2 + \delta \frac{1}{I} \sum_{i=1}^{I} (p_{i,t} - p_t)^2\right] = \frac{1}{1-\beta} (g_0^*)^2 \sigma_\lambda^2, \tag{93}$$

where g_0^* is given by Proposition 5 if $\phi_c \in \left[\frac{1}{2}, \infty\right)$ and g_0^* is given by Proposition 6 if $\phi_c \in \left(0, \frac{1}{2}\right)$. The loss in welfare relative to the efficient allocation is weakly decreasing in μ because the absolute value of g_0^* is weakly decreasing in μ. See Figure 3 for an illustration. The intuition is simple. When price setters' marginal cost of paying attention to markup shocks is larger, the central bank does not have to counteract markup shocks as much to discourage price setters from paying attention to these shocks that cause inefficient fluctuations.

In the model with an exogenous information structure and $\rho_\lambda = 0$, the value of the central bank's objective (48) at the optimal policy specified in Propositions 2 and 4 equals

$$\sum_{t=0}^{\infty} \beta^t E\left[(c_t - c_t^*)^2 + \delta \frac{1}{I} \sum_{i=1}^{I} (p_{i,t} - p_t)^2\right] = \frac{1}{1-\beta}\left[\left(\frac{\delta \phi_c \phi_\lambda}{\frac{\sigma_\zeta^2}{\sigma_\lambda^2} + \delta \phi_c^2}\right)^2 \sigma_\lambda^2 + \delta \left(\frac{\phi_\lambda}{\frac{\sigma_\zeta^2}{\sigma_\lambda^2} + \delta \phi_c^2}\right)^2 \sigma_\zeta^2\right]. \tag{94}$$

The loss in welfare relative to the efficient allocation is decreasing in the variance of noise σ_ζ^2. A larger variance of noise decreases inefficient consumption fluctuations because prices respond less strongly to markup shocks. On the other hand, a larger variance of noise may increase price dispersion because the signal concerning the desired markup is more noisy. It turns out that the first effect dominates, that is, the derivative of expression (94) with respect to σ_ζ^2 is strictly negative. Hence, the loss in welfare is decreasing in σ_ζ^2.

In summary, in the model with an endogenous information structure easier access to information concerning markup shocks reduces welfare, and in the model with an exogenous information structure a more precise private signal concerning markup shocks reduces welfare. The reason is that markup shocks cause inefficient consumption fluctuations.[6]

8.2 The value of commitment

It is well understood that in the Calvo model there is a value of commitment to future monetary policy when there are markup shocks. By contrast, in the model with exogenous imperfect information, there is no value of commitment to future monetary policy because firms set prices period by period. Finally, in the model with an endogenous information structure, there is a value of commitment to future monetary policy when there are markup shocks and $g_0^* < 0$. The reason is the following. In this case, only when the central bank can commit, price setters can trust the central bank that not paying attention to aggregate conditions is optimal. We find it interesting that the nature of the value of commitment in the rational inattention model is different from the nature of the value of commitment in the Calvo model.

8.3 More general signal structure

We have so far assumed that paying attention to aggregate productivity and paying attention to the desired markup are independent activities. Formally, in equation (55) we assume that the decision-maker in firm i who has to set a price receives two independent signals concerning aggregate productivity and the desired markup. The interpretation of this assumption is that paying attention to aggregate productivity and paying attention to the desired markup are independent activities.

[6] Baeriswyl and Cornand (2010) also argue that information about markup shocks is harmful. They study a model with an exogenous information structure.

We now relax this assumption. We assume that the decision-maker in firm i who has to set a price can pay attention to any variable that is a linear combination of a_t and λ_t. Formally, the signal that the price setter of good i receives in period t can be any signal of the form

$$s_{i,t} = \xi_a a_t + \xi_\lambda \lambda_t + \zeta_{i,t}. \tag{95}$$

In the new model with an endogenous information structure, the decision-maker in firm i chooses the coefficients $(\xi_a, \xi_\lambda) \in \mathbb{R}^2$ and the variance of noise $\sigma_\zeta^2 \in \mathbb{R}_+$.[7] We interpret the choice of (ξ_a, ξ_λ) as the choice of which variable to pay attention to (e.g., ξ_a and ξ_λ could be chosen such that $\xi_a a_t + \xi_\lambda \lambda_t$ equals the equilibrium price level or equilibrium consumption). We interpret the choice of σ_ζ^2 as the choice of how much attention to devote to this variable.

For the old signal structure (55), the information flow constraint (20) reduces to equation (65). For the new signal structure (95), this is no longer the case and we therefore have to work with the original formulation of the information flow constraint (20). Hence, in the new model with an endogenous information structure, the Ramsey problem (48)-(66) changes as follows. Equation (95) replaces equation (55), equation (20) replaces equation (65), and the decision-maker in firm i chooses $\left(\xi_a, \xi_\lambda, 1/\sigma_\zeta^2\right)$ rather than $\left(1/\sigma_\eta^2, 1/\sigma_\zeta^2\right)$.

In the case of $\rho_a = \rho_\lambda = 0$, we can solve this new Ramsey problem analytically. In particular, the optimal monetary policy is again the monetary policy specified in Propositions 3, 5, and 6. The reason is quite simple. When $\rho_a = \rho_\lambda = 0$, the optimal signal is a signal concerning the profit-maximizing price, that is, the optimal choice of (ξ_a, ξ_λ) is the (ξ_a, ξ_λ) with the property that $\xi_a a_t + \xi_\lambda \lambda_t$ equals the equilibrium profit-maximizing price. In addition, the optimal attention devoted to the profit-maximizing price equals

$$\kappa^* = \begin{cases} \frac{1}{2}\log_2\left(\frac{\omega\sigma_{p^*}^2 \ln(2)}{\mu}\right) & \text{if } \frac{\omega\sigma_{p^*}^2 \ln(2)}{\mu} \geq 1 \\ 0 & \text{otherwise} \end{cases}, \tag{96}$$

where $\sigma_{p^*}^2$ is the variance of the profit-maximizing price. The signal-to-noise ratio then equals

$$\frac{\sigma_{p^*}^2}{\sigma_\zeta^2} = 2^{2\kappa^*} - 1, \tag{97}$$

and the price set by firm i in period t equals

$$p_{i,t} = E\left[p_{i,t}^* | \mathcal{I}_{i,t}\right] = \left(1 - 2^{-2\kappa^*}\right)\left(p_{i,t}^* + \zeta_{i,t}\right). \tag{98}$$

[7] Adam (2007) and Mondria (2010) model the attention decision in a similar way.

Let us now turn to optimal monetary policy. First, consider the monetary policy response to aggregate productivity shocks. Suppose that the central bank commits to the monetary policy specified in Proposition 3. This policy yields the efficient response of composite consumption to aggregate productivity shocks. Furthermore, this policy yields a variance of the profit-maximizing price due to aggregate productivity shocks of zero. Thus, this policy is the monetary policy response to aggregate productivity shocks that yields the smallest κ^* and the smallest price dispersion. Moreover, a small κ^* is good because then prices respond less to markup shocks. For these reasons, the optimal monetary policy response to aggregate productivity shocks is the monetary policy specified in Proposition 3. Second, once the profit-maximizing price does not respond to aggregate productivity shocks, equation (96) reduces to equation (90). In other words, the firms' optimal allocation of attention is exactly the same as in the model with no aggregate productivity shocks. For this reason, the optimal monetary policy response to markup shocks is the monetary policy specified in Propositions 5 and 6.

8.4 More general shocks

We have so far studied two types of shocks: aggregate productivity shocks and markup shocks. We now show that the results from Sections 6 and 7 extend to a much larger class of shocks. Consider a more general exogenous variable z_t that may affect both efficient composite consumption and the profit-maximizing price:

$$c_t^* = \varphi z_t, \tag{99}$$

and

$$p_{i,t}^* = p_t + \phi_c c_t + \phi_z z_t, \tag{100}$$

where $\varphi \in \mathbb{R}$, $\phi_z \in \mathbb{R}_{++}$, and z_t follows a stationary Gaussian first-order autoregressive process. Two examples of the exogenous variable z_t are aggregate productivity and the desired markup. If $z_t = -a_t$ then $\phi_z = \phi_a$ and $\varphi = -(\phi_a/\phi_c)$. If $z_t = \lambda_t$ then $\phi_z = \phi_\lambda$ and $\varphi = 0$. Apart from introducing a more general exogenous variable by replacing equations (49) and (53) by (99) and (100), the Ramsey problem (48)-(66) remains unchanged.[8] In particular, each price setter chooses the precision of his/her signal knowing that a more precise signal requires more attention.

[8] For clarity of exposition, we now assume in equations (99)-(100) that there is only one exogenous variable.

Formally, $s_{i,t} = z_t + \eta_{i,t}$ and $\kappa = \frac{1}{2}\log_2\left(\frac{\sigma^2_{z|s_i^{t-1}}}{\sigma^2_{z|s_i^t}}\right)$. What is the optimal monetary policy response to an innovation in z_t?

First, note that if decision-makers in firms had perfect information then each firm would set the profit-maximizing price, implying that

$$c_t = -\frac{\phi_z}{\phi_c} z_t, \tag{101}$$

and

$$p_{i,t} - p_t = 0. \tag{102}$$

Second, consider the case of $-\frac{\phi_z}{\phi_c} = \varphi$. In this case, a z shock (i.e., an innovation in z_t) has the efficiency property: the response of the economy to a z shock under perfect information is efficient. Then, the reasoning of Sections 5 and 6 applies. By setting $m_t = c_t^*$ the central bank can replicate the perfect-information response of the economy to a z shock and this response is efficient. Hence, in the case of $-\frac{\phi_z}{\phi_c} = \varphi$, the optimal monetary policy response to a z shock is given by $m_t = c_t^*$. One example of a z shock with the property $-\frac{\phi_z}{\phi_c} = \varphi$ is an aggregate productivity shock in the model given in Section 2.

Third, consider the case of $-\frac{\phi_z}{\phi_c} < \varphi$. In this case, a z shock has the inefficiency property: the response of the economy to a z shock under perfect information is inefficient. More precisely, either the response of actual consumption and the response of efficient consumption have the same sign but the response of actual consumption is larger in magnitude (i.e., $-\frac{\phi_z}{\phi_c} < \varphi \leq 0$) or the response of actual consumption and the response of efficient consumption have the opposite sign (i.e., $\varphi > 0$). One example of a z shock with the property $-\frac{\phi_z}{\phi_c} < \varphi$ is a markup shock in the model given in Section 2 because the response of equilibrium consumption to a markup shock under perfect information is larger in magnitude than the response of efficient consumption to a markup shock. It turns out that Proposition 5 generalizes in a straightforward way from markup shocks to any z shock with the property $-\frac{\phi_z}{\phi_c} < \varphi$. In the beginning of the new proposition it says $\rho_z = 0$, $\sigma_z^2 > 0$, $m_t = g_0 z_t$, and $p_t = \theta z_t$. The only change in equations (81)-(87) is that ϕ_z and σ_z^2 replace ϕ_λ and σ_λ^2. Furthermore, equation (88) becomes

$$g_0^* = \begin{cases} \varphi & \text{if } \frac{\omega(\phi_c\varphi+\phi_z)^2\sigma_z^2\ln(2)}{\mu} \leq 1 \\ -\frac{\phi_z}{\phi_c} + \frac{1}{\phi_c}\sqrt{\frac{\mu}{\omega\sigma_z^2\ln(2)}} & \text{if } \frac{\omega(\phi_c\varphi+\phi_z)^2\sigma_z^2\ln(2)}{\mu} > 1 \end{cases}. \tag{103}$$

Most importantly, at the optimal monetary policy, decision-makers in firms who set prices devote no attention to the variable z_t, and the price level does not respond to an innovation in z_t. Hence, in the model with an endogenous information structure, complete price stabilization is optimal in response to any z shock with the property $-\frac{\phi_z}{\phi_c} < \varphi$.

Finally, consider the case of $\varphi < -\frac{\phi_z}{\phi_c}$. In this case, the response of actual consumption and the response of efficient consumption have the same sign but the response of efficient consumption is larger in magnitude. The proof of Proposition 5 does not generalize in a straightforward way from markup shocks to this case. Therefore, we do not know yet whether complete price stabilization is optimal in response to z shocks with the property $\varphi < -\frac{\phi_z}{\phi_c}$. This may or may not be the case.

In summary, our result of complete price stabilization extends from aggregate productivity shocks (i.e., $-\frac{\phi_z}{\phi_c} = \varphi$) and markup shocks (i.e., $\varphi = 0$) to any z shock with the property $-\frac{\phi_z}{\phi_c} \leq \varphi$ (i.e., any z shock with the property that equilibrium fluctuations under perfect information equal, are larger in magnitude, or have the opposite sign than efficient fluctuations).

8.5 Information of the central bank

So far we did not model the information choice of the central bank. We simply assumed that the central bank has perfect information and can therefore implement the optimal monetary policy. We now study the central bank's benefit of learning aggregate conditions. If the central bank has no information about aggregate conditions, the central bank cannot respond to aggregate productivity shocks and markup shocks. Figure 5 shows the central bank's benefit of learning aggregate conditions for the following parameter values: $\gamma = 1$, $\psi = 0$, $\alpha = (2/3)$, $\Lambda = (1/3)$, $\rho_a = \rho_\lambda = 0$, $\sigma_a^2 = \left[(0.0085)^2 / \left(1 - (0.95)^2\right)\right]$ and $\sigma_\lambda^2 = \left[(0.2)^2 / \left(1 - (0.9)^2\right)\right]$.[9] In particular, the upper panel shows the loss in welfare (compared to the efficient allocation) in the case of the optimal monetary policy response to aggregate productivity shocks and in the case of no policy response to aggregate productivity shocks. The lower panel of Figure 5 shows the loss in welfare (compared to the efficient allocation) in the case of the optimal monetary policy response to markup shocks and in the case of no policy response to markup shocks. For values of (μ/ω) between $0.1 * 10^{-3}$ and

[9] The parameter values $\rho_a = 0.95$ and $\sigma_\varepsilon = 0.0085$ are a standard calibration of the aggregate productivity process. We set $\rho_a = 0$ and $\sigma_a^2 = \left[(0.0085)^2 / \left(1 - (0.95)^2\right)\right]$ to make Figure 5 comparable to Figure 3. The figure looks similar for $\rho_a = 0.95$ and $\sigma_\varepsilon = 0.0085$, which again implies $\sigma_a^2 = \left[(0.0085)^2 / \left(1 - (0.95)^2\right)\right]$.

$0.9{*}10^{-3}$, the per-period welfare gain from implementing the optimal monetary policy is quite large: about half a percent of steady state consumption.[10] Hence, for these values of (μ/ω), the central bank has a substantial incentive to become informed about aggregate conditions to implement the optimal monetary policy. At the same time, this optimal monetary policy makes the variance of the profit-maximizing price due to aggregate shocks sufficiently small such that price setters in firms pay no attention to aggregate conditions.[11]

Furthermore, the central bank does not literally have to know aggregate productivity and the desired markup. Suppose that the central bank knows output, the output gap and the price level, where the output gap is defined as output minus efficient output. The central bank can then implement a monetary policy that is arbitrarily close to the optimal monetary policy. The idea is simple: if the central bank encourages firms to pay a little bit of attention to the desired markup, the price level reveals the desired markup to the central bank and nevertheless the loss in welfare due to deviations from the optimal monetary policy can be made arbitrarily small. Formally, consider the case of $\rho_\lambda = 0$, $\phi_c \in \left[\frac{1}{2}, \infty\right)$, and $\frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} > 1$.[12] Suppose that the central bank implements the following monetary policy

$$m_t = c_t^* + g_0 \frac{1 - (1 - \phi_c)\left(1 - 2^{-2\kappa^*}\right)}{(\phi_c g_0 + \phi_\lambda)\left(1 - 2^{-2\kappa^*}\right)} p_t. \tag{104}$$

Here c_t^* is efficient composite consumption, $g_0 \in (g_0^*, 0)$ where g_0^* is given by equation (88), κ^* is given by equations (81) and (83), and the ratio in front of the price level is the inverse of the ratio in equation (85). Since $g_0 > g_0^*$, price setters pay some attention to markup shocks and the price level reveals the desired markup to the central bank. At the same time, by making the difference $g_0 - g_0^*$ arbitrarily small, the central bank can approximate the optimal monetary policy arbitrarily well.

[10] Maćkowiak and Wiederholt (2010) solve a DSGE model with rational inattention and a Taylor rule. They find that for a value of (μ/ω) between $0.1 * 10^{-3}$ and $0.2 * 10^{-3}$ the model matches various empirical impulse responses of prices to shocks.

[11] In the welfare calculations, we are not taking into account that the optimal monetary policy has additional welfare benefits due to the fact that price setters do not have to pay attention to aggregate conditions and can thus focus on firm-specific conditions. Taking these additional welfare benefits into account would strengthen the case for complete price stabilization in response to aggregate shocks and would increase the welfare gain from optimal monetary policy.

[12] The case $\frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} > 1$ is the interesting case, because when $\frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} \leq 1$ the central bank needs no information about the desired markup to implement the optimal policy. See Proposition 5.

In the New Keynesian literature on optimal monetary policy, it is typically assumed that the central bank knows the output gap and inflation, where the output gap is defined as output minus efficient output. See for example Woodford (2003), Chapter 8, and Giannoni and Woodford (2010). Thus, when we assume that the central bank knows output, the output gap and the price level, we make essentially the same assumption about information of the central bank as the standard New Keynesian literature. Despite this, we obtain a markedly different result concerning optimal monetary policy: the optimal monetary policy is arbitrarily close to complete price stabilization in response to markup shocks and the only purpose of arbitrarily small fluctuations in the price level is to reveal the desired markup to the central bank.

We think it would be interesting to study optimal monetary policy when the central bank only observes noisy indicators of the output gap and inflation. Svensson and Woodford (2003, 2004) study this question in a New Keynesian model and Lorenzoni (2010) studies this question in a model with exogenous dispersed information. For studying this question, it will be useful to know what the central bank should do when the central bank has perfect information, which is the content of this paper.

8.6 Interest rate rule

We now assume that the central bank commits to an interest rate rule, rather than a money supply rule. Assuming that the central bank can commit to an interest rate rule of the form (11) instead of a money supply rule of the form (10) does not affect optimal monetary policy. This is because the set of equilibria that the central bank can implement with an interest rate rule of the form (11) equals the set of equilibria that the central bank can implement with a money supply rule of the form (10). To see this, note the following. We so far did not use the log-linearized consumption Euler equation in the Ramsey problem (48)-(66) because in the case of a money supply rule this equation only determines the equilibrium nominal interest rate. Now take a law of motion of the economy that is an equilibrium law of motion under some money supply rule of the form (10). One can then compute the equilibrium law of motion for the nominal interest rate from the consumption Euler equation and the central bank can commit to this law of motion as an interest rate rule. Similarly, take a law of motion of the economy that is an equilibrium law of motion under some interest rate rule of the form (11). One can then compute the equilibrium law of motion for the money supply from equation (50) and the central bank can commit to this law of motion as a money supply rule.[13]

9 Conclusion

This paper studies optimal monetary policy in a model with exogenous dispersed information and in a rational inattention model. In the model with exogenous dispersed information, complete stabilization of the price level is optimal after aggregate productivity shocks but not after markup shocks. By contrast, in the rational inattention model, complete stabilization of the price level is optimal both after aggregate productivity shocks and after markup shocks. Furthermore, in the model with exogenous dispersed information, there is no value from commitment to a future monetary policy, while in the rational inattention model there is value from commitment to a future monetary policy because then the private sector can trust the central bank that not paying attention to certain variables is optimal.

[13] One issue that arises when the central bank commits to an interest rate rule of the form (11) instead of a money supply rule of the form (10) is the following. When the policy rule specifies the nominal interest rate as a function of exogenous events, the equilibrium is typically not unique. One way to address this issue is to allow the nominal interest rate to depend on endogenous variables and to allow the policy rule to differ on and off the equilibrium path.

A Proof of Proposition 1

First, we introduce notation. The function u is given by equation (27). Let x_t denote the vector of all arguments of the function u that are endogenous variables

$$x_t = \begin{pmatrix} c_t & \hat{c}_{1,t} & \cdots & \hat{c}_{I-1,t} \end{pmatrix}'. \tag{105}$$

Let z_t denote the vector of all arguments of the function u that are exogenous variables

$$z_t = \begin{pmatrix} a_t & \lambda_t \end{pmatrix}'. \tag{106}$$

Second, we compute a log-quadratic approximation of expected utility (24) around the non-stochastic steady state. Let $\tilde{u}$ denote the second-order Taylor approximation to the function u at the origin. We have

$$\begin{aligned} & E\left[\sum_{t=0}^{\infty} \beta^t \tilde{u}(x_t, z_t)\right] \\ = \; & E\left[\sum_{t=0}^{\infty} \beta^t \left(u(0,0) + h_x' x_t + h_z' z_t + \frac{1}{2} x_t' H_x x_t + x_t' H_{xz} z_t + \frac{1}{2} z_t' H_z z_t\right)\right], \end{aligned} \tag{107}$$

where h_x is the vector of first derivatives of u with respect to x_t evaluated at the origin, h_z is the vector of first derivatives of u with respect to z_t evaluated at the origin, H_x is the matrix of second derivatives of u with respect to x_t evaluated at the origin, H_z is the matrix of second derivatives of u with respect to z_t evaluated at the origin, and H_{xz} is the matrix of second derivatives of u with respect to x_t and z_t evaluated at the origin. Third, we rewrite equation (107) using condition (31). Let ω_t denote the following vector

$$\omega_t = \begin{pmatrix} x_t' & z_t' & 1 \end{pmatrix}', \tag{108}$$

and let $\omega_{n,t}$ denote the nth element of ω_t. Condition (31) implies that

$$\sum_{t=0}^{\infty} \beta^t E\left| u(0,0) + h_x' x_t + h_z' z_t + \frac{1}{2} x_t' H_x x_t + x_t' H_{xz} z_t + \frac{1}{2} z_t' H_z z_t \right| < \infty. \tag{109}$$

It follows that one can change the order of integration and summation on the right-hand side of equation (107):

$$\begin{aligned} & E\left[\sum_{t=0}^{\infty} \beta^t \tilde{u}(x_t, z_t)\right] \\ = \; & \sum_{t=0}^{\infty} \beta^t E\left[u(0,0) + h_x' x_t + h_z' z_t + \frac{1}{2} x_t' H_x x_t + x_t' H_{xz} z_t + \frac{1}{2} z_t' H_z z_t\right]. \end{aligned} \tag{110}$$

See Rao (1973), p. 111. Condition (31) also implies that the infinite sum on the right-hand side of equation (110) converges to an element in $\mathbb{R}$. Fourth, we define the vector x_t^*. In each period $t \geq 0$, the vector x_t^* is defined by

$$h_x + H_x x_t^* + H_{xz} z_t = 0. \tag{111}$$

We will show below that H_x is an invertible matrix. Therefore, one can write the last equation as

$$x_t^* = -H_x^{-1} h_x - H_x^{-1} H_{xz} z_t. \tag{112}$$

Hence, x_t^* is uniquely determined and the vector ω_t with $x_t = x_t^*$ satisfies condition (31). Fifth, equation (110) implies that

$$\begin{aligned} & E\left[\sum_{t=0}^{\infty} \beta^t \tilde{u}\left(x_t, z_t\right)\right] - E\left[\sum_{t=0}^{\infty} \beta^t \tilde{u}\left(x_t^*, z_t\right)\right] \\ = & \sum_{t=0}^{\infty} \beta^t E\left[h_x'\left(x_t - x_t^*\right) + \frac{1}{2} x_t' H_x x_t - \frac{1}{2} x_t^{*\prime} H_x x_t^* + \left(x_t - x_t^*\right)' H_{xz} z_t\right]. \end{aligned} \tag{113}$$

Using equation (111) to substitute for $H_{xz} z_t$ in the last equation and rearranging yields

$$\begin{aligned} & E\left[\sum_{t=0}^{\infty} \beta^t \tilde{u}\left(x_t, z_t\right)\right] - E\left[\sum_{t=0}^{\infty} \beta^t \tilde{u}\left(x_t^*, z_t\right)\right] \\ = & \sum_{t=0}^{\infty} \beta^t E\left[\frac{1}{2}\left(x_t - x_t^*\right)' H_x \left(x_t - x_t^*\right)\right]. \end{aligned} \tag{114}$$

Sixth, we compute the vector of first derivatives and the matrices of second derivatives appearing in equations (112) and (114). We obtain

$$h_x = 0, \tag{115}$$

$$H_x = -C^{1-\gamma} \begin{bmatrix} \gamma - 1 + \frac{1}{\alpha}(1+\psi) & 0 & \cdots & \cdots & 0 \\ 0 & 2\frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} & \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} & \cdots & \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} \\ \vdots & \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} \\ 0 & \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} & \cdots & \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} & 2\frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} \end{bmatrix}, \tag{116}$$

and

$$H_{xz} = C^{1-\gamma} \begin{bmatrix} \frac{1}{\alpha}(1+\psi) & 0 \\ 0 & 0 \\ \vdots & \vdots \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}. \tag{117}$$

Seventh, substituting equations (115)-(117) into equation (111) yields the following system of I equations:

$$c_t^* = \frac{\frac{1}{\alpha}(1+\psi)}{\gamma - 1 + \frac{1}{\alpha}(1+\psi)} a_t, \tag{118}$$

and, for all $i = 1, \ldots, I-1$,

$$\hat{c}_{i,t}^* + \sum_{k=1}^{I-1} \hat{c}_{k,t}^* = 0. \tag{119}$$

Finally, we rewrite equation (119). Summing equation (119) over all $i \neq I$ yields

$$\sum_{i=1}^{I-1} \hat{c}_{i,t}^* = 0. \tag{120}$$

Substituting the last equation back into equation (119) yields

$$\hat{c}_{i,t}^* = 0. \tag{121}$$

Collecting equations (114), (116), (118) and (121), we arrive at Proposition 1.

B Proof of Proposition 4

Step 1: Substituting the cash-in-advance constraint (50), $a_t = 0$, the monetary policy $m_t = g_0 \lambda_t$, and $p_t = \theta \lambda_t$ into the equation for the profit-maximizing price (53) yields

$$p_{i,t}^* = \left[(1-\phi_c)\theta + \phi_c g_0 + \phi_\lambda\right] \lambda_t. \tag{122}$$

The price of good i in period t then equals

$$\begin{aligned} p_{i,t} &= \left[(1-\phi_c)\theta + \phi_c g_0 + \phi_\lambda\right] E\left[\lambda_t | \mathcal{I}_{i,t}\right] \\ &= \left[(1-\phi_c)\theta + \phi_c g_0 + \phi_\lambda\right] \frac{\sigma_\lambda^2}{\sigma_\lambda^2 + \sigma_\zeta^2} \left(\lambda_t + \zeta_{i,t}\right), \end{aligned} \tag{123}$$

and the price level in period t equals

$$p_t = \left[\left(1-\phi_c\right)\theta + \phi_c g_0 + \phi_\lambda\right]\frac{\sigma_\lambda^2}{\sigma_\lambda^2+\sigma_\zeta^2}\lambda_t. \tag{124}$$

Thus, the unique rational expectations equilibrium of the form $p_t = \theta\lambda_t$ is given by the solution to the following equation

$$\theta = \left[\left(1-\phi_c\right)\theta + \phi_c g_0 + \phi_\lambda\right]\frac{\sigma_\lambda^2}{\sigma_\lambda^2+\sigma_\zeta^2}.$$

Solving the last equation for θ yields

$$\theta = \frac{\phi_c g_0 + \phi_\lambda}{\phi_c + \frac{\sigma_\zeta^2}{\sigma_\lambda^2}}. \tag{125}$$

Substituting equation (125) into equations (123) and (124) yields

$$p_t = \frac{\phi_c g_0 + \phi_\lambda}{\phi_c + \frac{\sigma_\zeta^2}{\sigma_\lambda^2}}\lambda_t, \tag{126}$$

$$p_{i,t} - p_t = \frac{\phi_c g_0 + \phi_\lambda}{\phi_c + \frac{\sigma_\zeta^2}{\sigma_\lambda^2}}\zeta_{i,t}. \tag{127}$$

Finally, substituting the monetary policy $m_t = g_0\lambda_t$ and equation (126) into equation (50) yields

$$c_t = \frac{\frac{\sigma_\zeta^2}{\sigma_\lambda^2}g_0 - \phi_\lambda}{\phi_c + \frac{\sigma_\zeta^2}{\sigma_\lambda^2}}\lambda_t. \tag{128}$$

Step 2: Substituting equations (127), (128), (49) and $a_t = 0$ into the central bank's objective (48) yields

$$\frac{1}{1-\beta}\left[\left(\frac{\frac{\sigma_\zeta^2}{\sigma_\lambda^2}g_0 - \phi_\lambda}{\phi_c + \frac{\sigma_\zeta^2}{\sigma_\lambda^2}}\right)^2\sigma_\lambda^2 + \delta\left(\frac{\phi_c g_0 + \phi_\lambda}{\phi_c + \frac{\sigma_\zeta^2}{\sigma_\lambda^2}}\right)^2\sigma_\zeta^2\right]. \tag{129}$$

If $\sigma_\zeta^2 > 0$, the unique $g_0 \in \mathbb{R}$ that minimizes this expression is

$$g_0^* = \frac{\left(1-\delta\phi_c\right)\phi_\lambda}{\frac{\sigma_\zeta^2}{\sigma_\lambda^2} + \delta\phi_c^2}. \tag{130}$$

Step 3: Substituting the optimal monetary policy g_0^* into equations (126) and (128) yields

$$p_t = \frac{\phi_\lambda}{\frac{\sigma_\zeta^2}{\sigma_\lambda^2} + \delta\phi_c^2}\lambda_t, \tag{131}$$

$$c_t = -\frac{\delta\phi_c\phi_\lambda}{\frac{\sigma_\zeta^2}{\sigma_\lambda^2} + \delta\phi_c^2}\lambda_t. \tag{132}$$

C Proof of Propositions 5 and 6

Step 1: Characterizing equilibrium attention by two equations. We begin by rewriting the equation for the profit-maximizing price (53). Substituting the cash-in-advance constraint (50), $a_t = 0$, the monetary policy $m_t = g_0\lambda_t$, and $p_t = \theta\lambda_t$ into the equation for the profit-maximizing price (53) yields

$$p^*_{i,t} = \left[\left(1-\phi_c\right)\theta + \phi_c g_0 + \phi_\lambda\right]\lambda_t. \tag{133}$$

Since the profit-maximizing price is given by equation (133), the desired markup follows a white noise process, and $a_t = 0$, the attention problem of firm i reads

$$\min_{\kappa\in\mathbb{R}_+}\left\{\frac{\omega}{2}E\left[\left(p_{i,t}-p^*_{i,t}\right)^2\right]+\mu\kappa\right\},$$

subject to

$$p^*_{i,t} = \left[\left(1-\phi_c\right)\theta + \phi_c g_0 + \phi_\lambda\right]\lambda_t,$$

$$p_{i,t} = E\left[p^*_{i,t}|s_{\lambda,i,t}\right],$$

$$s_{\lambda,i,t} = \lambda_t + \zeta_{i,t},$$

and

$$\frac{1}{2}\log_2\left(\frac{\sigma^2_\lambda}{\sigma^2_{\lambda|s_\lambda}}\right) = \kappa.$$

Substituting the constraints into the objective, the attention problem of firm i can be expressed as

$$\min_{\kappa\in\mathbb{R}_+}\left\{\frac{\omega}{2}\left[\left(1-\phi_c\right)\theta + \phi_c g_0 + \phi_\lambda\right]^2\sigma^2_\lambda 2^{-2\kappa}+\mu\kappa\right\}. \tag{134}$$

The solution to this attention problem is

$$\kappa^* = \begin{cases} \frac{1}{2}\log_2\left(\frac{\omega\left[\left(1-\phi_c\right)\theta+\phi_c g_0+\phi_\lambda\right]^2\sigma^2_\lambda\ln(2)}{\mu}\right) & \text{if } \frac{\omega\left[\left(1-\phi_c\right)\theta+\phi_c g_0+\phi_\lambda\right]^2\sigma^2_\lambda\ln(2)}{\mu}\geq 1 \\ 0 & \text{otherwise} \end{cases}. \tag{135}$$

The price set by firm i in period t then equals

$$\begin{aligned} p_{i,t} &= \left[\left(1-\phi_c\right)\theta + \phi_c g_0 + \phi_\lambda\right]E\left[\lambda_t|s_{\lambda,i,t}\right] \\ &= \left[\left(1-\phi_c\right)\theta + \phi_c g_0 + \phi_\lambda\right]\frac{\frac{\sigma^2_\lambda}{\sigma^2_\zeta}}{\frac{\sigma^2_\lambda}{\sigma^2_\zeta}+1}\left(\lambda_t+\zeta_{i,t}\right), \end{aligned} \tag{136}$$

where

$$\frac{\sigma_\lambda^2}{\sigma_\zeta^2} = 2^{2\kappa^*} - 1. \tag{137}$$

The price level in period t equals

$$p_t = \left[(1-\phi_c)\,\theta + \phi_c g_0 + \phi_\lambda\right]\left(1 - 2^{-2\kappa^*}\right)\lambda_t. \tag{138}$$

Thus, the set of rational expectations equilibria of the form $p_t = \theta\lambda_t$ is given by the solutions to the following two equations:

$$\theta = \left[(1-\phi_c)\,\theta + \phi_c g_0 + \phi_\lambda\right]\left(1 - 2^{-2\kappa^*}\right), \tag{139}$$

and

$$\kappa^* = \begin{cases} \frac{1}{2}\log_2\left(\frac{\omega[(1-\phi_c)\theta+\phi_c g_0+\phi_\lambda]^2\sigma_\lambda^2\ln(2)}{\mu}\right) & \text{if } \frac{\omega[(1-\phi_c)\theta+\phi_c g_0+\phi_\lambda]^2\sigma_\lambda^2\ln(2)}{\mu} \geq 1 \\ 0 & \text{otherwise} \end{cases}. \tag{140}$$

Equation (139) determines θ (the responsiveness of the price level to the desired markup) as a function of κ^* (equilibrium attention), while equation (140) determines κ^* as a function of θ. Solving equation (139) for θ yields

$$\theta = \frac{(\phi_c g_0 + \phi_\lambda)\left(1 - 2^{-2\kappa^*}\right)}{1 - (1-\phi_c)\left(1 - 2^{-2\kappa^*}\right)}. \tag{141}$$

The set of rational expectations equilibria of the form $p_t = \theta\lambda_t$ for given monetary policy g_0 consists of the pairs (κ^*, θ) that solve equations (140)-(141).

Step 2: **Zero attention equilibrium**. We now study under which conditions there exists a solution to equations (140)-(141) with the property $\kappa^* = 0$. We call this a zero attention equilibrium. It follows from equation (141) that $\kappa^* = 0$ implies $\theta = 0$. Furthermore, it follows from equation (140) that at $\theta = 0$ we have $\kappa^* = 0$ if and only if

$$\frac{\omega\left(\phi_c g_0 + \phi_\lambda\right)^2 \sigma_\lambda^2 \ln(2)}{\mu} \leq 1. \tag{142}$$

Thus, there exists a rational expectations equilibrium of the form $p_t = \theta\lambda_t$ with $\kappa^* = 0$ if and only if condition (142) is satisfied. Note that the central bank can always ensure the existence of a zero attention equilibrium by making the term $\left(\phi_c g_0 + \phi_\lambda\right)^2$ sufficiently small through an appropriate choice of g_0.

Step 3: Interior attention equilibrium. Next we study under which conditions there exists a solution to equations (140)-(141) with the property

$$\kappa^* = \frac{1}{2}\log_2\left(\frac{\omega\left[\left(1-\phi_c\right)\theta+\phi_c g_0+\phi_\lambda\right]^2\sigma_\lambda^2\ln\left(2\right)}{\mu}\right). \tag{143}$$

We call this an interior attention equilibrium because in such an equilibrium the non-negativity constraint $\kappa \geq 0$ in the firms' attention problem (134) is not binding. Substituting equation (141) into equation (143) yields

$$\kappa^* = \frac{1}{2}\log_2\left(\frac{\omega\frac{\left(\phi_c g_0+\phi_\lambda\right)^2}{\left[1-\left(1-\phi_c\right)\left(1-2^{-2\kappa^*}\right)\right]^2}\sigma_\lambda^2\ln\left(2\right)}{\mu}\right). \tag{144}$$

Rearranging the last equation yields a quadratic equation in 2^{κ^*}:

$$\phi_c\left(2^{\kappa^*}\right)^2-\sqrt{\frac{\omega\left(\phi_c g_0+\phi_\lambda\right)^2\sigma_\lambda^2\ln\left(2\right)}{\mu}}2^{\kappa^*}+1-\phi_c=0. \tag{145}$$

Defining $x \equiv 2^{\kappa^*}$, the last equation can be written as

$$\phi_c x^2-\sqrt{\frac{\omega\left(\phi_c g_0+\phi_\lambda\right)^2\sigma_\lambda^2\ln\left(2\right)}{\mu}}x+1-\phi_c=0. \tag{146}$$

An interior attention equilibrium has to satisfy this quadratic equation as well as: $x \in \mathbb{R}$ and $x \geq 1$. Define

$$b \equiv \sqrt{\frac{\omega\left(\phi_c g_0+\phi_\lambda\right)^2\sigma_\lambda^2\ln\left(2\right)}{\mu}}. \tag{147}$$

The quadratic equation (146) has two solutions:

$$x_H = \frac{b+\sqrt{b^2-4\phi_c\left(1-\phi_c\right)}}{2\phi_c}, \tag{148}$$

and

$$x_L = \frac{b-\sqrt{b^2-4\phi_c\left(1-\phi_c\right)}}{2\phi_c}. \tag{149}$$

We now check whether these two solutions to the quadratic equation (146) satisfy: $x \in \mathbb{R}$ and $x \geq 1$. First, consider the case of $\phi_c \in \left(0, \frac{1}{2}\right]$. Then x_H and x_L are real if and only if $b \geq \sqrt{4\phi_c\left(1-\phi_c\right)}$. At $b = \sqrt{4\phi_c\left(1-\phi_c\right)}$, we have $x_H = x_L = \sqrt{\frac{1}{\phi_c}-1} \geq 1$. Furthermore, x_H is increasing in b and thus $x_H \geq 1$ for all $b \geq \sqrt{4\phi_c\left(1-\phi_c\right)}$, whereas x_L is decreasing in b and $x_L \geq 1$ for all $b \in \left[\sqrt{4\phi_c\left(1-\phi_c\right)}, 1\right]$. Hence, if $\phi_c \in \left(0, \frac{1}{2}\right]$, then x_H is an interior attention equilibrium so long as

$b \geq \sqrt{4\phi_c\left(1-\phi_c\right)}$, while x_L is an interior attention equilibrium so long as $b \in \left[\sqrt{4\phi_c\left(1-\phi_c\right)}, 1\right]$. Second, consider the case of $\phi_c \in \left(\frac{1}{2}, 1\right]$. Again x_H and x_L are real if and only if $b \geq \sqrt{4\phi_c\left(1-\phi_c\right)}$. At $b = \sqrt{4\phi_c\left(1-\phi_c\right)}$, we have $x_H = x_L = \sqrt{\frac{1}{\phi_c} - 1} < 1$. Furthermore, x_H is increasing in b and $x_H \geq 1$ for all $b \geq 1$, whereas x_L is non-increasing in b and thus $x_L < 1$ for all $b \geq \sqrt{4\phi_c\left(1-\phi_c\right)}$. Hence, if $\phi_c \in \left(\frac{1}{2}, 1\right]$, then x_H is an interior attention equilibrium so long as $b \geq 1$, while x_L is not an interior attention equilibrium. Finally, consider the case of $\phi_c > 1$. Then x_H and x_L are real for all $b \geq 0$. At $b = 0$, we have $x_H = \sqrt{1 - \frac{1}{\phi_c}} < 1$ and $x_L = -\sqrt{1 - \frac{1}{\phi_c}} < 0$. Furthermore, x_H is increasing in b and $x_H \geq 1$ for all $b \geq 1$, whereas $x_L < 0$ for all $b \geq 0$. Hence, if $\phi_c > 1$, then x_H is an interior attention equilibrium so long as $b \geq 1$, while x_L is not an interior attention equilibrium. In summary, if and only if either $\phi_c \in \left(0, \frac{1}{2}\right]$ and $b \geq \sqrt{4\phi_c\left(1-\phi_c\right)}$ or $\phi_c > \frac{1}{2}$ and $b \geq 1$, then x_H is an interior attention equilibrium. In addition, if and only if $\phi_c \in \left(0, \frac{1}{2}\right]$ and $b \in \left[\sqrt{4\phi_c\left(1-\phi_c\right)}, 1\right]$, then x_L is an interior attention equilibrium.

Step 4: Uniqueness and multiplicity of equilibria. When $\phi_c \geq \frac{1}{2}$, there exists a unique rational expectations equilibrium of the form $p_t = \theta\lambda_t$ for any monetary policy $g_0 \in \mathbb{R}$. In particular, if $b \in [0, 1)$ then $\kappa^* = 0$ is the unique equilibrium; if $b = 1$ then $\kappa^* = \log_2\left(x_H\right) = 0$ is the unique equilibrium; and if $b > 1$ then $\kappa^* = \log_2\left(x_H\right)$ is the unique equilibrium. By contrast, when $\phi_c \in \left(0, \frac{1}{2}\right)$, there exist multiple rational expectations equilibria of the form $p_t = \theta\lambda_t$ for some monetary policies $g_0 \in \mathbb{R}$. In particular, if $b \in \left[0, \sqrt{4\phi_c\left(1-\phi_c\right)}\right)$ then $\kappa^* = 0$ is the unique equilibrium; if $b = \sqrt{4\phi_c\left(1-\phi_c\right)}$ then $\kappa^* = 0$ and $\kappa^* = \log_2\left(x_L\right) = \log_2\left(x_H\right) = \log_2\left(\sqrt{\frac{1}{\phi_c} - 1}\right)$ are equilibria; if $b \in \left(\sqrt{4\phi_c\left(1-\phi_c\right)}, 1\right)$ then $\kappa^* = 0$, $\kappa^* = \log_2\left(x_L\right)$ and $\kappa^* = \log_2\left(x_H\right)$ are equilibria where x_L is decreasing in b and x_H is increasing in b; if $b = 1$ then $\kappa^* = \log_2\left(x_L\right) = 0$ and $\kappa^* = \log_2\left(x_H\right) = \log_2\left(\frac{1}{\phi_c} - 1\right)$ are equilibria; and if $b > 1$ then $\kappa^* = \log_2\left(x_H\right)$ is the unique equilibrium. See steps 2 and 3.

Step 5: Price dispersion and consumption variance. We now derive expressions for price dispersion and consumption variance at an equilibrium. First, we derive expressions for individual prices and the price level. Substituting equations (137) and (141) into equation (136) yields

$$p_{i,t} = \frac{\left(\phi_c g_0 + \phi_\lambda\right)\left(1 - 2^{-2\kappa^*}\right)}{1 - \left(1 - \phi_c\right)\left(1 - 2^{-2\kappa^*}\right)}\left(\lambda_t + \zeta_{i,t}\right). \tag{150}$$

Substituting equation (141) into equation (138) yields

$$p_t = \frac{\left(\phi_c g_0 + \phi_\lambda\right)\left(1 - 2^{-2\kappa^*}\right)}{1 - \left(1 - \phi_c\right)\left(1 - 2^{-2\kappa^*}\right)}\lambda_t. \tag{151}$$

Second, we derive a simple expression for price dispersion at an equilibrium. Consider the case of an equilibrium with $\kappa^* > 0$. An equilibrium with $\kappa^* > 0$ is an interior attention equilibrium and in an interior attention equilibrium equation (144) holds. Equations (150) and (151) imply

$$E\left[(p_{i,t} - p_t)^2\right] = \left[\frac{(\phi_c g_0 + \phi_\lambda)\left(1 - 2^{-2\kappa^*}\right)}{1 - (1 - \phi_c)\left(1 - 2^{-2\kappa^*}\right)}\right]^2 \sigma_\zeta^2.$$

Substituting equation (144) into the last equation yields

$$E\left[(p_{i,t} - p_t)^2\right] = \frac{\frac{\mu}{\omega}}{\sigma_\lambda^2 \ln(2)} 2^{2\kappa^*}\left(1 - 2^{-2\kappa^*}\right)^2 \sigma_\zeta^2.$$

Furthermore, substituting equation (137) into the last equation and rearranging yields

$$E\left[(p_{i,t} - p_t)^2\right] = \frac{\frac{\mu}{\omega}}{\ln(2)}\left(1 - 2^{-2\kappa^*}\right). \tag{152}$$

Next consider the case of an equilibrium with $\kappa^* = 0$. Equation (152) holds again because in an equilibrium with $\kappa^* = 0$ we have $E\left[(p_{i,t} - p_t)^2\right] = 0$. In summary, in any equilibrium, price dispersion is given by equation (152). It follows that equilibrium price dispersion is an increasing function of equilibrium attention. Third, we derive an expression for consumption variance at an equilibrium. Substituting the monetary policy $m_t = g_0 \lambda_t$ and the equation for the price level (151) into the cash-in-advance constraint (50) yields

$$c_t = \left[g_0 - \frac{(\phi_c g_0 + \phi_\lambda)\left(1 - 2^{-2\kappa^*}\right)}{1 - (1 - \phi_c)\left(1 - 2^{-2\kappa^*}\right)}\right]\lambda_t, \tag{153}$$

implying

$$E\left[c_t^2\right] = \left[g_0 - \frac{(\phi_c g_0 + \phi_\lambda)\left(1 - 2^{-2\kappa^*}\right)}{1 - (1 - \phi_c)\left(1 - 2^{-2\kappa^*}\right)}\right]^2 \sigma_\lambda^2. \tag{154}$$

The first term in square brackets in equation (153) equals the response of nominal spending to the desired markup, while the second term in square brackets in equation (153) equals the response of the price level to the desired markup. The difference between the two determines the response of composite consumption to the desired markup.

Step 6: Optimal monetary policy has to satisfy $g_0 \geq -\frac{\phi_\lambda}{\phi_c}$. Having characterized the set of rational expectations equilibria of the form $p_t = \theta \lambda_t$ for given monetary policy g_0 and having derived expressions for price dispersion and consumption variance, we now derive results concerning optimal monetary policy. We begin by showing that optimal monetary policy has to satisfy $g_0 \geq -\frac{\phi_\lambda}{\phi_c}$. The proof is as follows. First, at the monetary policy $g_0 = -\frac{\phi_\lambda}{\phi_c}$ we have $b = 0$ and thus the unique rational expectations equilibrium of the form $p_t = \theta\lambda_t$ is a zero attention equilibrium, implying that price dispersion equals zero and consumption variance equals $E\left[c_t^2\right] = \left(\frac{\phi_\lambda}{\phi_c}\right)^2 \sigma_\lambda^2$. Second, consider a monetary policy $g_0 < -\frac{\phi_\lambda}{\phi_c}$. Price dispersion at a monetary policy $g_0 < -\frac{\phi_\lambda}{\phi_c}$ is weakly larger than price dispersion at the monetary policy $g_0 = -\frac{\phi_\lambda}{\phi_c}$ because price dispersion is weakly larger than zero. Furthermore, consumption variance at a monetary policy $g_0 < -\frac{\phi_\lambda}{\phi_c}$ is strictly larger than consumption variance at the monetary policy $g_0 = -\frac{\phi_\lambda}{\phi_c}$. This result follows from the fact that consumption variance at an equilibrium is given by equation (154) and, for all $g_0 < -\frac{\phi_\lambda}{\phi_c}$, we have

$$g_0 - \frac{\left(\phi_c g_0 + \phi_\lambda\right)\left(1 - 2^{-2\kappa^*}\right)}{1 - \left(1 - \phi_c\right)\left(1 - 2^{-2\kappa^*}\right)} < -\frac{\phi_\lambda}{\phi_c} < 0. \tag{155}$$

In summary, a monetary policy $g_0 < -\frac{\phi_\lambda}{\phi_c}$ yields weakly larger price dispersion and strictly larger consumption variance than the monetary policy $g_0 = -\frac{\phi_\lambda}{\phi_c}$. Hence, a monetary policy $g_0 < -\frac{\phi_\lambda}{\phi_c}$ cannot be optimal. To see what this result means economically, note that the condition $g_0 \geq -\frac{\phi_\lambda}{\phi_c}$ can be written as $\phi_c g_0 + \phi_\lambda \geq 0$. Furthermore, substituting equation (141) into equation (133) yields the following equation for the profit-maximizing price at an equilibrium

$$p_{i,t}^* = \frac{\phi_c g_0 + \phi_\lambda}{1 - \left(1 - \phi_c\right)\left(1 - 2^{-2\kappa^*}\right)} \lambda_t. \tag{156}$$

The result that optimal monetary policy has to satisfy $g_0 \geq -\frac{\phi_\lambda}{\phi_c}$ means that at an optimal monetary policy the profit-maximizing price cannot be decreasing in the desired markup, implying that individual prices and the price level cannot be decreasing in the desired markup.

Step 7: Optimal monetary policy when $\phi_c \geq \frac{1}{2}$. First, when $\phi_c \geq \frac{1}{2}$, there exists a unique rational expectations equilibrium of the form $p_t = \theta\lambda_t$ for any monetary policy $g_0 \in \mathbb{R}$: if $b \in [0, 1)$ then $\kappa^* = 0$ is the unique equilibrium; if $b = 1$ then $\kappa^* = \log_2(x_H) = 0$ is the unique equilibrium; and if $b > 1$ then $\kappa^* = \log_2(x_H)$ is the unique equilibrium. See step 4. Second, in the derivation of optimal monetary policy we can focus on $g_0 \geq -\frac{\phi_\lambda}{\phi_c}$. See step 6. Furthermore, equation (147) implies that the variable b is an increasing function of the monetary policy g_0 for all $g_0 \geq -\frac{\phi_\lambda}{\phi_c}$. Define $\bar{g}_0$ as the value of $g_0 \in \left[-\frac{\phi_\lambda}{\phi_c}, \infty\right)$ at which $b = 1$. Equation (147) implies that

$$\bar{g}_0 = -\frac{\phi_\lambda}{\phi_c} + \frac{1}{\phi_c}\sqrt{\frac{\mu}{\omega\sigma_\lambda^2 \ln(2)}}. \tag{157}$$

If $g_0 \in \left[-\frac{\phi_\lambda}{\phi_c}, \bar{g}_0\right)$ then $b \in [0, 1)$ and $\kappa^* = 0$ is the unique equilibrium; if $g_0 = \bar{g}_0$ then $b = 1$ and $\kappa^* = \log_2(x_H) = 0$ is the unique equilibrium; and if $g_0 > \bar{g}_0$ then $b > 1$ and $\kappa^* = \log_2(x_H)$ is the unique equilibrium. Note that in the case of $\frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} \leq 1$ we have $\bar{g}_0 \geq 0$, whereas in the case of $\frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} > 1$ we have $\bar{g}_0 < 0$. Hence, when $\frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} \leq 1$ the monetary policy $g_0 = 0$ yields a zero attention equilibrium, whereas when $\frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} > 1$ the central bank has to lower the money supply after a positive markup shock to attain a zero attention equilibrium. Third, consider the case of $\phi_c \geq \frac{1}{2}$ and $\frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} \leq 1$. At the monetary policy $g_0 = 0$, we have $b \leq 1$ and therefore $\kappa^* = 0$ is the unique equilibrium, implying that price dispersion equals zero, $E\left[(p_{i,t} - p_t)^2\right] = 0$, and consumption variance equals zero, $E\left[c_t^2\right] = g_0^2\sigma_\lambda^2 = 0$. See equations (152) and (154). Thus, in the case of $\phi_c \geq \frac{1}{2}$ and $\frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} \leq 1$ the monetary policy $g_0 = 0$ attains the efficient allocation. Furthermore, any monetary policy $g_0 \neq 0$ does not attain the efficient allocation. If the equilibrium at the monetary policy $g_0 \neq 0$ is an equilibrium with $\kappa^* = 0$ then consumption variance is strictly positive, while if the equilibrium at the monetary policy $g_0 \neq 0$ is an equilibrium with $\kappa^* > 0$ then price dispersion is strictly positive. See equations (152) and (154). Hence, in the case of $\phi_c \geq \frac{1}{2}$ and $\frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} \leq 1$ the unique optimal monetary policy is $g_0^* = 0$. Fourth, consider the case of $\phi_c \geq \frac{1}{2}$ and $\frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} > 1$. We derive optimal monetary policy in this case by showing that the monetary policy minimizing objective (48) among all monetary policies $g_0 \in \left[-\frac{\phi_\lambda}{\phi_c}, \bar{g}_0\right]$ is $g_0 = \bar{g}_0$ and by showing that the monetary policy minimizing objective (48) among all monetary policies $g_0 \in [\bar{g}_0, \infty)$ is $g_0 = \bar{g}_0$. Combining results then yields that in the case of $\phi_c \geq \frac{1}{2}$ and $\frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} > 1$ the unique optimal monetary policy among all monetary policies $g_0 \in \mathbb{R}$ is $g_0 = \bar{g}_0$. For all $g_0 \in \left[-\frac{\phi_\lambda}{\phi_c}, \bar{g}_0\right]$, we have $b \leq 1$ and thus $\kappa^* = 0$ is the unique equilibrium, implying that price dispersion equals zero and consumption variance equals $E\left[c_t^2\right] = g_0^2\sigma_\lambda^2$. Furthermore, $\frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} > 1$ implies $\bar{g}_0 < 0$. It follows that in the case of $\phi_c \geq \frac{1}{2}$ and $\frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} > 1$ the monetary policy minimizing objective (48) among all monetary policies $g_0 \in \left[-\frac{\phi_\lambda}{\phi_c}, \bar{g}_0\right]$ is $g_0 = \bar{g}_0$. Next, for all $g_0 \in [\bar{g}_0, \infty)$, we have $b \geq 1$ and thus $\kappa^* = \log_2(x_H)$ is the unique equilibrium. Let us study price dispersion. Since equilibrium price dispersion is strictly increasing in κ^*, x_H is strictly increasing in b, and b is strictly increasing in g_0 for all $g_0 \geq \bar{g}_0$, it follows that price dispersion is strictly increasing in g_0 for all $g_0 \geq \bar{g}_0$. See equations (147), (148) and (152). Let us turn to consumption variance. Equilibrium consumption is given by equation (153). Furthermore, when $\kappa^* = \log_2(x_H)$, equation (144) holds. Rearranging equation (144) using $g_0 \geq -\frac{\phi_\lambda}{\phi_c}$ yields

$$\frac{\phi_c g_0 + \phi_\lambda}{1 - (1 - \phi_c)\left(1 - 2^{-2\kappa^*}\right)} = \sqrt{\frac{\mu}{\omega\sigma_\lambda^2\ln(2)}} 2^{\kappa^*}. \tag{158}$$

Substituting the last equation into equation (153) yields

$$c_t = \left[g_0 - \sqrt{\frac{\mu}{\omega \sigma_\lambda^2 \ln(2)}} \left(2^{\kappa^*} - 2^{-\kappa^*} \right) \right] \lambda_t. \tag{159}$$

In addition, solving equation (147) for g_0 using $g_0 \geq -\frac{\phi_\lambda}{\phi_c}$ yields

$$g_0 = -\frac{\phi_\lambda}{\phi_c} + \frac{b}{\phi_c} \sqrt{\frac{\mu}{\omega \sigma_\lambda^2 \ln(2)}}. \tag{160}$$

Substituting $\kappa^* = \log_2(x_H)$, equation (148) and equation (160) into equation (159) yields

$$c_t = \left[-\frac{\phi_\lambda}{\phi_c} + \sqrt{\frac{\mu}{\omega \sigma_\lambda^2 \ln(2)}} \left(\frac{b}{\phi_c} - \frac{b + \sqrt{b^2 - 4\phi_c(1-\phi_c)}}{2\phi_c} + \frac{1}{\frac{b + \sqrt{b^2 - 4\phi_c(1-\phi_c)}}{2\phi_c}} \right) \right] \lambda_t. \tag{161}$$

Rearranging the last equation yields

$$c_t = \left[-\frac{\phi_\lambda}{\phi_c} + \sqrt{\frac{\mu}{\omega \sigma_\lambda^2 \ln(2)}} \frac{2}{b + \sqrt{b^2 - 4\phi_c(1-\phi_c)}} \right] \lambda_t. \tag{162}$$

Hence, when $g_0 \geq -\frac{\phi_\lambda}{\phi_c}$ and $\kappa^* = \log_2(x_H)$, equilibrium consumption is given by equation (162). The term in square brackets in equation (162) is strictly decreasing in b for all $b \geq 1$. Moreover, in the case of $\phi_c \geq \frac{1}{2}$ and $\frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} > 1$, the term in square brackets in equation (162) is strictly negative at $b = 1$. Thus, in the case of $\phi_c \geq \frac{1}{2}$ and $\frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} > 1$, consumption variance is strictly increasing in g_0 for all $g_0 \geq \bar{g}_0$. It follows that, in the case of $\phi_c \geq \frac{1}{2}$ and $\frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} > 1$, the monetary policy minimizing objective (48) among all monetary policies $g_0 \in [\bar{g}_0, \infty)$ is $g_0 = \bar{g}_0$ because both price dispersion and consumption variance are strictly increasing in g_0 for all $g_0 \geq \bar{g}_0$. Combining results yields that in the case of $\phi_c \geq \frac{1}{2}$ and $\frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} > 1$ the unique optimal monetary policy among all monetary policies $g_0 \in \mathbb{R}$ is $g_0 = \bar{g}_0$.

Step 8: Optimal monetary policy when $\phi_c \in \left(0, \frac{1}{2}\right)$. First, when $\phi_c \in \left(0, \frac{1}{2}\right)$, there exist multiple rational expectations equilibria of the form $p_t = \theta \lambda_t$ for some monetary policies $g_0 \in \mathbb{R}$. We will use the following results below. If $b \in \left[0, \sqrt{4\phi_c(1-\phi_c)}\right]$ then $\kappa^* = 0$ is an equilibrium. If $b \geq 1$ then $\kappa^* = \log_2(x_H)$ is an equilibrium. Furthermore, if $b \in \left[0, \sqrt{4\phi_c(1-\phi_c)}\right)$ or $b > 1$ then there is a unique equilibrium, whereas if $b \in \left[\sqrt{4\phi_c(1-\phi_c)}, 1\right]$ then there exist multiple equilibria. See step 4. Second, in the derivation of optimal monetary policy we can focus on $g_0 \geq -\frac{\phi_\lambda}{\phi_c}$. See step 6. Furthermore, equation (147) implies that b is strictly increasing in g_0 for all $g_0 \geq -\frac{\phi_\lambda}{\phi_c}$.

Define $\hat{g}_0$ as the value of $g_0 \in \left[-\frac{\phi_\lambda}{\phi_c}, \infty\right)$ at which $b = \sqrt{4\phi_c(1-\phi_c)}$. Define $\bar{g}_0$ as the value of $g_0 \in \left[-\frac{\phi_\lambda}{\phi_c}, \infty\right)$ at which $b = 1$. Equation (147) implies that

$$\hat{g}_0 = -\frac{\phi_\lambda}{\phi_c} + \frac{\sqrt{4\phi_c(1-\phi_c)}}{\phi_c}\sqrt{\frac{\mu}{\omega\sigma_\lambda^2 \ln(2)}}, \tag{163}$$

and

$$\bar{g}_0 = -\frac{\phi_\lambda}{\phi_c} + \frac{1}{\phi_c}\sqrt{\frac{\mu}{\omega\sigma_\lambda^2 \ln(2)}}. \tag{164}$$

For all $g_0 \in \left[-\frac{\phi_\lambda}{\phi_c}, \hat{g}_0\right)$, $\kappa^* = 0$ is the unique equilibrium. For all $g_0 > \bar{g}_0$, $\kappa^* = \log_2(x_H)$ is the unique equilibrium. Note that the parameter restriction $\frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} < 4\phi_c(1-\phi_c)$ implies $\hat{g}_0 > 0$. Third, consider the case of $\phi_c \in \left(0, \frac{1}{2}\right)$ and $\frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} < 4\phi_c(1-\phi_c)$. At the monetary policy $g_0 = 0$, we have $g_0 < \hat{g}_0$ and thus $\kappa^* = 0$ is the unique equilibrium, implying that price dispersion equals zero and consumption variance equals zero, $E\left[c_t^2\right] = g_0^2\sigma_\lambda^2 = 0$. See equations (152) and (154). Thus, when $\phi_c \in \left(0, \frac{1}{2}\right)$ and $\frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} < 4\phi_c(1-\phi_c)$ the monetary policy $g_0 = 0$ attains the efficient allocation as the unique equilibrium allocation. Moreover, any monetary policy $g_0 \neq 0$ does not attain the efficient allocation. If the equilibrium at the monetary policy $g_0 \neq 0$ is an equilibrium with $\kappa^* = 0$ then consumption variance is strictly positive, while if the equilibrium at the monetary policy $g_0 \neq 0$ is an equilibrium with $\kappa^* > 0$ then price dispersion is strictly positive. See equations (152) and (154). Hence, when $\phi_c \in \left(0, \frac{1}{2}\right)$ and $\frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} < 4\phi_c(1-\phi_c)$ the unique optimal monetary policy is $g_0^* = 0$. Fourth, consider the case of $\phi_c \in \left(0, \frac{1}{2}\right)$ and $\frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} \geq 4\phi_c(1-\phi_c)$. For all monetary policies $g_0 \in \left[-\frac{\phi_\lambda}{\phi_c}, \hat{g}_0\right]$, $\kappa^* = 0$ is an equilibrium. Let us rank these zero attention equilibria for different monetary policies $g_0 \in \left[-\frac{\phi_\lambda}{\phi_c}, \hat{g}_0\right]$. In a zero attention equilibrium price dispersion equals zero and consumption variance equals $E\left[c_t^2\right] = g_0^2\sigma_\lambda^2$. The parameter restriction $\frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} \geq 4\phi_c(1-\phi_c)$ implies $\hat{g}_0 \leq 0$. Hence, in the case of $\phi_c \in \left(0, \frac{1}{2}\right)$ and $\frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} \geq 4\phi_c(1-\phi_c)$, the value of objective (48) at an equilibrium with $\kappa^* = 0$ is strictly decreasing in g_0 for all $g_0 \in \left[-\frac{\phi_\lambda}{\phi_c}, \hat{g}_0\right]$. Next, for all monetary policies $g_0 \geq \bar{g}_0$, $\kappa^* = \log_2(x_H)$ is an equilibrium. Let us rank these equilibria with $\kappa^* = \log_2(x_H)$ for different monetary policies $g_0 \geq \bar{g}_0$. It follows from equations (152), (147) and (148) that price dispersion at an equilibrium with $\kappa^* = \log_2(x_H)$ is strictly increasing in g_0 for all $g_0 \geq \bar{g}_0$. Furthermore, the same derivation as in step 7 yields that when $g_0 \geq -\frac{\phi_\lambda}{\phi_c}$ and $\kappa^* = \log_2(x_H)$ equilibrium consumption equals

$$c_t = \left[-\frac{\phi_\lambda}{\phi_c} + \sqrt{\frac{\mu}{\omega\sigma_\lambda^2\ln(2)}}\frac{2}{b + \sqrt{b^2 - 4\phi_c(1-\phi_c)}}\right]\lambda_t. \tag{165}$$

The term in square brackets in equation (165) is strictly decreasing in b for all $b \geq 1$. Moreover, in the case of $\phi_c \in \left(0, \frac{1}{2}\right)$ and $\frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} \geq 4\phi_c (1 - \phi_c)$, the term in square brackets in equation (165) is strictly negative at $b = 1$. Hence, in the case of $\phi_c \in \left(0, \frac{1}{2}\right)$ and $\frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} \geq 4\phi_c (1 - \phi_c)$, both price dispersion and consumption variance at an equilibrium with $\kappa^* = \log_2(x_H)$ are strictly increasing in g_0 for all $g_0 \geq \bar{g}_0$, implying that the value of objective (48) at an equilibrium with $\kappa^* = \log_2(x_H)$ is strictly increasing in g_0 for all $g_0 \geq \bar{g}_0$. Finally, in the case of $\phi_c \in \left(0, \frac{1}{2}\right)$ and $\frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} \geq 4\phi_c (1 - \phi_c)$, let us compare the zero attention equilibrium at $g_0 = \hat{g}_0$ to the equilibrium with $\kappa^* = \log_2(x_H)$ at $g_0 = \bar{g}_0$. At $g_0 = \bar{g}_0$ we have $b = 1$ and thus $\log_2(x_H) = \log_2\left(\frac{1}{\phi_c} - 1\right) > 0$ in the case of $\phi_c \in \left(0, \frac{1}{2}\right)$. See equation (148). It follows from equation (152) that price dispersion is strictly smaller in a zero attention equilibrium than in the equilibrium with $\kappa^* = \log_2(x_H)$ at $g_0 = \bar{g}_0$. In addition, consumption variance is strictly smaller in the zero attention equilibrium at $g_0 = \hat{g}_0$ than in the equilibrium with $\kappa^* = \log_2(x_H)$ at $g_0 = \bar{g}_0$ because the parameter restrictions $\phi_c \in \left(0, \frac{1}{2}\right)$ and $\frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} \geq 4\phi_c (1 - \phi_c)$ imply

$$\left[-\frac{\phi_\lambda}{\phi_c} + \frac{\sqrt{4\phi_c(1-\phi_c)}}{\phi_c}\sqrt{\frac{\mu}{\omega\sigma_\lambda^2 \ln(2)}}\right]^2 \sigma_\lambda^2 < \left[-\frac{\phi_\lambda}{\phi_c} + \sqrt{\frac{\mu}{\omega\sigma_\lambda^2 \ln(2)}}\frac{2}{1 + \sqrt{(1-2\phi_c)^2}}\right]^2 \sigma_\lambda^2.$$

Hence, in the case of $\phi_c \in \left(0, \frac{1}{2}\right)$ and $\frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} \geq 4\phi_c (1 - \phi_c)$, the zero attention equilibrium at $g_0 = \hat{g}_0$ yields a strictly smaller value of objective (48) than the equilibrium with $\kappa^* = \log_2(x_H)$ at $g_0 = \bar{g}_0$. In summary, in the case of $\phi_c \in \left(0, \frac{1}{2}\right)$ and $\frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} \geq 4\phi_c (1 - \phi_c)$, we have the following four results: (i) there is a unique rational expectations equilibrium of the form $p_t = \theta\lambda_t$ for all $g_0 \in \left[-\frac{\phi_\lambda}{\phi_c}, \hat{g}_0\right)$ and $g_0 > \bar{g}_0$, whereas there are multiple rational expectations equilibria of the form $p_t = \theta\lambda_t$ for all $g_0 \in [\hat{g}_0, \bar{g}_0]$, (ii) the value of objective (48) at an equilibrium with $\kappa^* = 0$ is strictly decreasing and continuous in g_0 for all $g_0 \in \left[-\frac{\phi_\lambda}{\phi_c}, \hat{g}_0\right]$, (iii) the value of objective (48) at an equilibrium with $\kappa^* = \log_2(x_H)$ is strictly increasing in g_0 for all $g_0 \geq \bar{g}_0$, and (iv) the zero attention equilibrium at $g_0 = \hat{g}_0$ yields a strictly smaller value of objective (48) than the equilibrium with $\kappa^* = \log_2(x_H)$ at $g_0 = \bar{g}_0$. It follows that, in the case of $\phi_c \in \left(0, \frac{1}{2}\right)$ and $\frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} \geq 4\phi_c (1 - \phi_c)$, the best the central bank can do among all monetary policies $g_0 \in \mathbb{R}$ if the central bank wants to obtain a unique equilibrium of the form $p_t = \theta\lambda_t$ is to choose a g_0 marginally below $\hat{g}_0$.

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Figure 1: Optimal Monetary Policy, Exogenous Information Structure

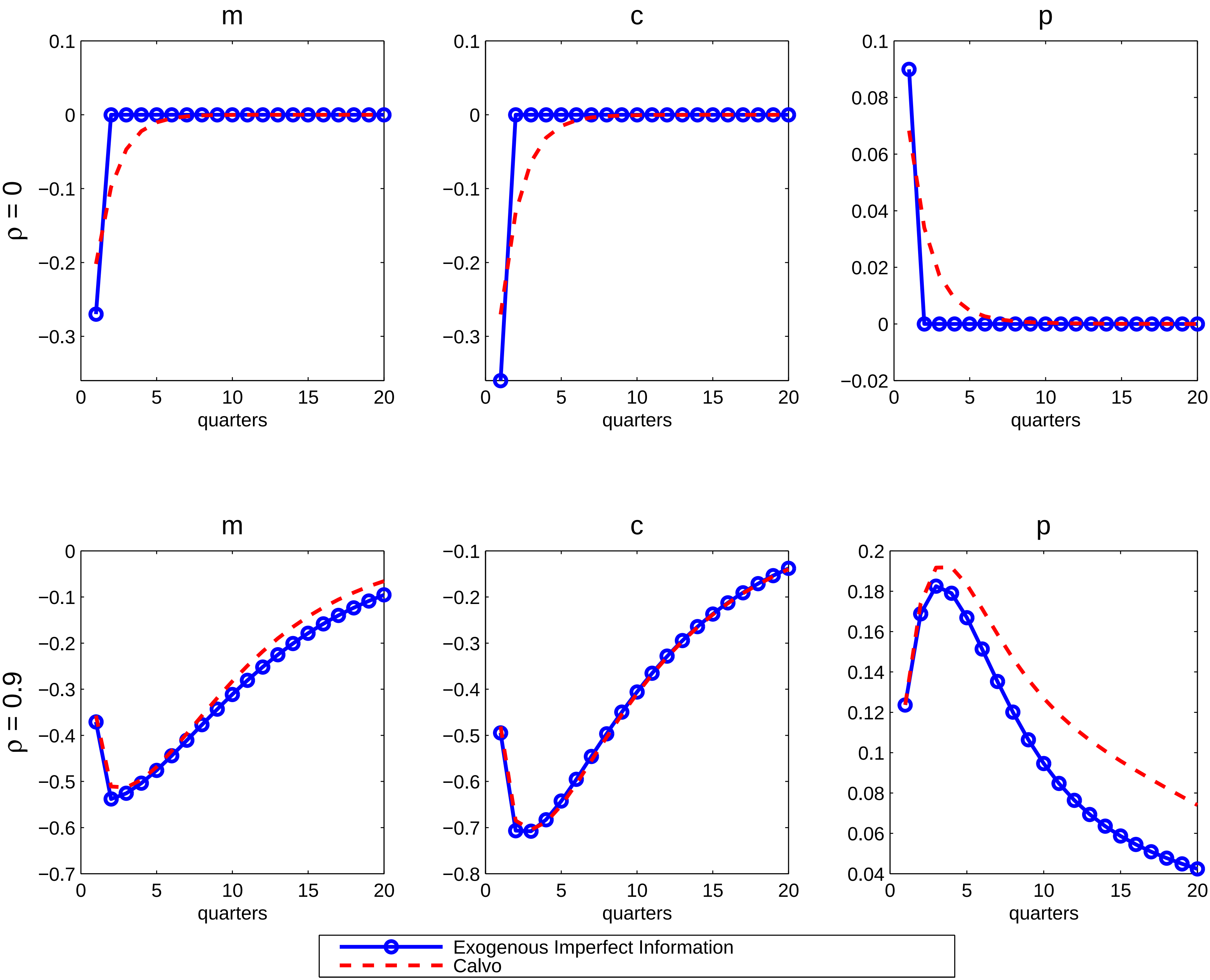

Figure 2: Equilibrium Attention as a Function of b

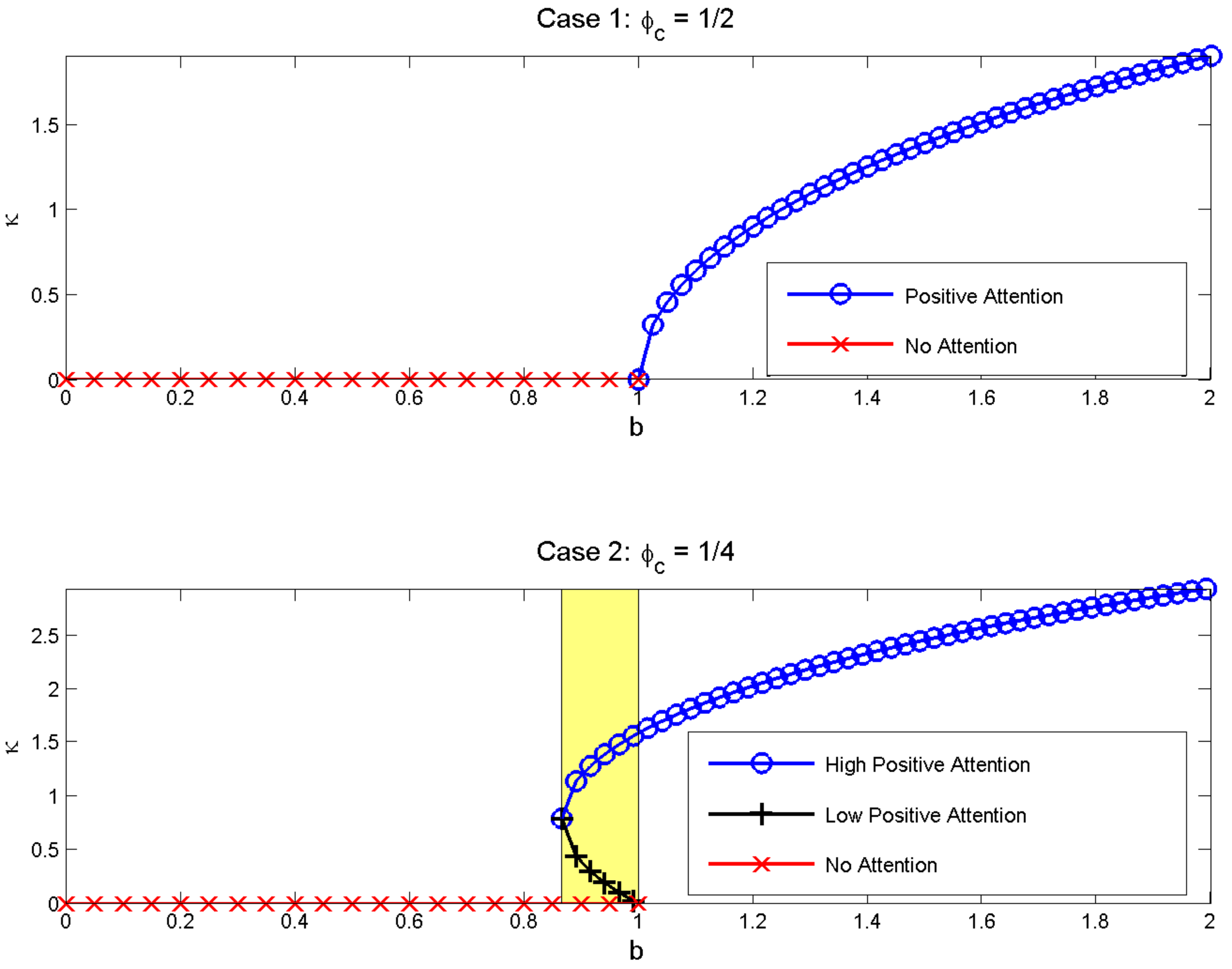

Figure 3: Optimal Monetary Policy, Endogenous Information Structure, iid Desired Markup

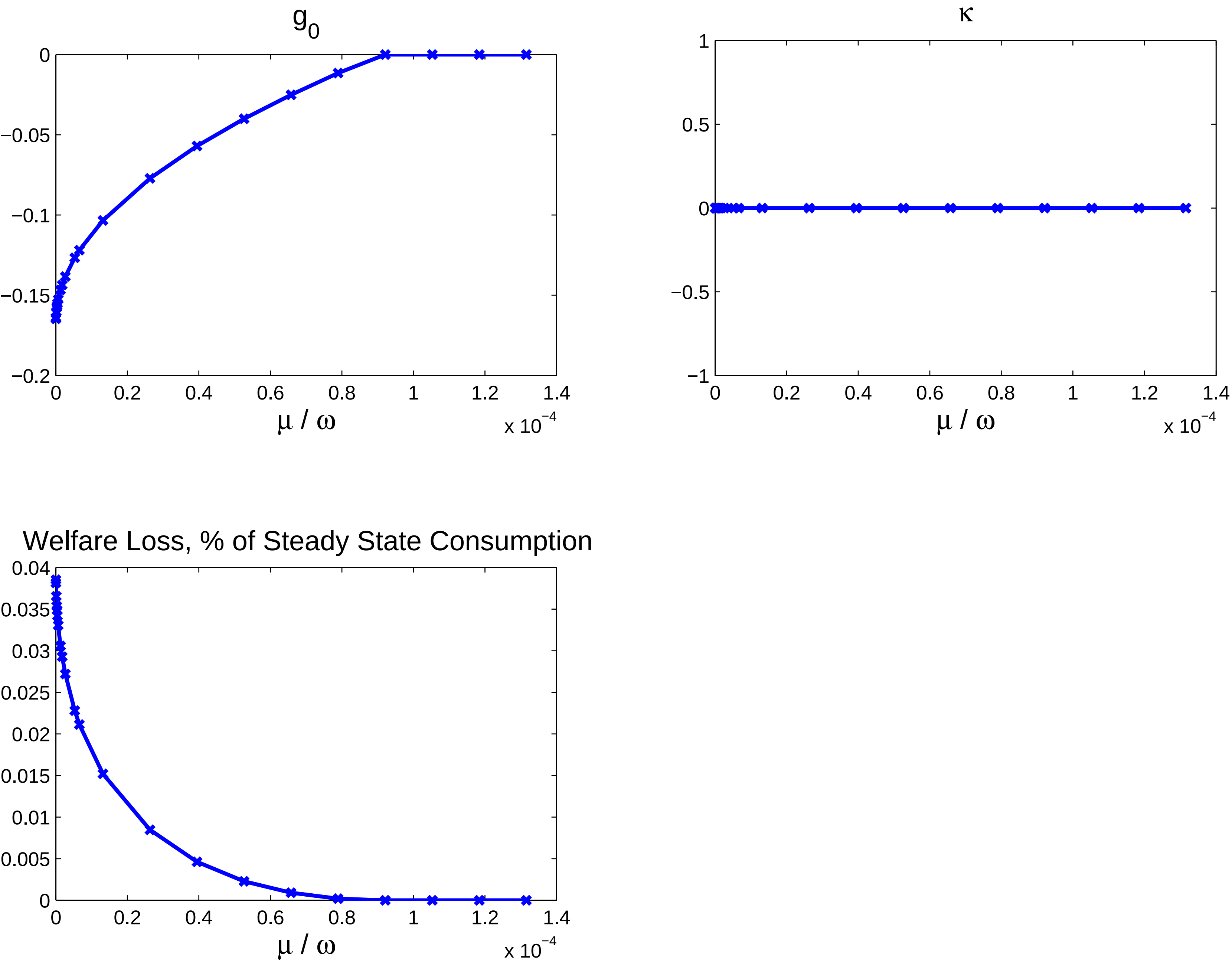

Figure 4: Optimal Monetary Policy, Endogenous Information Structure, Persistent Desired Markup

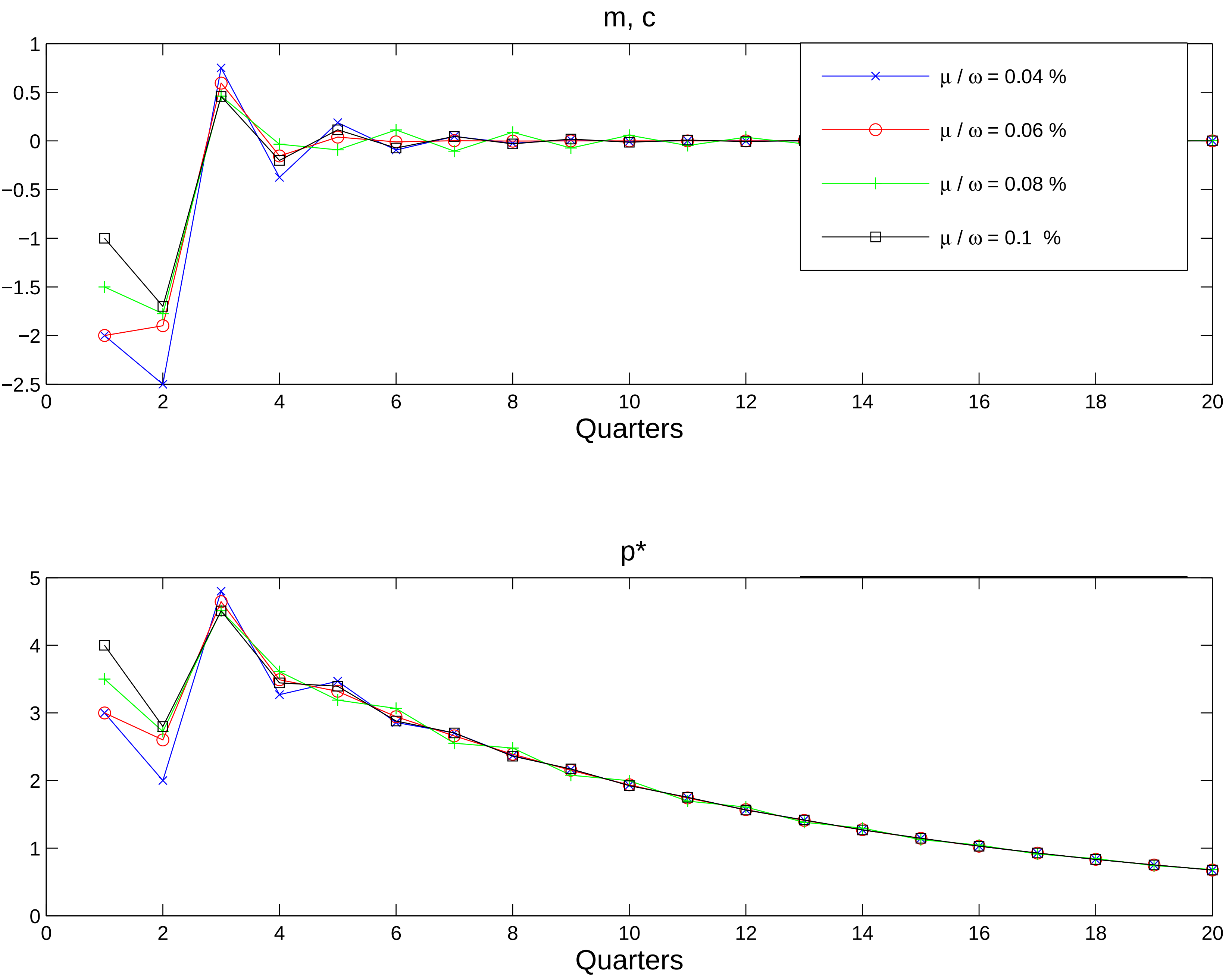

Figure 5: Welfare Losses under Endogenous Information for Alternative Policies