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Expansion of the supernova remnant 3C 10 (Tycho) and its implications for models of young remnants

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Summary. The remnant of Tycho Brahe's supernova (3C 10) was observed with the Westerbork telescope at 21 cm in 1971 and 1979, and these measurements have been used to determine the radial expansion rate. The average value obtained is 0.256 ± 0.026 arcsec yr^{-1}. Although this seems to be higher than that found optically, the individual radio and optical data points agree well within the errors. The apparent discrepancy is primarily the result of a low expansion speed at the position of the most prominent nebulosity, and we attribute this to deceleration caused by the higher density of material. While the expansion speed only marginally exceeds that predicted by the Sedov solution, the confirmation lent by the optical data suggests a possible real effect. We consider two plausible explanations: the remnant is not yet fully in the adiabatic phase; or the dynamics are being modified by the evaporation of neutral material behind the shock front. Whether either of these is the correct explanation, it is clear that swept-up material now dominates the dynamics of 3C 10. No significant change in flux density was detected at a level which favours models where particle acceleration/field amplification are occurring over that of simple adiabatic expansion.

1 Introduction

The determination of expansion rates of young supernova remnants (SNR) is vital to our understanding of their time evolution as well as the interaction of a strong shock with the interstellar medium. Most of the measurements to date have been based on optical observations. This is the case for the only two remnants of bona fide Type I supernovae: 3C 10 (Tycho – AD 1572) and 3C 358 (Kepler – AD 1604). In the former case, Kamper & van den Bergh (1978) find an average expansion rate for the period 1949–79 of 0.21 arcsec yr^{-1}, in apparent excellent agreement with that expected from the Sedov similarity solution for a blast wave (Sedov 1959). For 3C 358 the situation is less clear, with unexpectedly low velocities found for the nebulosity, many of which are not radially directed (van den Bergh & Kamper 1977).

Of other SNR, Trimble (1968) determined the expansion of the thermal filaments in the Crab nebula. Although the rate is larger than that expected if the Crab is associated with the supernova of AD 1054, it is widely believed that energy injected by the pulsar is accelerating the shell. The situation in Cas A is complex, the optical remnant consisting of fast moving knots and quasi-stationary flocculi. Kamper & van den Bergh (1976) measured the moving knots over the period 1951–75 and found a regular pattern of expansion velocities, but the lack of an independent estimate of the age of Cas A limits the value of this determination.

Cas A is the only remnant for which radio measurements of the expansion rate exist, but here the situation is complex. Bell (1977) analysed Cambridge synthesis observations at 6 cm made over a five year period and found that the compact peaks are indeed moving radially outward, but at a rate three times slower than the fast optical knots. This appears to have been confirmed by Tuffs' (1979) analysis of maps made with the 5-km telescope over a four year period. However, observations made with the NRAO three-element interferometer at 11 cm between 1967 and 1976 do not unequivocally show expansion (Dickel & Greisen 1979), and the Cambridge measurements indicate substantial tangential motions.

In contemplating an experiment of this nature, it is worthwhile considering the advantages and drawbacks of the optical and radio techniques. Optically, one usually has substantially higher resolution and a dense grid of bright reference points (distant stars) against which to gauge any motion found. Stars have the disadvantage, however, that they often have non-negligible proper motions. Furthermore, the nebulosity to be measured is often faint, usually quite irregular and in some SNR has been found to change in brightness over relatively short time-scales. Moreover, it may consist of circumstellar material still being accelerated which does not necessarily give a fair indication of the shock front velocity.

Radio observations of sufficiently high resolution usually have the advantage of studying a complete shell of high intensity (all young remnants are much more prominent radio objects than optical ones). Furthermore, there is every reason to believe that the synchrotron emission seen in the radio is intimately connected with the shock front. If an intense background source is conveniently located for use as a reference, it is almost certain to be extragalactic so its proper motion is negligible.

With these considerations in mind, we set out to complement Westerbork Synthesis Radio Telescope (WSRT) observations of 3C 10 made at 21 cm in 1971 (Strom & Duin 1973) with new measurements made in exactly the same telescope configuration. The potential advantage of a radio determination over the earlier optical one (Kamper & van den Bergh 1978) lies particularly in the regularity and intensity of the radio shell. Based on the optically determined expansion rate, we expect a radial expansion of 1.7 arcsec over the eight years separating our two observations.

2 Observations

All of the data used in our analysis were obtained with the WRST operating in the continuum at a wavelength of 21 cm. The 1971 observations have been analysed and discussed by Strom & Duin (1973). They were made using the then existing 20 interferometers (Baars & Hooghoudt 1974) and analogue receiver system (Casse & Muller 1974), and reduced in a standard way (Höbgom & Brouw 1974). The observing frequency was 1415 MHz with a bandwidth of 4.2 MHz and system noise temperature of about 260 K.

The diameter of 3C 10 (8 arcmin) requires that the first grating ring have a radius of 20 arcmin. In order to obtain this it was necessary to observe with two settings of the

Table 1. Observations of 3C 10.

Date	Hour angle Start	End	Baseline coverage (m)*	Missing spacings (m)
1971–5–3	−90°	−18°.5	36, 72, 1404	1332, 1404
1971–5–3	−11°.5	+90°	36, 72, 1404	1332, 1404
1971–8–30	−81°	+90°	72, 72, 1440	936†, 1008†
1979–7–16	−90°	+80°	36, 36, 1440	1332, 1368, 1404, 1440
1979–7–16	+83°	+90°	36, 36, 1440	1332, 1368, 1404, 1440

* Listed in order of shortest spacing, increment, longest baseline.
† Two of the four polarization channels were unusable.

movable telescopes. Salient properties of the 1971 measurements are summarized in Table 1.

A variety of changes and improvements have been made to the WSRT system between 1971 and 1979. The most important of these are the addition of two more movable telescopes, a new digital correlation line receiver, and a substantial decrease in the system noise temperature. When the 1979 observations were carried out, the new movable telescopes (C and D) were located on the same rails as the previously existing ones (A and B), at the end of the 1500 m array. It was thus possible in a single 12-hr measurement to obtain the same baselines as in 1971.

A change with more important implications for our observations was the replacement of the analogue correlator with a digital line backend (Bos, Raimond & van Someren Gréve 1981). Observations with this instrument can only be made at bandwidths of 10, 5, 2.5 MHz, etc. We chose to carry out the new measurements in the line mode and, by including an appropriate selection of adjacent bands, were able to obtain an effective bandwidth of 3.75 MHz centred at 1415 MHz. Neither the 12 per cent change in bandwidth which this represents (bandwidth decorrelation will cause a decrease in the measured surface brightness of 3C 10 not exceeding 0.02 per cent) nor the improved system temperature of 90 K should adversely affect our comparison of the two observations, provided one takes due account of the change in system temperature when calculating flux densities.

Because of receiver, telescope or other failures, several spacings and hour angle segments are missing; these are indicated in Table 1. As we are only interested in the total intensity distribution, the four polarization combinations we have measured will be summed to produce Stokes parameter I. The loss of two polarization channels on two baselines in one of the observations (Table 1) has minor consequences for the data reduction.

3 Reduction

Although the basic technique for calibrating the WSRT (van Someren Gréve 1974) has undergone no modification, details in the procedure have been improved and the set of calibration sources refined in the course of the last decade. For the type of analysis we have carried out, it is essential that the observations be carefully matched. We have therefore reanalysed the calibration for the 1971 measurements, updating telescope parameters so that they agree with those presently in use and adjusting calibration source positions and flux densities where necessary. The most significant changes were shifts of up to 0.5 arcsec in calibrator positions and an adjustment of the flux density scale to correct for the increase in system temperature caused by strong calibrators. This information has been used in standard reduction programs (van Someren Gréve 1974) to bring the calibration of the 1971 measurement in line with that of the 1979 one.

In the course of this reanalysis of the 1971 data, it became apparent that a serious error afflicted the original calibration: no gain correction was applied to the negative hour angle segment of the 36-m baseline (Table 1). As a result, the visibilities were low by roughly a factor of two. This depresses the brightness of the large scale structure in the maps previously published at 21 cm (Strom & Duin 1973; Duin & Strom 1975) and almost certainly explains the discrepancy in overall 21 cm surface brightness, and total flux density, deduced by Klein *et al.* (1979) and Henbest (1980).

The 1971 and 1979 observations were calibrated using parameters derived from measurements of calibration sources with positions based on 5-km telescope astrometry (Elsmore & Ryle 1976) and flux densities on the scale of Baars *et al.* (1977). For slightly different reasons it was necessary to correct both sets of measurements for the contribution which strong calibrators and 3C 10 (including the galactic background) make to the system temperature. In 1971 quasi-degenerate paramps and a linear detector were used (Casse & Muller 1974), while by 1979, although a square law detector had replaced the linear one, automatic monitoring of and correction for total power changes (Bos *et al.* 1981) had not yet been implemented. From the scatter observed in the calibrators we can ascertain that the typical uncertainty in our determination of the phase of each interferometer is 1–2°, and in the amplitude about 1 per cent. The former should give a position uncertainty not exceeding 0.2 arcsec.

Since the 1971 measurement consists of two separate observations done several months apart (Table 1), we have made the following check on data quality. A map was made of each observation and the position and flux density of the strong compact source 3C 11.1 determined in both. We find a discrepancy of 0.4 arcsec in right ascension, 0.1 arcsec in declination, and a difference in flux density of 4 per cent. Because the source is far removed from the centre of the 36 arcmin diameter primary beam (31 arcmin to the south-east, and hence attenuated by a factor of 6.6), quite modest pointing deviations could account for the discrepant flux densities. Otherwise, the position agreement is satisfactory and, given the calibration uncertainty mentioned above, probably a fair indication of the quality of our data.

After calibration, the 1971 and 1979 observations were processed in a standard map-making program, including only those parts of the visibility plane present in both measurements (Table 1). On the 936 m and 1008 m baselines the same two polarization channels were given double weight and included in both maps. This results in a small deviation, for this baseline, from the Stokes parameter I. Given the modest polarization present in 3C 10 at 21 cm (Strom & Duin 1973), this should be of little consequence to our comparison. The resulting maps have identical synthesized beams whose half power widths are 27.3×31.5 arcsec^2 (RA $\times$ Dec). At position angles corresponding to gaps in our visibility coverage, well-removed from the centre of the antenna pattern, there are significant differences caused by slightly different sampling points in the visibility plane. None of the deviations exceeds 1 per cent of the peak in the synthesized beam.

4 Comparison of the maps

The presence of the strong compact source 3C 11.1 in the 3C 10 field provides a useful check on the success of our calibration of the 1971 and 1979 measurements. Although the 12 arcsec size of 3C 11.1 (Rickard, private communication) means that it is slightly resolved and, as noted above, rather heavily attenuated by the primary beam (the measured signal strength was nevertheless 0.4 Jy), neither of these defects should prove consequential. We have used a standard program which fits the theoretical synthesized beam to the observed source brightness distribution to derive its position; the flux density was obtained by integrating over 3C 11.1. The results are given in Table 2, where we also include the most accurate published measurement known to us.

Table 2. Position and flux density of 3C 11.1 at 21 cm.

Observation	Position (epoch 1950.0) RA	Dec	S(Jy)
1979.5	$00^h\,26^m\,54^s.546$	63° 42′ 07″.5	2.98*
1971.5	$00^h\,26^m\,54^s.549$	63° 42′ 06″.7	2.83*
OVRO†	$00^h\,26^m\,54^s.2$	63° 42′ 08″.5	2.88

* Corrected by a factor 6.6 for primary beam attenuation.
† Fomalont & Moffet (1971) at 1425 MHz in 1965–66.

Fomalont & Moffet (1971) note that their observations of 3C 11.1 are confused by 3C 10 and consequently suffer from larger than normal uncertainties of several arcsec in position and 5 per cent in flux density. (In our synthesis the area around 3C 11.1 is free of grating responses from 3C 10 and we are therefore unaffected by this problem.) Given these uncertainties, the agreement between the WSRT and OVRO measurements is satisfactory. The recent VLA map (Rickard, private communication) also yields a position which agrees well with ours. An additional check on the flux density is the 1400 MHz value (Kellermann, Pauliny-Toth & Williams 1969) of 2.8 Jy, which is consistent with those in Table 2.

Crucial for the present comparison, however, is the consistency between the two WSRT observations. We do not attach much significance to the 5 per cent difference in flux density. Given the pointing uncertainties alluded to above, better agreement could probably not have been expected. The 0.8 arcsec discrepancy in position (almost entirely in declination) is, however, somewhat larger than one might have hoped for, especially given the deviation half that size found in comparing the halves of the 1971 measurement alone.

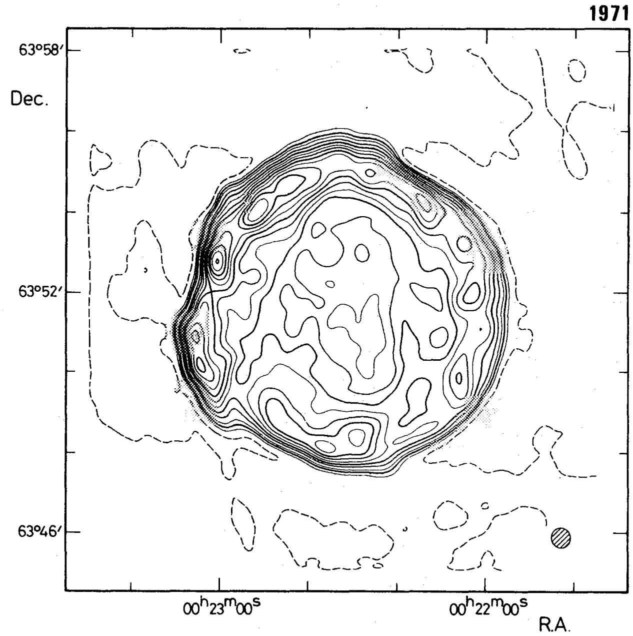
There are no instrumental effects known to us which can explain the discrepancy in position. Errors which are unlikely to have been entirely accounted for include differential precession and second order refraction (both tropospheric and ionospheric). Because of the high declination of 3C 10, refraction should be minimal. Although estimates of the differential precession also suggest a small contribution, it may be significant that only sources well removed from the field centre like 3C 11.1 will be affected. We conclude below after a detailed comparison of our maps, that the position offset for 3C 10 differs from that for 3C 11.1. If the effects of refraction, differential precession and instrumental imperfections should all add up the same direction, we might just be able to explain the magnitude of the discrepancy.

We have also tried to use other, much weaker background sources in the field as a check on positional uncertainties. Their very weakness compared to 3C 10 has not enabled us to improve upon the 0.8 arcsec difference. What we can say is that over the entire field there are no gross positional discrepancies (greater than 2 arcsec).

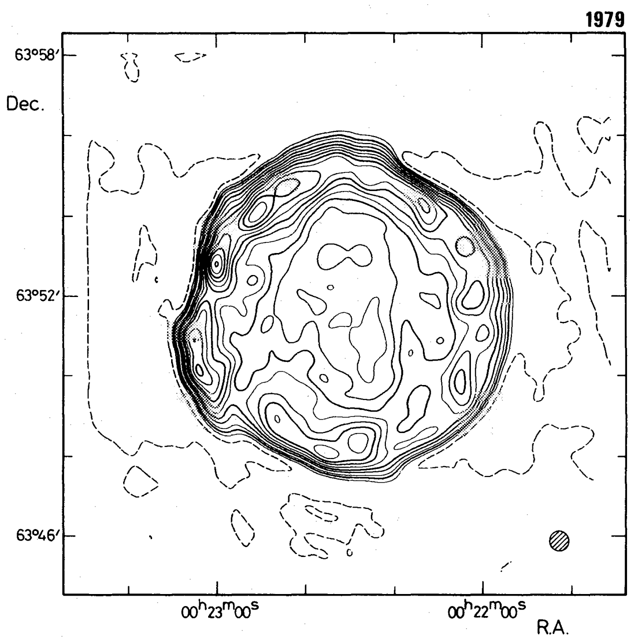
Certain errors, such as those caused by uncertainties in positioning the telescopes, in setting the frequency and in correcting for time constants, introduce defects which increase with distance from the field centre. We have used 3C 11.1 as a sensitive check on them, and in terms of their maximum expected effect on 3C 10 can draw the following conclusions about differences between our two observations:

(a) Flux density errors: maximum deviation in brightness at any point of 0.5 per cent, uncertainty in integrated flux density of 0.2 per cent;
(b) Scale error: well under 0.1 arcsec;
(c) Rotation error: not exceeding 0.2 arcsec.

Since expansion is indistinguishable from a scale difference, the most significant of these is the small limit we are able to place on any scale error. In view of the positional inconsistency



(a)

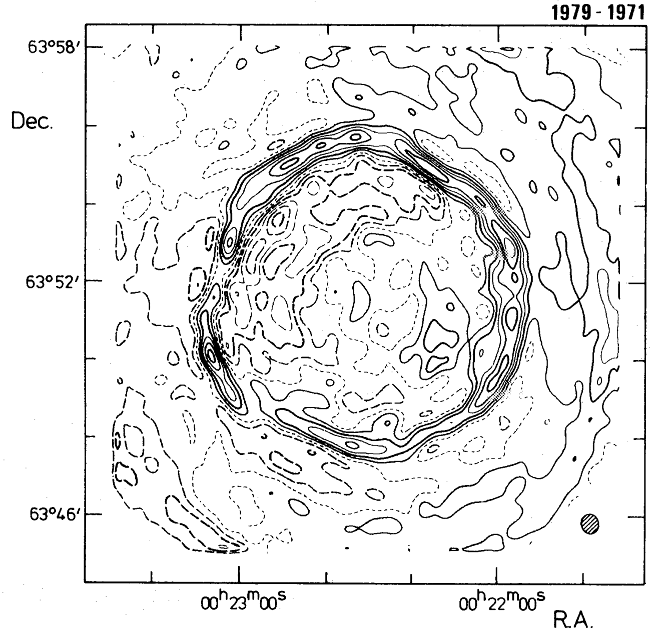


(b)

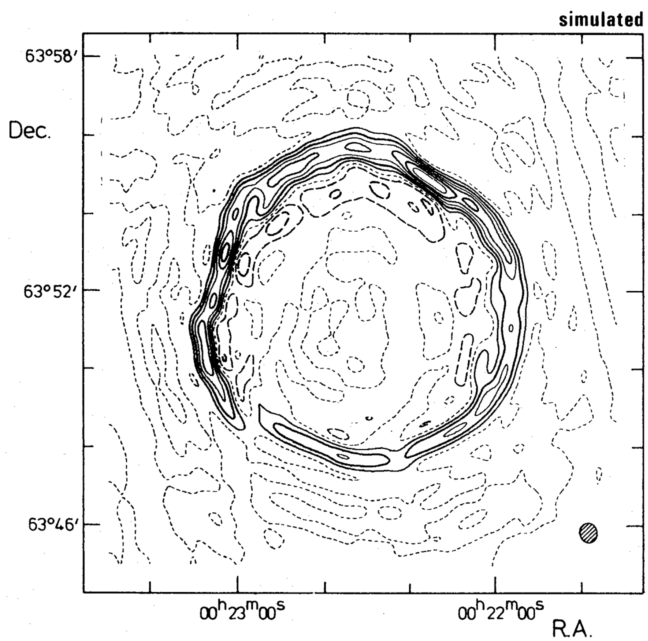
Figure 1. (a–b) The total intensity maps of 3C 10 obtained from the 1971 and 1979 WSRT 21 cm measurements. Contour values used for the 1979 map are: −10.5 (dashed), 21, 42, 63, 94.5, 126, ..., 346.6, 378.1 mJy beam^{-1}. For the 1971 map the same contour values have been used after decreasing the brightness values by a factor of 0.958 (see text). (a) 1971 data; (b) 1979 data.

which remains, we have not actually used 3C 11.1 to correct the maps, but have instead made a final adjustment based on our comparison of the 3C 10 maps and the assumption of a minimum deviation between them (see below).

The maps of 3C 10 observed in 1971 and 1979 are shown in Figs 1(a) and (b). They are presented here as obtained from the Fourier transform program. To diminish possible grating lobes from 3C 11.1, a point source with its position and peak flux was removed from the complex visibilities before computing the maps. The surface brightness observed in 1971 is



(a)



(b)

Figure 2. (a) The difference map obtained by subtracting the 1971 brightness distribution from the 1979 one, after adjusting the former down by 4 per cent (see text). The contours go from −26.25 to 31.5 mJy beam^{-1} in steps of 5.25 mJy beam^{-1} with the zero and negative contours dashed. (b) A simulation of the difference map shown in (a), obtained by applying the average expansion to the 1979 map and subtracting (see text). The same contour values have been used.

empirically found to be 4 per cent higher than that obtained in 1979 and we have consequently adjusted the contour levels accordingly; the significance of this difference will be discussed below. Similarities in the morphological details found in the two maps are particularly striking. Whatever changes may have taken place in the eight-year interval, they appear to have preserved the relative morphology and brightness.

A close inspection of the two maps reveals the expected overall expansion. The most vivid demonstration of this is obtained by blinking between video images of them. In lieu of this we present in Fig. 2(a) a difference map obtained by subtracting the brightness distribution observed in 1971 (with the intensities adjusted downward by 4 per cent) from that in 1979. It remains for us to explain how this qualitative demonstration of expansion can be translated into a velocity field for the rim of 3C 10.

There are several approaches one might adopt to obtain a quantitative estimate of the motion observed. The fact that the brightness distribution has hardly changed (apart from the overall decrease of 4 per cent) is a simplifying factor we will heavily rely on. Our method is an empirical one in which we try to match the radial scale of the maps over a range of expansion parameters, difference them, and determine which gives the best fit from the residuals. This is done for evenly spaced locations around the rim of 3C 10.

In detail, we have made a series of maps from the 1979 measurement with slightly different grid spacings. We have chosen an increment which corresponds to shrinking the map in steps of 0.4 arcsec at the average radius of 220 arcsec, covering the range 0.4 to 3.2 arcsec. The difference was then taken between each of these maps and the 1971 observation, and crosscuts were made through the centres every 20° in position angle (ϕ, measured eastward from north). It is clear that if the brightness distribution has remained precisely similar and the maps have been properly normalized, they will cancel perfectly when the expansion has been mimicked.

In practice some difference always remains, so we attempt to determine which expansion parameter comes closest to providing cancellation by the following methods. Two of us (RGS and WMG) have independently examined the profiles drawn through the difference maps for each ϕ and decided which gave, in his opinion, the best cancellation. Furthermore, we have used a goodness-of-fit criterion to determine which profile of each set deviated least from a straight line. In addition to doing this at specific position angles, we have averaged the brightness over small (typically 20°) carefully chosen segments, in the hope that some of the brightness differences might be smoothed out, and repeated the fitting exercise. The agreement between these various methods is generally quite good.

We are interested in the amount of expansion at precise intervals around the rim of the remnant, and so have averaged the determinations made by RGS, WMG and that obtained from the goodness of fit. An examination of these data shows a very striking sinusoidal-like variation such as might be caused by a coordinate shift between the two maps. The value at $\phi = 80°$ constitutes the most significant deviation from this pattern. Excluding this point (we give a more detailed justification below), we have made a least squares fit of the remaining data to a dependence of the form $A \sin(\phi + B) + C$. The result is a very good fit, the sinusoidal variation implying a shift between the maps of about 0.5 arcsec in both coordinates. With respect to the 1971 map, the 1979 observation appears to be low in right ascension by 0.57 arcsec and 0.52 arcsec high in declination. It is important at this juncture to emphasize that, although we are unable to distinguish between a coordinate error and a large scale variation in expansion, we do have a perfectly valid estimate of the average expansion. Our considered judgment is that the sinusoidal behaviour most likely originates in a coordinate shift for, while the discrepancy might be the result of a genuine variation in expansion speed, it does fall within the estimated positional accuracy of our data.

As a final check on our determination of the amount of expansion and the position discrepancy just derived, we have attempted to simulate the difference map (Fig. 2a) from one of the observations. The 1979 measurement was rescaled to decrease the average radius in accordance with our findings above, and the resulting map (which thus simulates the 1971 observation) subtracted from the original. It is shown in Fig. 2(b), using the same contour values as for Fig. 2(a). The agreement, in terms of both morphological details and brightness levels obtained, is quite good. It is clear that there is a large-scale east–west slope across Fig. 2(a), probably caused by a phase error in one or more short baselines of the original observations. Note that at $\phi = 80^\circ$, where the amount of expansion is anomalously low, the brightness in Fig. 2(b) is, as might be expected, much higher than in 2(a). From this exercise we conclude that our analysis probably has produced reliable estimates of the expansion in 3C 10 and in particular that the method is unlikely to suffer from gross systematic errors.

For the remainder of this paper we assume that the sinusoidal variation in the original expansion measurement represents a coordinate difference. As noted above, even if this should prove to be incorrect, our determination still gives a valid estimate of the average amount of expansion. The values obtained have been converted to an expansion rate, v_θ, by dividing by 8.04 yr, the mean time difference between the two sets of observations. To determine the errors, we have adopted a minimum of 0.025 arcsec yr^{-1} (corresponding to a shift of 0.2 arcsec), and to this have added in quadrature the deviation in the three measurements making up the average. Values of v_θ are listed in column 2 of Table 3 with the $A \sin(\phi + B)$ term found in the least squares fit removed. Errors are given in column 3. The average speed, $\langle v_\theta \rangle$ (parameter C), is 0.256 ± 0.026 arcsec yr^{-1}, the error being based strictly on the scatter of the data points. As noted earlier, the average brightness decreased by 4 per cent between 1971 and 1979. We have integrated the two maps to determine the flux density and, after correcting for primary beam attenuation as well as the zero level offsets in each map, find a flux density of 47.6 Jy for the 1971 measurement and 46.7 Jy for the 1979 one. Taking into account uncertainties in our flux density determination, we conclude that the flux density decreased by 1.8 ± 1.5 per cent over the eight-year interval.

5 Discussion

We will now compare the observational findings presented above with theoretical descriptions of the dynamics of young supernova remnants. Relevant models have been reviewed by Woltjer (1972). Given the age of 3C 10 and the fact that a substantial amount of material appears to have been swept up (Duin & Strom 1975; Henbest 1980), the remnant is probably close to or in the energy-conserving Phase II of evolution. The dynamics are governed by the outward motion of a strong shock wave which defines the remnant's periphery.

Table 3. Expansion rate for 3C 10, 1971–1979.

ϕ (°arc)	v_θ (arcsec yr^{-1})	Error
0	0.25	0.03
20	0.29	0.04
40	0.27	0.03
60	0.23	0.05
80	0.17	0.03
100	0.27	0.03
120	0.26	0.04
140	0.28	0.07
160	0.24	0.05
180	0.24	0.10
200	0.23	0.06
220	0.23	0.04
240	0.31	0.08
260	0.28	0.06
280	0.27	0.04
300	0.25	0.07
320	0.22	0.04
340	0.23	0.03

According to Sedov's (1959) similarity approach, the quantities r (radius), ρ_0 (external density), E_0 (initial energy) and t (time) form a unique dimensionless constant,

$$r^5 \rho_0 E_0^{-1} t^{-2}.$$

From this it follows that $r \propto t^{2/5}$ and by differentiation one obtains the well-known result

$$v = \frac{2}{5}\frac{r}{t} \tag{1}$$

where v is the shock velocity.

Our measurements give an estimate of v_θ. (Note that v and r can be replaced by v_θ and θ in equation (1), the distance cancelling out.) We can also determine θ and from historical records (Clark & Stephenson 1977) dating the supernova to 1572 November, we know t. Using the average radius found from our observations (221 arcsec), we obtain for the dimensionless velocity ratio,

$$\frac{v}{r/t} \equiv \eta = 0.47. \tag{2}$$

This marginally exceeds the two-fifths relationship expected from the Sedov solution, prompting us to consider why this might be so.

Rosenberg & Scheuer (1973) have made a series of numerical calculations to simulate the various stages of supernova remnant evolution. Using the dimensionless ratio η, we can locate 3C 10 in their evolutionary sequence. For their particular choice of initial parameters, this corresponds to an age of 2000 yr, which we then scale to the 400 yr old Tycho using the fact that time $\propto M_0^{5/6}/E_0^{1/2}\rho_0^{1/3}$. The quantities length $[\propto (M_0/\rho_0)^{1/3}]$ and velocity $[\propto (E_0/M_0)^{1/2}]$ can also be appropriately scaled, although they depend on an uncertain distance.

(In two recent determinations, Schwarz, Arnal & Goss (1980) and Albinson *et al.* (in preparation) obtain distances differing by about a factor of 2 from rather similar H I absorption spectra. This discrepancy arises from anomalous systematic motions in the Perseus arm leading to a double-valued velocity–distance relationship. Its ramifications for the distance determination of 3C 10 have been considered by Schwarz *et al.* (1980). In addition to the arguments they present favouring a minimum distance of 4 kpc, we note that the similarity between the 3C 11.1 and 3C 10 absorption profiles between 0 and $-55\,\mathrm{km\,s^{-1}}$ would, if 3C 10 were only 2 kpc away, imply a pathlength of at least 2 kpc in the galactic plane devoid of cold H I concentrations. On the basis of studies of such concentrations (Radhakrishnan & Goss 1972), at least $2.5\,\mathrm{kpc^{-1}}$ are expected. In the few instances where we must actually refer to a distance, we will consider the range 2.25 to 4.5 kpc.)

In Fig. 3 we show how the range of velocity and radius depends on initial energy and external density in the context of the Rosenberg & Scheuer (1973) model, for an assumed ejected mass of $0.5 M_\odot$. E_0, ρ_0 and M_0 can be scaled according to the relationships given above, so the values themselves are not uniquely determined by the model. The velocity ratio, η, does determine the ratio of swept-up to ejected mass. Our measurement gives $M = 7.8\, M_0$, suggesting that it is the swept-up rather than ejected material which is dominant.

An alternative explanation for our value of η is found in recent theoretical work. McKee & Ostriker (1977) have suggested that the evaporation of compact cloudlets by the hot material within a supernova remnant will modify the standard Sedov solution. For the case where evaporation dominates. Chièze & Lazareff (1981) obtain a self-similar solution. The

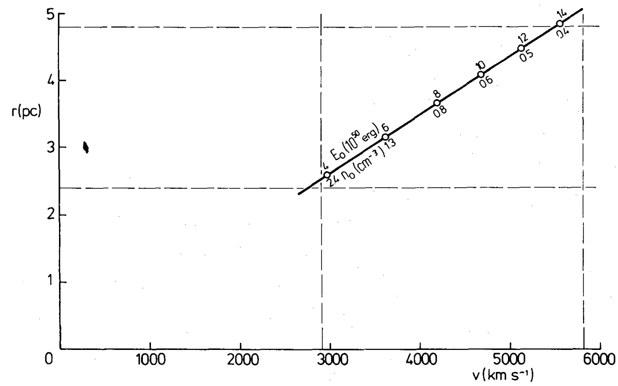


Figure 3. Values of initial energy, E_0, and interstellar density, n_0, shown as a function of expansion speed v and radius r for 3C 10. Calculations have been made according to the Rosenberg & Scheuer (1973) model for an ejected mass of $0.5\,M_\odot$. Dashed lines show limits based on the distance to 3C 10, 2.25–4.5 kpc.

evaporation rate is taken to be $\dot{\rho}_{\rm ev} = \Omega(\epsilon/\rho)^{5/2}$ where ϵ is the internal energy density and ρ the density of hot material. One can then combine the quantities r, Ω, E_0 and t to form the dimensionless expression $r^{10}\Omega E_0^{-1} t^{-6}$ from whence $r \propto t^{3/5}$ which gives for the velocity

$$v = \frac{3}{5}\frac{r}{t}. \qquad (3)$$

Equations (1) and (3) describe the shock motion for the evaporationless and evaporation dominated situations. For the general case, McKee & Ostriker (1977) show $r \propto t^{\eta}$, where η

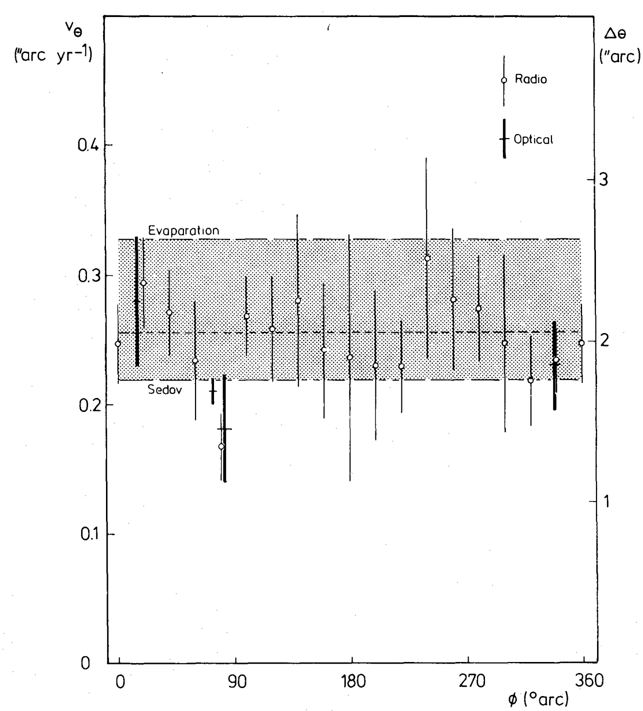


Figure 4. The expansion determined at 21 cm in 3C 10 shown as a function of azimuth, ϕ. The expansion is indicated both in terms of the rate, v_θ, and angular displacement, $\Delta\theta$. Optical values determined by Kamper & van den Bergh (1978) are given for comparison. The average radio value (short dashes) and range allowed by the evaporation model (shaded) are also indicated.

is a slowly varying function of r. One can then obtain, approximately, relationship (2), where from the limiting cases (1) and (3) we see $2/5 \leqslant \eta \leqslant 3/5$. In Fig. 4 our determination of v_θ (column 2 of Table 3) is shown as a function of position angle, ϕ. Because the scatter in the points is likely to be dominated by measuring uncertainties, we have used the average radius found from our observations (221 arcsec) to indicate the range of shock velocity permitted by equation (2). It is clear that all points, except the one at $\phi = 80°$, fall between the Sedov and evaporation limits.

We have examined the evaporation rate expected on the basis of equations derived by McKee & Ostriker, in the light of our parameters for 3C 10 and an X-ray determination of density and pressure (Pravdo & Smith 1979). For $\eta = 0.47$, an initial energy of 10^{51} erg and other 'reasonable' assumed values, one finds that evaporation occurs at only a fraction (typically 10 per cent) of the rate at which material is currently swept-up from the ambient tenuous medium. Indirect evidence for an irregular, if not positively 'cloudy', ambient medium has been obtained from radio polarization (Duin & Strom 1975) and morphological (Henbest 1980) considerations, and can also be construed from the irregular distribution of nebulosity and perhaps optical brightness variations (Kamper & van den Bergh 1978).

Let us now backtrack and briefly consider the elements upon which our determination of η through equation (2) is based. As noted earlier, there can be little doubt about the age of 3C 10 (e.g. Clark & Stephenson 1977). Slightly less certain is the value for θ, not because of indeterminacy in the radio SNR radius, but rather because we have assumed it to be the same as the shock radius. A better indicator is the X-ray SNR, where our assumption appears to be confirmed by the 3.8 arcmin radius found by Fabbiano *et al.* (1980). Higher resolution imaging data obtained with the *Einstein Observatory* (F. D. Seward, private communication; Fabbiano 1980) demonstrate the congruence of the radio and X-ray outer boundaries. Should subsequent analysis lead to revision of the shock-front radius, however, our conclusion may need to be altered.

The greatest uncertainty abides in our determination of v_θ for which we do have an independent check provided by the optical measurements (Kamper & van den Bergh 1978). The values, grouped into four ranges of ϕ and averaged, have been included in Fig. 4. It is clear that the agreement is very good, with the variations in the optical determination of v_θ closely mimicking those in the radio. Although the rate of expansion indicated by the radio and optical measurements in isolation only marginally exceeds the Sedov value, these two independent determinations do tend to confirm one another. We tentatively conclude that some modification of the simple adiabatic model, of which the two considered above are prime candidates, may prove necessary.

With respect to the optical measurements of Kamper & van den Bergh, we think it is no accident that the most prominent optical features coincide with an indentation in the radio shell as well as the lowest expansion velocity in 3C 10. Presumably, we are witnessing the interaction of the blast wave with a rather dense cloud several parsec in diameter. It was the anomalous behaviour of the shock front in this region which prompted the exclusion of the $\phi = 80°$ value from our derivation of the average expansion velocity earlier. It is clear that the optical determination is similarly affected, for the velocities found at $\phi = 15°$ and $338°$ agree well with the average radio value (Fig. 4). Ironically, the very factors contributing to a well-determined velocity near $\phi = 75°$ make the value itself unsuitable for determining the average expansion speed of the entire remnant.

Let us now briefly consider the situation in this anomalous region. The velocity found (0.167 arcsec yr^{-1}) is about two-thirds that of the Sedov speed for the entire remnant. Even making a fairly liberal allowance for the indentation and hence smaller radius in this sector, we still obtain a speed 15 per cent lower than the Sedov value. McKee & Cowie (1975) have considered the interaction between a blast wave and clouds and find that there will be a shock produced in the cloud, whose velocity v_c is approximately $v_b(\rho_0/\rho_c)^{1/2}$ where v_b is the blast-wave velocity, ρ_0 the density of the undisturbed medium and ρ_c that of the cloud. Taking v_b to be the average expansion velocity and v_c the value at $\phi = 80°$, we find $\rho_c = 2.3\,\rho_0$. Alternatively, in the case where the cloud has been struck by the dense material behind a blast wave, we have $v_c/v_b \simeq [1 + (\rho_c/\beta\rho_1)^{1/2}]^{-1}$ where ρ_1 is the density behind the shock. This gives $\rho_c \simeq 0.3\,\rho_1$ (where the numerical factor $\beta = 1$ at the shock). Henbest (1980) concludes that $\rho_c/\rho_0 > 12$, while combining his lower limit for ρ_0 with the X-ray estimate for ρ_1 yields $\rho_c/\rho_1 > 1$. (These various density ratios are essentially constant over the distance range we have been considering.) In both possible interactions, the ratio predicted by the model calculation falls below that obtained from the density estimates, although it should be emphasized that there is considerable uncertainty in the latter.

Finally, let us turn to the changes in radio brightness and flux density found in our measurements. The flux densities we obtain (Section 4) correspond to a rate $\Delta S/S\Delta t = -0.23 \pm 0.19$ per cent yr^{-1}. Measurements by other observers using pencil beam instruments have given rates of -0.4 ± 0.5 per cent yr^{-1} at 1400 MHz (Dickel & Spangler 1979) and -0.8 ± 0.1 per cent yr^{-1} at 952 MHz (Stankevich, Ivanov & Torkov 1973). Two of the three measurements are essentially consistent with no change at all, implying that sufficient particle acceleration is present to overcome the usual loss mechanisms. In a young object like 3C 10 expansion losses are likely to dominate as in the model originally proposed by Shklovsky (1960). The expected rate of change is,

$$\frac{\Delta S}{S} = 2(2\alpha - 1)\frac{\Delta\theta}{\theta}, \tag{4}$$

where α is the spectral index (defined in the sense, $S \propto \nu^{\alpha}$). For $\alpha = -0.55$ (Kellermann *et al.* 1969), our measurements of $\Delta\theta$ mean that we expect

$$\frac{\Delta S}{S\Delta t} = -0.49 \pm 0.05 \text{ per cent yr}^{-1}$$

(the error being based on the uncertainty in our determination of $\Delta\theta$). For the surface brightness (which is not independent since it depends on S and θ), one can show from equation (4) that its expected change is -5.78 ± 0.58 per cent over the eight-year interval separating the two observations.

Our measurements of essentially no change in flux density and a small decrease in surface brightness are marginally inconsistent with models in which expansion losses dominate. More satisfactory agreement could be obtained by incorporating a degree of particle acceleration and magnetic field amplification in the shock region as in the mechanisms originally discussed by Gull (1973). Reynolds & Chevalier (1981) have recently considered a model which predicts, in the case of 3C 10, a change of -0.25 per cent yr^{-1}. This is fully consistent with our determination.

6 Conclusions

According to the evolutionary sequence detailed by Woltjer (1972), a SNR is expected to pass from the initial stage in which the ejecta dominate the dynamics, through an energy-conserving (or adiabatic) phase and subsequently to a momentum-conserving one. We have described how one arrives at the well-known relationship

$$v = \frac{2}{5}\frac{r}{t}$$

for the adiabatic stage. In terms of the parameter η defined in equation (2), it is obvious that, in the initial phase, $\eta = 1$ as long as the swept-up material is negligible, while it is simple to show that for momentum-conservation a value of $1/4$ is expected. Thus, if one characterizes SNR evolution by the ratio of instantaneous to average velocity, we should expect the initial value of 1 to be followed by a stage in which it is $2/5$ and subsequently $1/4$.

Our determination of this ratio can clearly not be reconciled with either 1 or $1/4$, demonstrating (as most of us have long suspected) that 3C 10 is probably in the adiabatic phase. Thus, for this simplest (in structural appearance) of SNR, a straightforward application of the Sedov solution appears valid. One would obviously like to know if this is true of any other young remnants, as it is probably not the case for Cas A and the Crab nebula. Our work demonstrates that radio measurements can make a valuable contribution to this area of research.

The possible deviation from the Sedov $2/5$ relationship requires more scrutiny. Observationally, further radio measurements are desirable, and a new determination may be possible in the near future with planned WSRT 6-cm observations. An X-ray measurement of the shock radius is also essential, and this should be available shortly from existing data. It goes without saying that an X-ray determination of the expansion speed of the shock would be invaluable. Theoretically, the transition between the ejecta-dominated and adiabatic phases deserves more attention.

The uniformity of the expansion, and in particular the low speed found near the indentation on the eastern side, should also be the object of further research. As we may be witnessing the interaction with a compact cloud which will eventually end (the optical intensity may be diminishing), regular monitoring seems advisable. Finally, more work should be done on the flux density, to see if genuine changes can be detected.

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