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EXPECTED TIME TO RECRUITMENT IN AN ORGANIZATION WITH TWO-GRADES INVOLVING TWO THRESHOLDS

J. SRIDHARAN [1], P. SARANYA [2] and A. SRINIVASAN [3]

[1] Department of Mathematics, Government Arts College (Autonomous), Kumbakonam, Tamil Nadu, India.
[2] Department of Mathematics, Parisutham Institute of Technology and Science, Thanjavur,Tamil Nadu, India.
[3] Department of Mathematics, Bishop Heber College (Autonomous), Trichirappalli,Tamil Nadu, India.

Abstract : An organization with two grades subjected to loss of man-power due to the policy decisions taken by the organization is considered .Three mathematical models are constructed using an univariate recruitment policy, based on shock model approach involving optional and mandatory thresholds for the loss of man-power in each grade. The system performance measures namely mean and variance of time to recruitment are obtained for all the models when (i) the loss of man-hours form a sequence of independent and identically distributed exponential random variables (ii)the inter-decision times form an order statistics according as the optional thresholds follow exponential or extended exponential and the distribution of mandatory thresholds possess SCBZ property .The analytical results are numerical illustrated and the influence of nodal parameters on the performance measures is also reported.
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1. Introduction

Exit of personnel in other words known as wastage is a common phenomenon in any marketing organization .In the univariate recruitment policy, based on shock model approach, recruitment is made as and when the cumulative loss of man hours crosses a threshold. In [4],for a single grade man-power system with a mandatory exponential threshold for the loss of manpower ,the authors have obtained the system performance measures namely mean and variance of the time to recruitment when the inter-decision times form an order statistics .Since the number of exits in a policy decision making epoch is unpredictable and the time at which the cumulative loss of man-hours crossing a single threshold is probabilistic, the organization has left with no choice except making recruitment immediately upon threshold crossing. In [2] for a single graded system, the authors have considered a recruitment policy involving two thresholds for the loss of manpower in the organization in which one is optional and the other is mandatory and obtained the mean time to recruitment under different conditions on the nature of the thresholds according as the inter-decision times are independent and identically distributed random variables or the inter-decision times are exchangeable and constantly correlated exponential random variables . In [9-12], the authors have studied the problem of recruitment in a two-grade system according as the thresholds are

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exponential random variables or geometric random variables or SCBZ property possessing random variables or extended exponential random variables. Recently in [5-8], the authors have extended the results in [4] for a two-grade system involving two thresholds by assuming different distributions for thresholds. In [13], the performance measures are obtained when the inter –decision times are exchangeable and constantly correlated exponential random variables and the distributions of the thresholds have SCBZ property. The objective of the present paper is to obtain system performance measures when the optional thresholds follow either exponential or extended exponential distribution and the distribution of mandatory thresholds possess SCBZ property and there by extending the results in [4] for a two-grade man-power system.

This paper is organized as follows .In sections 2, 3 and 4 models I, II and III are described respectively and the analytical expressions for mean and variance of the time to recruitment are derived. In section 5, the influence of nodal parameters on the system performance measures is studied for all the models and relevant findings and conclusion are reported.

2. Model Description and Analysis of Model –I

Consider an organization with two grades taking decisions at random epoch in $(0, \infty)$ and at every decision epoch a random number of persons quit the organization. There is an associated loss of man-hours if a person quits. It is assumed that the loss of man-hours are linear and cumulative. Let X_i be the loss of man hours due to the i^{th} decision epoch , i=1,2,3… forming a sequence of independent and identically distributed exponential random variables with mean $\frac{1}{c}$ (c>0), probability density function g(.).Let U_i be a continuous random variable denoting inter-decision time between $(i-1)^{th}$ and i^{th} decision, i=1,2, 3…. with cumulative distribution function F(.), probability density function f(.) and mean $\frac{1}{\lambda}$ $(\lambda>0)$. Let $U_{(1)}$ ($U_{(k)}$) be the smallest (largest) order statistic with probability density function$f_{u(1)}(.), (f_{u(k)}(.))$. Let Y_1, Y_2 (Z_1, Z_2) be random variables denoting optional (mandatory) thresholds for the loss of man-hours in grades 1 and 2, with parameters $\theta_1, \theta_2, (\delta_3, \eta_3, \mu_3, \delta_4, \eta_4, \mu_4)$ respectively. It is assumed that $Y_1 < Z_1$ and $Y_2 < Z_2$ .Write Y = Max (Y_1, Y_2) and Z = Max (Z_1, Z_2) where Y (Z) is the optional (mandatory) threshold for the loss of man-hours in the organization. The loss of man-hours and the optional and mandatory thresholds are statistically independent. Let T be the time to recruitment in the organization with cumulative distribution function L (.), probability density function l (.), mean E (T) and variance V (T). Let $F_k(.)$ be the k fold convolution of F(.). Let $l^*(.)$, $f^*(.)$, and $g^*(.)$ be the Laplace transform of l(.), f(.), and g(.) respectively. Let $V_k(t)$ be the probability that there are exactly k decision epochs in (0, t]. It is known from Renewal theory [3] that $V_k(t) = F_k(t) - F_{k+1}(t)$ with $F_0(t) = 1$. Let p be the probability that the organization is not going for recruitment whenever the total loss of man-hours crosses optional threshold Y. The univariate recruitment policy employed in this paper is as follows: If the total loss of man-hours exceeds the optional threshold Y, the organization may or may not go for recruitment. But if the total loss of man-hours exceeds the mandatory threshold Z, the recruitment is necessary.

Main results

$$P(T > t) = \sum_{k=0}^{\infty} V_k(t) P\left(\sum_{i=1}^{k} X_i \le Y\right) + p \sum_{k=0}^{\infty} V_k(t) \times P\left(\sum_{i=1}^{k} X_i > Y\right) \times P\left(\sum_{i=1}^{k} X_i < Z\right) \qquad (1)$$

Case (i): If the distribution of optional thresholds follow exponential distribution and the distribution of mandatory thresholds possess SCBZ property

Using law of total probability it can be shown that

$$P\left(\sum_{i=1}^{k} X_i \le Y\right) = (D_1)^k + (D_2)^k - (D_3)^k \qquad (2)$$

and

$$P\left(\sum_{i=1}^{k} X_i < Z\right) = p_2(D_9)^k + q_2(D_{10})^k + p_1(D_{11})^k - p_1p_2(D_{12})^k - p_1q_2(D_{13})^k + q_1(D_{14})^k + -p_2q_1(D_{15})^k - q_1q_2(D_{16})^k$$

(3)

where $D_1 = g^*(\theta_1), D_2 = g^*(\theta_2), D_3 = g^*(\theta_1 + \theta_2), D_9 = g^*(\delta_4 + \mu_4), D_{10} = g^*(\eta_4)$

$D_{11} = g^*(\delta_3 + \mu_3), D_{12} = g^*(\delta_3 + \delta_4 + \mu_3 + \mu_4), D_{13} = g^*(\delta_3 + \eta_4 + \mu_3), D_{14} = g^*(\eta_3)$

$D_{15} = g^*(\eta_3 + \delta_4 + \mu_4), D_{16} = g^*(\eta_3 + \eta_4), p_1 = \dfrac{(\delta_3 - \eta_3)}{(\mu_3 + \delta_3 - \eta_3)}, q_1 = 1 - p_1,$

$$p_2 = \frac{(\delta_4 - \eta_4)}{(\mu_4 + \delta_4 - \eta_4)}, q_2 = 1 - p_2 \qquad (4)$$

For a =1,2,3,9,10,11,12,13,14,15,16, b=1, 2, 3 and d=9,10,11,12,13,14,15,16,write

$$E_a(t) = [1 - D_a] \sum_{k=1}^{\infty} F_k(t)(D_a)^{k-1}$$

and

$$E_{b,d}(t) = [1 - D_b D_d] \sum_{k=1}^{\infty} F_k(t)(D_b D_d)^{k-1} \qquad (5)$$

Since $l(t) = \frac{d}{dt} L(t), L(t) = 1 - P(T > t)$ and $l^*(s) = L(l(t))$

from (1), (2), (3), (4), and (5) and on simplification we get

$$l^*(s) = e_1^*(s) + e_2^*(s) - e_3^*(s) + p(p_2e_9^*(s) + q_2e_{10}^*(s) + p_1e_{11}^*(s) - p_1p_2e_{12}^*(s) - p_1q_2e_{13}^*(s) + q_1e_{14}^*(s) - p_2q_1e_{15}^*(s) - q_1q_2e_{16}^*(s) - p_2e_{1,9}^*(s) - q_2e_{1,10}^*(s) - p_1e_{1,11}^*(s) + p_1p_2e_{1,12}^*(s) + p_1q_2e_{1,13}^*(s) - q_1e_{1,14}^*(s) + p_2q_1e_{1,15}^*(s) + q_1q_2e_{1,16}^*(s) - p_2e_{2,9}^*(s) - q_2e_{2,10}^*(s) - p_1e_{2,11}^*(s) + p_1p_2e_{2,12}^*(s) + p_1q_2e_{2,13}^*(s) - q_1e_{2,14}^*(s) + p_2q_1e_{2,15}^*(s) + q_1q_2e_{2,16}^*(s) + p_2e_{3,9}^*(s) + q_2e_{3,10}^*(s) + p_1e_{3,11}^*(s) - p_1p_2e_{3,12}^*(s) - p_1q_2e_{3,13}^*(s) + q_1e_{3,14}^*(s) - p_2q_1e_{3,15}^*(s) - q_1q_2e_{3,16}^*(s)) \qquad (6)$$

where

$$e_a^*(s) = \frac{(1 - D_a)f^*(s)}{[1 - f^*(s)D_a]} \text{ and } e_{b,d}^*(s) = \frac{(1 - D_bD_d)f^*(s)}{[1 - f^*(s)D_bD_d]} \qquad (7)$$

Let $U_1, U_2, U_3, \ldots\ldots U_k$ be arranged in an increasing order so that we have sequence $U_{(1)}, U_{(2)}, U_{(3)}, \ldots\ldots U_{(k)}$ .Here $U_{(r)}$ is the r^{th} order statistics, r=1, 2,3…..k .The random variable $U_{(1)}, U_{(2)}, U_{(3)}, \ldots\ldots U_{(k)}$ are not independent. For r=1,2,3…k the probability density function of $U_{(r)}$ is given by

$$f_{u(r)}\ (t) = r\binom{k}{r}\left(F(t)\right)^{r-1}\ f(t)(1-F(t))^{k-r} \quad (8)$$

Suppose $f(t) = f_{u(1)}(t)$ .Then

$$f_{u(1)}{}^{*}(s) = \frac{k\lambda}{k\lambda+s} \quad (9)$$

Since $E(T) = \frac{-d}{ds}\left(l^{*}\ (s)\right)_{s=o}$ and $E(T^2) = \frac{d^2}{ds^2}\left(l^{*}(s)\right)_{s=0}$ (10)

Using (9),(10) in (6) we get

$$E(T) = C_1 + C_2 - C_3 + p(p_2C_9 + q_2C_{10} + p_1C_{11} - p_1p_2C_{12} - p_1q_2C_{13} + q_1C_{14} - p_2q_1C_{15} - q_1q_2C_{16} - p_2H_{1,9} - q_2H_{1,10} - p_1H_{1,11} + p_1p_2H_{1,12} + p_1q_2H_{1,13} - q_1H_{1,14} + p_2q_1H_{1,15} + q_1q_2H_{1,16} - p_2H_{2,9} - q_2H_{2,10} - p_1H_{2,11} + p_1p_2H_{2,12} + p_1q_2H_{2,13} - q_1H_{2,14} + p_2q_1H_{2,15} + q_1q_2H_{2,16} + p_2H_{3,9} + q_2H_{3,10} + p_1H_{3,11} + p_1p_2H_{3,12} - p_1q_2H_{3,13} + q_1H_{3,14} - p_2q_1H_{3,15} - q_1q_2H_{3,16})$$

(11)

$$E(T^2) = 2\left(C_1{}^2 + C_2{}^2 - C_3{}^2 + p(p_2C_9{}^2 + q_2C_{10}{}^2 + p_1C_{11}{}^2 - p_1p_2C_{12}{}^2 - p_1q_2C_{13}{}^2 + +q_1C_{14}{}^2 - p_2q_1C_{15}{}^2 - q_1q_2C_{16}{}^2 - p_2H_{1,9}{}^2 - q_2H_{1,10}{}^2 - p_1H_{1,11}{}^2 + p_1p_2H_{1,12}{}^2 + p_1q_2H_{1,13}{}^2 - q_1H_{1,14}{}^2 + p_2q_1H_{1,15}{}^2 + q_1q_2H_{1,16}{}^2 - p_2H_{2,9}{}^2 - q_2H_{2,10}{}^2 - p_1H_{2,11}{}^2 + p_1p_2H_{2,12}{}^2 + p_1q_2H_{2,13}{}^2 - q_1H_{2,14}{}^2 + p_2q_1H_{2,15}{}^2 + q_1q_2H_{2,16}{}^2 + p_2H_{3,9}{}^2 + q_2H_{3,10}{}^2 + p_1H_{3,11}{}^2 - p_1p_2H_{3,12}{}^2 - p_1q_2H_{3,13}{}^2 + q_1H_{3,14}{}^2 - p_2q_1H_{3,15}{}^2 - q_1q_2H_{3,16}{}^2)\right)$$

(12)

where for a =1, 2,3,9,10,11,12,13,14,15,16. b=1, 2, 3 and d=9, 10,11,12,13,14,15,16.

$$C_a = \frac{1}{\lambda(1-D_a)} \quad \text{and} \quad H_{b,d} = \frac{1}{\lambda(1-D_bD_d)} \quad (13)$$

Since $V(T) = E(T^2) - \left(E(T)\right)^2$, the variance of the time to recruitment can be calculated from (12) and (13).

Suppose $f(t) = f_{u(k)}(t)$ .Then

$$f^{*}_{u(k)}\ (s) = \frac{k!\lambda^k}{(s+\lambda)(s+2\lambda)\ldots.(s+(k-1)\lambda)(s+k\lambda)} \quad (14)$$

From (6), (10), (14) and on simplification it can be shown that

$$E(T) = P_1 + P_2 - P_3 + p(p_2P_9 + q_2P_{10} + p_1P_{11} - p_1p_2P_{12} - p_1q_2P_{13} + q_1P_{14} - p_2q_1P_{15} - q_1q_2P_{16} - p_2Q_{1,9} - q_2Q_{1,10} - p_1Q_{1,11} + p_1p_2Q_{1,12} + p_1q_2Q_{1,13} - q_1Q_{1,14} + p_2q_1Q_{1,15} + q_1q_2Q_{1,16} - p_2Q_{2,9} - q_2Q_{2,10} - p_1Q_{2,11} + p_1p_2Q_{2,12} + p_1q_2Q_{2,13} - q_1Q_{2,14} + p_2q_1Q_{2,15} + q_1q_2Q_{2,16} + p_2Q_{3,9} + q_2Q_{3,10} + p_1Q_{3,11} - p_1p_2Q_{3,12} - p_1q_2Q_{3,13} + q_1Q_{3,14} - p_2q_1Q_{3,15} - q_1q_2Q_{3,16}) \quad (15)$$

$$E(T^2) = 2\Big(P_1{}^2 + P_2{}^2 - P_3{}^2 + p(p_2P_9{}^2 + q_2P_{10}{}^2 + p_1P_{11}{}^2 - p_1p_2P_{12}{}^2 - p_1q_2P_{13}{}^2 + q_1P_{14}{}^2 - p_2q_1P_{15}{}^2 - q_1q_2P_{16}{}^2 - p_2Q_{1,9}{}^2 - q_2Q_{1,10}{}^2 - p_1Q_{1,11}{}^2 + p_1p_2Q_{1,12}{}^2 + p_1q_2Q_{1,13}{}^2 - q_1Q_{1,14}{}^2 + p_2q_1Q_{1,15}{}^2 + q_1q_2Q_{1,16}{}^2 - p_2Q_{2,9}{}^2 - q_2Q_{2,10}{}^2 - p_1Q_{2,11}{}^2 + p_1p_2Q_{2,12}{}^2 + p_1q_2Q_{2,13}{}^2 - q_1Q_{2,14}{}^2 + p_2q_1Q_{2,15}{}^2 + q_1q_2Q_{2,16}{}^2 + p_2Q_{3,9}{}^2 + q_2Q_{3,10}{}^2 + p_1Q_{3,11}{}^2 - p_1p_2Q_{3,12}{}^2 - p_1q_2Q_{3,13}{}^2 + q_1Q_{3,14}{}^2 - p_2q_1Q_{3,15}{}^2 - q_2q_1Q_{3,16}{}^2)\Big) - \frac{1}{\lambda^2}(N^2 - M)\Bigg(\Big(\frac{1}{1-D_1} + \frac{1}{1-D_2} - \frac{1}{1-D_3}\Big) + p\Big(\frac{p_2}{1-D_9} + \frac{q_2}{1-D_{10}} + \frac{p_1}{1-D_{11}} - \frac{p_1p_2}{1-D_{12}} - \frac{p_1q_2}{1-D_{13}} + \frac{q_1}{1-D_{14}} - \frac{p_2q_1}{1-D_{15}} - \frac{q_1q_2}{1-D_{16}} - \frac{p_2}{1-D_1D_9} - \frac{q_2}{1-D_1D_{10}} - \frac{p_1}{1-D_1D_{11}} + \frac{p_1p_2}{1-D_1D_{12}} + \frac{p_1q_2}{1-D_1D_{13}} - \frac{q_1}{1-D_1D_{14}} + \frac{p_2q_1}{1-D_1D_{15}} + \frac{q_1q_2}{1-D_1D_{16}} - \frac{p_2}{1-D_2D_9} - \frac{q_2}{1-D_2D_{10}} - \frac{p_1}{1-D_2D_{11}} + \frac{p_1p_2}{1-D_2D_{12}} + \frac{p_1q_2}{1-D_2D_{13}} - \frac{q_1}{1-D_2D_{14}} + \frac{p_2q_1}{1-D_2D_{15}} + \frac{q_1q_2}{1-D_2D_{16}} + \frac{p_2}{1-D_3D_9} + \frac{q_2}{1-D_3D_{10}} + \frac{p_1}{1-D_3D_{11}} - \frac{p_1p_2}{1-D_3D_{12}} - \frac{p_1q_2}{1-D_3D_{13}} + \frac{q_1}{1-D_3D_{14}} - \frac{p_2q_1}{1-D_3D_{15}} - \frac{q_1q_2}{1-D_3D_{16}}\Big)\Bigg) \quad (16)$$

where for a =1, 2,3,9,10,11,12,13,14,15,16, b=1, 2, 3 and d=9,10,11,12,13,14,15,16.

$$P_a = \frac{N}{\lambda(1-D_a)} \quad \text{and} \quad Q_{b,d} = \frac{N}{\lambda(1-D_bD_d)}, N = \sum_{n=1}^{k}\frac{1}{n} \text{ and} M = \sum_{n=1}^{k}\frac{1}{n^2} \quad (17)$$

Since $V(T) = E(T^2) - \big(E(T)\big)^2$, the variance of the time to recruitment can be calculated from (15) and (16).

Case (ii): If the distribution of optional thresholds follow extended exponential distribution and the distribution of mandatory thresholds possess SCBZ property.

$$P\Big(\sum_{i=1}^{k} X_i \le Y\Big) = 2(D_1)^k + 2(D_2)^k + 2(D_4)^k + 2(D_5)^k - (D_6)^k - (D_7)^k - (D_8)^k - 4(D_3)^k \quad (18)$$

where

$$D_4 = g^*(2\theta_1 + \theta_2), D_5 = g^*(\theta_1 + 2\theta_2), D_6 = g^*(2\theta_1), D_7 = g^*(2\theta_2), D_8 = g^*(2\theta_1 + 2\theta_2)$$

Using (3) and (18) in (1) and on simplification we get

Suppose $f(t) = f_{u(1)}(t)$.Then

$$\mathrm{E(T)} = 2C_1 + 2C_2 - 4C_3 + 2C_4 + 2C_5 - C_6 - C_7 - C_8 + p(p_2C_9 + q_2C_{10} + p_1C_{11} - p_1p_2C_{12} - q_2p_1C_{13} + q_1C_{14} - p_2q_1C_{15} - q_2q_1C_{16} - 2p_2H_{1,9} - 2q_2H_{1,10} - 2p_1H_{1,11} + 2p_2p_1H_{1,12} + 2p_1q_2H_{1,13} - 2q_1H_{1,14} + 2p_2q_1H_{1,15} + 2q_1q_2H_{1,16} - 2p_2H_{2,9} - 2q_2H_{2,10} - 2p_1H_{2,11} + 2p_2p_1H_{2,12} + 2p_1q_2H_{2,13} - 2q_1H_{2,14} + 2p_2q_1H_{2,15} + 2q_1q_2H_{2,16} + 4p_2H_{3,9} + 4q_2H_{3,10} + 4p_1H_{3,11} - 4p_2p_1H_{3,12} - 4p_1q_2H_{3,13} + 4q_1H_{3,14} - 4p_2q_1H_{3,15} - 4q_1q_2H_{3,16} - 2p_2H_{4,9} - 2q_2H_{4,10} - 2p_1H_{4,11} + 2p_2p_1H_{4,12} + 2p_1q_2H_{4,13} - 2q_1H_{4,14} + 2p_2q_1H_{4,15} + 2q_1q_2H_{4,16} - 2p_2H_{5,9} - 2q_2H_{5,10} - 2p_1H_{5,11} + 2p_2p_1H_{5,12} + 2p_1q_2H_{5,13} - 2q_1H_{5,14} + 2p_2q_1H_{5,15} + 2q_1q_2H_{5,16} + p_2H_{6,9} + q_2H_{6,10} + p_1H_{6,11} - p_1p_2H_{6,12} - q_2p_1H_{6,13} + q_1H_{6,14} - p_2q_1H_{6,15} - q_2q_1H_{6,16} + p_2H_{7,9} + q_2H_{7,10} + p_1H_{7,11} - p_1p_2H_{7,12} - q_2p_1H_{7,13} + q_1H_{7,14} - p_2q_1H_{7,15} - q_2q_1H_{7,16} + p_2H_{8,9} + q_2H_{8,10} + p_1H_{8,11} - p_1p_2H_{8,12} - q_2p_1H_{8,13} + q_1H_{8,14} - p_2q_1H_{8,15} - q_2q_1H_{8,16}) \quad (19)$$

$$E(T^2) = 2\big(2C_1^2 + 2C_2^2 - 4C_3^2 + 2C_4^2 + 2C_5^2 - C_6^2 - C_7^2 - C_8^2 + p\big(p_2C_9^2 + q_2C_{10}^2 + p_1C_{11}^2 - p_1p_2C_{12}^2 - q_2p_1C_{13}^2 + q_1C_{14}^2 - p_2q_1C_{15}^2 - q_2q_1C_{16}^2 - 2p_2H_{1,9}^2 - 2q_2H_{1,10}^2 - 2p_1H_{1,11}^2 + 2p_2p_1H_{1,12}^2 + 2p_1q_2H_{1,13}^2 - 2q_1H_{1,14}^2 + 2p_2q_1H_{1,15}^2 + 2q_1q_2H_{1,16}^2 - 2p_2H_{2,9}^2 - 2q_2H_{2,10}^2 - 2p_1H_{2,11}^2 + 2p_2p_1H_{2,12}^2 + 2p_1q_2H_{2,13}^2 - 2q_1H_{2,14}^2 + 2p_2q_1H_{2,15}^2 + 2q_1q_2H_{2,16}^2 + 4p_2H_{3,9}^2 + 4q_2H_{3,10}^2 + 4p_1H_{3,11}^2 - 4p_2p_1H_{3,12}^2 - 4p_1q_2H_{3,13}^2 + 4q_1H_{3,14}^2 - 4p_2q_1H_{3,15}^2 - 4q_1q_2H_{3,16}^2 - 2p_2H_{4,9}^2 - 2q_2H_{4,10}^2 - 2p_1H_{4,11}^2 + 2p_2p_1H_{4,12}^2 + 2p_1q_2H_{4,13}^2 - 2q_1H_{4,14}^2 + 2p_2q_1H_{4,15}^2 + 2q_1q_2H_{4,16}^2 - 2p_2H_{5,9}^2 - 2q_2H_{5,10}^2 - 2p_1H_{5,11}^2 + 2p_2p_1H_{5,12}^2 + 2p_1q_2H_{5,13}^2 - 2q_1H_{5,14}^2 + 2p_2q_1H_{5,15}^2 + 2q_1q_2H_{5,16}^2 + p_2H_{6,9}^2 + q_2H_{6,10}^2 + p_1H_{6,11}^2 - p_1p_2H_{6,12}^2 - q_2p_1H_{6,13}^2 + q_1H_{6,14}^2 - p_2q_1H_{6,15}^2 - q_2q_1H_{6,16}^2 + p_2H_{7,9}^2 + q_2H_{7,10}^2 + p_1H_{7,11}^2 - p_1p_2H_{7,12}^2 - q_2p_1H_{7,13}^2 + q_1H_{7,14}^2 - p_2q_1H_{7,15}^2 - q_2q_1H_{7,16}^2 + p_2H_{8,9}^2 + q_2H_{8,10}^2 + p_1H_{8,11}^2 - p_1p_2H_{8,12}^2 - q_2p_1H_{8,13}^2 + q_1H_{8,14}^2 - p_2q_1H_{8,15}^2 - q_2q_1H_{8,16}^2\big)\big) \quad (20)$$

Since $V(T) = E(T^2) - \big(E(T)\big)^2$, the variance of the time to recruitment can be calculated from (19) and (20).

Suppose $\mathrm{f(t)} = \mathrm{f_{u(k)}(t)}$ .Then

$$E(T) = 2P_1 + 2P_2 - 4P_3 + 2P_4 + 2P_5 - P_6 - P_7 - P_8 + p(p_2P_9 + q_2P_{10} + p_1P_{11} - p_1p_2P_{12} - q_2p_1P_{13} + q_1P_{14} - p_2q_1P_{15} - q_2q_1P_{16} - 2p_2Q_{1,9} - 2q_2Q_{1,10} - 2p_1Q_{1,11} + 2p_2p_1Q_{1,12} + 2p_1q_2Q_{1,13} - 2q_1Q_{1,14} + 2p_2q_1Q_{1,15} + 2q_1q_2Q_{1,16} - 2p_2Q_{2,9} - 2q_2Q_{2,10} - 2p_1Q_{2,11} + 2p_2p_1Q_{2,12} + 2p_1q_2Q_{2,13} - 2q_1Q_{2,14} + 2p_2q_1Q_{2,15} + 2q_1q_2Q_{2,16} + 4p_2Q_{3,9} + 4q_2Q_{3,10} + 4p_1Q_{3,11} - 4p_2p_1Q_{3,12} - 4p_1q_2Q_{3,13} + 4q_1Q_{3,14} - 4p_2q_1Q_{3,15} - 4q_1q_2Q_{3,16} - 2p_2Q_{4,9} - 2q_2Q_{4,10} - 2p_1Q_{4,11} + 2p_2p_1Q_{4,12} + 2p_1q_2Q_{4,13} - 2q_1Q_{4,14} + 2p_2q_1Q_{4,15} + 2q_1q_2Q_{4,16} - 2p_2Q_{5,9} - 2q_2Q_{5,10} - 2p_1Q_{5,11} + 2p_2p_1Q_{5,12} + 2p_1q_2Q_{5,13} - 2q_1Q_{5,14} + 2p_2q_1Q_{5,15} + 2q_1q_2Q_{5,16} + p_2Q_{6,9} + q_2Q_{6,10} + p_1Q_{6,11} - p_1p_2Q_{6,12} - q_2p_1Q_{6,13} + q_1Q_{6,14} - p_2q_1Q_{6,15} - q_2q_1Q_{6,16} + p_2Q_{7,9} + q_2Q_{7,10} + p_1Q_{7,11} - p_1p_2Q_{7,12} - q_2p_1Q_{7,13} + q_1Q_{7,14} - p_2q_1Q_{7,15} - q_2q_1Q_{7,16} + p_2Q_{8,9} + q_2Q_{8,10} + p_1Q_{8,11} - p_1p_2Q_{8,12} - q_2p_1Q_{8,13} + q_1Q_{8,14} - p_2q_1Q_{8,15} - q_2q_1Q_{8,16}) \quad (21)$$

$$E(T^2) = 2(2P_1^2 + 2P_2^2 - 4P_3^2 + 2P_4^2 + 2P_5^2 - P_6^2 - P_7^2 - P_8^2 + p(p_2P_9^2 + q_2P_{10}^2 + p_1P_{11}^2 - p_1p_2P_{12}^2 - q_2p_1P_{13}^2 + q_1P_{14}^2 - p_2q_1P_{15}^2 - q_2q_1P_{16}^2 - 2p_2Q_{1,9}^2 - 2q_2Q_{1,10}^2 - 2p_1Q_{1,11}^2 + 2p_2p_1Q_{1,12}^2 + 2p_1q_2Q_{1,13}^2 - 2q_1Q_{1,14}^2 + 2p_2q_1Q_{1,15}^2 + 2q_1q_2Q_{1,16}^2 - 2p_2Q_{2,9}^2 - 2q_2Q_{2,10}^2 - 2p_1Q_{2,11}^2 + 2p_2p_1Q_{2,12}^2 + 2p_1q_2Q_{2,13}^2 - 2q_1Q_{2,14}^2 + 2p_2q_1Q_{2,15}^2 + 2q_1q_2Q_{2,16}^2 + 4p_2Q_{3,9}^2 + 4q_2Q_{3,10}^2 + 4p_1Q_{3,11}^2 - 4p_2p_1Q_{3,12}^2 - 4p_1q_2Q_{3,13}^2 + 4q_1Q_{3,14}^2 - 4p_2q_1Q_{3,15}^2 - 4q_1q_2Q_{3,16}^2 - 2p_2Q_{4,9}^2 - 2q_2Q_{4,10}^2 - 2p_1Q_{4,11}^2 + 2p_2p_1Q_{4,12}^2 + 2p_1q_2Q_{4,13}^2 - 2q_1Q_{4,14}^2 + 2p_2q_1Q_{4,15}^2 + 2q_1q_2Q_{4,16}^2 - 2p_2Q_{5,9}^2 - 2q_2Q_{5,10}^2$$

$$
\begin{aligned}
&-2p_1Q_{5,11}{}^2 + 2p_2p_1Q_{5,12}{}^2 + 2p_1q_2Q_{5,13}{}^2 - 2q_1Q_{5,14}{}^2 + 2p_2q_1Q_{5,15}{}^2 + 2q_1q_2Q_{5,16}{}^2 \\
&\quad + p_2Q_{6,9}{}^2 + q_2Q_{6,10}{}^2 + p_1Q_{6,11}{}^2 - p_1p_2Q_{6,12}{}^2 \\
&\quad - q_2p_1Q_{6,13}{}^2 + q_1Q_{6,14}{}^2 - p_2q_1Q_{6,15}{}^2 - q_2q_1Q_{6,16}{}^2 + p_2Q_{7,9}{}^2 + q_2Q_{7,10}{}^2 \\
&\quad + p_1Q_{7,11}{}^2 - p_1p_2Q_{7,12}{}^2 - q_2p_1Q_{7,13}{}^2 + q_1Q_{7,14}{}^2 - p_2q_1Q_{7,15}{}^2 - q_2q_1Q_{7,16}{}^2 \\
&\quad + p_2Q_{8,9}{}^2 + q_2Q_{8,10}{}^2 + p_1Q_{8,11}{}^2 - p_1p_2Q_{8,12}{}^2 \\
&\quad - q_2p_1Q_{8,13}{}^2 + q_1Q_{8,14}{}^2 - p_2q_1Q_{8,15}{}^2 - q_2q_1Q_{8,16}{}^2)) \\
&\quad - \frac{1}{\lambda^2}\Big\{(N^2 - M)\ \Big(\frac{2}{1-D_1} + \frac{2}{1-D_2} - \frac{4}{1-D_3} + \frac{2}{1-D_4} + \frac{2}{1-D_5} - \frac{1}{1-D_6} \\
&\quad - \frac{1}{1-D_7} - \frac{1}{1-D_8}\Big)\Big\} \\
&\quad - \frac{p}{\lambda^2}\Big\{(N^2 - M)\ \Big(\frac{p_2}{1-D_9} + \frac{q_2}{1-D_{10}} + \frac{p_1}{1-D_{11}} - \frac{p_1p_2}{1-D_{12}} - \frac{q_2p_1}{1-D_{13}} + \frac{q_1}{1-D_{14}} \\
&\quad - \frac{p_2q_1}{1-D_{15}} - \frac{q_2q_1}{1-D_{16}} - \frac{2p_2}{1-D_1D_9} - \frac{2q_2}{1-D_1D_{10}} - \frac{2p_1}{1-D_1D_{11}} + \frac{2p_2p_1}{1-D_1D_{12}} \\
&\quad + \frac{2q_2p_1}{1-D_1D_{13}} - \frac{2q_1}{1-D_1D_{14}} + \frac{2p_2q_1}{1-D_1D_{15}} + \frac{2q_1q_2}{1-D_1D_{16}} - \frac{2p_2}{1-D_2D_9} - \frac{2q_2}{1-D_2D_{10}} \\
&\quad - \frac{2p_1}{1-D_2D_{11}} + \frac{2p_2p_1}{1-D_2D_{12}} + \frac{2q_2p_1}{1-D_2D_{13}} - \frac{2q_1}{1-D_2D_{14}} + \frac{2p_2q_1}{1-D_2D_{15}} \\
&\quad + \frac{2q_1q_2}{1-D_2D_{16}} + \frac{4p_2}{1-D_3D_9} + \frac{4q_2}{1-D_3D_{10}} + \frac{4p_1}{1-D_3D_{11}} - \frac{4p_2p_1}{1-D_3D_{12}} - \frac{4q_2p_1}{1-D_3D_{13}} \\
&\quad + \frac{4q_1}{1-D_3D_{14}} - \frac{4p_2q_1}{1-D_3D_{15}} - \frac{4q_1q_2}{1-D_3D_{16}} - \frac{2p_2}{1-D_4D_9} - \frac{2q_2}{1-D_4D_{10}} - \frac{2p_1}{1-D_4D_{11}} \\
&\quad + \frac{2p_2p_1}{1-D_4D_{12}} + \frac{2q_2p_1}{1-D_4D_{13}} - \frac{2q_1}{1-D_4D_{14}} + \frac{2p_2q_1}{1-D_4D_{15}} + \frac{2q_1q_2}{1-D_{14}D_{16}} \\
&\quad - \frac{2p_2}{1-D_5D_9} - \frac{2q_2}{1-D_5D_{10}} - \frac{2p_1}{1-D_5D_{11}} + \frac{2p_2p_1}{1-D_5D_{12}} + \frac{2p_1q_2}{1-D_5D_{13}} - \frac{2q_1}{1-D_5D_{14}} \\
&\quad + \frac{2p_2q_1}{1-D_5D_{15}} + \frac{2q_1q_2}{1-D_5D_{16}} + \frac{p_2}{1-D_6D_9} + \frac{q_2}{1-D_6D_{10}} + \frac{p_1}{1-D_6D_{11}} - \frac{p_2p_1}{1-D_6D_{12}} \\
&\quad - \frac{p_1q_2}{1-D_6D_{13}} + \frac{q_1}{1-D_6D_{14}} - \frac{p_2q_1}{1-D_6D_{15}} - \frac{q_1q_2}{1-D_6D_{16}} + \frac{p_2}{1-D_7D_9} + \frac{q_2}{1-D_7D_{10}} \\
&\quad + \frac{p_1}{1-D_7D_{11}} - \frac{p_2p_1}{1-D_7D_{12}} - \frac{q_2p_1}{1-D_7D_{13}} + \frac{q_1}{1-D_7D_{14}} - \frac{p_2q_1}{1-D_7D_{15}} \\
&\quad - \frac{q_1q_2}{1-D_7D_{16}} + \frac{p_2}{1-D_8D_9} + \frac{q_2}{1-D_8D_{10}} + \frac{p_1}{1-D_8D_{11}} - \frac{p_2p_1}{1-D_8D_{12}} - \frac{p_1q_2}{1-D_8D_{13}} \\
&\quad + \frac{q_1}{1-D_8D_{14}} - \frac{p_2q_1}{1-D_8D_{15}} - \frac{q_1q_2}{1-D_8D_{16}}\Big)\Big\} \qquad (22)
\end{aligned}
$$

Since $V(T) = E(T^2) - \left(E(T)\right)^2$, the variance of the time to recruitment can be calculated from (21) and (22).

3. Model description and Analysis of Model-II

For this model, the optional and mandatory thresholds for the loss of man-hours in the organization are taken as Y=min (Y_1, Y_2) and Z=min (Z_1, Z_2). All the other assumptions and notations are as in model-I.

Proceeding as in model-I, it can be shown for the present model that

$$l^*(s) = e_3{}^*(s) + p\,(p_1p_2e_{12}{}^*(s) + p_1q_2e_{13}{}^*(s) + p_2q_1e_{15}{}^*(s) + q_1q_2e_{16}{}^*(s) - p_1p_2e_{3,12}{}^*(s) - p_1q_2e_{3,13}{}^*(s) - p_2q_1e_{3,15}{}^*(s) - q_1q_2e_{3,16}{}^*(s)) \quad (23)$$

Case (i): If the distribution of optional thresholds follow exponential distribution and the distribution of mandatory thresholds possess SCBZ property

Suppose $f(t) = f_{u(1)}(t)$ .Then

$$E(T) = C_3 + p\,(p_1p_2C_{12} + p_1q_2C_{13} + p_2q_1C_{15} + q_1q_2C_{16} - p_1p_2H_{3,12} - p_1q_2H_{3,13} - p_2q_1H_{3,15} - q_1q_2H_{3,16})$$

(24)

$$E(T^2) = 2\left(C_3{}^2 + p\left(p_1p_2C_{12}{}^2 + p_1q_2C_{13}{}^2 + p_2q_1C_{15}{}^2 + q_1q_2C_{16}{}^2 - p_1p_2H_{3,12}{}^2 - p_1q_2H_{3,13}{}^2 - p_2q_1H_{3,15}{}^2 - q_1q_2H_{3,16}{}^2\right)\right) \quad (25)$$

where $C_a, H_{b,d}$ are given by equation (13) for a=3,12,13,15,16 ,b=3 and d=12,13,15,16.

Since $V(T) = E(T^2) - (E(T))^2$, the variance of the time to recruitment can be calculated from (24) and (25).

Suppose $f(t) = f_{u(k)}(t)$ .Then

$$E(T) = P_3 + p\,(p_1p_2P_{12} + p_1q_2P_{13} + p_2q_1P_{15} + q_1q_2P_{16} - p_1p_2Q_{3,12} - p_1q_2Q_{3,13} - p_2q_1Q_{3,15} - q_1q_2Q_{3,16})$$

(26)

$$E(T^2) = 2\left(P_3{}^2 + p\left(p_1p_2P_{12}{}^2 + p_1q_2P_{13}{}^2 + p_2q_1P_{15}{}^2 + q_1q_2P_{16}{}^2 - p_1p_2Q_{3,12}{}^2 - p_1q_2Q_{3,13}{}^2 - p_2q_1Q_{3,15}{}^2 - q_1q_2Q_{3,16}{}^2\right)\right) - \frac{1}{\lambda^2}(N^2 - M)\left[\left(\frac{1}{1-D_3}\right) + p\left(\frac{p_1p_2}{1-D_{12}} + \frac{p_1q_2}{1-D_{13}} + \frac{p_2q_1}{1-D_{15}} + \frac{q_1q_2}{1-D_{16}} - \frac{p_1p_2}{1-D_3D_{12}} - \frac{p_1q_2}{1-D_3D_{13}} - \frac{p_2q_1}{1-D_3D_{15}} - \frac{q_1q_2}{1-D_3D_{16}}\right)\right] \quad (27)$$

where $P_a, Q_{b,d}$ are given by equation (17) for a=3,12,13,15,16 ,b=3 and d=12,13,15,16.

Since $V(T) = E(T^2) - (E(T))^2$, the variance of the time to recruitment can be calculated from (26) and (27).

Case (ii): If the distribution of optional thresholds follow extended exponential distribution and the distribution of mandatory thresholds possess SCBZ property .

Assume $f(t) = f_{u(1)}(t)$ .Then

$$E(T) = 4C_3 - 2C_4 - 2C_5 + C_8 + p(p_1p_2C_{12} - 4p_1p_2H_{3,12} + 2p_1p_2H_{4,12} + 2p_1p_2H_{5,12} - p_1p_2H_{8,12} + q_1p_2C_{15} - 4q_1p_2H_{3,15} + 2q_1p_2H_{4,15} + 2q_1p_2H_{5,15} - q_1p_2H_{8,15} + q_2p_1C_{13} - 4q_2p_1H_{3,13} + 2q_2p_1H_{4,13} + 2q_2p_1H_{5,13} - q_2p_1H_{8,13} + q_1q_2C_{16} - 4q_1q_2H_{3,16} + 2q_1q_2H_{4,16} + 2q_1q_2H_{5,16} - q_1q_2H_{8,16}) \quad (28)$$

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$$E(T^2) = 2\big(4C_3{}^2 - 2C_4{}^2 - 2C_5{}^2 + C_8{}^2 + p\big(p_1p_2C_{12}{}^2 - 4p_1p_2H_{3,12}{}^2 + 2p_1p_2H_{4,12}{}^2 + 2p_1p_2H_{5,12}{}^2 - p_1p_2H_{8,12}{}^2 + q_1p_2C_{15}{}^2 - 4q_1p_2H_{3,15}{}^2 + 2q_1p_2H_{4,15}{}^2 + 2q_1p_2H_{5,15}{}^2 - q_1p_2H_{8,15}{}^2 + q_2p_1C_{13}{}^2 - 4q_2p_1H_{3,13}{}^2 + 2q_2p_1H_{4,13}{}^2 + 2q_2p_1H_{5,13}{}^2 - q_2p_1H_{8,13}{}^2 + q_1q_2C_{16}{}^2 - 4q_1q_2H_{3,16}{}^2 + 2q_1q_2H_{4,16}{}^2 + 2q_1q_2H_{5,16}{}^2 - q_1q_2H_{8,16}{}^2\big)\big) \quad (29)$$

Since $V(T) = E(T^2) - \big(E(T)\big)^2$, the variance of the time to recruitment can be calculated from (28) and (29).

Assume $f(t) = f_{u(k)}(t)$ .Then

$$E(T) = 4P_3 - 2P_4 - 2P_5 + P_8 + p(p_1p_2P_{12} - 4p_1p_2Q_{3,12} + 2p_1p_2Q_{4,12} + 2p_1p_2Q_{5,12} - p_1p_2Q_{8,12} + q_1p_2P_{15} - 4q_1p_2Q_{3,15} + 2q_1p_2Q_{4,15} + 2q_1p_2Q_{5,15} - q_1p_2Q_{8,15} + q_2p_1P_{13} - 4q_2p_1Q_{3,13} + 2q_2p_1Q_{4,13} + 2q_2p_1Q_{5,13} - q_2p_1Q_{8,13} + q_1q_2P_{16} - 4q_1q_2Q_{3,16} + 2q_1q_2Q_{4,16} + 2q_1q_2Q_{5,16} - q_1q_2Q_{8,16}) \quad (30)$$

$$E(T^2) = 2\big(4P_3{}^2 - 2P_4{}^2 - 2P_5{}^2 + P_8{}^2 + p\big(p_1p_2P_{12}{}^2 - 4p_1p_2Q_{3,12}{}^2 + 2p_1p_2Q_{4,12}{}^2 + 2p_1p_2Q_{5,12}{}^2 - p_1p_2Q_{8,12}{}^2 + q_1p_2P_{15}{}^2 - 4q_1p_2Q_{3,15}{}^2 + 2q_1p_2Q_{4,15}{}^2 + 2q_1p_2Q_{5,15}{}^2 - q_1p_2Q_{8,15}{}^2 + q_2p_1P_{13}{}^2 - 4q_2p_1Q_{3,13}{}^2 + 2q_2p_1Q_{4,13}{}^2 + 2q_2p_1Q_{5,13}{}^2 - q_2p_1Q_{8,13}{}^2 + q_1q_2P_{16}{}^2 - 4q_1q_2Q_{3,16}{}^2 + 2q_1q_2Q_{4,16}{}^2 + 2q_1q_2Q_{5,16}{}^2 - q_1q_2Q_{8,16}{}^2\big)\big) - \frac{1}{\lambda^2}(N^2 - M)\left(\frac{4}{1-D_3} - \frac{2}{1-D_4} - \frac{2}{1-D_5} + \frac{1}{1-D_8} + p\left[\frac{p_1p_2}{1-D_{12}} - \frac{4p_1p_2}{1-D_3D_{12}} + \frac{2p_1p_2}{1-D_4D_{12}} + \frac{2p_1p_2}{1-D_5D_{12}} - \frac{p_1p_2}{1-D_8D_{12}} + \frac{q_1p_2}{1-D_{15}} - \frac{4q_1p_2}{1-D_3D_{15}} + \frac{2q_1p_2}{1-D_4D_{15}} + \frac{2q_1p_2}{1-D_5D_{15}} - \frac{q_1p_2}{1-D_8D_{15}} + \frac{q_2p_1}{1-D_{13}} - \frac{4q_2p_1}{1-D_3D_{13}} + \frac{2q_2p_1}{1-D_4D_{13}} + \frac{2q_2p_1}{1-D_5D_{13}} - \frac{q_2p_1}{1-D_8D_{13}} + \frac{q_1q_2}{1-D_{16}} - \frac{4q_1q_2}{1-D_3D_{16}} + \frac{2q_1q_2}{1-D_4D_{16}} + \frac{2q_1q_2}{1-D_5D_{16}} - \frac{q_1q_2}{1-D_8D_{16}}\right]\right) \quad (31)$$

4. Model description And Analysis of Model-III

For this model, the optional and mandatory thresholds for the loss of man-hours in the organization are taken as $Y=Y_1+Y_2$ and $Z=Z_1+ Z_2$ .All the other assumptions and notations are as in model-I.

Proceeding as in model-I, it can be shown for the present model that

Case (i): If the distribution of optional thresholds follow exponential distribution and the distribution of mandatory thresholds possess SCBZ property

Suppose $f(t) = f_{u(1)}(t)$ .Then

$$E(T) = A_2C_2 - A_1C_1 + p[(A_3 + A_4)C_9 + (A_5 + A_6)C_{10} - (A_7 + A_8)C_{11} - (A_9 + A_{10})C_{14} - A_2(A_3 + A_4)H_{2,9} - A_2(A_5 + A_6)H_{2,10} + A_2(A_7 + A_8)H_{2,11} + A_2(A_9 + A_{10})H_{2,14} + A_1(A_3 + A_4)H_{1,9} + A_1(A_5 + A_6)H_{1,10} - A_1(A_7 + A_8)H_{1,11} - A_1(A_9 + A_{10})H_{1,14}] \quad (32)$$

$$E(T^2) = 2\big(A_2{C_2}^2 - A_1{C_1}^2 + p\big[(A_3 + A_4){C_9}^2 + (A_5 + A_6){C_{10}}^2 - (A_7 + A_8){C_{11}}^2 - (A_9 + A_{10}){C_{14}}^2 - A_2(A_3 + A_4){H_{2,9}}^2 - A_2(A_5 + A_6){H_{2,10}}^2 + A_2(A_7 + A_8){H_{2,11}}^2 + A_2(A_9 + A_{10}){H_{2,14}}^2 + A_1(A_3 + A_4){H_{1,9}}^2 + A_1(A_5 + A_6){H_{1,10}}^2 - A_1(A_7 + A_8){H_{1,11}}^2 - A_1(A_9 + A_{10}){H_{1,14}}^2\big]\big) \quad (33)$$

Since $V(T) = E(T^2) - \big(E(T)\big)^2$, the variance of the time to recruitment can be calculated from (32) and (33).

Assume $f(t) = f_{u(k)}(t)$ .Then

$$E(T) = 2[A_2P_2 - A_1P_1 + p[(A_3 + A_4)P_9 + (A_5 + A_6)P_{10} - (A_7 + A_8)P_{11} - (A_9 + A_{10})P_{14} - A_2(A_3 + A_4)Q_{2,9} - A_2(A_5 + A_6)Q_{2,10} + A_2(A_7 + A_8)Q_{2,11} + A_2(A_9 + A_{10})Q_{2,14} + A_1(A_3 + A_4)Q_{1,9} + A_1(A_5 + A_6)Q_{1,10} - A_1(A_7 + A_8)Q_{1,11} - A_1(A_9 + A_{10})Q_{1,14}]] \quad (34)$$

$$E(T^2) = 2\big[A_2{P_2}^2 - A_1{P_1}^2 + p\big[(A_3 + A_4){P_9}^2 + (A_5 + A_6){P_{10}}^2 - (A_7 + A_8){P_{11}}^2 - (A_9 + A_{10}){P_{14}}^2 - A_2(A_3 + A_4){Q_{2,9}}^2 - A_2(A_5 + A_6){Q_{2,10}}^2 + A_2(A_7 + A_8){Q_{2,11}}^2 + A_2(A_9 + A_{10}){Q_{2,14}}^2 + A_1(A_3 + A_4){Q_{1,9}}^2 + A_1(A_5 + A_6){Q_{1,10}}^2 - A_1(A_7 + A_8){Q_{1,11}}^2 - A_1(A_9 + A_{10}){Q_{1,14}}^2\big]\big] - \frac{1}{\lambda^2}(N^2 - M)\Bigg(\frac{A_2}{1 - D_2} - \frac{A_1}{1 - D_1} + p\Bigg(\frac{(A_3 + A_4)}{1 - D_9} + \frac{(A_5 + A_6)}{1 - D_{10}} - \frac{(A_7 + A_8)}{1 - D_{11}} - \frac{(A_9 + A_{10})}{1 - D_{14}} - \frac{A_2(A_3 + A_4)}{1 - D_2D_9} - \frac{A_2(A_5 + A_6)}{1 - D_2D_{10}} + \frac{A_2(A_7 + A_8)}{1 - D_2D_{11}} + \frac{A_2(A_9 + A_{10})}{1 - D_2D_{14}} + \frac{A_1(A_3 + A_4)}{1 - D_1D_9} + \frac{A_1(A_5 + A_6)}{1 - D_1D_{10}} - \frac{A_1(A_7 + A_8)}{1 - D_1D_{11}} - \frac{A_1(A_9 + A_{10})}{1 - D_1D_{14}}\Bigg)\Bigg) \quad (35)$$

$A_3 = \frac{(\delta_3+\mu_3)p_1p_2}{(\delta_3+\mu_3-\delta_4-\mu_4)}, A_4 = \frac{\eta_3p_2q_1}{(\eta_3-\delta_4-\mu_4)} A_5 = \frac{(\delta_3+\mu_3)p_1q_2}{(\delta_3+\mu_3-\eta_4)}, A_6 = \frac{\eta_3q_1q_2}{(\eta_3-\eta_4)}$

$A_7 = \frac{(\delta_4+\mu_4)p_1p_2}{(\delta_3+\mu_3-\delta_4-\mu_4)}, A_8 = \frac{\eta_4p_1q_2}{(\delta_3+\mu_3-\eta_4)}, A_9 = \frac{(\delta_4+\mu_4)q_1p_2}{(\eta_3-\delta_4-\mu_4)}, A_{10} = \frac{\eta_4q_1q_2}{(\eta_3-\eta_4)}$

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Since $V(T) = E(T^2) - \left(E(T)\right)^2$, the variance of the time to recruitment can be calculated from (34) and (35).

Case (ii): If the distribution of optional thresholds follow extended exponential distribution and the distribution of mandatory thresholds possess SCBZ property

Assume $f(t) = f_{u(1)}(t)$ .Then

$$E(T) = S_1C_1 + S_3C_2 - S_4C_7 - S_2C_6 + p[(A_3 + A_4)C_9 + (A_5 + A_6)C_{10} - (A_7 + A_8)C_{11} - (A_9 + A_{10})C_{14} - S_1(A_3 + A_4)H_{1,9} - S_1(A_5 + A_6)H_{1,10} + S_1(A_7 + A_8)H_{1,11} + S_1(A_9 + A_{10})H_{1,14} - S_3(A_3 + A_4)H_{2,9} - S_3(A_5 + A_6)H_{2,10} + S_3(A_7 + A_8)H_{2,11} + S_3(A_9 + A_{10})H_{2,14} + S_4(A_3 + A_4)H_{7,9} + S_4(A_5 + A_6)H_{7,10} - S_4(A_7 + A_8)H_{7,11} - S_4(A_9 + A_{10})H_{7,14} + S_2(A_3 + A_4)H_{6,9} + S_2(A_5 + A_6)H_{6,10} - S_2(A_7 + A_8)H_{6,11} - S_2(A_9 + A_{10})H_{6,14}] \quad (36)$$

$$E(T^2) = 2\Big[S_1C_1{}^2 + S_3C_2{}^2 - S_4C_7{}^2 - S_2C_6{}^2 + p\big[(A_3 + A_4)C_9{}^2 + (A_5 + A_6)C_{10}{}^2 - (A_7 + A_8)C_{11}{}^2 - (A_9 + A_{10})C_{14}{}^2 - S_1(A_3 + A_4)H_{1,9}{}^2 - S_1(A_5 + A_6)H_{1,10}{}^2 + S_1(A_7 + A_8)H_{1,11}{}^2 + S_1(A_9 + A_{10})H_{1,14}{}^2 - S_3(A_3 + A_4)H_{2,9}{}^2 - S_3(A_5 + A_6)H_{2,10}{}^2 + S_3(A_7 + A_8)H_{2,11}{}^2 + S_3(A_9 + A_{10})H_{2,14}{}^2 + S_4(A_3 + A_4)H_{7,9}{}^2 + S_4(A_5 + A_6)H_{7,10}{}^2 - S_4(A_7 + A_8)H_{7,11}{}^2 - S_4(A_9 + A_{10})H_{7,14}{}^2 + S_2(A_3 + A_4)H_{6,9}{}^2 + S_2(A_5 + A_6)H_{6,10}{}^2 - S_2(A_7 + A_8)H_{6,11}{}^2 - S_2(A_9 + A_{10})H_{6,14}{}^2\big]\Big] \quad (37)$$

where $S_1 = \frac{4\theta_2{}^2}{(\theta_1-\theta_2)(\theta_1-2\theta_2)}$, $S_2 = \frac{\theta_2{}^2}{(\theta_1-\theta_2)(2\theta_1-\theta_2)}$, $S_1 = \frac{4\theta_1{}^2}{(\theta_1-\theta_2)(2\theta_1-\theta_2)}$ $and\ S_4 = \frac{\theta_1{}^2}{(\theta_1-\theta_2)(\theta_1-2\theta_2)}$

Suppose $f(t) = f_{u(k)}(t)$ .Then

$$E(T) = S_1P_1 + S_3P_2 - S_4P_7 - S_2P_6 + p[(A_3 + A_4)P_9 + (A_5 + A_6)P_{10} - (A_7 + A_8)P_{11} - (A_9 + A_{10})P_{14} - S_1(A_3 + A_4)Q_{1,9} - S_1(A_5 + A_6)Q_{1,10} + S_1(A_7 + A_8)Q_{1,11} + S_1(A_9 + A_{10})Q_{1,14} - S_3(A_3 + A_4)Q_{2,9} - S_3(A_5 + A_6)Q_{2,10} + S_3(A_7 + A_8)Q_{2,11} + S_3(A_9 + A_{10})Q_{2,14} + S_4(A_3 + A_4)Q_{7,9} + S_4(A_5 + A_6)Q_{7,10} - S_4(A_7 + A_8)Q_{7,11} - S_4(A_9 + A_{10})Q_{7,14} + S_2(A_3 + A_4)Q_{6,9} + S_2(A_5 + A_6)Q_{6,10} - S_2(A_7 + A_8)Q_{6,11} - S_2(A_9 + A_{10})Q_{6,14}] \quad (39)$$

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$$
\begin{aligned}
E(T^2) = 2\Big[&S_1P_1{}^2 + S_3P_2{}^2 - S_4P_7{}^2 - S_2P_6{}^2 + p\big[(A_3 + A_4)P_9{}^2 + (A_5 + A_6)P_{10}{}^2 \\
&- (A_7 + A_8)P_{11}{}^2 - (A_9 + A_{10})P_{14}{}^2 - S_1(A_3 + A_4)Q_{1,9}{}^2 - S_1(A_5 + A_6)Q_{1,10}{}^2 \\
&+ S_1(A_7 + A_8)Q_{1,11}{}^2 + S_1(A_9 + A_{10})Q_{1,14}{}^2 \\
&- S_3(A_3 + A_4)Q_{2,9}{}^2 - S_3(A_5 + A_6)Q_{2,10}{}^2 + S_3(A_7 + A_8)Q_{2,11}{}^2 \\
&+ S_3(A_9 + A_{10})Q_{2,14}{}^2 + S_4(A_3 + A_4)Q_{7,9}{}^2 + S_4(A_5 + A_6)Q_{7,10}{}^2 \\
&- S_4(A_7 + A_8)Q_{7,11}{}^2 - S_4(A_9 + A_{10})Q_{7,14}{}^2 \\
&+ S_2(A_3 + A_4)Q_{6,9}{}^2 + S_2(A_5 + A_6)Q_{6,10}{}^2 - S_2(A_7 + A_8)Q_{6,11}{}^2 \\
&- S_2(A_9 + A_{10})Q_{6,14}{}^2\big]\Big] \\
&- \frac{1}{\lambda^2}(N^2 - M)\Bigg(\left(\frac{S_1}{1 - D_1} + \frac{S_3}{1 - D_2} - \frac{S_4}{1 - D_7} - \frac{S_2}{1 - D_6}\right) \\
&+ p\Bigg(\frac{(A_3 + A_4)}{1 - D_9} + \frac{(A_5 + A_6)}{1 - D_{10}} - \frac{(A_7 + A_8)}{1 - D_{11}} - \frac{(A_9 + A_{10})}{1 - D_{14}} - \frac{S_1(A_3 + A_4)}{1 - D_1D_9} \\
&- \frac{S_1(A_5 + A_6)}{1 - D_1D_{10}} + \frac{S_1(A_7 + A_8)}{1 - D_1D_{11}} + \frac{S_1(A_9 + A_{10})}{1 - D_1D_{14}} - \frac{S_3(A_3 + A_4)}{1 - D_2D_9} - \frac{S_3(A_5 + A_6)}{1 - D_2D_{10}} \\
&+ \frac{S_3(A_7 + A_8)}{1 - D_2D_{11}} + \frac{S_3(A_9 + A_{10})}{1 - D_2D_{14}} + \frac{S_4(A_3 + A_4)}{1 - D_7D_9} + \frac{S_4(A_5 + A_6)}{1 - D_7D_{10}} - \frac{S_4(A_7 + A_8)}{1 - D_7D_{11}} \\
&- \frac{S_4(A_9 + A_{10})}{1 - D_7D_{14}} + \frac{S_2(A_3 + A_4)}{1 - D_6D_9} + \frac{S_2(A_5 + A_6)}{1 - D_6D_{10}} \\
&- \frac{S_2(A_7 + A_8)}{1 - D_6D_{11}} - \frac{S_2(A_9 + A_{10})}{1 - D_6D_{14}}\Bigg)\Bigg) \qquad (40)
\end{aligned}
$$

Since $V(T) = E(T^2) - \big(E(T)\big)^2$, the variance of the time to recruitment can be calculated from (39) and (40).

5. Numerical Illustrations

The mean and variance of the time to recruitment for the above models are given in the following tables for the cases (i),(ii) respectively by keeping p fixed and varying c, k, and λ one at a time and the results are tabulated below.

Table I: **(Effect of c, k, λ on the performance measures E (T) and V (T))**
$(\delta_3 = 0.3\ ,\eta_3 = 0.4\ ,\delta_3 = 0.5\ \ \eta_4 = 0.6,\ \theta_1 = 0.1\ ,\theta_2 = 0.2, \mu_3 = 0.9, \mu_4 = 1\ and\ p = 0.2)$

MODEL-I			
Case –I		Case –II	
$f(t) = f_{u(1)}(t)$	$f(t) = f_{u(k)}(t)$	$f(t) = f_{u(1)}(t)$	$f(t) = f_{u(k)}(t)$

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k	c	λ	E(T)	V(T)	E(T)	V(T)	E(T)	V(T)	E(T)	V(T)
1	0.5	1	6.9061	35.0660	6.9061	35.0660	9.2829	45.1911	9.2829	45.1911
2	0.5	1	3.4531	8.7665	10.3592	71.9924	4.6414	11.2978	13.9243	92.3971
3	0.5	1	2.3020	3.8962	12.6610	104.0442	3.0943	5.0212	17.0183	133.3211
2	0.5	1	3.4531	8.7665	10.3592	71.9924	4.6414	11.2978	13.9243	92.3971
2	1	1	6.3890	28.6330	19.1670	244.9191	8.7710	36.2740	26.3129	308.924
2	1.5	1	9.3244	59.8402	27.9732	519.9133	12.9008	75.1827	38.7024	650.8427
2	0.5	1	3.4531	8.7665	10.3592	71.9924	4.6414	11.2978	13.9243	92.3971
2	0.5	2	1.7265	2.1916	5.1796	17.9981	2.3207	2.8244	6.9622	23.0993
2	0.5	3	1.1510	0.9741	3.4531	7.9992	1.5471	1.2553	4.6414	10.2663

Table II: **(Effect of c, k, λ on the performance measures E (T) and V (T))**

$(\delta_3 = 0.3\ , \eta_3 = 0.4\ , \delta_3 = 0.5\ \ \eta_4 = 0.6,\ \theta_1 = 0.1\ , \theta_2 = 0.2, \mu_3 = 0.9, \mu_4 = 1\ and\ p = 0.2)$

MODEL-II										
Case –I							Case –II			
$f(t) = f_{u(1)}(t)$					$f(t) = f_{u(k)}(t)$		$f(t) = f_{u(1)}(t)$		$f(t) = f_{u(k)}(t)$	
k	C	λ	E(T)	V(T)	E(T)	V(T)	E(T)	V(T)	E(T)	V(T)
1	0.5	1	2.7199	8.2841	2.7199	8.2841	4.0280	11.3073	4.0280	11.3073
2	0.5	1	1.36	2.0710	4.0799	16.2882	2.0140	2.8268	6.0420	21.4135

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3	0.5	1	0.9066	0.9205	4.9864	23.1407	1.3427	1.2564	7.3845	29.9479
2	0.5	1	1.36	2.071	4.0799	16.2882	2.0140	2.8268	6.0420	21.4135
2	1	1	2.2092	5.2838	6.6275	43.6284	3.5180	7.5622	10.5541	61.0240
2	1.5	1	3.0570	9.9531	9.1709	84.0813	5.0216	14.4464	15.0649	120.1542
2	0.5	1	1.3600	2.071	4.0799	16.2882	2.0140	2.8268	6.0420	21.4135
2	0.5	2	0.6800	0.5178	2.0399	4.0721	1.007	0.7067	3.0210	5.3534
2	0.5	3	0.4533	0.2301	1.36	1.8098	0.6713	0.3141	2.0140	2.3793

Table III: **(Effect of c, k, λ on the performance measures E (T) and V (T))**
Case (i) ($\delta_3 = 0.3$,$\eta_3 = 0.4$,$\delta_3 = 0.5$ $\eta_4 = 0.6,$ $\theta_1 = 0.1$,$\theta_2 = 0.2, \mu_3 = 0.9, \mu_4 = 1$ *and* $p =$ 0.2)Case(ii):
($\delta_3 = 0.3$,$\eta_3 = 0.4$,$\delta_3 = 0.5$ $\eta_4 = 0.6,$ $\theta_1 = 0.6$,$\theta_2 = 0.4, \mu_3 = 0.9, \mu_4 = 1$ *and* $p =$ 0.2)

MODEL-III										
Case –I							Case –II			
$f(t) = f_{u(1)}(t)$					$f(t) = f_{u(k)}(t)$		$f(t) = f_{u(1)}(t)$		$f(t) = f_{u(k)}(t)$	
k	λ	C	E(T)	V(T)	E(T)	V(T)	E(T)	V(T)	E(T)	V(T)
1	0.5	1	8.5803	46.7284	8.5803	46.7284	4.2902	10.1460	4.2902	10.1460
2	0.5	1	4.2902	11.6821	12.8705	96.5595	2.1451	2.5365	6.4353	18.5383
3	0.5	1	2.8601	5.1920	15.7303	139.8948	1.4301	1.1273	7.8652	25.5202
2	0.5	1	4.2902	11.6821	12.8705	96.5595	2.1451	2.5365	6.4353	18.5383

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2	1	1	8.0623	38.5433	24.1870	330.7669	3.7607	6.2420	11.2821	48.6562
2	1.5	1	11.8341	80.8518	35.5024	704.0014	5.3742	11.3673	16.1226	91.5574
2	0.5	1	4.2902	11.6821	12.8705	96.5595	2.1451	2.5365	6.4353	18.5383
2	0.5	2	2.1451	2.9205	6.4353	24.1399	1.0726	0.6341	3.2177	4.6346
2	0.5	3	1.4301	1.2980	4.2902	10.7288	0.7150	0.2818	2.1451	2.0598

FINDINGS

The influence of nodal parameters on the performance measures namely mean and variance of the time to recruitment for all the models are reported below.

i. It is observed that if k, the number of decision epochs in (0, t] increases, the mean and variance of the time to recruitment of all these models decreases ,when the probability density function of inter-decision times follows probability density function of first order statistics and increases when it follows probability density function of k-th order statistics .

ii. If c increases, the average number of exits increases, which, in turn, implies that mean and variance of the time to recruitment increase for both the models.

iii. As λ increases, the average inter-decision time decreases, which, in turn, shows that frequent decisions are made on average and hence mean and variance of the time to recruitment decrease for both the models.

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