Vol. 56 (1992), No. 2, pp. 157–184

Journal of Economics
Zeitschrift für Nationalökonomie

Expectations, Substitution, and Scrapping in a Putty-Clay Model

By

Erik Biørn and **Petter Frenger,** Oslo, Norway

(Received May 14, 1991; revised version received May 19, 1992)

The paper presents a framework for analyzing the effect of changing expectations about future prices on a firm's choice of technique, and on its anticipated scrapping of capital equipment. Assuming a putty-clay technology, particular attention is paid to the way in which the scrapping age depends on the degree of ex ante input substitution. Numerical illustrations — based on data for Norwegian manufacturing for the years 1964–1983, an ex ante technology represented by a Generalized Leontief cost function in materials, energy, labor, and capital, and an ARMA representation of the price expectation mechanism — are presented. The results indicate that the price changes in this period may have had a substantial impact on planned scrapping, and on the chosen production techniques.

1. Introduction

Input and output prices often show sharp, unanticipated changes over time. Such price changes, for instance unanticipated changes in energy prices, may affect not only the firms' current input structure and their decisions about investment in new fixed capital, but also their plans for scrapping of old capital goods. The form of the production technology — in particular the degree of input substitution — is a key factor in explaining how the scrapping plans may respond to price changes. Input substitution from this point of view has two aspects: substitution between capital and other inputs and substitution between capital goods installed at different points in time.

In this paper, we address the general problem of analyzing how expectations about future input and output prices and unanticipated changes in these prices can affect the firms' investment decisions and scrapping plans. We focus on the relationship between price expectations, choice of technique, and planned scrapping age of capital equipment for a profit maximizing firm. In particular, we show that the degree of input substitution may crucially affect the planned service life of new capital equipment. Little attention has been given to this point in the literature. To illustrate the theoretical conclusions, we report some tentative numerical results for a producer with a four-factor technology based on data for Norwegian manufacturing for the years 1964–1983 in a general setting in which energy price changes, and changes in the wage rate and in the price of non-energy material inputs, are taken into account.[1] From an econometric point of view, a serious problem arises due to the virtually complete non-existence of data on price expectations and, more generally, the paucity of information about the mechanisms which link expected with observed prices. The numerical results in the paper will therefore mainly be of an explorative character.

The technology is represented by a vintage production model of the putty-clay type. Ex ante — i.e., before an investment is made — the firm is assumed to face a neoclassical technology with one type of capital and one or more variable inputs. Ex post — i.e., after the investment has taken its specific physical form — all inputs must be used in fixed proportions. This model — originally proposed by Johansen (1959, 1972) — is well suited to deal with the relationship between price expectations, price shocks, and capital formation since it implies non-myopic decision rules.[2] This is in contrast to neoclassical (putty-putty) models which, by assuming the same degree of ex ante and ex post substitution and a capital stock which is completely malleable, makes it possible to change the factor input combinations at any time, and implies that capital never will be scrapped since there always will be some input combination with which it will be profitable to use it. With a putty-clay technology, decisions taken today will strongly depend on expectations about the future development of prices. Further, the rigidities which exist in the adjustment of factor proportions are represented, in a consistent way, by the model's distinction between ex ante and ex post optimal factor proportions. Finally, since it is a vintage model, it is well suited to analyzing the endogeneity of the

---

[1] The energy-capital substitution and the relationship between energy price shocks and capital service life is discussed in some more detail in Biørn (1986).

[2] Johansen, probably due to his emphasis on the planning context, did not elaborate on these aspects of the model either in his presentation or in the applications, though he did mention in rather general terms the role played by expectational variables: cf. Johansen (1972, pp. 33, 201, and 225). See also Johansen (1967).

scrapping decisions. The latter property has been utilized by, inter alia, Ando et al. (1974), Malcomson (1975, 1979), and Malcomson and Prior (1979). The problem of choice of technique is analyzed in Hjalmarsson (1974), and Førsund and Hjalmarsson (1986) in the context of an expanding industrial sector with increasing returns to scale, but under the assumption that each production plant is infinitely long lived, thus avoiding the problem of scrapping.

The paper is organized as follows. Section 2 describes the theoretical framework in terms of a general ex ante production technology and introduces the terminal quasirent function and the life cycle output and input prices: two concepts which are basic to the rest of the analysis. We discuss the way in which the ex ante service life of the capital is related to the form of the quasirent function and the life cycle prices for each vintage, and how the planned scrapping age depends on the degree of ex ante input substitution. Section 3 introduces specific assumptions about the decline in the efficiency of the capital stock and about the price expectation process, and provides a decomposition of the total effect of price changes on the scrapping age and choice of technique, highlighting the role of the ex ante substitution possibilities. After a brief presentation of the data and the chosen functional form for the ex ante production function, Sect. 4 then presents some simulation results which illustrate the joint determination of the scrapping age and the choice of technique, and the role played by the degree of input substitution in this process. Concluding remarks are given in Sect. 5.

2. The General Model

Consider a producer who, at time t, is in the process of investing in a new capital vintage. Let the ex ante technology — i.e., the set of blueprints of techniques from which he can choose — be described by the linear homogeneous production function

$$y = f(x_1, \ldots, x_m, K, t) \ , \tag{1}$$

where $(x_1, \ldots, x_m)$ is the vector of variable inputs and K is the quantity of capital invested. Technological change, represented by the time index t, is assumed to be embodied in the vintage, and thus to affect the ex ante technology only. The deterioration of the capital stock is described by the *efficiency function* $B(\tau)$, which represents the share of the original efficiency of a capital unit which remains at age τ, with $B(0) = 1$, $B(\infty) = 0$, and $B'(\tau) \leqq 0$. The form of $B(\tau)$ is a technological datum which represents the decline in efficiency with age.[3] It is not affected by the firm's decision regarding the scrapping of capital. The potential capital input at age τ will then be $K(\tau) = B(\tau)K$.

The ex post technology is characterized by fixed factor proportions between the inputs. This implies that the input of the i'th variable factor at age τ is equal to $x_i(\tau) = B(\tau)x_i$ and, since the technology is linear homogeneous, that potential output at age τ is

$$y(\tau) = B(\tau)y \ .$$

Let $q(t+\tau, t)$ and $p_i(t+\tau, t)$ denote the output price and the price of the i'th input, $i = 1, \dots, m$, respectively, which at time t the producer expects to prevail at a future time $t+\tau$.[4] These expectations are assumed to hold with certainty, but may be subject to revisions, as indicated by the double time subscript.[5] The *ex ante quasirent* from vintage t at time $t + \tau$, i.e., the expected difference between the output value and the cost of the m variable inputs, can then be written as

$$\begin{aligned} v(t+\tau, t) &= q(t+\tau, t)\, y(\tau) - \sum_{i=1}^{m} p_i(t+\tau, t)\, x_i(\tau) \\ &= B(\tau)\left[q(t+\tau, t)\, y - \sum_{i=1}^{m} p_i(t+\tau, t)\, x_i \right] \ . \end{aligned}$$

The total expected profit from vintage t is equal to the discounted value of the ex ante quasirents from age 0 to the scrapping age s, less the initial investment cost. Considered as a function of the scrapping age, it is given by

$$\begin{aligned} V(t, s) &= \int_0^s e^{-r(t)\tau}\, v(t+\tau, t)\, \mathrm{d}\tau - p_K(t)K \\ &= q^*(t, s)\, y - \sum_{i=1}^{m} p_i^*(t, s)\, x_i - p_K(t)K \ , \end{aligned}$$

[3] For a further discussion see Biørn (1989, Sect. 7.2.1.).

[4] We are making the simplifying assumption that producers form independent expectations about output and input prices. Endogenous output prices are discussed in Koizumi (1969) and Malcomson (1975).

[5] Some consequences of (stochastically specified) price uncertainty are discussed, within the framework of a simple putty-clay model, by Moene (1985) and Førsund and Hjalmarsson (1987, pp. 26–30).

where $p_K(t)$ is the purchase price of capital at time t, and

$$\left.\begin{aligned} q^* &= q^*(t,s) = \int_0^s e^{-r(t)\tau}\, B(\tau)\, q(t+\tau,t)\,\mathrm{d}\tau \ , \\ p_i^* &= p_i^*(t,s) = \int_0^s e^{-r(t)\tau}\, B(\tau)\, p_i(t+\tau,t)\,\mathrm{d}\tau \ , \\ &\qquad\qquad i = 1,\ldots,m \ . \end{aligned}\right\} \tag{2}$$

The latter expressions can be interpreted as the *life cycle prices* of output and inputs from age 0 to age s. The prevailing rate of discount, $r(t)$, is assumed to remain constant from time t up to the horizon.

Consider now the problem of choosing the profit maximizing technique, i.e., the input vector which for an exogenously given output y and the price expectations held at time t maximizes the ex ante life cycle profit $V(t,s)$. This maximization problem can conveniently be divided into two stages:

(i) maximization with respect to $x_1,\ldots,x_m$, and K for given s, and

(ii) maximization of the resulting profit function, $\Pi(t,s)$, with respect to s.

Problem (i) is formally equivalent to a neoclassical restricted profit maximization problem for a producer who is a price taker in all markets, since the life cycle prices, conditional on s, can be regarded as exogenous variables corresponding to the market prices in the equivalent neoclassical problem. Its first order conditions, subject to (1), are

$$\left.\begin{aligned} p_i^*(t,s) &= \lambda(t,s)\, f_i(x_1,\ldots,x_m,K,t) \ , \\ &\qquad\qquad i = 1,\ldots,m \ , \\ p_K(t) &= \lambda(t,s)\, f_K(x_1,\ldots,x_m,K,t) \ , \end{aligned}\right\} \tag{3}$$

where $f_i(x_1,\ldots,x_m,K,t)$, $i = 1,\ldots,m,K$, are the partial derivatives of f with respect to the i'th input, and $\lambda(t,s)$ is the Lagrangian multiplier associated with the constraint (1). The solution to (3) is implicitly defined by the *life cycle cost function* dual to (1)

$$\begin{aligned} &C(y,p_1^*,\ldots,p_m^*,p_K,t) = \\ &= \min_{x_1,\ldots,x_m,K} \left\{ \sum_{i=1}^m p_i^* x_i + p_K K \mid y = f(x_1,\ldots,x_m,K,t) \right\} \\ &= y\, c(p_1^*,\ldots,p_m^*,p_K,t) \ , \end{aligned} \tag{4}$$

the second equality following from the linear homogeneity of f, c being the unit cost function.

Application of Shephard's lemma to c gives the optimal input coefficients

$$\left.\begin{aligned} a_i &= \frac{x_i}{y} = \frac{\partial}{\partial p_i^*} c\big(p_1^*(t,s),\ldots,p_m^*(t,s),p_K(t),t\big) \ , \\ & \qquad\qquad i = 1,\ldots,m \ , \\ a_K &= \frac{K}{y} = \frac{\partial}{\partial p_K} c\big(p_1^*(t,s),\ldots,p_m^*(t,s),p_K(t),t\big) \ , \end{aligned}\right\} \quad (5)$$

conditional upon the service life s. The solution to problem (i) then defines the function

$$\begin{aligned} \Pi(t,s) &= \max_{x_1,\ldots,x_m,K} \left[\int_0^s e^{-r(t)\tau}\, v(t+\tau,t)\,\mathrm{d}\tau - p_K(t)K\right] \\ &= y\left[q^*(t,s) - c\big(p_1^*(t,s),\ldots,p_m^*(t,s),p_K(t),t\big)\right] \ , \quad (6) \end{aligned}$$

which represents the maximum profit attainable for vintage t, given its initial capacity y and an assumed service life of s years.

Associated with problem (i) we also define the *terminal quasirent function* of vintage t

$$\begin{aligned} R(t,s) &= \frac{1}{e^{-rs}\,B(s)y}\,\frac{\partial}{\partial s}\Pi(t,s) \\ &= q(t+s,t) - \\ &\quad - \sum_{i=1}^{m} p_i(t+s,t)\, a_i\big(p_1^*(t,s),\ldots,p_m^*(t,s),p_K(t),t\big) \ , \quad (7) \end{aligned}$$

which represents the current value of the quasirent per unit of output on the equipment installed in year t and planned to be scrapped in year $t+s$, the last year of its service life. The terminal quasirent function is an ex ante concept, and a change in s will result in a change in technique. This contrasts with the usual quasirent function, which is an ex post construct and takes the technique as given.

Problem (ii) reduces to solving

$$\Pi(t) = \max_s \Pi(t,s) \ ,$$

and in the process the life cycle prices become endogenous variables. Note that both $\Pi(t,s)$ and $\Pi(t)$ are functionals, being functions of the expected price paths. The first order condition for a profit maximum is equivalent to the condition that the terminal quasirent is zero at the time of scrapping, i.e., that

$$R(t,S) = 0 \ ,$$

and this relation defines implicitly the scrapping age, S, as the profit maximizing value of the service life s. Using (7), this scrapping condition may be written

$$q(t+S,t) = \\ = \sum_{i=1}^{m} p_i(t+S,t)\, a_i\big(p_1^*(t,S),\dots,p_m^*(t,S),p_K(t),t\big) \ , \tag{8}$$

with S as the single unknown variable. The condition states that vintage t is planned to be taken out of operation when the expected per unit cost of the variable inputs equals the expected output price. Whether this equation in fact has a solution or not will depend on the current prices and their expected growth paths.

This two stage argument thus permits us to start with the life cycle cost function — with life cycle prices as arguments — as a description of the ex ante technology, and then appeal to duality theory to ensure the existence of the primal production function.[6] This is in fact the route we will follow in the empirical part of this paper.

Substituting the life cycle prices (2) into the terminal quasirent function (7) and differentiating with respect to s gives[7]

---

[6] This approach is also used in Fuss (1977, 1978) in describing his putty-semiputty technology. He also uses life cycle prices, but needs cross products of the expected price paths due to the flexibility of the ex post technology. The GL model we use below is in fact his putty-clay model, but he assumes that the planning horizon (service life) is exogenously given and constant.

[7] Whether (8) in fact gives a maximum must be checked by computing the second derivative of Π with respect to s, which, using (7) and (9), becomes

$$\Pi_{ss}(t,s) = -\left[r - \frac{B'(s)}{B(s)}\right] e^{-rs}\, B(s)\, yR(t,s) + e^{-rs}\, B(s)\, yR_s(t,s) \ .$$

At a critical point, $R(t,s) = 0$: thus we have a maximum at $s = S$ if $\Pi_{ss}(t,S) < 0$, or equivalently if $R_s(t,S) < 0$, i.e., $R_s(t,S) < 0$ is a necessary condition for profit maximization at $s = S$.

$$\begin{aligned} R_s(t,s) &= \frac{\partial}{\partial s} R(t,s) \qquad (9) \\ &= \frac{\partial q(t+s,t)}{\partial s} - \sum_{i=1}^{m} \frac{\partial p_i(t+s,t)}{\partial s} a_i - \\ &\quad - e^{-rs} B(s) \sum_{i=1}^{m} \sum_{j=1}^{m} c_{ij}\, p_i(t+s,t)\, p_j(t+s,t) \; . \end{aligned}$$

This derivative describes the effect on the terminal quasirent function of a lengthening of the expected service life, an effect which can be interpreted as the sum of a direct price effect and a substitution effect. The term

$$\bar{R}_s(t,s) = \frac{\partial q(t+s,t)}{\partial s} - \sum_{i=1}^{m} \frac{\partial p_i(t+s,t)}{\partial s} a_i \qquad (10)$$

describes the direct price effect and shows the change in the terminal quasirent which would follow from a change in the service life if the technique were held fixed, while

$$- e^{-rs} B(s) \sum_{i=1}^{m} \sum_{j=1}^{m} c_{ij}\, p_i(t+s,t)\, p_j(t+s,t)$$

reflects the indirect effect, i.e., the change in technique induced by a lengthening of the service life. The concavity of the cost function implies that the quadratic form is negative semidefinite, and thus that the substitution effect is always nonnegative. This quadratic form measures the curvature of the factor price frontier in the direction of the price change vector $(\partial p_1^*/\partial s, \ldots, \partial p_m^*/\partial s)$ induced by a change in the expected service life. Suitably normalized, it may be interpreted as a directional shadow elasticity of substitution, and (9) shows that R will fall more slowly as a function of the anticipated service life the greater is this substitution effect.[8] This implies that a change in the service life will have a smaller impact on the profitability of investment the more easily the technology can be adjusted to the changing prices.

[8] See Frenger (1985) for a more detailed exposition of this argument.

To see how a change in the service life affects the chosen technology, define the *technique derivatives*

$$
\begin{aligned}
c_{is} &= \frac{\partial a_i}{\partial s} = \sum_{j=1}^{m} c_{ij} \frac{\partial p_j^*(t,s)}{\partial s} = \\
&= e^{-rs} B(s) \sum_{j=1}^{m} c_{ij} p_j(t+s,t), \qquad i = 1,\dots,m,K \ ,
\end{aligned} \tag{11}
$$

which measure the effect on the i'th input coefficient of a lengthening of the anticipated service life. Few general conclusions can be stated about the sign of c_{is}, $i = 1,\dots,m$. It will be negative if all inputs are substitutes ($c_{ij} > 0$ for $i, j = 1,\dots,m,K$, $j \neq i$) and $p_i^*(t,s)$ and $p_i(t+s,t)$ are roughly proportional. (This follows from the linear homogeneity of the cost function and the fact that $c_{ii} < 0$.) In this case, a lengthening of the service life will lead to the use of a technique which is less intensive in the use of the variable inputs. On the other hand, if the i'th input is complementary to capital ($c_{iK} < 0$), then there will be a tendency, depending on the behavior of the prices, to use more of that input as the service life is increased. The c_{Ks} term will be positive if all inputs are substitutes to capital, but it could be negative for some price constellations if some input is complementary to capital.

The second part of the optimization procedure is illustrated in Fig. 1, which shows the terminal quasirent function $R(t,s)$ and the input coefficients as functions of the anticipated service life.[9] For each anticipated service life s, f. ex. $s = 10$, we obtain the life cycle prices and the input coefficients a_M, a_E, a_L, and a_K, which in turn determine the terminal quasirent which will be earned per unit of output in year s. At $s = 10$ the expected terminal quasirent is positive, and it will be profitable to keep on producing. As we increase the expected service life of the equipment, life cycle prices change and a different technique is chosen, more intensive in the use of capital and less intensive in the use of labor. The input coefficients for materials and energy remain essentially unchanged. This adjustment continues until at $s = S = 14.7$ the terminal quasirent becomes 0. The figure then shows that the optimal input coefficients are $a_M = 0.63$, $a_E = 0.03$, $a_L = 0.20$, and $a_K = 0.64$. The producer would then choose a plant which uses the

[9] The example is the base alternative of the technology which we will use below, except that the scaling factor α is set equal to 5. The four inputs are materials (M), energy (E), labor (L), and capital (K).

factors in these proportions. Whether he then would operate it for 14.7 years would depend on whether his expectations turn out to be right.

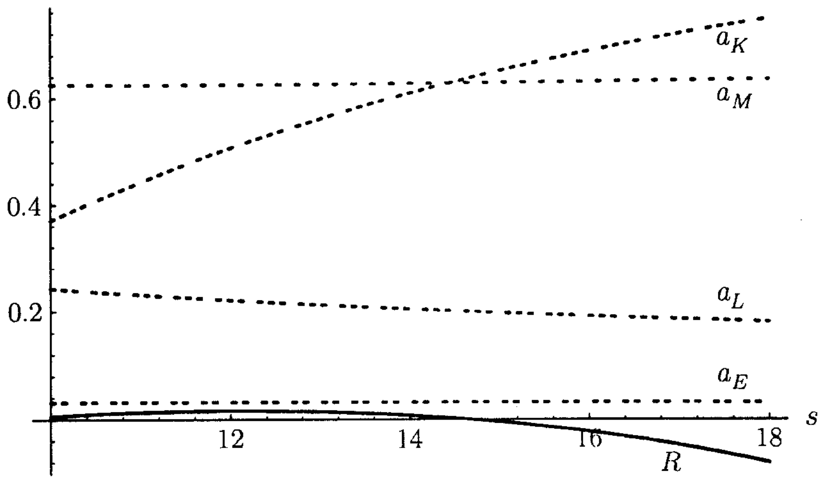

Fig. 1: The terminal quasirent function and the choice of technique

3. Scrapping Plans and Choice of Technique

In this section, we discuss the effect of changes in prices and price expectations on the scrapping plans and on the choice of technique, and present a decomposition of these changes. For this purpose, it is necessary to parametrize the efficiency function and the price expectation functions. Assume that the efficiency of capital follows the exponential function

$$B(\tau) = e^{-\delta\tau}, \qquad \delta \geqq 0 ,$$

with $\delta = 0$ representing the case with constant efficiency of capital, and that the output and input prices are expected to grow from time t at the rates $\pi_q = \pi_q(t)$ and $\pi_i = \pi_i(t)$, $i = 1, \ldots, m$, respectively, i.e.,[10]

$$q(t+\tau) = q(t+\tau, t) = e^{\pi_q \tau} q(t) ,$$

$$p_i(t+\tau) = p_i(t+\tau, t) = e^{\pi_i \tau} p_i(t) , \qquad i = 1, \ldots, m ,$$

where $q(t)$ and $p_i(t)$ are the prices observed at time t. The life cycle output and input prices (2) then become

$$\left.\begin{aligned} q^*(t,s) &= q(t)\int_0^s e^{-(r+\delta-\pi_q)\tau}\,\mathrm{d}\tau = \\ &= \frac{q(t)}{r+\delta-\pi_q}\left[1-e^{-(r+\delta-\pi_q)s}\right] , \\ p_i^*(t,s) &= p_i(t)\int_0^s e^{-(r+\delta-\pi_i)\tau}\,\mathrm{d}\tau = \\ &= \frac{p_i(t)}{r+\delta-\pi_i}\left[1-e^{-(r+\delta-\pi_i)s}\right] . \end{aligned}\right\} \tag{12}$$

The scrapping condition (8) now takes the form

$$\begin{aligned} & e^{\pi_q(t)S} q(t) = \\ & \quad = \sum_{i=1}^{m} e^{\pi_i(t)S}\, p_i(t)\, a_i\big(p_1^*(t,S),\ldots,p_m^*(t,S),p_K(t),t\big) . \end{aligned} \tag{13}$$

Solving the scrapping condition (13) for S, and substituting for S in the profit function (6) and the factor demand equations (5) gives the solution to the output constrained profit maximization problem, determining profit, scrapping age, and the choice of technique as functions of the level and rate of change of the output and variable input prices, the level of the investment price, the interest rate, and the level of the technology. Formally, this can be written

$$\begin{aligned} \Pi &= \tilde{\Pi}(q,\pi_q,p_1,\ldots,p_m,\pi_1,\ldots,\pi_m,p_K,r,t) , \\ S &= \tilde{S}(q,\pi_q,p_1,\ldots,p_m,\pi_1,\ldots,\pi_m,p_K,r,t) , \\ a_i &= \tilde{a}_i(q,\pi_q,p_1,\ldots,p_m,\pi_1,\ldots,\pi_m,p_K,r,t) , \qquad i=1,\ldots,m , \\ a_K &= \tilde{a}_K(q,\pi_q,p_1,\ldots,p_m,\pi_1,\ldots,\pi_m,p_K,r,t) . \end{aligned}$$

The functions $\tilde{S}$, $\tilde{a}_1,\ldots,\tilde{a}_m$, and $\tilde{a}_K$ are homogeneous of degree zero

---

[10] From now on we suppress the second index t on the expected price variables, i.e., $q(t+S)$ is a short hand for $q(t+S,t)$, etc. We will also often disregard the argument (t,s) on the life cycle prices $p_i^*(t,s)$.

in q, $p_1, \ldots, p_m$, and p_K, and their values are unaffected by equal changes in π_q, $\pi_1, \ldots, \pi_m$, and r.

It will in general be impossible to determine these functions explicitly. We will instead characterize them by expressing their derivatives in terms of the derivatives of the cost function and the life cycle price functions. The change in the scrapping age brought about by a change in q, π_q, p_i, π_i, p_K, r, and t, respectively, is found by totally differentiating the scrapping condition (13), which gives[11]

$$\left.\begin{aligned}
\frac{\mathrm{d}S}{\mathrm{d}q} &= -\frac{q(t+S)}{q(t)\,R_s(t,S)} ,\\
\frac{\mathrm{d}S}{\mathrm{d}\pi_q} &= -\frac{S\,q(t+S)}{R_s(t,S)} ,\\
\frac{\mathrm{d}S}{\mathrm{d}p_i} &= \frac{1}{p_i(t)\,R_s(t,S)}\left[p_i(t+S)a_i + e^{(r+\delta)S}\,c_{is}\,p_i^*\right] ,\\
&\qquad i = 1,\ldots,m ,\\
\frac{\mathrm{d}S}{\mathrm{d}\pi_i} &= \frac{1}{R_s(t,S)}\left[S\,p_i(t+S)\,a_i + e^{(r+\delta)S}\,c_{is}\,\frac{\partial p_i^*}{\partial \pi_i}\right] ,\\
&\qquad i = 1,\ldots,m ,\\
\frac{\mathrm{d}S}{\mathrm{d}p_K} &= \frac{1}{R_s(t,S)} e^{(r+\delta)S}\,c_{Ks} ,\\
\frac{\mathrm{d}S}{\mathrm{d}r} &= \frac{e^{(r+\delta)S}}{R_s(t,S)}\sum_{i=1}^{m} c_{is}\,\frac{\partial p_i^*}{\partial r} ,\\
\frac{\mathrm{d}S}{\mathrm{d}t} &= \frac{1}{R_s(t,S)}\sum_{i=1}^{m} c_{it}\,p_i(t+S) ,
\end{aligned}\right\} \quad (14)$$

where c_{is}, representing the response of the technique to a change in the service life, is given by (11). Note the key role played by $R_s(t,S)$ in these expressions. If $R_s(t,S)$ is large, i.e., if a change in the service life has a large effect on the terminal quasirent, then changes in the prices and their rate of increase, and in the interest rate will have a small effect on the scrapping age.

Differentiating the service life with respect to r is equivalent to

[11] Remember that $R_s(t,S)<0$ is a necessary condition for profit maximum.

differentiating with respect to δ, i.e., a change in the rate of interest and a change in the rate of deterioration have the same effect on the scrapping age. Further, an equal change in all expected growth rates of prices, $\pi_q, \pi_1, \ldots, \pi_m$, will also have the same effect as a change in r, but with opposite sign. Hence $-\mathrm{d}S/\mathrm{d}r$ may be taken as a measure of the effect of a change in the inflationary expectations on the planned scrapping age. The derivative with respect to t reflects the effect of the (embodied) technical change only.

The signs of the derivatives are in most cases ambiguous. Only the effect of an increase in the output price or in its growth rate are predictable: both will lead to a lengthening of the scrapping age. An increase in an input price or in its growth rate will tend to decrease the scrapping age if the substitution possibilities are small. The derivatives $\mathrm{d}S/\mathrm{d}p_i$ and $\mathrm{d}S/\mathrm{d}\pi_i$ will always be negative if $c_{is} > 0$. The effect of an increase in the price of investment goods is unambiguously negative if all variable inputs are substitutes to the capital good. Only in extreme cases of complementarity and with very divergent price changes would it seem possible for an increase in the investment price to lead to a lengthening of the scrapping age. The effect of technical progress depends on its specific pattern, but if it is Hicks neutral, which implies that $c_{it} < 0$, then technical change will always lead to a lengthening of the planned scrapping age, since it reduces production costs under constant input and output prices.

Consider next the induced changes in the input coefficients a_i, $i = 1, \ldots, m, K$. From (5) it follows that

$$\begin{aligned} \frac{\mathrm{d}a_i}{\mathrm{d}q} &= c_{is}\frac{\mathrm{d}S}{\mathrm{d}q}\,, \qquad && i = 1, \ldots, m, K\,, \\ \frac{\mathrm{d}a_i}{\mathrm{d}\pi_q} &= c_{is}\frac{\mathrm{d}S}{\mathrm{d}\pi_q}\,, \qquad && i = 1, \ldots, m, K\,, \end{aligned} \tag{15}$$

i.e., the effect of output price changes on the input coefficients is due entirely to the induced change in the scrapping age. The effect of a change in an input price or in its expected growth rate is more complicated. It is given by

$$\begin{aligned} \frac{\mathrm{d}a_i}{\mathrm{d}p_k} &= c_{ik}\frac{p_k^*}{p_k} + c_{is}\frac{\mathrm{d}S}{\mathrm{d}p_k}\,, \qquad && i = 1, \ldots, m, K, \\ \frac{\mathrm{d}a_i}{\mathrm{d}\pi_k} &= c_{ik}\frac{\partial p_k^*}{\partial \pi_k} + c_{is}\frac{\mathrm{d}S}{\mathrm{d}\pi_k}\,, \qquad && k = 1, \ldots, m. \end{aligned} \tag{16}$$

The first term of these expressions represents the direct substitution effect of an increase in p_k or π_k on the input coefficient a_i, with the scrapping age held constant. The second term represents the indirect effects via the induced change in the scrapping age. The effects of a change in the investment price, in the interest rate, or in the technology are given by

$$\left.\begin{aligned} \frac{\mathrm{d}a_i}{\mathrm{d}p_K} &= c_{iK} + c_{is}\frac{\mathrm{d}S}{\mathrm{d}p_K}\ , \\ \frac{\mathrm{d}a_i}{\mathrm{d}r} &= \sum_{j=1}^{m} c_{ij}\frac{\partial p_j^*}{\partial r} + c_{is}\frac{\mathrm{d}S}{\mathrm{d}r}\ , \qquad i = 1,\ldots,m,K\ , \\ \frac{\mathrm{d}a_i}{\mathrm{d}t} &= c_{it} + c_{is}\frac{\mathrm{d}S}{\mathrm{d}t}\ . \end{aligned}\right\} \tag{17}$$

All sets of derivatives (16) and (17) have two components: (i) a direct substitution effect brought about by the price change, the interest change, or the technical change, respectively, with the scrapping age kept constant, and (ii) an indirect effect brought about by the induced effect on the scrapping age. These direct effects are all, except for the own derivatives c_{ii}, uncertain as to sign, and even c_{it} can have either sign as long as the pattern of the technical change is unspecified. The signs of the indirect effects are also indeterminate since c_{is} may have either sign.

Most analyses of the choice of technique in putty-clay models treat the service life as exogenously given (cf., e.g., Fuss, 1977, 1978; and Berndt and Wood, 1984). These studies thus ignore entirely the effects in (15) and the last terms of (16) and (17). Particularly extreme is the neglect of the effect (15), with the consequence that a change in the path of the output price will not affect the choice of technique.

The two stage optimization discussed in Sect. 2 led naturally to the decomposition of the change in the input coefficients into the two effects we described above, one representing the effect obtained when the service life is held constant, the other reflecting induced changes via changes in the service life. We will now consider a similar decomposition of the effect which a price change has on the service life by decomposing it into (i) a direct effect with the technique held constant, and (ii) an indirect effect via the induced change in technique.

The expressions for the change in the scrapping age (14) can, by using (9), (10), and (15)–(17), be decomposed as follows

$$\left.\begin{aligned}
\frac{\mathrm{d}S}{\mathrm{d}q} &= -\frac{q(t+S)}{q(t)\,\bar{R}_s} + \frac{1}{\bar{R}_s}\sum_{j=1}^{m} p_j(t+S)\frac{\mathrm{d}a_j}{\mathrm{d}q} \;, \\
\frac{\mathrm{d}S}{\mathrm{d}\pi_q} &= -\frac{S\,q(t+S)}{\bar{R}_s} + \frac{1}{\bar{R}_s}\sum_{j=1}^{m} p_j(t+S)\frac{\mathrm{d}a_j}{\mathrm{d}\pi_q} \;, \\
\frac{\mathrm{d}S}{\mathrm{d}p_i} &= \frac{p_i(t+S)\,a_i}{p_i(t)\,\bar{R}_s} + \frac{1}{\bar{R}_s}\sum_{j=1}^{m} p_j(t+S)\frac{\mathrm{d}a_j}{\mathrm{d}p_i} \;, \\
& \qquad i = 1,\dots,m \;, \\
\frac{\mathrm{d}S}{\mathrm{d}\pi_i} &= \frac{S\,p_i(t+S)\,a_i}{\bar{R}_s} + \frac{1}{\bar{R}_s}\sum_{j=1}^{m} p_j(t+S)\frac{\mathrm{d}a_j}{\mathrm{d}\pi_i} \;, \\
& \qquad i = 1,\dots,m \;.
\end{aligned}\right\} \qquad (18)$$

The first terms represent effect (i). They are positive for the output price and negative for the input prices, and represent the effects of price changes on the scrapping age under a fixed coefficient ex ante (i.e., clay-clay) technology. With such a technology, an increase in an input price, or in its growth rate, would always lead to a reduction in the scrapping age. The second terms in (18) represent effect (ii), i.e., the additional response of the technique to the price changes. Further,

$$\left.\begin{aligned}
\frac{\mathrm{d}S}{\mathrm{d}p_K} &= \frac{1}{\bar{R}_s}\sum_{j=1}^{m} p_j(t+S)\frac{\mathrm{d}a_j}{\mathrm{d}p_K} \;, \\
\frac{\mathrm{d}S}{\mathrm{d}r} &= \frac{1}{\bar{R}_s}\sum_{j=1}^{m} p_j(t+S)\frac{\mathrm{d}a_j}{\mathrm{d}r} \;, \\
\frac{\mathrm{d}S}{\mathrm{d}t} &= \frac{1}{\bar{R}_s}\sum_{j=1}^{m} p_j(t+S)\frac{\mathrm{d}a_j}{\mathrm{d}t} \;.
\end{aligned}\right\} \qquad (19)$$

An increase in the investment price, in the rate of interest, or in the technology embodied in vintage t has no effect on the scrapping age when the technique is held constant. The only effect is the secondary effect representing the response of the technique to the price changes. This effect may be of either sign.

4. Simulation Experiments

In this section we present some simulation experiments illustrating the theoretical conclusions and the decompositions of Sect. 2 and 3. The experiments are based partly on time series data for the Norwegian manufacturing sector and partly on a priori given parameter values.

Assumptions and Data

We utilize a technology with four inputs (i.e., $m = 3$): materials (M), energy (E), labor (L), and capital (K), and make the following assumptions:

Let p_t denote an arbitrary input price or the output price in year t and let $\pi_t = p_t/p_{t-1} - 1$ denote its rate of increase in year t. We assume that the typical producer forms his price anticipations by adaptive expectations by smoothing the observed rates of price increase by means of an ARMA$(1,1)$ process so that

$$\pi_t^* = (1-\gamma)\pi_{t-1}^* + \gamma[\mu\pi_t + (1-\mu)\pi_{t-1}] \tag{20}$$

is the expected increase in prices in year t. Here γ and μ are constants between zero and one. We consider four values for (μ, γ): $(0.1, 1.0)$, $(0.2, 1.0)$, $(0.5, 0.5)$, and $(1.0, 1.0)$, of which we regard the second, which implies a mean lag between π_t and π_t^* of 4 years, as our base specification.[12] We convert π_t^* into a continuous rate by means of $\pi(t) = \log(1 + \pi_t^*)$. The interest rate is converted in a similar way, but we assume that expectations about this variable are adjusted instantaneously.

We assume that the linear homogeneous ex ante technology can be represented by the Generalized Leontief (GL) unit cost function (see Diewert, 1971)

$$c(p^*, t) = \tag{21}$$
$$= e^{-\varepsilon t}\left[\sum_{i=1}^{m}\sum_{j=1}^{m} b_{ij}(p_i^* p_j^*)^{1/2} + 2\sum_{j=1}^{m} b_{jK}(p_j^* p_K)^{1/2} + b_{KK}p_K\right],$$

[12] This price process agrees, to some extent, with estimates based on expectations data from British manufacturing industries (see Pesaran, 1985, Table 2 A). Note that the smoothing implied by (20) is one way of taking into account the uncertainty with respect to future prices.

where p^* is the vector of life cycle prices, $B = [b_{ij}]$ is a matrix of coefficients, and ε is the rate of (Hicks neutral) technical change. Substituting the input coefficient equations of the GL technology [see (5)] into the scrapping condition (13) gives

$$e^{\pi_q(t)S}\, q(t) =$$
$$= e^{-\varepsilon t} \sum_{i=1}^{m} e^{\pi_i(t)S}\, p_i(t) \left[\sum_{j=1}^{m} b_{ij} \left(\frac{p_j^*(t,S)}{p_i^*(t,S)} \right)^{\frac{1}{2}} + b_{iK} \left(\frac{p_K(t)}{p_i^*(t,S)} \right)^{\frac{1}{2}} \right] .$$

This equation together with (12) gives a set of $m+1$ equations in the unknown variables S, $p_1^*, \ldots, p_m^*$, and substituting for the life cycle prices gives a single equation in the unknown S.[13]

Our numerical examples are based on estimates of the b_{ij} coefficients from a neoclassical model with a homothetic technology with Hicks neutral technical change, derived from national accounts data, i.e., data aggregated across vintages, for the years 1962–1981 (see Bye and Frenger, 1985). We parametrize our ex ante model by assuming that its second order properties in 1980 are the same as those obtained by Bye and Frenger for their base year 1981, and described by the shadow elasticities of substitution (SES) presented in Table 1.

Table 1: 1980 Shadow elasticities of substitution (SES) implied by the estimated GL technology

	SES		
	E	L	K
M	0.3600	0.9036	0.5474
E		0.4205	0.2789
L			0.7580

Since vintage data are unavailable, a problem arises in the determination of the level of the cost structure of new investment. We have decided to impose an exogenously given profit rate in the base year t_0

[13] In actually determining and understanding the solution, we found the terminal quasirent function (7) very useful.

by writing

$$q^*(t_0) = c\big(p_1^*(t_0, S), \ldots, p_m^*(t_0, S), p_K(t_0), t_0\big) \ .$$

This determines the "efficiency parameter" of the ex ante cost function, and the scaling of the input coefficients. For the other vintages, a non-zero profit will, of course, normally occur.

The above estimates, based on average data, are likely to seriously underestimate the elasticities of substitution of the ex ante technology. To compensate for this, we also consider specifications with higher values for these elasticities. Technically, these have been computed by magnifying all second order derivatives of the cost function at the base point by a scaling factor α, while holding the first derivatives, i.e., the input coefficients, constant.[14]

A decline in efficiency at a (continuous) rate of 10 per cent ($\delta = 0.10$) is assumed over the capital's life cycle.[15]

Properties of Base Year Technology

Let us first consider the behavior of the model in the base year 1980, when the scaling factor α is unity, using the base specification of the price expectations. The anticipated scrapping age of new equipment installed in 1980 was 14.7 years.

The input coefficients are given in the first column of Table 2. The second column presents the elasticity of the technology with respect to the service life. An increase in the scrapping age S will change the life cycle prices, which induces a change in the optimal technique. The directional shadow elasticity of substitution in the direction of the induced change is 0.52. This brings about a substantial substitution of

---

[14] Since the second order derivatives of the cost function (21) are linear in the off-diagonal GL coefficients, this rescaling is equivalent to multiplying these coefficients by the factor α,

$$b_{ij}(\alpha) = \alpha\, b_{ij} \ , \qquad i, j = 1, \ldots, m, K, \quad j \neq i \ ,$$

and then determining the rescaled diagonal coefficients $b_{ii}(\alpha)$ residually.

[15] Since only the sum of the retirement rate δ and the interest rate r occurs in the model [cf. (12)], a non-zero value of δ may be interpreted as including a risk premium claimed by the firm for undertaking an uncertain investment project. Then $r + \delta$ can be reinterpreted as the sum of the market interest rate on approximately risk-free assets, the rate of retirement (decline in efficiency), and the risk premium.

capital for labor, while the input coefficients for materials and energy change relatively little. Note, however, the signs of the elasticities for materials and energy: we get a larger use of materials and a smaller use of energy, despite the complementarity between energy and capital, and despite the tendency to use less of the variable factors as S increases, because of the substantially lower growth rate expected for the price of materials than for the prices of energy and labor.

Table 2: 1980 input coefficients and technique elasticities; base alternative for price expectations; $\alpha = 1$

i	Input coef. a_i	Technique elast.[a]
M	0.6289	0.0081
E	0.0321	−0.0103
L	0.2000	−0.0995
K	0.6371	0.1893

[a] Elasticity of input coefficients with respect to service life. Cf. (11).

Let us now apply the decomposition presented in Sect. 3 to get a better understanding of the changes in the input structure and the scrapping implied by the model. Table 3 (in two parts) presents a decomposition of the change in technique, based on (15)–(17). Column A shows the effect which would obtain if the scrapping age were held constant, column B gives the adjustments induced by the change in the scrapping age. The total effect is given in the third column. The sign pattern in column A (the primary effects) is the same as the one that would be observed in a corresponding neo-classical model: all own price effects are negative, while the cross price effects are positive for substitutes and negative for complements (i.e., energy and capital). A change in the output price has no effect on the technique since it does not affect the relative life cycle input prices when the service life is held constant [cf. (15)]. Increasing the rate of interest will lead to a less capital intensive technique, while technical change will affect all input coefficients proportionately. The sign pattern in column B (the secondary effects) depends on the sign of the technique elasticities (see Table 2) and on the $\mathrm{d}S/\mathrm{d}p_i$ and $\mathrm{d}S/\mathrm{d}\pi_i$ terms [see (15)–(17)]. An increase in the output price, or in its rate of growth, leads to a more material and capital intensive technique and a lower energy and labor intensity. This is due to the secondary effect via the scrapping age, the

Table 3: 1980 elasticities[a] of input coefficients; decomposition; base alternative for price expectations; $\alpha = 1$, $\varepsilon = 0.01$;
A. elasticity with no change in service life;
B. correction due to change in service life

Elasticity of	A	B	Total
a_M w.r.t.			
q	0.0000	0.0534	0.0534
p_M	−0.2688	−0.0351	−0.3039
p_E	0.0251	−0.0039	0.0212
p_L	0.2038	−0.0116	0.1922
p_K	0.0398	−0.0027	0.0371
π_q	0.0000	0.7816	0.7816
π_M	−1.4068	−0.5064	−1.9133
π_E	0.1547	−0.0584	0.0963
π_L	1.1372	−0.2012	0.9360
r	0.1150	−0.0155	0.0995
t	−0.0100	0.0005	−0.0095
a_E w.r.t.			
q	0.0000	−0.0683	−0.0683
p_M	0.3569	0.0449	0.4018
p_E	−0.3162	0.0050	−0.3111
p_L	0.1140	0.0149	0.1289
p_K	−0.1547	0.0035	−0.1513
π_q	0.0000	−1.0008	−1.0008
π_M	1.8681	0.6485	2.5166
π_E	−1.9463	0.0747	−1.8716
π_L	0.6359	0.2577	0.8936
r	−0.5577	0.0199	−0.5379
t	−0.0100	−0.0007	−0.0107

[a] Elasticities for q, p_M, p_E, p_L, and p_K, derivatives of logarithms for π_q, π_M, π_E, π_L, r, and t.

primary effect being zero, and the sign of the effect is determined by the technique elasticities of Table 2.

Table 4 presents a similar decomposition of the effect of price changes etc. on the scrapping age, based on (18) and (19). In column A, the technique (i.e., the input coefficients) is held constant, column B gives the changes in the scrapping age which are induced by changes in technique, and the last column shows the total effect. Column A thus gives the effect which would have been obtained if the technology had been Leontief (clay-clay), with coefficients equal to those observed in the base year, and shows that in this case increasing the output price

Table 3 *(continued)*

Elasticity of	A	B	Total
a_L w.r.t.			
q	0.0000	−0.6565	−0.6565
p_M	0.5692	0.4316	1.0008
p_E	0.0224	0.0485	0.0709
p_L	−0.7232	0.1429	−0.5803
p_K	0.1316	0.0335	0.1651
π_q	0.0000	−9.6168	−9.6168
π_M	2.9794	6.2316	9.2110
π_E	0.1379	0.7182	0.8561
π_L	−4.0348	2.4762	−1.5586
r	0.9175	0.1908	1.1083
t	−0.0100	−0.0066	−0.0166
a_K w.r.t.			
q	0.0000	1.2494	1.2494
p_M	0.2667	−0.8213	−0.5546
p_E	−0.0730	−0.0923	−0.1653
p_L	0.3156	−0.2719	0.0436
p_K	−0.5093	−0.0638	−0.5731
π_q	0.0000	18.3017	18.3017
π_M	1.3961	−11.8594	−10.4632
π_E	−0.4491	−1.3667	−1.8158
π_L	1.7605	−4.7125	−2.9520
r	−2.7075	−0.3631	−3.0706
t	−0.0100	0.0125	0.0025

would increase the scrapping age, while increases in the input prices would reduce it. The secondary effects reinforce the primary effects, with one exception: the change in technique induced by an increase in the wage rate has a positive secondary effect on the scrapping age.

Consider, as an example, an increase in the expected rate of growth of the price of materials, π_M, by one percentage point. This will bring about a more labor intensive technique (3.0 %) when holding S constant (Table 3). Allowing for the induced reduction in the scrapping age by 63 %, or 9.2 years (Table 4), which leads to increased labor demand ($c_{Ls} < 0$) (Table 2), we get a secondary effect on the labor coefficient of 6.2 %, so that the total effect is an increase in the labor coefficient by 9.2 % (Table 3). The secondary effect thus exceeds by far the primary effect in this case. Consider next the effect of the increase in π_M on the capital stock. Holding S constant leads to an *increased* demand

Table 4: 1980 elasticities[a] of service life; decomposition; base alternative for price expectations; $\alpha = 1$, $\varepsilon = 0.01$;
A. elasticity with no change in input coefficients;
B. correction due to change in input coefficients

Elasticity of S w.r.t.	A	B	Total
q	5.7104	0.8907	6.6011
p_M	−3.6705	−0.6689	−4.3394
p_E	−0.4296	−0.0583	−0.4879
p_L	−1.6103	0.1736	−1.4367
p_K	0.0000	−0.3371	−0.3371
π_q	83.6508	13.0473	96.6980
π_M	−53.7686	−8.8910	−62.6596
π_E	−6.2931	−0.9281	−7.2211
π_L	−23.5891	−1.3095	−24.8986
r	0.0000	−1.9187	−1.9187
t	0.0000	0.0660	0.0660

[a] Elasticities for q, p_M, p_E, p_L, and p_K, derivatives of logarithms for π_q, π_M, π_E, π_L, r, and t.

for capital (primary effect equal to 1.4 %), but when we allow for the induced reduction in S, this effect is *reversed* (secondary effect equal to −11.9 %), and we end up with a more than 10 % less capital intensive technique. The end result is thus in both cases markedly different from what would have been predicted from a neoclassical model.

Let us now increase the scaling parameter α, thus increasing the ex ante elasticities of substitution. The terminal quasirent function becomes flatter, i.e., $R_s(t, S)$ decreases in absolute value. The elasticities do not, however, increase pari passu with α. In fact, the sign of some of the effects for $\alpha = 5$ differs from the sign when $\alpha = 1$ or 3.[16] A notable example occurs for the labor price: $(\partial S/\partial p_L)(p_L/S)$ is equal to −0.85 for $\alpha = 3$ and 0.70 for $\alpha = 5$. In the latter case, the high degree of substitution possibilities makes it profitable to substitute the fixed factor capital for labor to such an extent that variable costs at scrapping actually fall. This leads to an increase in the scrapping age, and to the use of a technique which is much less labor intensive

[16] In the base year, we obtain a critical value for $\bar{\alpha} = 7.41$. The derivative of the terminal quasirent function $R_s(t, s)$ [see (9)] would become positive if α were larger than this critical value and our base point would cease to be a profit maximizing point.

$[(\partial a_L/\partial p_L)(p_L/a_L) = -3.96]$ and substantially more capital intensive $[(\partial a_K/\partial p_L)(p_L/a_K) = 2.24]$.

Simulation Results for the Years 1964–1983

Let us now consider the behavior of the model when simulated over the entire observation period, 1964–1983.[17] Part A of Table 5 presents simulated values of the ex ante service life, using the base specification of the price expectation process ($\gamma = 0.2$, $\mu = 1$), for scaling factors $\alpha = 1$, 2, and 3, and the rate of technical change $\varepsilon = 0$ and 1 %.[18] The service life shows substantial cyclical variations. On the whole, the year to year changes tend to be larger, the larger is the scaling factor, i.e., the higher is the overall degree of ex ante substitution between the inputs. The service life is quite sensitive to variations in the rate of technical change; the more efficient is a vintage, the longer is its profitable service period. A 1 % rate of technical change from 1980 to 1983 will, for instance, increase the ex ante service life of the 1983 vintage from 14.2 years to 17.0 years as compared with a situation with no technical change. Part B of the table shows, not surprisingly, that the year to year fluctuations are smaller the smoother is the price expectation process.

Particularly interesting is the behavior of the service life in the years 1973–1980, which contain the two OPEC induced energy price shocks (1973/74, 1979/80), the sharp rise in the international raw material prices (1973/75), years with a substantial rise in the Norwegian labor cost (1974/75), and years in which a wage and prize freeze was in effect in Norway (1978/79). From 1973 to 1974, the estimated ex ante service life is reduced from 12.6 to 6.9 years if the producers are assumed to react with no lag in their price expectations ($\gamma = \mu = 1$), even with the scaling factor α set as low as unity (last column of Table 5, part B). It is reduced from 14.2 to 11.8 years when the more sluggish process implied by the base alternative is assumed. With a higher degree of ex ante substitution, represented by the more realistic value $\alpha = 3$, we find that the planned service life may drop by more than 50 % from 1973 to 1974, and increase again in the following year. The behavior of the planned service life from 1973 to 1975 also reflects, to a large

[17] Due to space limitations, we only report the main results. A more detailed documentation is available from the authors on request.

[18] For the base vintage, 1980, the service life, $S = 14.7$ years, is independent of the value of α. This follows from the way in which the GL cost function parameters have been constructed.

Table 5, part A: Ex ante service life in years. Variation with scaling factor α for $\gamma = 0.2$, $\mu = 1.0$

Vintage	Scaling factor / Rate of technical change			
	$\alpha = 1$, $\varepsilon = 0$	$\alpha = 2$, $\varepsilon = 0$	$\alpha = 3$, $\varepsilon = 0$	$\alpha = 1$, $\varepsilon = 0.01$
1964	22.50	19.58	17.30	11.89
1965	20.82	18.40	16.41	10.73
1966	18.37	16.15	14.18	8.89
1967	15.27	13.18	10.87	6.77
1968	15.30	13.59	11.28	7.80
1969	15.44	13.85	11.60	8.27
1970	17.82	16.68	15.19	10.73
1971	14.93	14.14	12.71	8.43
1972	16.74	18.30	24.20	11.38
1973	14.15	15.56	22.36	9.07
1974	11.83	11.12	9.79	6.90
1975	14.03	15.92	23.95	10.00
1976	11.37	12.68	*	7.97
1977	10.25	10.65	13.59	7.52
1978	10.00	9.62	*	8.10
1979	19.76	22.33	26.44	18.75
1980	14.65	14.65	14.65	14.65
1981	14.28	14.41	14.59	15.23
1982	14.11	13.94	13.68	16.08
1983	14.22	15.03	16.42	17.03

* No (positive and finite) profit maximizing service life exists.

extent, the difference between the rates of increase of the output and the material prices in these years. A similar effect of the price changes occurs in the years 1978/1980.

These revisions of the expected service life are accompanied by substantial changes in the input coefficients. Let us give a few examples for the years 1973/1974, assuming the base specification of the price expectations. For $\alpha = 1$, the simulated input coefficient for capital (a_K) declines from 0.630 to 0.618, for $\alpha = 2$ it declines from 0.655 to 0.575, and for $\alpha = 3$ the decline is from 0.821 to 0.469, i.e., by more than 40 %. At the same time, the material intensity (a_M) is somewhat reduced, the energy coefficient (a_E) is substantially reduced — for $\alpha = 3$ for instance by more than 30 % — while the input coefficient of labor (a_L) is increased — for $\alpha = 3$ for instance by more than 180 % (from 0.098 to 0.278).

Table 5, part B: Ex ante service life in years. Variation with price expectation process for $\alpha = 1$

Vintage	Parameters in ARMA process for price expectations $\gamma = 0.1$, $\mu = 1.0$	$\gamma = 0.2$, $\mu = 1.0$	$\gamma = 0.5$, $\mu = 0.5$	$\gamma = 1.0$, $\mu = 1.0$
1964	21.47	22.50	21.97	50.08
1965	20.50	20.82	22.26	21.33
1966	18.92	18.37	16.79	17.03
1967	16.73	15.27	12.54	12.96
1968	16.37	15.30	12.12	20.41
1969	16.03	15.44	13.78	25.24
1970	17.02	17.82	18.00	49.86
1971	15.25	14.93	16.35	13.30
1972	16.00	16.74	15.89	27.57
1973	14.43	14.15	14.57	12.61
1974	13.20	11.83	8.05	6.89
1975	13.88	14.03	9.38	*
1976	12.01	11.37	10.43	12.36
1977	11.02	10.25	7.13	15.14
1978	10.38	10.00	7.00	22.51
1979	15.06	19.76	23.85	*
1980	13.45	14.65	22.33	8.74
1981	13.43	14.28	10.65	18.82
1982	13.07	14.11	10.99	25.81
1983	13.06	14.22	12.38	21.00

* No (positive and finite) profit maximizing service life exists.

Changes in technology of this order of magnitude, which at a first glance may seem surprisingly large, are quite reasonable when we recall that they reflect the changes in the relative life cycle prices which are induced by the price changes through the changes in the ex ante service lives and the real interest rates for a marginal vintage. Table 6 indicates that dramatic changes in the relative life cycle prices occur from 1973 to 1975 and from 1978 to 1980 (the life cycle capital price of a vintage being by assumption its investment price).

From 1973 to 1974, for instance, the energy/capital life cycle price ratio shows a substantially larger increase than the corresponding annual price ratio. This is the net effect of the dramatic rise in both the level and the rate of increase of the energy price, and the reduction in the service life. In 1974, capital is, in comparison with labor, a substantially more expensive input in the ex ante life cycle sense than it was in 1973 (in

Table 6: Annual rate of increase of life cycle prices, per cent; $\varepsilon = 0$

A. Base specification of price expectations

		1973/74	1974/75	1978/79	1979/80
Materials,	$\alpha = 1$	33.6	15.5	37.6	9.4
	$\alpha = 2$	25.1	26.2	43.4	6.9
	$\alpha = 3$	6.3	55.1	*	4.5
Energy,	$\alpha = 1$	101.2	19.3	57.6	32.6
	$\alpha = 2$	85.9	35.4	67.9	27.5
	$\alpha = 3$	53.8	86.6	*	21.9
Labor,	$\alpha = 1$	5.4	45.9	35.8	−5.3
	$\alpha = 2$	−4.8	66.5	44.7	−8.9
	$\alpha = 3$	−28.6	134.9	*	−12.7
Capital		15.5	10.3	9.1	5.1

B. Instantaneous adjustment of price expectations, 1973/74

		Life cycle price	Annual price
Materials,	$\alpha = 1$	30.4	23.9
Energy,	$\alpha = 1$	245.4	48.8
Labor,	$\alpha = 1$	−24.1	16.4
Capital		15.5	15.5

* No (positive and finite) profit maximizing service life exists.

particular for $\alpha = 3$), and the producers would have found it profitable to operate the former vintage with a more labor intensive technique than the latter.

5. Conclusion

In this paper, we have been concerned with the relationship between the price expectations for output and inputs, the degree of input substitution, and the firm's plans for scrapping of capital goods. The putty-clay framework used, with the ex ante substitution represented by a Generalized Leontief technology in the empirical part of the paper, has proved to be a convenient framework for this kind of investigation. We have shown that the *interaction* between the input substitution and the planned scrapping age, via the life cycle output and input prices, is important. This interaction implies that *partial* analyses, which disregard (i) input substitution when analyzing the response of the scrapping age to price changes and (ii) the endogeneity of the scrapping plans when dealing with the response of the technique to price changes, miss essential points. The empirical illustrations have demonstrated that, for quite reasonable parameter values, the "secondary" effects may dominate the "primary" ones.

The main limitations of the paper are two. First, the analysis is confined to the *ex ante* scrapping plans; *ex post* decisions are not brought into focus. Second, the simulation experiments of the paper are based on aggregate market data with no vintage dimension specified. Ideally, we should have had access to genuine micro data for estimating the ex ante technology and, maybe most importantly, genuine data on price anticipations.

References

Ando, A. K., Modigliani, F., Rasche, R., and Turnovsky, S. J. (1974): "On the Role of Expectations of Price and Technological Change in an Investment Function." *International Economic Review* 15: 384–414.

Berndt, E. R., and Wood, D. O. (1984): "Energy Price Changes and the Induced Revaluation of Durable Capital in the U.S. Manufacturing during the OPEC Decade." M.I.T. Energy Lab Report no. 84-003. Massachusetts Institute of Technology, Center for Energy Policy Research (March).

Biørn, E. (1986): "Energy Price Changes, and Induced Scrapping and Revaluation of Capital – A Putty-Clay Approach." Discussion Paper no. 16, Central Bureau of Statistics, Oslo.

—— (1989): *Taxation, Technology and the User Cost of Capital.* Amsterdam: North-Holland.

Bye, T., and Frenger, P. (1985): "Factor Substitution, Non-Homotheticity, and Technical Change in the Norwegian Production Sectors." Unpublished paper (May).

Diewert, W. E. (1971): "An Application of the Shephard Duality Theorem: A Generalized Leontief Production Function." *Journal of Political Economy* 79: 481–507.

Frenger, P. (1985): "Using the Directional Shadow Elasticity of Substitution to Measure the Concavity of the Cost Function." *Economics Letters* 19: 171–175.

Fuss, M. (1977): "The Structure of Technology over Time: A Model for Testing the 'Putty-Clay' Hypothesis." *Econometrica* 45: 1797–1821.

—— (1978): "Factor Substitution in Electricity Generation: A Test of the Putty-Clay Hypothesis." In *Production Economics: A Dual Approach to Theory and Applications, Vol. 2*, edited by M. Fuss and D. McFadden. Amsterdam: North-Holland.

Førsund, F. R., and Hjalmarsson, L. (1986): "Technique Choice and Energy Demand in a Vintage Model." Unpublished paper (March).

—— (1987): *Analyses of Industrial structure: A Putty-Clay Approach.* Stockholm: The Industrial Institute for Economic and Social Research.

Hjalmarsson, L. (1974): "The Size Distribution of Establishments and Firms Derived from an Optimal Process of Capacity Expansion." *European Economic Review* 5: 123–140.

Johansen, L. (1959): "Substitution versus Fixed Production Coefficients in the Theory of Economic Growth: A Synthesis." *Econometrica* 27: 157–176.

—— (1967): "Some Problems of Pricing and Optimal Choice of Factor Proportions in a Dynamic Setting." *Economica* 34: 131–152.

—— (1972): *Production Functions*. Amsterdam: North-Holland.

Koizumi, S. (1969): "Technical Progress and Investment." *International Economic Review* 10: 68–81.

Malcomson, J. M. (1975): "Replacement and the Rental Value of Capital Equipment Subject to Obsolescence." *Journal of Economic Theory* 10: 24–41.

—— (1979): "Optimal Replacement Policy and Approximate Replacement Rules." *Applied Economics* 11: 405–414.

Malcolmson, J. M., and Prior, M. J. (1979): "The Estimation of a Vintage Model of Production for U.K. Manufacturing." *Review of Economic Studies* 46: 719–736.

Moene, K. O. (1985): "Fluctuations and Factor Proportions: Putty-Clay Investments Under Uncertainty." In *Production, Multi-Sectoral Growth and Planning*, edited by F. R. Førsund, M. Hoel, and S. Longva. Amsterdam: North-Holland.

Pesaran, M. H. (1985): "Formation of Inflation Expectations in British Manufacturing Industries." *Economic Journal* 95: 948–975.

Address of authors: Prof. Erik Biørn and Petter Frenger, Ph. D., University of Oslo, Department of Economics, P.O. Box 1095 Blindern, N-0317 Oslo 3, Norway.