Experiencing a Flipped Mathematics Class

by

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Abstract

The flipped classroom is an old concept that has recently been redefined through the emergence of new technologies that allow teachers to deliver content out of class time. As such, various approaches for implementation exist. Most prominently, teachers are able to use class time in a student-centered manner, which allows students to experience the classroom in diverse ways. This study focuses on describing these experiences in a particular flipped adult upgrading mathematics class. Students in the class are surveyed and interviewed about their experiences in relation to autonomy, goals, self-efficacy, and anxiety. It is found that students can bifurcate into engaging in the class completely and engaging in it in a self-paced manner. Key interrelated factors in this bifurcation include election of cognitive autonomy, goal orientation, and attendance. This study also indicates that self-efficacy can be improved within a collaborative learning environment that provides students with autonomy.

Keywords: flipped classroom; autonomy; goals; self-efficacy; anxiety; classroom experience

For my husband,

who has supported me in pursuing my dreams

and has tirelessly put up with my desire to hide

in my office while writing this thesis.

For my parents,

who dedicated their lives to educating me.

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1. Introduction

In 1926, John Dewey asked, “Why is it, in spite of the fact that teaching by pouring in and learning by passive absorption, are universally condemned, that they are still so entrenched in practice?” (p. 46). Educators’ primary mode of content delivery is still through teacher-centered lecture, which is considered “an inefficient way of encouraging learning” (Penson, 2012, p. 72). Elen, Clarebout, Léonard, & Lowyck (2007) note that “a teacher-centered learning environment . . . is said to discourage students from adopting a deep approach to study” (p. 105) and claim that evidence is found in recent literature supporting the desirability of student-centered learning environments. Implementing a successful student-centered learning environment can be challenging.

As an undergraduate mathematics student at the University of the Fraser Valley (UFV), I often found it difficult to completely understand material when it was presented in a teacher-centered lecture format. The deeper connections among mathematical topics that I made occurred when I studied the material outside of class time either at home individually or in discussion with other mathematics students at UFV’s Math Center. I also found the extensive hours of high school level private mathematics tutoring that I provided during my undergraduate degree in mathematics extremely beneficial in developing my own deeper understanding of mathematics. Tutoring motivated me to become engaged in making deeper connections in mathematical material so that I could help others understand topics more effectively. Learning undergraduate level mathematics such as calculus, linear algebra, real analysis, modern algebra, etc. during my mathematics degree gave me a new lens with which to view the high school curriculum. I began to understand the reasoning behind the various high school mathematics topics that I was previously unaware of. I also developed an awareness of the connections between various high school mathematics topics that had previously seemed unrelated and disconnected such as sets and functions. Through tutoring, I found that students retained material more often when I provided them with reasoning and mathematical background for the topics they were learning. Most prominently, my own growth in mathematical understanding and engagement didn't occur in a teacher-centered classroom, but rather in a setting where I was in the role of a facilitator.

After completing my undergraduate degree in mathematics, I continued my role as a tutor due to my enjoyment of the engagement I experienced in helping students learn mathematics. I was soon hired by UFV as a mathematics upgrading instructor in the Upgrading and University Preparation (UUP) department. I have now taught various mathematics upgrading courses in this department for four years. The student population that I teach consists of adult students who are returning to learning mathematics because they need a certain level of high school mathematics as a prerequisite for a program path of their choice. Mathematics courses in this department are either taught in a multilevel format or in a single level format. The multilevel format can contain up to six levels of students in one room where students tend to learn material individually. The single level format is designated for advanced and provincial level courses where content is typically delivered through periods of lecture and periods of practice. These courses are predominantly delivered in a teacher-centered manner.

Up until entering this Master's degree program, I taught these advanced level courses predominantly in a teacher-centered manner. I tried to be engaging, but somehow I always seemed to encounter a sea of blank faces staring back at me by the end of a lesson. During a lecture, I found that only some of my students were engaged. The others were either bored because they already understood the material or behind with the material and only in class to copy down notes. I always felt as though I was doing the unengaged students a disservice when I set the pace according to the engaged group of students. However, even those who were engaged during class time often came to tell me that although they grasped the material in class, they became confused and frustrated with the material at home. This reflects the notion that engagement does not necessarily imply understanding.

As I embarked on my journey through this Master's degree program, I began to develop ideas for creating an engaging student-centered learning environment. However, I struggled with incorporating these ideas without compromising delivery of the curriculum. Wang (2011) reasons that "time is a crucial factor for teachers' pedagogical decisions" (p. 157). He further notes that "student-centered teaching tends to be more time-consuming and unpredictable than whole-class lecturing" and that "teachers working under a fixed curriculum and schedule are inclined to organize the class in a more teacher-centered manner to secure the completion of required tasks" (Wang, 2011, p. 157). The challenge of removing this time constraint in order to provide my students with more ownership over their learning in the hopes of developing their understanding led me to consider the implementation of a flipped classroom model because it allowed for the delivery of content out of class time, leaving class time dedicated to various student-centered approaches to learning.

2. Related Literature

The idea of a flipped classroom is not new, but it is recently gaining more attention in field of mathematics education. As such, I first outline the existing literature in the emergent field of research related to flipped classrooms. As noted above, my initial motivation for implementing a flipped classroom was the potential affordance it had on the development of student understanding. However, as further clarified in Section 2.8 Research Questions, this research does not focus on how students developed understanding in the flipped classroom, but rather how their experiences through the presence of autonomy were influenced by factors such as goals, self-efficacy, and anxiety in relation to the learning of mathematics. Nonetheless, literature on understanding is reviewed for the purposes of building clarity to the context in which the flipped classroom was implemented as well as forming the basis of terminology used in the analysis of the classroom setting. Literature on autonomy is also discussed due to its instrumental role in analyzing the flipped classroom context and the diverse ways in which students used it to engage in the flipped classroom. Finally, in order to more thoroughly address the key interest of this study, which relates to diverse student experiences of the flipped classroom, literature on goals, self-efficacy, and anxiety is reviewed and situated in the field of mathematics education.

2.1. Flipped Classrooms

Bergmann & Sams (2012) have most often been credited with coining the phrase ‘flipped classroom,’ which refers to “the use of technology to remove passive, one-way lecturing as the only means of teaching” (para. 6). Video lectures watched at home and activities completed during class time are the defining characteristics of a flipped classroom, which is also referred to as the ‘inverted classroom’ (Lage, Platt, & Treglia, 2000; Lage & Platt, 2000). However, Johnson (2013) reminded us in his recent study that the flipped classroom “should be viewed as a mindset rather than a pedagogy” and that “educators are continuing to experiment with the flipped classroom strategies to meet their curricular needs” (p. 76). The concept of reversing content delivery and practice time is not a new phenomenon in education, but it is recently being redefined and improved along with the emergence of new technologies (Kachka, 2012). Kachka (2012) noted that "the increase of teacher-student interaction during class time is what characterizes [the flipped classroom model's] success" (para. 6). Similarly, Bergmann and Sams (2012) claimed that "redirecting attention away from the teacher and putting attention on the learner and the learning" is the most important feature of the flipped classroom (Chapter 1, para. 2). Teachers are capitalizing on the affordances created by technology to deliver content specific videos available for asynchronous viewing online in order to dedicate classroom time for student-centered and inquiry-based learning (Bergmann & Sams, 2008). Students are provided with opportunities to engage in content prior to class time as well as opportunities to engage in learning activities during class time. Teachers can become facilitators of learning by directing engaging classroom activities and guiding students to participate in a collaborative classroom environment. Collaboration is an important component of a student-centered learning environment, and is therefore a key element of many flipped classrooms. Most importantly, the flipped classroom model provides an opportunity to create a classroom that promotes understanding.

The flipped classroom model has recently gained a lot of attention in the media due to the increase in accessibility to technology that can be used for its purpose. Media outlets such as USA Today (Toppo, 2011), Washington Post (Strauss, 2012), and CNN (Green, 2012) have covered experiences and opinions regarding the flipped classroom. However, research based literature pertaining to the flipped classroom is still limited, especially in a context of mathematics education (Johnson, 2013). Several studies report increased student achievement under the implementation of a flipped classroom model (Day & Foley, 2006; Green, 2011; Johnson, 2013; Kirch, 2012; Mussallam, 2010), but few of them relate specifically to mathematics education. The most notable studies conducted within a mathematics education context in relation to the flipped classroom model have focused on student perceptions of the flipped classroom model in both an undergraduate level statistics course (Strayer, 2008) and a set of high school level mathematics classes (Johnson, 2013). Strayer (2008) compared student responses from a flipped classroom version of his statistics course with that of a traditional classroom version and noted that students in a flipped classroom can experience higher levels of innovation and cooperation than those in a traditional classroom. However, students in a flipped classroom can experience feelings of unsettledness due to the unpredictability of class time and the variety of learning activities, an experience that students in the traditional classroom do not generally encounter. Strayer (2008) also discovered that students find a flipped classroom model difficult to accustom to. In contrast, Johnson (2013) studied student perceptions of a flipped classroom model designed towards mastery learning, student-centered learning, and self-pacing in a high school mathematics context. He found that students value his version of the flipped classroom model due to the ability to use time more efficiently and flexibly, the enjoyment of learning that is afforded through classroom learning activities, the frequent interaction with their teacher and peers, and the reduction of time spent on homework out of class time (Johnson, 2013). Johnson (2013) discovered that students learning under a flipped classroom model showed an increase in their perceptions of their own engagement, communication, and understanding. These two studies provide evidence of varying and almost contradictory results, which may be due to various methods of implementation.

As stated earlier, the flipped classroom is a mindset rather than a pedagogy, which allows teachers to implement it according to their perceptions of a successful learning environment. In this study, the flipped classroom, which is described more thoroughly in Section 3.3 Setting: Math 084 Flipped, was designed according to the goal of developing student understanding through the provision of engaging opportunities for collaboration and exploration in mathematics.

2.2. Understanding

As already mentioned, the initial motivation for implementing a flipped classroom in this study was to establish an environment conducive to building student understanding. Although this is not the focus of this research, literature on understanding is reviewed for the purposes of providing a framework for analyzing the context of the flipped classroom environment.

The notion of teaching and learning for understanding has long been present, but has not always been of precedence. Early in the 20th century, philosopher John Dewey cautioned against teaching through mindless repetition of procedures without development of understanding because doing so would damage students' ability to have intelligent consideration for what they were doing (Dewey, 1910, cited in Hiebert, 1997). During the 1930's and 1940's, theories of learning with meaning developed along with reasons for learning with meaning such as "assurance of retention," increased "likelihood that arithmetical ideas and skills will be used," prevention of "answers that are mathematically absurd," development of independence and confidence in new quantitative situations, and consideration of mathematics as a subject "worthy of respect" (Brownell, 1947, pp. 263-264).

The notion of understanding was further classified by Skemp (1976) during an era of behaviourist dominance and measurable objectives into two dimensions: *relational understanding* and *instrumental understanding*. Skemp (1976) defined relational understanding as knowing the reasoning behind what one is doing and instrumental understanding as simply knowing how to do it. Skemp (1976) also noted that although instrumental understanding can be more immediately rewarding, relational understanding is more adaptable and removes the need for extrinsic motivators.

In the early 1990's, the *Standards* were developed to emphasize relational understanding in the teaching and learning of mathematics (National Council of Teachers of Mathematics, 1989, 1991, 1995). Despite this call for understanding, mathematics is still often viewed as a set of isolated facts and skills that are learned in school and are assessed according to instrumental understanding with little concern for transferability and applicabilty. It is relational understanding that is of particular interest in this study.

It should be noted that Sierpinska (1990) further distinguished understanding as an act that is often influenced by overcoming epistemological obstacles and manifested through explaining. Sierpinska (1990) distinguished her definition of understanding from Skemp's (1976) by claiming that "Skemp classifies acts of understanding according to the styles of knowing they produce" (p. 27) whereas she treats it more as "an act involved in a process of interpretation" (p. 26). She also noted that understanding can be a difficult phenomenon to capture. Nonetheless, she provided a framework for evaluating the "depth of understanding" through her methodology for an epistemological analysis of the levels of understanding: identification, discrimination, generalization, and synthesis of ideas (Sierpinska, 1990, p. 35). Given that the analysis of student understanding is not a focus of this research, this framework will not be used to analyze the development of student understanding. Rather, understanding will be noted according to the style of knowing that a student evidences according to Skemp's (1978) classification of relational and procedural understanding.

In this research, understanding serves as the context because the flipped classroom implemented in this study is designed with a focus on developing student understanding. Sierpinska (1990) noted that "understanding of a concept is not normally reached through reading a single text [and] it demands being involved in certain activities, problem situations, dialogues and discussions, and the interpretation of many different texts" (p. 26). This implies a need for student collaboration in the learning environment in order to help students achieve relational understanding. As such, Hiebert's (1997) framework for the examination and facilitation of classroom environments that are conducive to developing understanding is particularly pertinent to this research.

For Hiebert (1997), "understanding is crucial because things learned with understanding can be used flexibly, adapted to new situations, and used to learn new things" (p. 1). Further, those "who work to develop understanding are likely to experience the kind of internal rewards that keep them engaged" and that "understanding breeds confidence" while "not understanding leads to disillusionment" (Hiebert, 1997, p. 2). As such, Hiebert (1997) regarded understanding to be the foundational goal of mathematics education. With this motivation in mind, Hiebert (1997) developed his definition of understanding based on several other definitions of understanding in the context of mathematics education (Carpenter & Lehrer, 1996; Davis, 1992; Pirie & Kieren, 1994; Putnam, Lampert, & Peterson, 1990) as the ability to see how a concept is related or connected to other things one knows. This definition is in alignment with Skemp's (1976) notion of relational understanding. In fact, Hiebert (1999) distinguished between conceptual understanding and procedural skill in the same way that Skemp (1976) differentiated between relational understanding and instrumental understanding, both referring to ways of knowing rather than the process of attaining understanding. In developing his framework for a classroom conducive to promoting understanding, Hiebert (1997) used the term *understanding* in the sense of conceptual or relational understanding. Therefore, every reference to understanding in the rest of this thesis will imply relational understanding.

Hiebert (1997) emphasized that understanding needs to be rooted in the psychological principles of *reflecting* (a principle rooted in cognitive psychology) and *communicating* (a principle rooted in social cognition). Reflection is the process of "conciously thinking about your experiences" and lends itself to the ability to build "relationships between ideas or facts," while communication involves "participating in social interaction, sharing thoughts with others and listening to others share their ideas" and "allows us to challenge each other's ideas and ask for clarification and further explanation" (p. 5). Utilizing reflection and communication together allows for building mathematical connections (Hiebert, 1997). Hiebert (1997) pointed out that if reflection and communication allow for mathematical connections to be made, then a classroom built for developing understanding should contribute to fostering strong reflection and communication practices. Based on these psychological principles, Hiebert (1997) designed a framework consisting of five dimensions.

These are: "[1] the nature of the learning *tasks*, [2] the role of the *teacher*, [3] the social *culture* of the classroom, [4] the kind of mathematical *tools* that are available, and [5] the *accessibility* of mathematics for every student" (Hiebert, 1997, p. 2, numbering added for organizational purposes). These five dimensions work together as a system and are interdependent in the sense that if one dimension is changed, the others will also change.

2.2.1. Nature of the Learning Tasks

Hiebert (1997) stated that "the kinds of tasks that students are asked to perform set the foundation for the system of instruction that is created" (p. 7). In order to foster opportunities for student reflection and communication, tasks need to be genuine problems that allow for exploration of mathematical concepts without one perceived correct solution nor any memorized rules (Hiebert, 1997). Further, students should find the tasks interesting, engaging, and connected to concepts they are already familiar with (Hiebert, 1997).

2.2.2. *Role of the Teacher*

In addition to the utilization of such rich learning tasks, Hiebert (1997) suggested that teachers act as facilitators of understanding without acting as centers of mathematical information. They can achieve this by selecting appropriate tasks, framing problems as opportunities for learning, and developing a classroom culture of reflection and communication (Hiebert, 1997). Teachers should be involved in student learning enough to help them build understanding without restricting student initiative and creativity, a difficult balance to achieve (Ball, 1993; Dewey, 1933; Hiebert, 1997; Lampert, 1991).

2.2.3. *Social Culture of the Classroom*

Along with implementation of genuine learning tasks and teacher facilitation of understanding emerges a social culture of learning for understanding within a classroom. To further examine the social culture of a classroom that encourages norms of reflection and communication, Hiebert (1997) identified four core features: “ideas are the currency of the classroom,” students have autonomy over exploring alternative methods for solving problems, mistakes are appreciated as sites for learning, and “persuasiveness of an explanation or the correctness of a solution depends on the mathematical sense it makes” (pp. 9-10). A classroom that exemplifies these core features should be synonymous with a collaborative community of learners who reflect and communicate with each other respectfully, with the ultimate goal of understanding the subject being explored.

2.2.4. *Mathematical Tools*

In order to better such a community, teachers should utilize a variety of tools in order to support learning for understanding. Tools can include “physical materials, . . . oral language, written notation, and any other tools with which students can think about mathematics” (Hiebert, 1997, p. 10). Hiebert (1997) noted that “different tools may encourage different understandings” and that “students must create meaning for them” in order for them to be useful (pp. 10-11). It is important to note that the choice of tool can influence the way in which students understand a particular concept.

2.2.5. Accessibility of Mathematics

Finally, Hiebert (1997) reminded that “every student has the right to reflect on, and communicate about , mathematics [and it] is not just the privilege of the high-achieving group” (p. 11). Understanding can have positive effects on the performance of any student regardless of their level (Carey, Fennema, Carpenter, & Franke, 1993; Hiebert & Wearne, 1993; Hiebert, Wearner, & Taber, 1991). Hiebert (1997) noted that the best way to encourage equitable respect for individuals of all levels is to carefully listen to each student’s ideas with genuine interest and embrace what they say by giving meaning to their ideas. He further noted that a “rich, fully functioning community requires everyone’s participation” so that every student’s ideas are heard (Hiebert, 1997, p. 12). This can be achieved through paying attention to the equity of reflection and communication in the classroom.

Overall, the most important feature of Hiebert’s (1997) five critical dimensions of a classroom built for understanding is that the dimensions work together as a system; it is important to consider all of the dimensions as integral to the success of the classroom environment in fostering understanding.

2.3. Autonomy

In the search for identifying classroom conditions that are conducive to developing understanding, Stefanou, Perencevich, DiCinto, & Turner (2004) turned to student *autonomy support*. Studies have shown that students of autonomy supportive teachers experience more classroom engagement, positive emotion, self-esteem, creativity, intrinsic motivation, psychological well-being, persistence in school, academic achievement, and conceptual understanding (Assor, Kaplan, & Roth, 2002; Benware & Deci, 1984; Deci & Ryan, 1985, 1987; Deci, Nezlek, & Sheinman, 1981; Hardre & Reeve, 2003; Koestner & Ryan, 1984; Reeve & Jang, 2006; Reeve, 2009; Ryan & Grolnick, 1986; Vallerand, Fortier, & Guay, 1997). Therefore, it is important to consider the role of autonomy and its implications in classrooms.

Autonomy was defined by Deci & Ryan (1987) as “action that is chosen; action for which one is responsible” (p. 1025). Deci & Ryan (2000) consider autonomy to be the most important of the three basic psychological needs (autonomy, competence, and relatedness) because of its fundamental role in promoting more intrinsically motivated behaviours that are driven by interest and enjoyment and lead to deepened engagement and understanding. Deci & Ryan’s (2000) self-determination theory posits that although intrinsic motivation is the prototype of self-determined activity, there is a continuum of extrinsically motivated behaviours varying from less self-determined to more self-determined and that extrinsically motivated behaviours can be internalized and self-determined given enough autonomy. In general, autonomy is viewed as the availability of choice, which is evident in Black & Deci's (2000) definition: autonomy is supported by providing students with “pertinent information and opportunities for choice, while minimizing the use of pressures and demands” (p. 742). However, it is important to note that many educators hold misconceptions regarding the role of autonomy in classrooms (Boggiano & Katz, 1991). It is not enough to simply give students autonomy. That autonomy also needs to be supported. Assor et al. (2002) claimed that in satisfying the need for autonomy, “the role of freedom of action is less important than the extent to which one’s actions reflect one’s personal goals, interests or values” (p. 272). Reeve (2009) noted that autonomy supportive teaching should “adopt the students’ perspective, welcome students’ thoughts, feelings, and behaviors, and support students’ motivational development and capacity for autonomous self-regulation” (p. 162). To further classify autonomy support, Stefanou et al. (2004) offered three distinct ways in which it can be manifested within a classroom: [1] *organizational* autonomy support, [2] *procedural* autonomy support, and [3] *cognitive* autonomy support.

Organizational autonomy support allows students to control their environment by directing them to choose classroom rules, the pace at which they learn, due dates which they set, students with whom they work, and ways in which they are evaluated (Stefanou et al., 2004). Meanwhile, procedural autonomy support allows students to control the form in which they present their work by inciting them to choose materials they use for a project, the ways in which they display work, and the ways in which their materials are handled (Stefanou et al., 2004). Finally, cognitive autonomy support allows for students to control their learning by encouraging them to generate their own distinct solutions, justify their solutions according to mathematical principles, evaluate their own work, evaluate work of their peers, discuss multiple approaches, debate ideas freely, ask questions, and formulate personal goals (Stefanou et al., 2004). Stefanou et al. (2004) argued that:

> Organizational and procedural autonomy support alone may not facilitate truly adaptive learning and motivation. Rather, the characteristics of ownership and justification of ideas, the construction of meaning, and the intentional self-reliance used in critical thinking are at the heart of learning and motivation in the classroom. We suggest that, although choice and decision making are fundamental, more than simple choices about tasks or roles are necessary to influence students' decisions to become cognitively engaged in academic tasks. Activities that support organizational or procedural autonomy may be necessary but insufficient to promote student engagement and intrinsic motivation. Cognitive autonomy support may be the essential ingredient without which motivation and engagement may not be maximized. (p. 109)

Therefore, it is important for educators to understand that it is cognitive autonomy support in particular that allows for positive educational benefits to occur in the provision of autonomy support, and not necessarily procedural or organizational autonomy support.

In fact, there is variance in results for the necessity of student autonomy over procedural and organizational dimensions of classroom organization. Stefanou et al. (2004) did not clearly indicate whether organizational and procedural autonomy support is essential in facilitating learning. Past studies, such as Decharms (1984), have shown that classroom engagement is facilitated best under conditions of high autonomy support and moderate structure. In such studies, structure is defined similarly to Jang, Reeve, & Deci's (2010) definition of structure, which is similar to the procedural and organizational dimensions defined earlier:

> Structure refers to the amount and clarity of information that teachers provide to students about expectations and ways of effectively achieving desired educational outcomes [by] establishing order, introducing procedures, communicating policies about how to get things done, and minimizing misbehaviour while encouraging engagement and achievement. (p. 589)

It is important to note that structure is often confused with control; Jang et al. (2010) noted that although "in many cases there is a confluence between the concepts of structure and control [where] structure can be used in controlling ways and often is, control is by no means essential to structure” (p. 590). Controlling teachers “adopt only the teacher’s perspective, intrude into students’ thoughts, feelings, or actions, and pressure students to think, feel, or behave in particular ways” (Reeve, 2009, p. 160). Controlling practices of teachers lead to negative effects on student learning (Sierens, Vansteenkiste, Goossens, Soenens, & Dochy, 2009). Jang et al. (2010) caution against using structure in a controlling manner, but rather in an autonomy supportive manner, which means that a teacher maintains a degree of respect for student thoughts, feelings and actions within the structure they provide. Unlike other classroom based studies, Jang et al. (2010) found that students across classes exhibit highest levels of engagement when teachers provide high autonomy support and high levels of structure. However, high levels of structure were only correlated positively with observed measures of engagement and not with self-reported measures of engagement (Jang et al., 2010). They suggested that a highly structured learning environment may not support student engagement unless it is structured in an manner that is supportive of student autonomy. Structure, when used in an autonomy supportive way, can help students develop perceived competence, self-efficacy, internal locus of control, and mastery motivation (Skinner, Furrer, Marchand, & Kindermann, 2008). Therefore, students should be provided with a learning environment that is structured in an autonomy supportive manner.

Predominantly, the literature reveals that the most essential factor in a classroom environment conducive to understanding is the presence of cognitive autonomy support. Cognitive autonomy support “encourages student ownership of the learning and can include teacher behaviours such as asking students to justify or argue for their point, asking students to generate their own solution paths, or asking students to evaluate their own and others’ solutions or ideas” (Stefanou et al., 2004). Jang et al. (2010) defined autonomy support as “building instruction around students’ interests, preferences, personal goals, choice making, and sense of challenge and curiosity” (p. 589), which is similar to cognitive autonomy support (Stefanou et al., 2004) with regard to personalization of learning. Both of these studies revealed the importance of cognitive autonomy support in promoting student engagement and deeper relational understanding.

2.4. Goals

Goal orientation can have a positive influence on performance and motivation in the face of a challenging task, such as that of learning mathematics (Grant & Dweck, 2003). In general, motivation, which is the "inclination to do certain things and avoid doing some others" (Hannula, 2006, p. 165), is viewed in the literature in two ways: *Self-Determination Theory* and *Achievement Goal Theory*. Self-Determination Theory regards motivation as a reflection of the satisfaction of basic psychological needs for autonomy, competence, and social belonging in the determination for extrinsic or intrinsic levels of motivation (Deci & Ryan, 2000). Achievement Goal Theory is rooted in the belief of intelligence as being either fixed or malleable giving rise to either performance (self-enhancing) or learning (mastery) goal orientations, leading to various motivation driven behaviour patterns that depend on self-efficacy beliefs (Dweck, 1986; Pintrich, 2000). Performance goal orientations can be seen as manifestations of instrumental understanding and learning goal orientations can be seen as manifestations of relational understanding. Again, Skemp (1976) noted that although instrumental understanding is often more immediately rewarding, relational understanding is more adaptable and removes the need for extrinsic motivators. Therefore, students with higher learning goal orientations should be more likely to exhibit intrinsic motivation. However, performance and learning goal orientations are often intermingled. In fact, Hannula (2006) offered that performance and learning goal orientations are not necessarily mutually exclusive and that goals are realisations of psychological needs and reflections of emotions encountered throughout the learning process.

Nonetheless, it is interesting to consider goal orientations individually before considering them together. Dweck (1986) defined the learning (mastery) goal orientation as one where "individuals seek to increase their competence, to understand or master something new," and the performance goal orientation as one where "individuals seek to gain favorable judgements of their competence or avoid negative judgments of their competence" (p. 1040). Grant & Dweck (2003) provided evidence that a learning (mastery) goal orientation positively impacts performance and motivation in the face of challenge while performance goal orientation only positively impacts performance and motivation if no challenge is present. Dweck (1986) posited that "performance goals work against the pursuit of challenge by requiring that [student's] perceptions of their ability be high (and remain high) before [they desire to] challenge [a] task" and that if a student's perception of ability is low, they may try to conceal their abilities and "protect [themselves] from negative evaluation" by choosing "personally easy tasks" (p. 1041). Conversely, goals of learning (mastery) allow students to willingly display "ignorance in order to acquire skills and knowledge" (Dweck, 1986, p. 1042). In support of Dweck's theory, Middleton & Spanias (1999) confirmed that students who are more intrinsically motivated towards academic tasks have tendencies towards learning (mastery) goal orientations and most often demonstrate higher achievement than those with performance goal orientations. It is, however, important to note that Dweck's socio-cognitive theory of motivation (*Achievement Goal Theory*) was originally developed with school-aged children. When tested on returning to school adults, the theory was only partially manifested: "mastery goals had a positive impact on learning activities and outcomes" and "performance goals or . . . work avoidance had negative influence on learning and achievement" but "the predicted effects of implicit theories of intelligence on goal orientation and cognitive engagement in learning . . . failed to emerge" (Dupeyrat & Mariné, 2005, p. 43).

In extension of Dweck's (1986) theory, Dupeyrat & Mariné (2005) discovered that "mastery goals [in adults returning to school have] a positive influence on academic achievement through the mediation of effort expenditure" (p.43). It is important to note here that there is ample consistency in the literature among positive effects of learning (mastery) goal orientations, but not as strong of a consistency in relation to the effects of performance goal orientations: Grant & Dweck (2003) and Dweck (1986) claimed that negative impacts of performance goal orientations only occur when students are faced with difficult situations, whereas Dupeyrat & Mariné (2005) claimed that students with performance goal orientations consistently experience negative effects on learning. Further, O'Shea, Cleary, & Breen (2010) discovered that the majority of the 182 first year undergraduate level participants taking mathematics courses classified themselves as oriented towards learning (mastery) goals. This may be influenced by the fact that these students have already satisfied many prerequisites in order to be enrolled at the undergraduate level. Also, it is difficult for adults to analyze their present situation and claim that it is negative, especially when they know that learning for mastery is more desirable than learning for performance. It is also possible for students to view themselves as holding both mastery and performance goal orientations. In fact, literature on goals has indicated that goal orientations are not mutually exclusive and can co-exist (Boekaerts, 1999; Dweck, 2002; Hannula, 2006).

In particular, Hannula (2006) focused on the interplay of goal orientations rather than on differentiating between them. Hannula (2006) attested that the psychological needs for autonomy, competence, and social belonging "are the most significant determinants of goal choices" positing that student needs within academic settings breed more specific learning goals (p. 165). Hannula (2006) further noted that the "realization of needs as goals in mathematics classrooms is greatly influenced by students' beliefs of themselves, mathematics, and learning as well as school context, the social and sociomathematical norms" (p. 167). This brings light to the importance of classroom environment in the development of goal orientations:

> In a teacher-centred mathematics classroom that emphasises rules and routines and individual drilling, there is little room to meet the students' needs for autonomy or social belonging within the context of mathematics learning. More student-centered classrooms with a lot of teamwork going on, and where the emphasis is on meaning making, there may be many opportunities to meet different needs; such approaches, by definition, as it were, rely on students exhibiting their autonomy and social interactions. (Hannula, 2006, p. 167)

Moreover, emotions and beliefs can have powerful effects on which goals students decide to pursue. If students feel that one or more of their psychological needs are under threat, they may fall into panic, and may shift from mastery goal orientations to performance goal orientations almost instantly (Boekaerts, 1999; Hannula, 2006). Further, self-efficacy beliefs can affect perceived accessibility of particular goals, causing predisposition towards certain goal orientations (Hannula, 2006; Philippou & Christour, 2002). Hannula (2006) offered a noteworthy example of a student who held both performance and mastery goal orientations, but with a dominating performance goal orientation. The student only saw himself successful at mathematics if he was able to perform the correct steps fluently. He also strongly valued his teacher's views due to his dominating performance goal orientation, and since his teacher did not interject to tell him that fluency in mathematics does not always occur immediately, he continued to pursue fluency in performing mathematical procedures. Ironically, the student also claimed to have goals of mastery, but his idea of mastery was that of fluency. The student's dominant goal orientation was affected by the learning environment he was in. Had his teacher placed more value on understanding rather than becoming fluent in completing procedures, the student may have adopted a focus on mastery of content rather than on performance in content.

Most importantly, Hannula (2006) showed that "students may have multiple simultaneous goals and [that] choices between them are made" (p. 175). He claimed that motivation is structured through the mediation of needs and goals with emotions and that a desired balance of goals can be promoted by offering students a safe learning environment that focuses "on mathematical processes rather than products" (Hannula, 2006, p. 176). Such an environment can be created through the provision of cognitive autonomy support and is conducive to the development of understanding.

2.5. Self-Efficacy

Adults returning to learning mathematics at the college level are often burdened with negative experiences from past mathematics classrooms. In turn, they often enter mathematics upgrading classes with relatively low self-efficacy, which is a specific measure of one's belief in one's ability to succeed at performing a particular task and is often affected by related past experiences, observations, or verbal persuasions (Zimmerman, 2000). Self-efficacy is formally defined as a "personal judgement of one's capabilities to organize and execute courses of action to attain designated goals" (Bandura, 1977, cited in Zimmerman, 2000, p. 87). Zimmerman (2000) clarified that self-efficacy involves "performance capabilities rather than personal qualities," is "multidimensional in form," depends on "a mastery criterion of performance rather than on normative criteria," and refers to "future functioning . . . assessed before students perform the relevant activities" (p. 83-84).

Self-efficacy is instrumental in academic functioning (Bandura, Barbaranelli, Caprara, & Pastorelli, 1996; Albert Bandura, 1989; Pajares, 1996; Schunk, 1989; Vieira & Grantham, 2011; Zimmerman, 2000). Students with positive self-efficacy "participate more readily [in challenging tasks], work harder, persist longer, and have fewer adverse emotional reactions when they encounter difficulties than do those who doubt their capabilities" (Bandura, 1997, cited in Zimmerman, 2000, p. 86). Further, self-efficacy correlates highly with intrinsic interest, effort expenditure, and success in course work (Zimmerman, 2000). In terms of emotions, self-efficacy is found to be more powerful than math anxiety in predicting math performance (Siegel, Galassi, & Ware, 1985). Self-efficacy is also a desirable trait in promoting motivation towards successful actions in learning environments.

Therefore, it is important to consider the factors that contribute to more positive self-efficacy beliefs. Zimmerman (2000) claimed that self-efficacy beliefs are "readily influenced by four types of experience: enactive attainment, vicarious experience, verbal persuasion, and physiological states" (p. 88). Enactive attainment refers to the most influential predictor of self-efficacy beliefs, which is past personal experience. Past negative experience with an action leads to lower self-efficacy beliefs related to that particular action. Likewise, past positive experience with an action leads to higher self-efficacy beliefs pertaining to that action. Next, vicarious experience refers to an observed experience. If one observes another experiencing an action in a particular way, they may be swayed to believe that they also will experience the action similarly. Of less influence is verbal persuasion, which refers to an experience that is described to an individual through verbal transmission. Without physically experiencing the action, one's self-efficacy beliefs can only be slightly influenced by another's statement unless the issuer of the statement is highly respected by the listener. Finally, self-efficacy beliefs can also be influenced by physiological reactions, which refer to stress, fatigue, or other emotions that indicate physical incapability. If one is not feeling physically prepared for an action, it is less likely that they will believe in their ability to perform the action at that particular time. It is quite often that students experience one or more of such sources of self-efficacy beliefs within mathematics education.

Furthermore, a more recent study on the self-efficacy and goal setting of undergraduate university students in an autonomous context found that "trait autonomy indirectly and positively predicted setting difficult goals through trait self-efficacy and a person's willingness to engage in important tasks" (Vieira & Grantham, 2011, p. 141). Therefore, it is interesting to consider the relationship between teacher-provided cognitive autonomy support and that of student self-efficacy beliefs.

2.6. Anxiety

North American culture fosters attitudes towards mathematics that contribute to anxiety with mathematics, which is commonly defined as “a feeling of tension, apprehension, or fear that interferes with math performance” (Ashcraft, 2002, p. 181). McLeod (1992) identified mathematics anxiety as an element of the affective domain that produces a hot and rapidly changing emotional reaction in response to frustration with mathematics. Further, Wigfield & Meece (1988) determined that mathematics anxiety can be considered as having two components: a cognitive component referred to as *worry*, which consists of “self-depreciatory thoughts about one’s performance,” and an affective component of anxiety referred to as *emotionality,* which includes “feelings of nervousness, tension, and unpleasant physiological reactions to testing situations” (p. 210). Interestingly, Wigfield & Meece (1988) concluded that the affective component of mathematics anxiety is “more strongly and negatively [correlated with student] ability perceptions, performance perceptions, and math performance” than the cognitive component and that the cognitive component is “more strongly and positively [correlated with] the importance that [students] attach to math and their reported actual effort in math” (p. 210). This bi-dimensionality of mathematics anxiety is important due to its specific effects on components related to the learning of mathematics. Anxiety in mathematics most often produces undesirable results and it is interesting to consider how it can be prevented.

Many studies have documented the negative effects of mathematics anxiety on mathematics achievement (Ashcraft & Krause, 2007; Ashcraft, 2002; Clute, 1984; Hopko, Ashcraft, & Gute, 1998; McLeod, 1992; Richardson & Suinn, 1972). Past failures in mathematics often lead to anxieties relating to mathematics, which in turn lead to more failures (Wadlington & Wadlington, 2008). The result is that people with mathematics anxiety tend to avoid mathematics in general and there does not seem to be a simple remedy for mathematics anxiety (Ashcraft & Krause, 2007; Ashcraft, 2002; Morris, 1981; Tobias, 1993; Wigfield & Meece, 1988). Bessant (1995) noted that mathematics anxiety is a complex phenomenon that interacts with a variety of cognitive variables. One cognitive variable that has recently received attention from scholars, is working memory. It has been empirically proven that “mathematics anxiety compromises the functioning of working memory," which in turn prevents one from completing complicated mathematical operations, ultimately leading to failure (Ashcraft & Krause, 2007, p. 243). The effects of mathematics anxiety are undesirable and unconducive to learning.

Therefore, it is important to consider what sorts of classroom environments are conducive to relieving mathematics anxiety. To begin with, it is common to find students experiencing anxiety and avoidance of mathematics in classrooms where teachers have a "very high demand for correctness but provide little cognitive or motivational support" (Turner et al., 2002, cited in Ashcraft, 2002, p. 184). Biggs (1985) claimed that a deep intrinsically motivated approach to learning fosters more "affectively satisfying outcomes" and more surface extrinsically motivated approach to study "leads to factually specific outcomes, which are often associated too with negative affect" (p. 187, 202). This implies that a classroom with more support for autonomy should contribute to lower levels of anxiety. Since it is common for adult students returning to school to hold anxieties towards mathematics, it is interesting to consider the effects of a flipped classroom model on such affective measures.

2.7. Summary

Overall, the impetus for this study is the need for developing and studying an environment conducive to promoting deeper understanding of mathematical concepts within an adult upgrading mathematics course. The flipped classroom model offers the availability of classroom time for providing opportunities to build such understanding. The type of understanding that is most desirable is relational understanding as defined by Skemp (1976). Literature on understanding reveals that reflection and communication are essential elements of a classroom environment conducive to building such understanding (Hiebert, 1997). In providing opportunities for reflection and communication through engaging learning tasks, the facilitative role of the teacher, positive classroom culture, strong learning tools, and accessibility of material (Hiebert, 1997), students are provided with various types of autonomy. In particular, cognitive, procedural, and organizational autonomy can be provided to various extents depending on the teacher and their perceptions of a positive classroom learning environment (Stefanou et al., 2004). Some literature (Jang et al., 2010) indicates that structuring the procedural and organizational components of a classroom in an autonomous way along with the provision of cognitive autonomy is the most desirable combination for increasing student engagement in course material.

However, students can experience a learning environment in various ways. Experiences can be determined according to student goal orientations, which are categorized in the literature as performance or learning (mastery) related (Dweck, 1986). Students can also hold various tendencies towards displaying their knowledge as instrumental or relational understanding (Skemp, 1976). Adult students in particular can have a variety of past experiences with mathematics, leading to various levels of self-efficacy and anxiety with mathematics. A classroom environment developed with the goal of promoting understanding should encourage more positive affective experiences with mathematics. Primarily, students can experience a learning environment in various ways depending on their goals and past experiences.

2.8. Research Questions

As already alluded, the population under study consists of adult students who have generally experienced low mathematical performance and understanding in their past encounters with mathematics. The flipped classroom offers a different mode of delivery of the same content students have likely already encountered. This allows them to experience learning the content in a different way. Despite the desirability of determining how a flipped classroom affects student performance and understanding, such outcomes cannot be adequately asserted in this small scale study without the room for a control group. Instead, this initial inquiry into the flipped classroom has more to do with exploring how students experience this new mode of delivery. Therefore, the key research question guiding this study is *how do students experience a flipped classroom?*

More particularly, the adult student population under study tends to experience low levels of self-efficacy and high levels of anxiety in relation to mathematics. This is largely due to singular negative past experiences with learning the subject. As such, the goal of implementing the flipped classroom in this study has a lot to do with attending to student goals, self-efficacy and anxiety in hopes of helping them develop a more positive relationship with mathematics. Therefore, a further research question in this study is *how are factors such as goals, self-efficacy, and anxiety interrelated with student experiences in a flipped classroom?*

3. Methodology

In order to further understand student experiences of a flipped classroom in an adult mathematics upgrading context, the research questions are pursued through case studies where attention is drawn to student understanding, autonomy, goals, self-efficacy, and anxiety. In what follows, the setting, participants, data collection, case construction, analysis, and risks and limitations of this study are detailed.

3.1. Setting: Upgrading and University Preparation Department

This research study has been conducted within the Upgrading and University Preparation (UUP) Department at the University of the Fraser Valley (UFV). UFV is a fully accredited public university located in the Fraser Valley just east of Vancouver, British Columbia, Canada. It has multiple campuses and locations within the Fraser Valley: Abbotsford, Chilliwack, Mission, Hope, and Agassiz, as well as a presence in Chandigarh, India. Originally, it was founded in 1974 as Fraser Valley College in response to a need for vocational training in the Fraser Valley. It then became a university college in 1988. Finally, it gained university status in 2008. UFV now caters to a diverse population of approximately 16,000 students each year including local and international students. It has a mandate to serve post-secondary educational needs of its region through offering diverse programs and small class sizes. UFV offers over 100 programs in post-secondary academic, trades, technical, career, continuing studies and adult basic education areas that lead to certificates, diplomas, and degrees at both the master's and the undergraduate level. Although priority is given to serving student educational needs and goals, faculty and staff are also engaged in scholarly research activities that support their program areas.

The UUP department at UFV offers programs in Adult Basic Education (ABE) for adults of all backgrounds and ages who want to meet their educational goals such as completing prerequisites for post-secondary programs, earning the BC adult graduation diploma, or improving skills for personal benefit. ABE courses offered at UUP follow guidelines and requirements of the BC Ministry of Advanced Education. UUP offers courses in English, Mathematics, Biology, Education and Career Planning, First Nations Studies, and Indigenous People's Knowledge. Courses are offered at the fundamental level (up to Grade 8 level), intermediate level (up to Grade 10 level), advanced level (Grade 11 equivalency), and provincial level (Grade 12 equivalency). Fundamental and intermediate level mathematics courses are offered as multilevel learning center environments while advanced and provincial level mathematics courses are offered as single level structured courses. Students are placed in an appropriate level either by satisfying certain prerequisites or by completing an assessment.

3.2. Setting: Math 084

This research study has been conducted within Math 084, which is the first of two provincially articulated advanced level mathematics courses in the UUP ABE program. Math 084 serves as a requirement for the Dogwood Diploma as well as a prerequisite for Math 085, which is the other advanced level mathematics course. Math 084 and Math 085 together serve as equivalent to Pre-Calculus 11, the high school course most often used as a prerequisite for undergraduate program admission for programs that lead to career paths such as teaching, nursing, business diplomas, etc. The official calendar description for Math 084 reveals that a variety of mathematics topics are taught in the course in order to prepare students for various disciplines of study.

> This course reviews operations with real numbers and the solution of linear equations. It introduces linear inequalities; the solution of quadratic, rational, and radical equations; operations with polynomial, rational, and radical expressions; and the graphing of equations, particularly linear equations. It also introduces function notation and applies basic geometry concepts such as volume and surface area of various 3D shapes as well as right angle trigonometry to solve practical problems.
>
> (University of the Fraser Valley, 2012)

The textbook used for all Math 084 sections is *Introductory Algebra and Trigonometry: Custom Edition* by Tussy & Gustafson (2011). Math 084 is a full-term course which is offered over 15 weeks, with two three hour class time periods per week, a total of 90 contact hours. It is traditionally taught with 60 lecture hours and 30 individual or group work hours. During each period, most instructors give a lecture on a particular section and follow it up with a short period of completing practice questions. Usually, an instructor will have time for two iterations of this process depending on the topics scheduled for that class. Sometimes, there is student interaction during class, but generally, students try to complete practice problems individually and ask the instructor for help if they are confused. Outside of class time, students complete assigned problems and can refer to notes they took in class. It is not often that students interact with each other outside of class time. Primarily, students work towards attaining a grade in the course through completing graded assignments from the textbook, unit tests, a midterm exam, and a final exam. It is common practice for instructors to assign due dates and implement policies on late work. As mentioned in the introduction, I taught this course for several years using the same teacher-centered manner described above. That is, until I decided to implement a flipped classroom model, which allowed me to grow from teaching in a teacher-centered manner to a student-centered manner.

3.3. Setting: Math 084 Flipped

As detailed in the introduction, the flipped classroom model allowed me to shift from teaching in a teacher-centered manner to a student-centered manner because of the additional class time it provided for collaborative learning opportunities. When I taught in a teacher-centered manner, even though students claimed to be engaged, they still struggled to understand the material well enough to be able to complete their homework. I wanted to utilize various student-centered teaching strategies that I was learning about in the course work for the Secondary Mathematics Education Master’s Program at SFU, such as problem based learning, student collaboration, and student-led discussions in an effort to promote student understanding. The flipped classroom model afforded the time for the implementation of such teaching strategies.

I implemented the flipped classroom model during my Winter 2013 term (January through April) section of Math 084. The section was listed as a hybrid course, which indicates to students when they register that they will be required to access course materials online out of class time in addition to attending class. In the Winter 2013 term, there were two sections of Math 084 offered: one in Abbotsford and one in Chilliwack. The Chilliwack section was taught in a traditional manner by another instructor and was not designated as a hybrid course. I taught the Abbotsford hybrid section according to the general flipped classroom guidelines of delivering content out of class time and dedicating class time to student-centered learning opportunities as rooted in the flipped classroom literature discussed in Section 2.1 Flipped Classrooms. Essentially, what was traditionally done during class time was removed and placed out of class time in an online environment using the Learning Management System (LMS) Blackboard Learn. This system allowed for the posting of video lecture lessons, online quizzes, announcements, and practice problems. The removal of content delivery from class time provided time to conduct content discussions, group learning activities, practice time, and assessments during class. Further, anything that contributed to a student's final grade, with few exceptions, was completed and submitted during class time. In essence, the in-class workload and the out-of-class workloads were swapped or flipped as compared to a traditional class. Student grades were still broken down in a traditional manner with 10% for graded assignments, 25% for the eight unit tests, 25% for the midterm, and 40% for the final exam. Although it is desirable to align an assessment system with a new teaching structure, I was unable to implement such a change completely without prior experience in conducting a flipped classroom. Ideally, standards based grading, as detailed by O'Connor (1999), would align best with the flipped classroom because of its focus on assessing completion of learning outcomes rather than assignments with the intention of measuring knowledge rather than organization. In another flipped mathematics classroom, Johnson (2013) implemented mastery learning, an assessment strategy where students were required to attain 70% or higher on formative quizzes in order to proceed with new material. If they did not meet this requirement, the teacher intervened and helped them prepare for another assessment on the same content. In an effort to more closely measure and motivate student learning than in a traditional scheduled testing structure, I encouraged my students to only take a unit test if they were adequately prepared, providing them with opportunities to reschedule tests.

3.3.1. Out-of-Class Time

At the beginning of the term, students were given a tentative schedule of topics that were going to be covered on each day. Students were asked to preview each topic out-of-class time before the day on which it was scheduled to be discussed. They were provided with a variety of learning materials including videos, online practice quizzes, and textbook problems for each topic. I created each topic video using screen capture software (Camtasia) that recorded my voice and pen annotations over a PowerPoint presentation using a tablet PC and a microphone (See *Figure 1* for video screenshot).

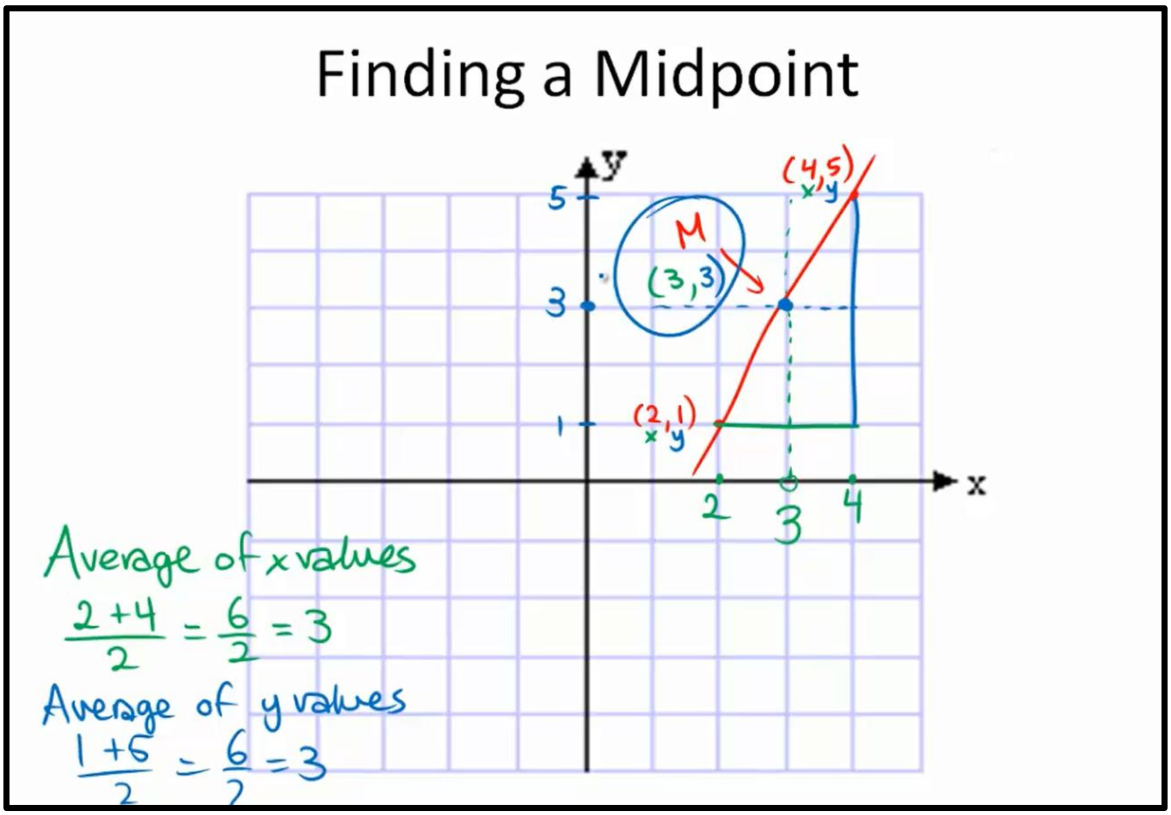

Figure 1: Screenshot of Video

Videos ranged from ten to thirty minutes in length and were organized by topic. There were 50 topics in the entire course. It is suggested that videos are clear, concise, and relatively short in length to allow "students to review them at separate times rather than in a single session" (Bull, Ferster, & Kjellstrom, 2012, p. 10). It is also suggested that videos do not merely "teach the procedure, but also represent the important underlying conceptual ideas" (Tucker, 2012, p. 82). I designed the videos with these criteria in mind.

In general, the videos included a conceptual introduction to the topic followed by a few worked through examples. Students could access the videos through Blackboard Learn at times convenient for them (See *Figure 2* for Blackboard Learn lesson homepage screenshot).

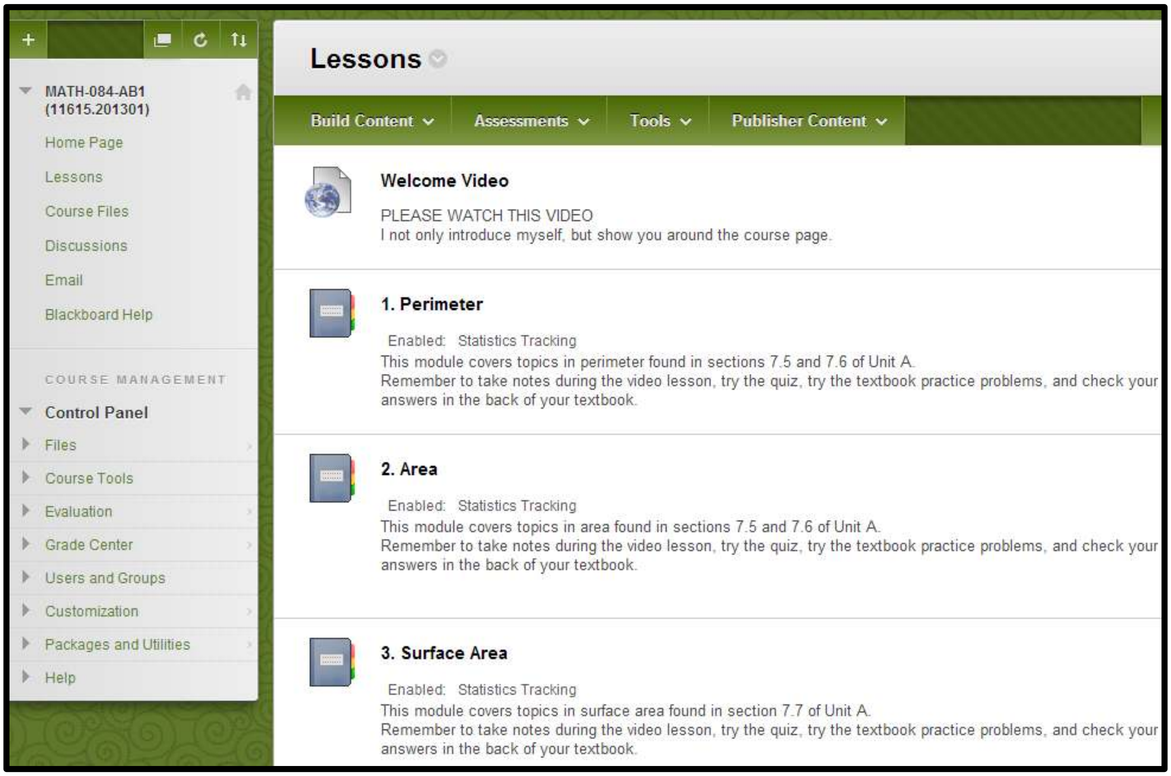

Figure 2: Screenshot of Blackboard Learn Lesson Homepage

If students had difficulties accessing material, they had the option of requesting to obtain the video files via USB or Dropbox. They were also able to view the videos on a computer as well as various mobile devices. Further, they had the liberty of pausing, rewinding, fast forwarding, and replaying the videos. Students also had the option to choose what capacity of note taking they would employ. Once they were finished with a video, they were provided with an online three-question quiz through Blackboard Learn relating to the topic, which was intended to help them reflect on what they had learned in the video (See *Figure 3* for online quiz screenshot).

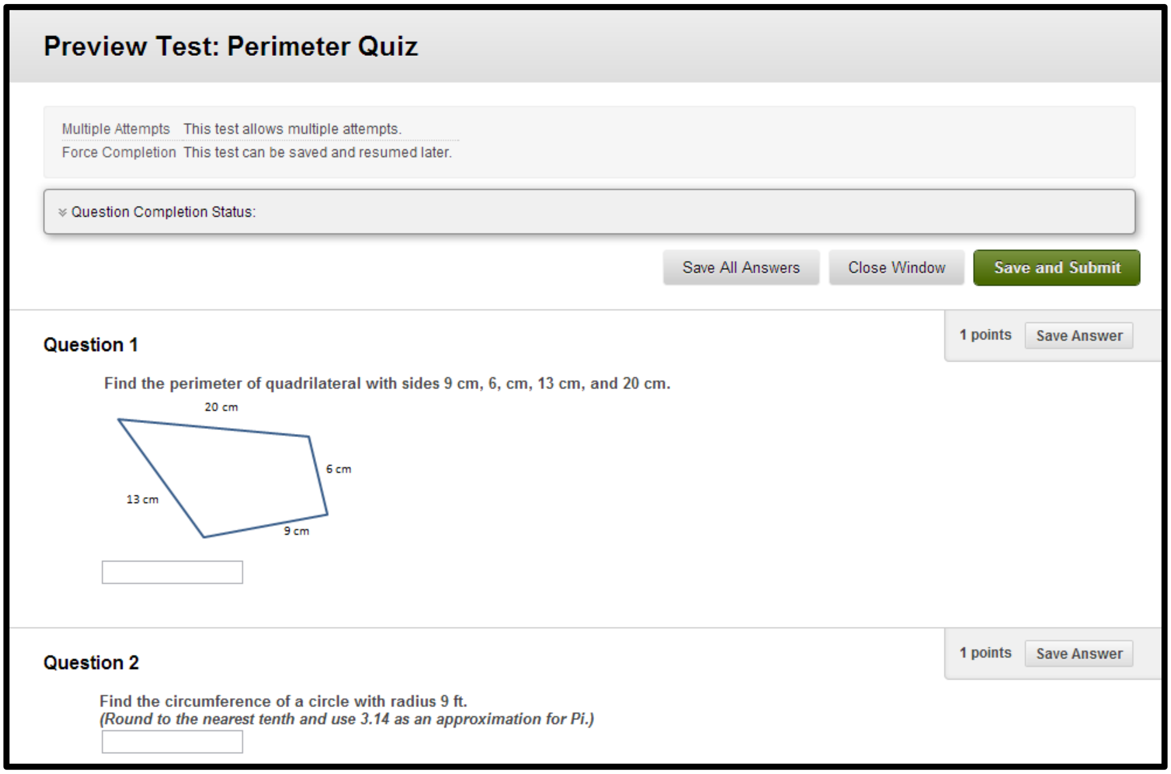

Figure 3: Screenshot of Online Quiz

I programmed the quizzes so that students obtained instant feedback regarding the question they answered. Answers could be entered either as a numerical value or a multiple choice selection and feedback included the correct answer as well as some hints on how to proceed with the problem or where to find an explanation of the concept. Students could repeat the quizzes as many times as they chose to, and whenever they chose to, and they did not contribute to their final grade. Finally, I provided students with a list of suggested practice problems from the textbook on that topic. The suggested practice problems had answers in the textbook that students could refer to, they were not collected, and they did not contribute to students' final grades. The learning tools provided out of class time, which included videos, problems given in the videos, quizzes, and suggested textbook problems, were provided with the intent that students would use them as a platform to ask questions about, and to develop a sense of, what they need help with. Most importantly, students had autonomous and equitable choice over a variety of learning materials they could engage with out of class time to prepare for class time discussions and activities.

3.3.2. *Class Time*

Class time was generally structured so that the first half of the class (80-90 min.) was spent either discussing the previewed material, working collaboratively on an activity relating to the previewed material, or completing a unit test. Discussions were student-generated and were based on material that was previewed prior to the class. I facilitated these discussions and prompted students to think about the connections throughout the material in order to give opportunities for the development of understanding. Opportunities for developing understanding were also present due to the collaborative nature of the activities. These often consisted of engaging open ended problems and collaborative group review discussions. Some activities and assessments also encouraged student-generated examples[1]. Various learning tools, including physical objects, were used to promote engagement and understanding throughout these periods of student collaboration.

An example of an engaging open ended problem used in the course was the National Council of Teachers of Mathematics (2008) Barbie Bungee Activity. During this activity, students were asked to find the maximum number of rubber bands required to allow a Barbie doll to 'bungee jump' from a certain height without hitting her head. Students, in random groups, were given rubber bands and a Barbie doll and were asked to make the prediction for the number of rubber bands required. Eventually, through discussion, students noted the linear relationship between the number of rubber bands and the measure of the doll's descent. This led to further discussion on linear equations and slopes.

Another example of an activity that was completed in the class was that of student-generated examples. As noted, this is not referring to Watson & Mason's (2005, 2002) concept development approach to learner-generated examples, but rather the opportunity for students to generate examples for purposes of involvement in the learning process. One instance of student-generated examples was when I provided students with a collection of 3-dimensional geometric objects and asked them to build a new object

[1] Student-generated examples (SGEs) are used colloquially here in the sense that students were asked to generate examples for the purposes of assessment or engagement and not in the more defined sense that Watson & Mason (2005, 2002) indicate in respect to constructive concept development.

composed of two or more smaller objects. They were then asked to give their new composite object to another group to find the surface area and the volume of the given composite structure. This activity led to some interesting discussion and even a Google search regarding the surface area of a cone because it was not provided in the course textbook. Another instance of student-generated examples was for a unit on exponential expressions. Students were asked to use whiteboards to develop exponential expressions that needed to be simplified. They were then asked to pass the problems to another group to simplify. Interesting examples arose from such activities. One example in particular was that of a student who created a complicated exponential expression but created it so that the entire expression was taken to the power of zero indicating that he understood the implication of a power of zero (See *Figure 5* in section 4.3.1 Case of Mark).

Other activities were group concept review sessions. For example, students used whiteboards to develop reasoning for why certain properties exist, such as the rules for simplifying exponential expressions. Products from review sessions were often documented with a camera and posted on the course website under resources to help provide study materials for students in preparing for tests.

The purpose of using 2” by 3” whiteboards in each group during periods of collaboration was to encourage collaboration, transparency, and visibility. Ideally, whiteboards should have been used as vertical writing spaces around the room to promote mobility of knowledge; however, there were limitations in the room such as concrete walls and classes booked into the room directly prior and directly after class time. The 2” by 3” whiteboards were portable enough to bring in to each class period. One whiteboard was used horizontally in each group of two to three students. Each student was encouraged to add their ideas to the whiteboard collaboratively; however, it was often that one student in the group dominated the whiteboard. The whiteboards also offered a non-permanent surface on which students could feel more comfortable trying out ideas. No names were attached to student work on whiteboards, which allowed for work to be taken at face value. The product of the group could be visibly shown on a whiteboard to others in the room in a non-judgemental manner.

In general, activities provided students with the ability to engage actively with the material as well as with others in the classroom, helping build a positive learning environment. Equally important to the choice of activities in the promotion of engagement and understanding was the method of grouping students so that they would productively collaborate. Liljedahl (in press) asserts that visibly random groups lead to positive observable changes such as “an elimination of social barriers, . . . [an increase in] mobility of knowledge between students, . . . [a decrease in] reliance on the teacher for answers, . . . [and an increase in] engagement” (p. 4). During the first half of the term, I always grouped students together randomly to increase the likelihood of students working in some capacity with as many other students as possible in alignment with Liljedahl’s (in press) suggestions for student grouping. Eventually, students found their favorite peers to work with and they settled in to working in their preferred groups. Even then, I still randomly grouped them once in a while so they could work with peers they didn’t normally interact with. Random grouping fostered the development of a community where students felt comfortable sharing ideas and contributed to the creation of a positive learning environment.

The second half of each class was dedicated towards completing graded assignment problems from the textbook and clarifying concepts students were still struggling with so that they could have more success with test questions. Students were allowed to work on whatever they needed to in whatever groups they wanted to and were given various opportunities for presenting their work. However, each student submitted their graded assignment problems individually into their own portfolios before they left for the day. This allowed for collaboration while maintaining the responsibility of each student to submit their own graded assignment problems. If a student was unable to complete all of the required graded problems for that day, the remaining problems were to be completed for homework and submitted the following day. I made sure to mark anything that was handed into the portfolios by the next class period. I marked the graded problems by checking off problems that were completed correctly with all work shown, commenting on any interesting solutions that students revealed, and circling problems that were completed incorrectly, often making a note of what they could try to fix. Students were encouraged to show all of their work so that this level of feedback would be available to them. When they received the portfolios with feedback included during the following class period, they were given the opportunity to earn credit for completing their corrections as long as they showed full reasoning. The portfolios contained a progress sheet with all graded assignment problems listed for each section on which I would either check off the question number if it was completed correctly or circle it if it needed correction. If a student completed a correction correctly, I checked it off as correct on the progress sheet. At the end of the term, I was able to look at the progress sheet and assign the student a grade for their assignment problems (10% of final grade). More importantly, each student was able to see which topics they struggled with and was encouraged to go back to those topics before the final exam. The intent of this process was to make students understand that I wanted them to master each topic to the fullest of their capabilities. I also gave journal prompts and encouraged students to reflect on their learning processes. These were also submitted into the students' portfolios. If students completed everything that was required for that class period, they were encouraged to begin learning the next upcoming topic by watching the next videos on a laptop, mobile device, or at the library. Some students chose to leave early if they completed everything they were required to complete. Most importantly, students had the option to receive individualized attention during this part of the class. There was time designated during class for students to ask questions both of myself and of their peers. Students often used this time to teach each other concepts they had learned. Benware & Deci (1984) have found that students who learn material in order to teach it exhibit higher intrinsic motivation, greater conceptual understanding, and feel more active in the learning process. Therefore, I made sure to encourage such actions during this time. Although the second half of class time was exceptionally unstructured, students had individual opportunities to inquire about particular areas of struggle in order to develop a better understanding if they so desired.

3.4. Flipped Student Interaction

Given that students in the flipped classroom version of Math 084 were given a diversity of learning tools, students interacted with the class in various ways. Although most students tried all the available tools provided during the beginning of the term, they tended to focus on their favorite learning tools as the term progressed. Some focused on utilizing class time completely in order to gain a better understanding of the topics. These students willingly participated in all classroom activities. Others, focused on the out-of-class learning materials such as the videos and the textbook. These students often attended less regularly or opted out of participating in the activities during class time. If they were behind in the material, they most often excluded themselves from collaborative activities and worked on their own in the classroom. Some simply did not show up when they were behind with the material. Finally, there were those who varied their focus between classroom activities and out-of-class learning tools depending on other life related issues that affected the amount of time they had to work on the material. Attendance varied among students depending on both their motivation to succeed in the course and other interfering life events that occurred.

3.5. Participants

Participants of the research study were students enrolled in the Winter 2013 term offering of Math 084, which is detailed in Section 3.2 Setting: Math 084. As alluded to in Section 2.7 Research Questions, students enrolling in this class have often endured difficult life circumstances as well as negative experiences with mathematics. Enjoyment of mathematics is hardly a goal for any of the students in the class because the main focus is on obtaining the required prerequisites.

The course started with 25 total students enrolled, 18 of whom completed the course. It should be noted that low completion rates are very common in these courses. Many students often stop showing up due to life circumstances, or they never show up to the class in the first place. Out of the 18 students who completed the course, two were registered, but were completing the course at a distance, and therefore were not part of the flipped classroom routine. This leaves 16 students who participated in the flipped classroom, 14 of whom gave consent to participate in the research study. All 14 of these students seemed to be in their twenties at similar stages of life.

As required by UFV's Research Ethics Board (REB), an external co-investigator was required to collect any data that was obtained directly from the students. Danica, another faculty member in my department, offered to collect data directly from the students. During these periods of data collection, I was away from the room as mandated by the REB. On the first day of data collection, Danica invited students to participate in the research study by giving them a description of the study and a consent form to sign as accepted by the Office of Research Ethics (ORE) from Simon Fraser University (SFU) and the Research Ethics Board (REB) from UFV.

After preliminary investigation of the data collected, I carefully selected six participants who represented various levels of interaction with the flipped classroom as described in 3.4 Flipped Student Interaction and used them as cases for the research study (two male and four female). These participants represented three groups as follows: two students who participated completely in both in-class and out of class components of the flipped classroom, two students who at first participated completely with the flipped classroom model but later fell behind and chose only to participate in out of class components, and two students who tried participating in the flipped classroom model completely, but quickly participated only in what was absolutely required in the course. These students were representative of all students completing the course and are further detailed in Chapter 4 Results and Case Analyses

3.6. Data

Data collected directly from the students during the term consisted of surveys and interviews and was collected by Danica while I was away from the room on three occasions (Week 3: January 24, 2013; Week 8: March 7, 2013; and Week 14: April 18, 2013). Students were reminded that none of the data would be seen by me until after grades were posted so that they would not think that their responses would affect their grade. After the term was over and grades were posted (April 26, 2013), I followed up with the chosen participants via an email survey and clarified any responses I needed to clarify via email in order to ensure validity and reliability. Throughout the entire term, I also collected observational data on each student after every class.

Overall, data collected consisted of the following:

- Observational data was collected by myself the researcher and instructor on each student after each class in relation to classroom interaction, goals statements, self-efficacy, anxiety levels, etc. tabulated in an Excel spreadsheet document for ease of analysis.
- All interviews were conducted and voice recorded by Danica with students who volunteered to participate in the interviews while I was away from the room. Interview guideline questions were developed based on observational

data for the purposes of gaining clarity on student interactions in class and are included in Appendices B, D, & F. It should be noted that I was blind to who was being interviewed and what specific questions they were being asked during the time of the interviews.

- Surveys were collected by Danica on each of the abovementioned dates. Surveys consisted of the following:
- Questions rooted in observational data with the intention to gain clarity on things that were observed in the class (see Appendix A, C, and E)
- MRBQ survey items (See Appendix E)
  - The MRBQ (Mathematics Related Beliefs Questionnaire) was originally developed by Op't Eynde & De Corte (2003). MRBQ items were used to measure mathematical self-efficacy of participants and their overall attitudes towards mathematics.
- MARS-R survey items (See Appendix E)
  - The MARS (Mathematics Anxiety Rating Scale) was originally developed by Richardson & Suinn (1972) as a 98 item rating scale in 1972. Since its creation, it has been modified several times. Items from the 24 item MARS-R (Mathematics Anxiety Rating Scale – Revised) were used to measure mathematics anxiety in participants.

- Follow-up surveys were emailed to the six selected participants after grades were posted and data collected by Danica was received on April 26th, 2013. The follow-up survey was the same for all participants and the questions were developed in order to clarify trends observed in the preliminary analysis of all other data. The follow-up survey is included in Appendix G.

The variety of data sources created a large data set that contributed positively to maintaining validity and reliability even though some participants did not complete all surveys or interviews. These data were aggregated into cases for six select participants as referenced in Section 3.5 Participants and detailed in Chapter 4 Results and Case Analyses.

3.7. Analysis

Once cases were constructed for the six participants selected, they were then analyzed through the method of analytic induction which is rooted in grounded theory.

> Analytic induction offers a specific form of inductive analysis that begins deductively, by formulating propositions or [theory driven] hypotheses, and then examines a particular case in depth to determine if the facts of the

> case support the hypothesis. It if fits, another case is studied, and so forth, in search of generalizations. (Patton, 2002, p. 94)

Much like in grounded theory, the inductive analyst recursively codes the data looking for themes to emerge; however, analytic induction allows for an a priori proposition or theory driven hypothesis to be used as a lens to deductively analyze the data in contrast to grounded theory which begins inductively through open coding (Glaser & Strauss, 1967, cited in Patton, 2002). Patton (2002) notes that "after or alongside this deductive phase of [inductive] analysis, the researcher strives to look at the data afresh for undiscovered patterns and emergent understandings" (p. 454).

In the case of this research study, the key *a priori* theories used in the deductive phase of the analysis were that of Stefanou et al.'s (2004) distinction between types of autonomy support and Hiebert's (1997) five critical features of classrooms built for understanding. Other theories used in the analysis pertained to goals, self-efficacy, and anxiety in the context of mathematics education (Ashcraft, 2002; Bandura, 1997; Biggs, 1985; Dweck, 1986; Hannula, 2006; Jang et al., 2010; McLeod, 1992; Zimmerman, 2000). Once each case was analyzed and coded according to the *a priori* theories, a cross-case analysis was performed inductively to derive common themes across the data.

3.8. Risks and Limitations

The main risk in this research study pertained to the duality of my role as both instructor and researcher in the course. This risk was minimized by strict adherence to the process mandated by UFV's Research Ethics Board (REB) for an external co-investigator to collect data from participants during the term while I was away from the room. Students were assured that I would not have access to any data and would not be aware of who was participating until grades were posted so that they could feel more comfortable expressing themselves truthfully without the risk of their opinions affecting their course grade. However, minimizing this risk caused a limitation on the quantity of data that was collected during the term. The co-investigator was only available to collect data three times over the course of the term for about an hour each time. Also, I had no control over which participants were interviewed nor could I probe those participants further about particular areas of interest I might have had specifically regarding their interaction with the course. However, this limitation was accounted for by the observational data and the follow-up surveys. The benefits of having an external co-investigator collect data during the course outweighed the limitations mostly because students were able to more freely discuss their opinions about the course without being worried about affecting their course grade.

A major limitation in this study was the small sample size. The class size was small to begin with, and not all members of the class consented to participate in the study. This limitation was accounted for by providing a case study analysis in which cases were carefully selected to represent as wide of a population in the classroom as possible so that all successful members of the class were accounted for in the analysis through analysis of these particular cases.

Another risk during the course of the study was also related to my presence as both instructor and researcher. I developed strong bonds with some of the students, as is usual for me when I teach, but it caused participants to hear me refer to my ideals of mathematics education through aside comments that I might have occasionally made. This may have impacted some participants' beliefs and interactions with the course as they adopted my ideals through the trust they developed in me. This risk was of course minimized by the various sources of data and by the presence of an external co-investigator during data collection throughout the term.

Further, my role as both instructor and analyst created a risk in the analysis of the implementation of the desired flipped classroom model. It is difficult to analyze one's own actions truthfully without leaning towards the perception that the actions desired were in fact achieved. This risk was minimized by consistent reflection on the events occurring within each class through the process of observational data collection.

Finally, as with many qualitative studies, there is always the risk pertaining to the analysis and interpretation of data being dependent on my perception as the researcher. In this study, this risk was minimized by strict adherence to the analysis process outlined in the analysis section of this chapter.

4. Results and Case Analyses

As mentioned, after preliminary analysis of data, six participants were selected as cases to represent the three categories of interaction in the flipped classroom as detailed in *Table 1* below.

Table 1: Case Construction

Group 1	Students who completely engaged in both in-class and out-of-class components.	Alexa (A)
		Kristy (A+)
Group 2	Students who at first engaged in both in-class and out-of-class components, but chose to opt out of class time activities near the end of the term.	Mark (A+)
		Ryan (A-)
Group 3	Students who tried engaging in both in-class and out-of-class components, but as soon as they could opt out of the activities, they did.	Lindsay (B+)
		Vanessa (A-)

Note. All names are pseudonyms to maintain anonymity.

The equal distribution of cases among categories of interaction represented a relatively equal distribution of the 14 participants who completed the course via the flipped classroom among the categories: five students were categorized as Group 1, five students were categorized as Group 2, and four students were categorized as Group 3.

Further, the grades obtained by these cases were within the grade range obtained by the majority of the students in the class. This is evident in *Figure 4* below, which shows that students who completed the course most commonly attained a B+ or higher (11 students), only 3 students received a grade lower than that, and no students attained lower than a B-.

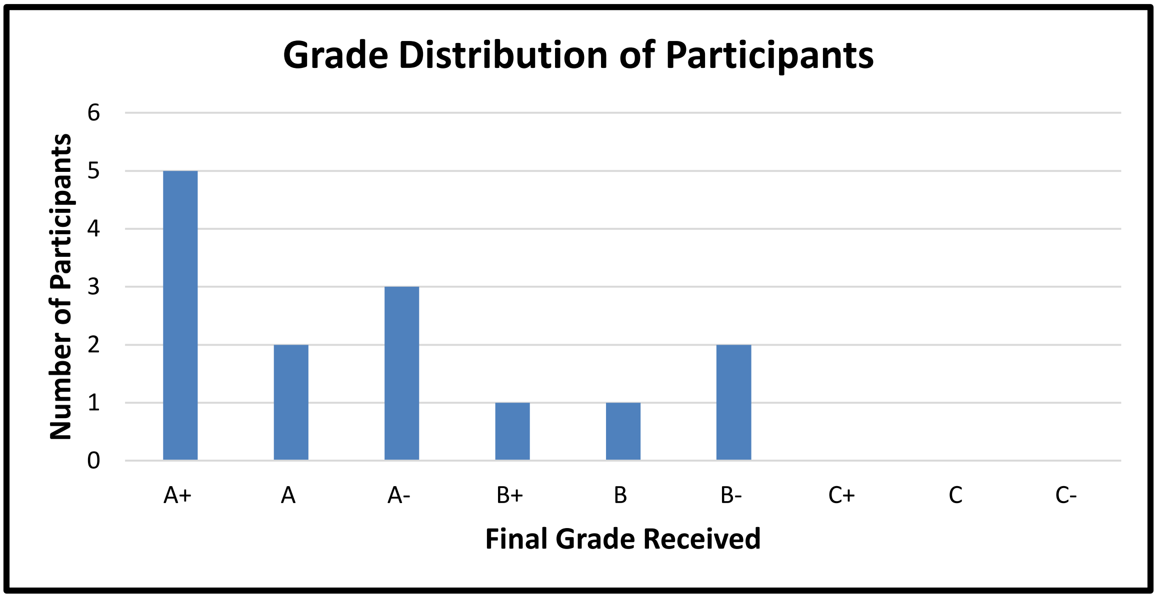

Figure 4: Grade Distribution of Participants

All six of the participants selected as cases successfully completed the course with a B+ or higher (See *Table 1*). This reflects the fact that students can be successful in a course and still experience it in various ways and at different levels of engagement.

For each case, I present key information as found in the data followed by the analysis of the case according to the conceptual framework presented earlier in Chapter 2 Related Literature. In particular, the analyses are informed by Skemp's (1976) definition of relational *understanding*, Hiebert's (1997) five critical features of a classroom built for understanding (*tasks*, *teacher as facilitator*, *culture*, *tools*, and *accessibility*) and the bases of these features (*communication* and *reflection*), Stefanou et al.'s (2004) three types of autonomy (*procedural*, *organizational*, and *cognitive* autonomy), Jang et al.'s (2010) idea of *structure* in the provision of autonomy, Dweck's (1986) theory of goal orientations (*learning goal orientation* and *performance goal orientation*), Bandura's (1997) definition of *self-efficacy*, and McLeod's (1992) definition of *anxiety*. As such, these terms will be presented using italics throughout the analyses of the cases to indicate that the terms are rooted in literature.

4.1. Alexa

Alexa was selected as a case in Group 1 because she completely engaged in both in-class and out-of-class components. She also attended every class and experienced the classroom environment positively. At the beginning of the course she introduced herself as someone who likes music, being outdoors, and singing loudly in her house.

4.1.1. Case of Alexa

Alexa enrolled in Math 084 to satisfy a prerequisite for the Radiation Technologist Program at BCIT. She had heard positive things about me as a teacher and was excited to work towards understanding mathematics even though she was still nervous from her past experiences with the subject.

Alexa described her past experiences with math as “scary, uninteresting, irritating, and frustrating” (Week 8 survey) which she attributes to having “bad experiences with math and math teachers” (Week 8 survey). In her interview with Danica (Week 8) she explained that she used to enjoy mathematics in elementary school when they had activities and colorful visuals, but after that period, her enjoyment and achievement in mathematics declined. She completed Math 10 eight years ago and failed Math 11. Her Math 11 teacher did not seem to care about taking the time to explain things to her and she lost interest. She tried to succeed, but found that she couldn’t keep up with material being explained on the board and she was not given room to test out her ideas. Receiving poor results and not being able to learn from her results, she developed a negative attitude towards mathematics based on her negative attitude towards her teacher. She was then placed in Math 11 Essentials, which she succeeded at and found to be extremely useful in everyday life. She claims that she would not have known how to do her taxes otherwise.

From my observations, Alexa thrived in the flipped classroom environment. She attended every class, participated completely in all in-class tasks, was often the leader of group discussions, and worked a lot on the material out of class time. Through her complete interaction in the flipped classroom, she was able to develop a new interest in mathematics. During her interview with Danica, she said, “I actually love my math class because of the way it’s taught this way, I look forward to school which I wasn’t thinking would happen" (Week 8 interview). The flipped classroom environment seemed to have a positive effect on her learning.

On the Week 8 survey, Alexa noted that out of class time she watches most of the videos, uses textbook examples, and completes suggested practice problems. She also noted in her Week 8 interview that she loved having the ability to pause and fast forward the videos depending on which topics she needed to understand more completely.

> I like these videos, it's on my own time, and I don't have to stare at the teacher's back. I have a really hard time reading the board half the time so I just sit there going, ok I'm bored, and I don't really understand this and some people are ahead of other people and some people are way behind. This way, I'm actually really caught up but I get to kind of go ahead or fall back as I like. (Week 8 interview)

On the follow-up survey, she continued with these thoughts.

> One section that I had trouble with, I constantly watched the videos over and over until I understood what was being taught. If it was an average classroom setting, I would only have had the textbook to refer to, which I find confusing at times. Plus I learn mathematics best by doing or watching someone else do equations, not by reading it. (Follow-up survey)

Alexa also agreed on the follow-up survey that the videos helped her understand the mathematics rather just helping her complete her homework.

> On some topics, I did take notes from the videos, but on most, I just understood what was being done and had no problems . . . [The videos] made understanding the material much easier because instead of asking a question that may have been asked over and over in class, I watched the videos to help me. (Follow-up survey)

When I talked with her during class in the middle of the term, Alexa explained to me how she developed her own method of studying over the course of the term. She noticed that she wasn't understanding concepts well enough when she didn't do enough practice questions at home, so she began to complete more practice questions than suggested because she wanted to understand and be prepared to ask questions in class. She told me that she found class time to be extremely useful. She also noted that most times, she liked watching the videos before class because they made her feel more confident with the material. However, something that was really notable about my observations of Alexa was that even when she hadn't watched the assigned videos before a class, she still engaged in the class time completely. For example, there was one lesson on factoring polynomials for which she hadn't watched any videos. She participated in the activity and exhibited confidence in the ability to learn how to factor polynomials. On a survey collected near the end of the term, she wrote, "I find the class time helpful to work with others and to brainstorm ideas on how to solve problems; sometimes I haven't watched all the videos so the group activity is a good start" (Week 14 survey). It was observed that any time that she was slightly behind, she exhibited strong motivation to get caught up as soon as possible in order to not miss out on any learning opportunities.

Alexa noted on the follow-up survey that she found the ability to use class time to clarify concepts with others to be the best part of the flipped classroom setting. She claimed, "I did my learning at home and came to class with questions to ask my classmates and then the teacher. I seemed to learn much faster this way" (Follow-up survey). She seemed to enjoy the independence she had with her own learning.

> When in studying for a lesson, I watch the videos and do practice questions. When I get stuck, I refer to the examples in the book, then I do graded questions. If I'm really stuck, I do everything to my best guess, bring it to class and compare with my peers. If we are all stuck, we ask Judy. (Week 8 survey)

Alexa also greatly enjoyed the social support that she developed in the classroom setting.

> I enjoyed being encouraged to sit beside different people in class in the beginning. I ended up meeting new people who we enjoyed each other's company and learned who wasn't exactly my favourite people to learn with. (Follow-up survey)

However, unlike some others in the class, Alexa found ways to learn even when she wasn't placed in a desirable group. That is, she did not become confused by other approaches during group work.

> If I was confused about anything, we would explain everything in great detail and have debates about it . . . I learned different ways to solve problems during the activities and others learned from me. This was fantastic. (Follow-up survey)

A good example of Alexa's ability to learn even when placed in a non-engaged group was when the class worked on an NCTM activity that involves figuring out how many rubber bands are needed to safely let a Barbie doll "bungee jump" from a certain height.

> I enjoyed Barbie Bungee Jump Activity because I was in a group that was distracted by the actual Barbie's and elastic bands. They decided to just give up and wait for the end of the activity. I was a little confused and frustrated until a little light bulb came on and I figured out how to make an equation all by myself. I ended up only having one elastic too many for poor Barbie. I then explained it to my teammates who kind of got the concept but just were not "into" learning math at that time. (Follow-up survey)

Alexa later expressed how impactful teaching others during group activities was on her self-efficacy.

> [Teaching others during group activities] really made me feel better about myself since I was the slower one in high school classes. I realized that I really can do these hard questions, most with no problem. (Follow-up survey)

Alexa also claimed that the activities helped her develop a stronger understanding of mathematics, in part, because she got to "talk it out" and in turn found that she understood more than she thought she did. The activities gave her room to try out ideas and to teach her peers what she has learned. I observed her teaching and collaborating with others in the classroom on several occasions throughout the term. This experience seemed to have impacted her greatly.

> When I first started this course, I just wanted it over and done with and to get through with a grade good enough to get into the next course . . . Since I failed Math 11 in high school, I felt stupid about math. I kind of gave up and always said "I suck at math". I've done much better than I thought here. Now I say, "I can figure this out, even if I have to Google it" . . . I've done much better than I thought here. I believe this flipped classroom setting contributed to this the most . . . I have honestly never enjoyed learning more. It was a very comfortable setting. I found it so low stress . . . I am actually excited, and a bit nervous to test my boundaries [in future mathematics courses]. I am only nervous because I will be going back to a normal classroom setting and I am not sure how well I will handle that. [However], this class overall made me feel much more confident in myself mathematically. (Follow-up survey)

Overall, Alexa attained an A in the course and serves as an excellent representation of someone who experienced the flipped classroom to the fullest extent. She engaged in all components both during class time and out of class time. It is evident in the data that Alexa benefitted from the flipped classroom environment. She also never exhibited or claimed to feel much anxiety about mathematics and noted that the environment was "stress-free" even when grouped with students not interested in learning mathematics. She was independent, had a strong work ethic, and exhibited motivation to succeed within this learning environment despite her past failures with the subject.

4.1.2. Analysis of Alexa's Case

Alexa participated completely in all components of the flipped classroom throughout the entire term and benefitted from the social environment of the flipped classroom. Over the course of the term, she *communicated* with others during genuine classroom learning tasks and participated in the classroom *culture* through helping others around her to *understand* mathematical concepts. In helping others, she found herself gaining a better *understanding*. In turn, she indicated a positive change in her *self-efficacy* in mathematics. Most importantly, she used the *cognitive autonomy support* that I provided her with constructively and respected my role as a *facilitator*. During times when she was provided with *organizational and procedural autonomy*, she *communicated* with others in the classroom if she needed help or if they needed help, and she *reflected* on her work before asking me for help. She also engaged in group learning *tasks* even if she had not watched the assigned videos because she found them helpful in her learning regardless of where she was in the material. She had the motivation to learn on her own and only required my help when she had exhausted all her other *tools* for learning. She highly appreciated the variety and accessibility of *tools* that were provided to help her learn. Outside of class time, she *reflected* on her work and devised a strategy for learning with the *tools* she was given. The *accessibility* of the tools provided her with o*rganizational and procedural autonomy support* out of class time. She used the resources to the fullest extent to which she needed to use them in order to learn. This is indicated by her strategy to complete more than the suggested number of problems in areas she felt she needed more understanding. It also indicates her strong *learning goal orientation*. However, she did not initially enter with this goal. When she entered the course, she just wanted to satisfy prerequisites for a career path. She was burdened by negative experiences with learning mathematics, and had low *self-efficacy*. She did not experience much *anxiety* throughout the term, but her *self-efficacy* definitely increased through the opportunities for engagement in the flipped classroom setting. She attributed the increase in *self-efficacy* with the relaxed *social environment* as well as all the *tools* that were provided in the flipped classroom.

4.2. Kristy

Kristy was selected as a case in Group 1 because she engaged completely in both in-class and out-of-class components. She also attended regularly and became a helpful member of the classroom community. At the beginning of the course she introduced herself as someone who likes sewing, sketch booking, and playing video games.

4.2.1. Case of Kristy

Kristy decided to upgrade her mathematics credentials because she thought she needed it towards her Bachelor of Fine Arts Program. It later turned out that she did not need the course as a prerequisite, but she decided to continue regardless. She was initially placed in Math 085 at the beginning of the term, but she chose to downgrade to taking Math 084 because she found Math 085 too challenging and she wanted to better understand the material. Initially, it was observed that she was shy and nervous about being in the class, but she soon seemed to find the environment to be comfortable and conducive to learning. On the follow-up survey, she noted that she now looks forward to taking Math 085 in the following term even though she does not need it as a prerequisite.

Kristy claimed in her initial survey that “up until this term, [she had] never liked mathematics and never grasped the concept.” She noted that in high school, she kept falling behind with notes, didn’t receive enough individual attention and was not shown things in a kinesthetic manner, which resulted in poor achievement. In an interview with Danica conducted near the beginning of the term she noted that she is “not a slow learner but a hands on learner.” Unfortunately, Kristy expressed on a survey that she had negative past experiences with learning mathematics in high school.

> It looked like a teacher talking – way too fast – for the duration of the class. I could never follow what he/she was teaching because I was too busy rushing notes to the paper. Because I was rushed, I not only didn't understand in class but also didn't understand my poorly written notes . . . I expected Math 084 to be just like the rest [of my past math classes] and I would have to struggle my way through it.
> (Week 3 survey)

Kristy did not find her teachers helpful in helping her understand the mathematics.

> Math was always my least favorite subject . . . The teachers never had time during lectures to assist students one on one. They basically spoke gibberish to us all class and sent us home to figure out what was meant.
> (Week 8 survey)

Based on my observations, Kristy thrived in the flipped classroom environment. She attended almost every class (except for legitimately excused absences), participated completely in all in-class tasks, was often the leader of group discussions, and worked a lot on the material out of class time. Through her complete interaction in the flipped classroom, she was able to develop a new interest in mathematics.

> Honestly, my goal was to pass. I hoped for a B, but I didn't expect it. I shot past my goal! I am walking out of this class with a high mark AND understanding math. I honestly never thought I would be able to understand math. I actually enjoy math now. (Follow-up survey)

Throughout the surveys, Kristy attributed her success in the course to several factors including the ability to progress through lectures at her own pace, the time available to discuss concepts that were troubling during class, the opportunity to teach others in the class what she had learned, and all the positive and constructive feedback she received in the course. In fact, she mentioned that her previous teachers "would only give feedback when [she] was not doing well, which made wanting to lean very difficult" (Follow-up survey). She claimed that both positive feedback and her ability to teach others in the class contributed to her increase in self-efficacy towards mathematics.

On the Week 8 survey, Kristy noted that out of class time, she watched all of the videos (sometimes more than once) and took detailed notes from them. She also noted that she re-watched the videos if she got stuck on a concept, and if she couldn't figure something out, she made note of it and moved on knowing that she could ask about it during the next class.

> Having the lectures in video form allowed me to study them at my own pace and take notes a lot more accurately. The option of being able to pause or rewind the video instead of asking the instructor to stop or repeat was great as well since it does not stop anyone else's learning process . . . . Since I am a visual learner, the detailed images in the video also made concepts much easier to understand, unlike white board chicken scratch . . . I never really knew what trigonometry was until I watched your videos and now it's easy! (Follow-up survey)

Most importantly, Kristy used the resources available to the fullest extent in order to engage with the material and be prepared to work more on it during class time. She made decisions about how much practice she needed to complete on each section claiming on a survey during the middle of the course, “If I feel strong on a concept I don’t do all the examples and if I feel weak I do more than the given” (Week 8 survey). She completed all of her assigned graded problems and completed all corrections diligently.

> I did all of my corrections but not all for the same reason. Some sections, yes, I did them for the marks only because they were silly errors not problems with grasping the concept. The other corrections I would have done even if they were not for marks because going over errors is a valuable way of learning. (Follow-up survey)

Not only was Kristy completely engaged with the material out of class time, it was also observed that she completely engaged with the material during class time. She only missed class when she had legitimate reasons and she never left early when she was in class. During class, she did express some concern to me regarding flexible deadlines and how it left room for unmotivated students to procrastinate and fall behind, but she was very motivated to stay on top of the material and didn’t, herself, fall behind. On a survey taken during the middle of the term, she wrote, “If I finish the work early, I have the opportunity to skim the next chapter and ask questions about areas I think I will struggle with” (Week 8 survey). Never once was it observed that she disengaged from the activities to work individually as others tended to do throughout the latter half of the term. Although she initially expressed concern about doing things the “right” way during the activities, she soon discovered that seeing multiple approaches is beneficial to understanding the concept. During the term it was observed that as her self-efficacy increased (likely due to positive feedback and the ability to teach others), she was more open to trying out various approaches to solving a problem. She summarized her engagement in classroom activities on the follow-up survey.

> Although I want to say that the at home lectures were the most valuable part of the class, the group activities played an equal role in how well I learned the mathematical concepts. Being forced (I use the term lightly) into group activities during class allowed me to get to know my classmates, which made me feel a lot more comfortable asking any questions I had. Secondly, the other ideas and approaches that students had towards problems allowed me to see different ways of understanding the questions and different techniques to use when finding an answer . . . [One of my favorite group activities was] the geometric shape creations and measurements [activity]. It is very easy to forget that there are six sides being added when finding the surface area (I used to always forget the top and bottom surfaces) . . . I took valuable information from every activity . . . Your Math 084 class actually allowed me to learn the concepts. The way I see it is, solving a problem is great, but being able to explain how the problem works means you truly understand it. I was almost testing myself by teaching others. It became another way of studying for me . . . I think I became much more outgoing as we did more and more activities because I was starting to get to know the class a lot better. Also, I was beginning to "know" math. I was starting to truly understand the concepts because I was able to study as much as I needed to since the lectures were always available to me. (Follow-up survey)

Kristy also commented on her, now, lack of anxiety.

> As for my anxiety, I now have none. In fact I am now very excited about my next math course starting in September. At the beginning of the term, math terrified me. Math was sort of one of those things that was "there to make everyone feel stupid." I honestly thought that math classes were only meant for super geniuses, and now I feel like one of those super geniuses. (Follow-up survey)

Overall, Kristy attained an A+ in the course and serves as a fantastic representation of someone who experienced the flipped classroom to the fullest extent. She engaged in all components in the classroom and out of the classroom. It is evident in the data that Kristy benefitted from the flipped classroom environment. Even near the beginning of the course, Kristy claimed in an interview, “I feel like I’m walking out of the classroom knowing something, I’m not just wasting my time trying to get a letter grade, I’m actually taking something away from the class too” (Week 3 interview). Watching this progression in Kristy was incredible. Regardless of the struggles she encountered throughout the term, she stayed motivated to keep on top of material. By the end of the term, she was pretty much my classroom assistant because she was on top of the material enough to use her class time to teach others and to help them succeed.

4.2.2. *Analysis of Kristy's Case*

Kristy participated completely in all components of the flipped classroom throughout the entire term. She benefitted greatly from the ability to *reflect* and *communicate* about mathematics during class time. Kristy was shy, highly *anxious*, and exhibited low *self-efficacy* near the beginning of the course. She noted that she greatly appreciated being placed in random groups (Liljedahl, in press) at the beginning of the course (an organizational *structure* I provided) because otherwise she would have been too shy to *communicate* with others. However, through her active interaction with others during genuine classroom learning *tasks* under my direction as a *facilitator*, she was able to seize opportunities to teach others. Through teaching others and by receiving positive feedback for her hard work, she developed positive *self-efficacy* and her *anxiety* greatly decreased. She became part of the classroom *culture* and encouraged others to learn the material by *communicating* her points of view to them. In this way, she utilized the *cognitive autonomy support* that I provided her with constructively, exhibiting a strong *learning goal orientation*. This *learning goal orientation* was new for her. She entered the class wanting to satisfy prerequisites for a program path, indicating a *performance goal orientation*. However, even after she found out she didn't need the course as a prerequisite, she decided to continue due to interest, indicating a *learning goal orientation*. Further, she always used class time completely, taking advantage of the *organizational and procedural autonomy support* I provided during the second part of class time. If she was finished the required work for the day, she would start on the next day's material.

Although Kristy responded well to *organizational autonomy support*, she did not initially respond well to *procedural and cognitive autonomy support*. At first, she was uncomfortable with not knowing how I wanted her assignment to look. She also reacted negatively when I probed her to think on her own. However, this quickly diffused as she came to understand my intent and her *goal* in the course shifted towards *understanding*. Eventually, she would come up with her own ways of solving problems and taught others before even asking me. In addition, she made good use of the variety of *tools* and *accessibility* to material provided. She watched every single video, sometimes repeating it, and always taking detailed notes. This was another thing she really appreciated because in high school, her main problem was keeping up with taking notes. This way, she could pause the video and think about the problems before continuing, which aligned with her new found *learning goal orientation*. If she was sick, she could use the videos to help her catch up. The flipped classroom experience was truly empowering for her.

4.3. Mark

Mark was selected as a case of Group 2 because he first engaged in both in-class and out-of-class components, but chose to opt out of class time activities near the end of the term when he wanted to get farther ahead with the material more efficiently. At the beginning of the course he introduced himself as someone who likes physics, playing guitar, rock climbing, and camping.

4.3.1. Case of Mark

Mark noted on his initial survey that he chose to take Math 084 "to get a better understanding of Math" because he is "just fascinated by how it works" (Week 3 survey). Even though he completed Math 11 and 12 in high school eight years ago, he noted that he did not find it enjoyable at the time and he found that he had forgotten too much of it when he recently attempted to complete a first year calculus course. In a survey taken in the middle of the term, he talked about this, as well as what had changed for him.

> Math really was one of my least favorite classes, mostly because of the way it was taught. I kept falling behind, so I didn't like it all that much . . . My interaction with math has changed greatly from high school. I wasn't as focused as I am now. Probably why it's hard to remember all the math from high school. But this class has filled in lots of gaps in my math knowledge. (Week 8 survey)

On the follow-up survey, Mark noted that he found the flipped classroom beneficial to his learning mostly due to the freedom to learn the content at his own pace and out of class time paired with the ability to bring questions for discussion in class. During the first half of the course, it was observed that Mark participated completely in all the components of the flipped classroom and was very inquisitive and engaged during class time. After about the middle of the term, he got a really bad case of the flu, which caused him to be absent for over a week. Prior to this, he had also missed a few classes due to missing his bus. He was still able to keep up with the videos, but he did so in his own way and not necessarily together with the other class members.

On the Week 8 survey, Mark detailed that out of class time he watched all the videos, took notes, re-watched parts he didn't understand, read through textbook examples, tried out the online quizzes, and completed assigned problems. Upon talking to him during class, I noted that he was actively engaged in course content out of class time throughout the term and that he was able to develop his own method of studying for a test by taking questions from each section and making a mock test for himself.

> The thing I find most enjoyable about the class is I don't feel pressure or anxiety about anything. All the content is posted and available and there is plenty of in class time for questions and clarifying. The online tests alleviate the stress of tests also by putting me in a test environment. (Week 3 survey)

At the end of the course, Mark agreed in the follow-up survey that he loved the videos because he could work at his own pace, the videos helped him understand the mathematics rather than just helping him complete his homework, and that the flipped classroom model allowed him to set his own goals and complete them.

> Being able to work at home through the material at my own pace was the best part about this class. [I took detailed notes from the videos] because I found it easier to follow the material and reference back to it. I focused on understanding the procedures and then I just memorized [them] because I understood them so well.
> (Follow-up survey)

The videos allowed Mark to keep up with the material in the course even when he missed classes. However, it was observed that when he began to miss classes, he began to use class time to write make-up tests. Once this started, he began to work more individually during class time. In a survey completed near the end of the term, he noted, "Yes I used class time more for doing homework [as the term progressed] so that I could ask questions" (Week 14 survey). He clarified this later claiming, "Near the end of the term, the topics we were doing I was very familiar with and I wanted to get ahead on my homework so that I could go back and check and think of any questions I could ask before the final" (Follow-up survey). It is interesting to note that some days I observed Mark being slightly behind and not participating in an activity because he hadn't yet watched the videos, while other days I observed that he was slightly ahead and also not participating in an activity because he wanted to make sure he found questions he may want to ask about in future material. It is possible that the class was a little too easy for him given that he had already completed higher mathematics levels. However, he noted in passing during one class that the extra time he was able to spend on topics that had troubled him in the past provided an extremely valuable opportunity.

To showcase Mark's search for understanding, it is worthy to mention a few of his observed classroom interactions during the first half of the term. One of these was when he completed his assigned work early and started working on a mathematics contest problem in class that he had found posted in the hallway together with another classmate. These problems are posted every month for mathematics students throughout the university. The particular problem was of finding the shortest distance that a spider, who is located in the middle of a 10' x 10' wall of a 10' x 10' x 25' rectangular room, has to travel in order to reach a fly that is in the middle of the opposite wall to the spider, one foot above the floor. Mark and the other classmate created a paper model of the problem so that they could think it through more easily. Although they did not complete the problem, the thinking process they underwent in trying to understand the problem was notable.

Further, Mark always asked questions that demonstrated his desire to test his own conjectures and search for generalisations. One example of such a question, noted in the observational data, was when he inquired about whether there existed a general method for finding the domain and range of any function after he determined the domain and range for a few rudimentary functions.

Mark was also often observed attempting to complete activity problems in several different ways and working collaboratively with others, encouraging them to think in various ways. In his follow-up survey, he noted that his favorite type of activity was "one that allows you to come to the same solution but with multiple paths" (Follow-up survey). Based on my observations, he thrived within activity problems that were open-ended because he worked towards creating difficult scenarios in order to challenge himself. One example of this was when he created a very complicated three dimensional shape consisting of a cone nested within a cylinder with a half-sphere on top (See *Figure 5*). He then encouraged his group to figure out the volume and surface area of the shape. We hadn't learned how to find the surface area of a cone, so it led the class to learn more than was expected. Combining several shapes also gave students the opportunity to learn how to alter formulas they had learned.

Figure 5: Student Generated Example

Another example of Mark's tendency towards creating interesting examples was when he came up with a complicated expression that simplified easily because the exponent was zero (see *Figure 6*). This indicated his comprehension of the outcome of taking an expression to the power of zero.

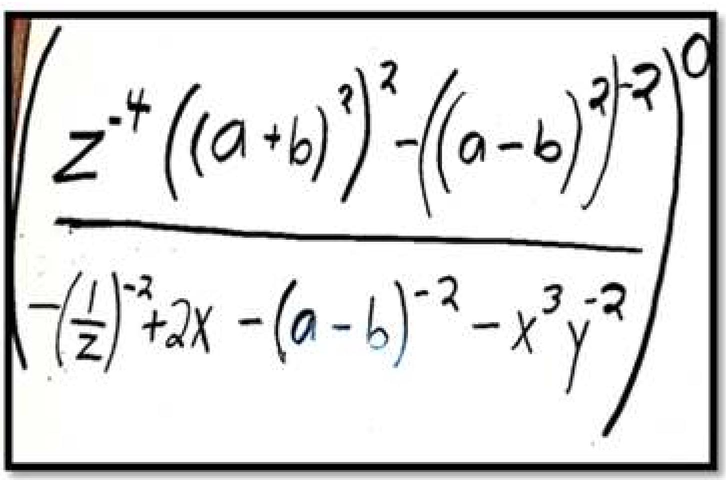

Figure 6: Student Generated Example

Mark was also interested in developing reasoning. In a survey during the early part of the term, Mark reflected on an activity that asked students to justify reasoning for various exponent rules on the whiteboards in groups. He claimed that the activity was "very helpful in understanding the way rules for exponents work instead of just memorizing them" and that that is his "favorite way to learn things" (Week 3 survey).

As mentioned, after a series of absences due to being sick in the latter part of the term, it was observed that Mark began to opt out of activities and worked on his own in the back of the classroom. During these times, he took the liberty to choose when to engage in the entire class and when to engage in his own work. He did this by looking up when something interesting was happening and looking down at his work when he felt he didn't need to be paying attention. It was noticed that he began to take his own learning into his own hands, setting his own goals, and then achieving them.

Further, it should be noted that Mark expressed in his follow-up survey that he began to feel more confident in his ability to successfully complete mathematics problems mainly because he was able "to pull a problem from any unit and complete it," and if he got stuck, he had "instant help in class while it was in the forefront of [his mind]" (Follow-up survey). This can be seen as an increase in self-efficacy.

Overall, Mark attained an A+ in the course. He showed complete interaction with the flipped classroom during the beginning of the term, but became more motivated to work individually after missing a few classes in the second part of the term. However, it is evident in the data that Mark benefitted from the flipped classroom environment. His favorite part about the flipped classroom model as stated on his follow-up survey was that he could "come to class with questions and actually get the questions answered instead of being stuck out of class time" (Follow-up survey). Due to his inquisitiveness, Mark was a pleasure to work with even though he was at times absent and often chose to opt out of class time activities during the latter part of the term.

4.3.2. Analysis of Mark's Case

Mark participated in all components of the flipped classroom until about two thirds of the way through the course, when he began to opt out of class time activities. Interestingly, Mark exhibited a *learning goal orientation* right from the beginning with his original intent for taking the course being to get a better *understanding* of mathematics.

He also had positive *self-efficacy* and low *anxiety*, neither of which changed over the course of the term. During the beginning of the course, he readily *communicated* with others and was intrigued by the genuine classroom learning *tasks*, using all *tools* that were available to him. He participated in the classroom *culture* by proposing interesting ideas to others and helping them with their work. He truly engaged in the *cognitive autonomy* that was provided. He also found my role as *facilitator* to be particularly useful because he could ask me about concepts that he didn't understand, which often resulted in an interesting mathematical discussion that others would listen in to. Further, Mark engaged in the *organizational and procedural autonomy* that was provided because he became more and more independent in his thinking and learning throughout the term.

As mentioned, after being sick for a while and being away from class, he began to come to class without engaging in classroom *tasks*. Due to his absences and low classroom involvement, I perceived his actions as that of someone who had fallen behind in his work and needed to catch up. However, Mark was actually moving ahead. He wanted to learn further material so that he would know what to ask questions about. He made good use of the out of class *tools* such as the videos that were readily *accessible* to him in order to engage autonomously with the material through *reflection*. However, he began to participate less and less in the genuine classroom learning *tasks* and classroom *culture* as the term neared completion. His *goals* were still predominantly of learning, but he did present some goals of *performance* within the need to complete the course requirements. Overall, Mark's experiences with the course were very positive because even when he was sick and had to miss class, he was not greatly inconvenienced by it because of the *accessibility* of learning *tools*. The environment allowed him to hone his mathematical skills and made it possible for him to maintain his high *self-efficacy* and low *anxiety*. He attributed these benefits to the ability he was provided with to work at his own pace and to ask questions of his peers or his teacher right when he became confused with something while it was still fresh in his mind.

4.4. Ryan

Ryan was selected as a case in Group 2 because he first engaged in both in-class and out-of-class components, but then chose to opt out of class time activities near the end of the term after becoming overdrawn by other priorities in his life. At the beginning of the course he did not provide information regarding his hobbies, but he noted his interest in psychology.

4.4.1. *Case of Ryan*

Ryan enrolled in Math 084 to satisfy a prerequisite for an undergraduate statistics course in order to get into a psychology degree program. He noted in an interview with Danica in the middle of the term that he takes interest in applications of mathematics to other fields and even considers working towards a minor in mathematics because of how valuable of an asset he sees it in getting a job. Although he expressed interest in mathematics, he also noted that he is highly motivated by grades and deadlines. Ironically, this meant that he was not successful in high school mathematics.

In the interview, Ryan also noted that it takes him longer to work through material than the average student and that this caused him to fall behind a lot in high school mathematics courses even though he had no problems with mathematics in elementary school. He further noted that his high school mathematics teachers moved too quickly through the material and he couldn't keep up.

> I disliked math in high school. The reason is because I felt I was very bad at it. I never could keep up with the concepts and it often felt like there was no continuity between concepts. (Week 8 survey)

On the follow-up survey, he claimed that the only reason why he passed Math 11 in high school was because "there was little focus on tests and the largest share of the grade was tied up in simply completing assignments" (Follow-up survey). However, he often didn't complete assignments if he didn't understand the material.

In the interview conducted with Danica in the middle of the term, Ryan attributed his success in the flipped classroom to his ability to learn at his own pace. At the end of the course, he claimed that his "experience with math has gone far better than in the past" (Follow-up survey). Ironically, it was observed that he resorted to only completing what was absolutely required by the latter half of the term when he realized that he didn't have to complete all in-class activities to obtain his grade. He did have a few setbacks during the term with the passing of his grandfather and English assignments taking precedence over his mathematics work. This caused him to shift from using the classroom environment completely to treating it more like a self-paced learning environment.

At the beginning of the term, Ryan was observed to be very inquisitive and completely engaged in all components of the flipped classroom including group work activities during class time. He seemed to enjoy working with others. He asked constructive and insightful questions, such as inquiring about why line equations are written in slope-intercept form or why an exponent of zero makes an expression equal to one. On a survey collected early in the term, he wrote, “I do find myself understanding math concepts that have eluded me in the past” (Week 3 survey). Early in the term, he told me about how invaluable the ability to stop lectures and listen to them at his own speed was. He spoke about how there was less wasted time with this model and on a survey in the middle of the term he wrote, “most importantly, [the flipped classroom] allows students to do their own time management by skipping through easy concepts and taking time on more difficult ones” (Week 8 survey). By the middle of the term, he noted that he was only watching some of the videos, scrolling through the examples without taking notes, and it was observed that he was only completing the required graded problems and not any practice problems.

> At first I would watch the videos and overview the homework. However, now I only watch the videos if I’m really stuck on a concept. I normally just read through the step by step break down in the text. As it is a lot faster and normally more convenient than having to access and then skim the videos to find the part that I need help with. (Week 3 survey)

This strategy led Ryan to “very slowly [fall] behind” (Follow-up survey). On the follow-up survey he agreed that he tended to avoid coming to class when he was behind because he “felt [he] could use [his] time more effectively outside of class, rather than covering more material [that he] would not understand” (Follow-up survey). It was noted that Ryan began to participate less in classroom activities as he fell behind with the material. In an interview with Danica conducted at the beginning of the term, he noted that he liked class time because it allowed him to work with others. In an interview with Danica conducted in the middle of the term, he mentioned that group work was enjoyable only when everyone was participating, and that he found that some of the groups he was in didn’t have solid enough group members. He expressed on the follow-up survey that some activities were more relevant than others and that he “didn’t really enjoy the activities that involved physically building things or that had hypothesis testing as much” (Follow-up survey). When asked about how he felt about working with others, he claimed, “I could take it or leave it. I would say how informal and the pacing was the best part” (Follow-up survey). I often observed that he preferred to work on paper with another mathematically strong member rather than in a random group on a white board dealing with an abstract problem. I also didn’t notice him explaining concepts to other students very often as much as some of the other students in the class did.

As the term progressed, it was observed that Mark began to focus solely on completing graded problems during class time rather than participating in activities he claimed that he was not ready for. This happened as Mark slowly fell behind with the material. By the latter part of the term, he mostly asked questions that clarified procedures he knew he had to learn and tended to work on his own, only communicating with me if he needed to. Near the end of the term, he didn’t always come to class, often missing a few lessons in a row. When he did come to class, he read examples out of the textbook and then tried to attempt the corresponding graded problems. By the end of the term, he noted that “it feels like I’m memorizing the steps rather than understanding them” (Week 14 survey). He also wrote in the survey that “at some point there should be a cut off to provide at least some extra motivation to people who put things off” (Week 14 survey). He expressed worry to Danica in an interview conducted late in the term about the flexible deadlines and how they are the cause of people falling behind.

Even though he was a couple units behind at the end of the term, he made a plan with me to catch up with his work and reschedule some of his tests. He followed through with these plans even completing the final exam a day early. During this time, he worked on his own a lot and predominantly used the textbook examples and some of the videos if he needed help in understanding a concept. I was impressed by his spike in motivation during the last week of the course. He seemed to realize that he needed to fulfill the requirements and held the belief that he was able to complete the material.

Overall, Ryan attained an A- in the course. He evidenced complete interaction with the flipped classroom during the beginning of the term, but became less motivated to complete unnecessary components of the class once he fell behind with the material. In the follow-up survey, he deemed his experience in the class to be better than past classes, but noted that he thinks he could have done better had there been a bit more structure in the course. He also claimed in the follow-up survey that he never experienced anxiety with learning mathematics, but rather experienced a disappointment in his ability to understand the concepts. He noted that he used to think he was bad at mathematics, but now feels better about it and believes that he can succeed in future mathematics courses.

4.4.2. Analysis of Ryan's Case

Ryan participated in all components of the flipped classroom until about half way through the course, when he began to opt out of class time activities. This occurred after some absences, which caused him to slowly fall behind with the material. Due to his predominant *performance goal orientation*, he began to use the *tools* available and the class time in a manner in which he saw most efficient to completing the course successfully.

Ironically, Ryan initially exhibited a *learning goal orientation* in the beginning of the course. Initially, he was participating in all the components of the flipped classroom, *communicating* with others during classroom *tasks*, engaging *cognitively* with the genuine learning *tasks* provided, and he was utilizing my role as *facilitator* by asking me deeper questions that allowed him to make connections between mathematical concepts. During this time, he was making good use of the *cognitive autonomy support* he was being provided with. He participated in the classroom *culture* of learning, but mentioned that he was indifferent to the ability to work with others. The *organizational structure* provided him with the opportunity to *communicate* with others during the beginning of the term. However, as he fell behind with the material, he utilized the *organizational and procedural liberties* as well as the *accessibility* of learning *tools* to work independently through *reflection*, but with little *communication*.

Ryan's ultimate goals of achieving the prerequisite he needed became clear when he began to opt out of components of the class that he found would not affect his grade. In this case, that meant he began to avoid coming to class when he was behind, and when he did come to class, he opted out of group activities, preferring to work on his own and ask for help when needed.

Meanwhile, in his out of class time, Ryan used the *tools* that were *accessible* in an *organizationally autonomous* manner because he only watched videos on topics he needed more clarity on, and didn't take notes unless he needed to. He no longer experienced falling behind during a lecture as he did in high school because he could pause the videos and pick out the parts he needed help with. Most importantly, he found the flipped classroom more efficient because of the *accessibility* and variety of learning *tools*.

Further, he did not have any *anxiety* to begin with. This remained, and he was able to slightly improve his *self-efficacy*, indicated by his reference to a stronger belief in his ability to succeed in future mathematics courses.

4.5. Lindsay

Lindsay was selected as a case of Group 3 because even though she initially tried engaging in both in-class and out-of-class components, she soon opted out of class activities after falling behind with the material and realizing that the activities were not required towards course completion. At the beginning of the course she introduced herself as someone who likes writing, hiking, travelling, and spending time with animals.

4.5.1. Case of Lindsay

Lindsay enrolled in Math 084 to satisfy a prerequisite for the animal health technology program at Douglas College. She had recently taken the preceding intermediate level mathematics upgrading course in our department, which was offered in a multi-level learning center environment, and noted in an initial survey that she was looking forward to having more structure in this course than in the previous one.

Early in the term, it was physically observed that Lindsay held high anxiety towards mathematics. Through conversations with Lindsay, it was also observed that she had low perceived ability and a discomfort in asking questions.

> In high school, I hated math and took very little of it. It was just the subject I felt the least good at. Some units were better than others, but I just felt that math was frustrating and confusing. (Week 8 survey)

Lindsay noted on the follow-up survey that although her primary goal with Math 084 was to get a good grade and complete her prerequisite requirements, the flipped classroom environment was beneficial for her because it helped her learn how to ask questions and provided her with enough material out of class time to work through and catch up with when she fell behind.

Lindsay noted that she enjoyed learning from the videos out of class time because she was able to "go through [each video] slowly and do the example questions one step at a time" (Week 8 survey). She also noted that she really appreciated the opportunity to "pause and rewind the video whenever" she needed to (Week 3 survey). At the end of the term, Lindsay wrote, "The ability to watch lessons at home and at [my] own pace was probably the thing I liked the most about the class" (Week 14 survey). On the Week 8 survey, she noted that she watched every video in great detail, took notes from the videos, and paused the videos in order to try the example questions on her own before proceeding with the video. She also noted on this survey that she referred to textbook examples often and tested her understanding by completing the online quizzes. It was observed that when she didn't understand a concept well, she gravitated towards re-watching the videos before asking any questions. This was evidenced by the reasoning she provided me when she left class early, which was to go watch videos in the library.

It was observed that Lindsay tended to work individually and as a quiet observer during class time. She tried engaging in the group activities (as described in 3.3.2 Class Time) during the first third of the course, but it just so happened that she was randomly placed into a group of weaker students several times over the first few weeks. Like Lindsay, the students in these groups were also catching up with the material and were not necessarily benefiting as much from the activities than those groups who were more on track with the material. This did not contribute well to Lindsay's perceived usefulness of the group activities as evidenced by the quote from her Week 14 survey below. Lindsay always seemed overwhelmed during group activities and tended to act as an observer. When placed randomly into a productive group, I observed that she would quietly listen to others writing down their ideas on the white board without speaking much. It was as if she interacted with them inaudibly. When I was in her proximity, she would often ask probing questions seeking confirmation of the work her group was doing. She didn't seem confident in her ability to do mathematics.

During the latter part of the course, it was observed that Lindsay began to use class time even more individually. As the material became more difficult, Lindsay began to be absent more often. She soon fell behind with the material and began to treat the flipped classroom as a place to learn individually. During one set of consecutive absences, she emailed me to explain that she needed to stay home because she wanted more time to go over the videos and complete missing graded problems. When she did come to class, I often encouraged her to participate in activities with others in the class. She would do so, but would act as a passive observer of the group without really interacting. If I didn't encourage her to work with a group, she would sit in the back of the class working individually on her mathematics work in the textbook. Because she was often catching up with material, and rescheduling tests, she didn't seem confident enough with the current material to participate in the group activities completely. Her avoidance of group work appeared to have caused her avoidance of attending class.

> One thing I didn't really like was the amount of group work we had to do. Sometimes it was helpful but sometimes it seemed to complicate things because not everyone understood the unit . . . [As the term progressed], I used class time to hand in work, work on graded problems and do tests. I [made] sure when I [got] stuck on something to ask for help. (Week 14 survey)

Notably, it was observed that Lindsay began to exhibit more confidence in her abilities as she asked more questions and received positive feedback on her work. On a survey taken during the middle of the term she noted, "This class has helped me realize that asking for help more when I need it is OK" (Week 8 survey). During the latter part of the term, I noted that she would watch the videos in great detail and then would come to class to clarify concepts that she struggled with. I observed that most of her clarifications pertained to implementation strategies of the various procedures outlined in the videos and used in the textbook problems. These clarifications were very important for her.

> I feel like I have been doing much better in this class than I originally thought I would and I feel it is due to the flipped classroom format . . . There are a few things I have learned that I didn't know before. I think the online videos and review of them in class has helped most. (Week 8 survey)

Remarkably, even though it was noted that she was absent a lot during the last third of the term, she was able to complete the course with a B+ by watching the videos, completing examples from the videos, and completing assigned graded textbook problems. Based on observational data pertaining to physical evidence of anxiety during mathematical interactions with Lindsay, it was noted that she seemed to exhibit anxiety with mathematical situations less frequently near the end of the term than at the beginning of the term. It was also noted that in mathematical conversations with Lindsay, she seemed to hold slightly more positive beliefs of her abilities near the end of the term because she came across to be more confident and self-determined in talking about mathematical problems. This indicates a slight increase in self-efficacy. Although Lindsay engaged in the course in an individual manner, it proved to be more beneficial for her than the other completely individually paced course she had previously taken because she had a greater variety of resources available and was able to use class time to get help with her mathematics work not only from her instructor, but also from her peers when placed in an appropriate setting.

4.5.2. Analysis of Lindsay's Case

Although Lindsay tried to engage in all components of the flipped classroom, she quickly began to avoid components that required *cognitive autonomy*, such as the genuine learning *tasks*. Upon entering the class, Lindsay had negative past experiences with learning mathematics which contributed to her low *self-efficacy* and high *anxiety*. She also held a strong *performance goal orientation* with her main reason for engaging in the class being to satisfy a career prerequisite. Lindsay was also extremely shy and did not *communicate* much unless she absolutely needed to. The organizational *structure* of requiring students to work in random groups at the beginning of the term allowed her to experience a *culture* of learning, a *teacher* in the role of a *facilitator*, and genuine classroom learning *tasks* through some exposure to *communication* with others. However, during the times when she was asked to work with others, she tended to observe the others in the group rather than initiate discussion. She seemed uncomfortable with the *cognitive autonomy support* that was provided and often became confused by other students' approaches to solving problems. This was at times frustrating for her and it may have interfered with her *performance goal orientation* because it compromised the efficiency of learning the material. As soon as more *organizational and procedural autonomy support* was available, she chose to focus on the videos as her main learning *tool* and was grateful for their *accessibility*. She was able to *reflect* on her work on her own time, but rarely wanted to *communicate* about it during class. This caused her to avoid class time when she was behind in the material because she did not want to participate in activities for which she may not have been prepared for. Although she missed a lot of class time, she was able to complete the course successfully due to the availability of the videos. The flipped classroom seemed to be beneficial for her because as she noted, it helped her learn how to ask questions, and it contributed to slightly improving her *self-efficacy* and slightly decreasing her *anxiety* with mathematics based on observational data.

4.6. Vanessa

Vanessa was selected as a case of Group 3 because even though she initially tried engaging in both in-class and out-of-class components, she soon learned that she could opt out of class activities without affecting her grade, and so she did. At the beginning of the course she introduced herself as someone who likes running, doing yoga, playing soccer, and making crafts.

4.6.1. Case of Vanessa

Vanessa enrolled in Math 084 to satisfy a prerequisite for a nursing or dietician program. She initially openly expressed high anxiety towards mathematics, low perceived ability, and a strongly negative attitude towards mathematics in general. The last time she had been in a mathematics class was in high school five years prior.

> [Math classes were] a lot of textbook notes, awful teachers and no visual learning videos very boring and 90% of the math teachers I had weren't very helpful . . . I expected this class to be awful. I did not expect the online videos, which I find very helpful. Honestly, I thought it was going to be a terrible experience, based on my past math classes . . . Personally, I hate math, I've never understood it, but I'm trying to. (Week 3 survey)

Vanessa noted on the follow-up survey that although her primary goal with Math 084 was to obtain a decent grade and satisfy prerequisite requirements, she benefitted from the flipped classroom environment because of the social support she received and the relaxed atmosphere she encountered. It was noted that Vanessa was almost always present in class throughout the term, but tended to use class time individually. She did not fall behind much likely because of how goal driven she seemed to be. It was noted that she often commented in class to me and to her peers about how hard she worked at home. She expressed her preference for working on her own or with her close friends in class early on in the term, often avoiding group activities and discussions.

Vanessa told me in class that she diligently watched all the videos, took detailed notes from the videos, and worked through as many textbook practice problems as she possibly could. She commented about how helpful she found the videos. I observed that she made note of questions she had completed at home that she didn't understand and would ask me to clarify them in class. When I looked through her work, it was very neat, orderly, and contained a series of the same types of problems. She seemed to develop confidence by completing several of the same types of problems.

During class time, Vanessa seemed to learn best on her own. She did not seem to grasp anything that I reviewed on the board and the group activities often confused her. As mentioned, she chose to opt out of group activities early in the term and when she did participate in them, she felt very lost.

> I don't always enjoy the group activities. It's not because I'm anti-social, I just find I usually have more important work to complete so I don't really pay attention anyways . . . I just stop paying attention during the activity, and then I look down at the paper and I just yawn because it just decreases productivity . . . What I find most useful in class time is a controlled discussion about the latest material we've learned. What I mean by controlled is no shouting out answers and no discussion about "interesting" math that is irrelevant to what we're learning, I find it very confusing. I like having lots of time to work on practice and graded assignments. I like working on my own and asking questions when I need help . . . I really enjoy the videos and practice quizzes, and I love how helpful and encouraging Judy is. I also really like the freedom [in the class because] I feel more relaxed . . . I work very hard and do as much extra work as I can. I'm hoping all my hard work will pay off come exams. (Week 3 survey)

Vanessa did not seem to learn anything until she completed a problem related to it on her own and by herself. She seemed greatly focused on memorizing procedures that she learned in the videos. When topics did not have set procedures, she expressed having troubles pursuing them. For example, there was one unit that had a section on word problems. She openly claimed to me and to her close friends that she did not like word problems. It was noted that she completely avoided working through them, often telling herself that she cannot do them, and that she will just opt out of them on the test. However, it was observed that when she encountered a procedure based concept such as long division of polynomials, she worked extremely hard at honing her technique by practicing a lot of the problems on her own time.

Nonetheless, it was observed that she still became highly anxious on tests and often "blanked out" even on a topic that she mastered through extended practice, like long division of polynomials. During such times, she did not hesitate to ask me questions that would potentially remind her of the process she needed to apply. In an effort to relieve her anxiety, I gave her small hints. Often, the slightest hint helped her complete the question. Throughout the term, she required a lot of frequent reassurance that she was doing the mathematics correctly. She was very focused on learning efficiently in the sense that she strictly worked towards completing tests successfully.

Although Vanessa worked extremely diligently throughout the term, she maintained a strong negative attitude towards mathematics in general. This was prominent because she often openly made statements of negative attitude towards mathematics in front of me and her close friends in the class. Statements I heard her say included "I hate math," "I hate word problems, they suck," "I just want this to be over," "I'm horrible at this stuff," "I just don't think I'm confident enough," "I hate the activities," "I don't like not doing well on tests, makes me angry," etc. Nonetheless, she seemed to have enough drive to stay on schedule with the material and successfully complete the course.

Something that I think may have contributed to this new found motivation to work at mathematics was the healthy competition and social support I observed her receiving from the two other girls in the course to whom she gravitated. I noticed that they were the only ones she was able to work with without becoming confused. During class time, I often observed them comparing each other's work and explaining their thinking to each other. They also told me that they would get together out of class time to work on mathematics. This social support along with the videos and relaxed classroom atmosphere seemed to contribute to Vanessa's strong work ethic.

> My goal was to get a B in this class, you are by far the best teacher I've ever had. Honestly, I'm terrible at math but somehow you brought it out in me. I found the videos were very helpful and how relaxed you were helped too . . . The amount of time and effort I put into succeeding in this class astonishes me. I truly tried my best. Unfortunately, I'm not so good at tests so my mark is lower than what it should be considering I can help/teach others the work but freeze up during a test. (Week 14 survey)

Overall, Vanessa succeeded in the course with an A- through her tireless perseverance with completing lots of textbook practice problems and studying the videos. She avoided working on any problems that didn't seem like they'd be on a test and avoided working with others who she worried would confuse her. However, she did interact with a select group of students and studied required procedures with them. Her anxiety seemed to interfere with completing tests without forgetting some of the procedures she had learned. This seemed to get slightly better over the term as she seemed to build a little more confidence in her abilities in response to receiving positive feedback and the relaxed atmosphere of the class. However, she never seemed completely satisfied with her test results, even though they were at a B grade level. She also never seemed to completely extinguish her anxiety during tests. She did not appear to learn well in situations where she was not doing the work herself. Despite her negative attitude towards mathematics, Vanessa stayed motivated to complete tasks on time in order to successfully complete the course albeit without evidencing any pursuit for a deeper understanding of mathematics.

4.6.2. Analysis of Vanessa's Case

Although Vanessa tried to engage in all components of the flipped classroom, she quickly began to avoid components that required *cognitive autonomy*, such as the genuine learning *tasks* that she was invited to partake in. Upon entering the class, Vanessa had negative past experiences with learning mathematics that contributed to her low *self-efficacy* and high *anxiety*. She also held a *strong performance goal orientation* with her main reason for engaging in the class being to satisfy a career prerequisite. Further, she was verbally explicit with her negative attitude towards mathematics, often expressing negative attitudes when asked to engage in *cognitively autonomous tasks*. Although not socially shy, she was often shy with *communicating* her ideas to other classmates in random group situations. She only enjoyed working with a select few others in the class, and this is where she benefitted from *communication* and partial participation in classroom *culture*. She did not respond well to the *organizational structure* that I used in the class during the beginning of the term. She often refused to work on the group problems if she was placed in an unproductive group and she did not like to listen to whole class discussions because they compromised the efficiency of her learning. Instead, she seemed to prefer to work on her own.

Even though Vanessa avoided participating in classroom learning *tasks*, she attended class time very regularly and exhibited strong motivation to succeed in the course by using the videos, textbook practice problems, and her select group of classmates as tools for driving her learning. Although she sometimes was interested in *understanding* the material, she was mostly focused on memorizing the procedures shown in the videos and in the textbook. She often asked me clarifying *procedural* questions during class and appreciated my role as a *facilitator*. Her *performance goal orientation* proved to be somewhat of a barrier for her due to her high *anxiety* with mathematics, particularly during test situations. During tests, her working memory would be taken up by her *anxiety* (Ashcraft & Krause, 2007) and she would forget all the procedures that she had practiced so diligently. This would frustrate her and she was never satisfied with her results, often comparing them with her friends. Even though she performed well, she was unhappy with her results. She wanted to obtain perfect results because of how hard she worked.

Out of the class, Vanessa made good use of the *accessibility* of the videos as learning *tools*, watching them diligently and taking clear notes from them. She was astonished by how invested she became in succeeding in this course. This may have in part been due to the positive social support she received from her select classmates, and by the *organizational and procedural autonomy* she received to work at her own pace. Even though she worked at her own pace, she stayed on top of the material. She was highly motivated to obtain as high of a mark as she could possibly get, and therefore appreciated the efficiency of learning that was provided by the *organizational and procedural autonomy.* As alluded to in the presentation of Vanessa's case, she seemed to benefit from the flipped classroom by rebuilding some of her *self-efficacy*; however, her test *anxiety* did not seem to improve over the course of the term based on my observations.

5. Cross-Case Analysis

It is evident from the aforementioned cases that the flipped classroom implemented in this research study afforded students with an opportunity to engage in a variety of classroom elements. Some students participated completely, while others chose to utilize only certain elements of the classroom to aid their learning. As a result, students experienced the flipped classroom in different ways. In an effort to understand these varying experiences, it is necessary to consider the classroom features, the kinds of autonomy support, the types of goals, the levels of self-efficacy, and the intensities of anxiety experienced by the cases. In addition, an emergent factor highly related to how students experienced the flipped classroom that arose from the analysis of the cases was absence. This factor is also discussed. Finally, connections between the theories are considered.

5.1. Classroom Features

The flipped classroom implemented in this study as detailed in Section 3.3 Setting: Math 084 Flipped can be viewed as a manifestation of Hiebert's (1997) classroom that is conducive to building understanding. Hiebert (1997) asserted that a classroom designed for understanding needs to foster reflection and communication in learning through the implementation of [1] genuine learning tasks, [2] a teacher in the role of a facilitator, [3] a classroom culture of learning, [4] useful learning tools, and [5] equitable accessibility of all elements of the classroom. All of these features were made available to students in the flipped classroom; however, only a portion of the students chose to engage in all of these components. Therefore, each feature was experienced by participants in different ways.

5.1.1. Learning Tasks

Learning tasks that were provided for students consisted of in-class group activities and practice problems. In-class group activities (detailed in Section 3.3.2 Class Time) were genuine learning tasks as defined by Hiebert (1997) because they were assigned in the context of a group learning environment where communication and reflection could be facilitated. All students were encouraged to participate in these tasks during class time. Alexa and Kristy represent cases of those students who engaged completely in these genuine learning tasks, indicating them to be beneficial to their learning. They both found the opportunity to collaborate with others exceptionally beneficial to their learning and success in the course. However, not all students experienced these tasks in this way.

Some students eventually chose to opt out of group learning tasks (even though they enjoyed them initially). This often occurred after a series of absences. Mark and Ryan represent such cases. After returning from a series of excused absences, Mark opted out of group learning activities with the intent of moving ahead in the material whereas Ryan did so in order to catch up with the material. Mark still engaged in opportunities for collaboration within the class even when he opted out of certain activities. He chose to engage in learning tasks that helped him build meaning to the mathematics he was learning. This was unlike Ryan, who along with avoiding group learning activities, also avoided opportunities for collaboration and communication with others. His learning tasks became less and less conducive to developing meaning as he tended towards efficiency of learning. Aside from group learning activities, students also used textbook practice problems as learning tasks. These problems did not offer the same opportunities for students to explore mathematical ideas as did the group learning tasks. Rather, they provided students with problems on which they could apply routines they acquired from content sources such as the videos or the textbook. If these students did not chose to ask conceptual questions in class, they did not have the opportunity to attain a deep conceptual understanding of the mathematics they were performing. This was particularly the case with those students who avoided group learning activities from the beginning of the term.

From very early in the course, both Lindsay and Vanessa avoided periods of collaboration and focused on learning the associated mathematical procedures for each topic from the videos and the textbook practice problems. Lindsay's avoidance was revealed through her absences, while Vanessa's avoidance was exposed through her verbal negative attitudes. The way in which students engaged with genuine learning tasks determined their learning experiences in the class.

5.1.2. *Facilitative Role of Teacher*

My role as a teacher was also experienced in different ways. Those who attended class regularly (represented by Alexa, Kristy, and Vanessa) experienced my role as a facilitator more prominently than those who were often absent during the second half of the course (represented by Mark, Ryan, and Lindsay). When absent, students experienced my role as that of one who delivers content through videos. It was during class time students could experience my role as a facilitator of learning. When students asked questions, I facilitated their development of understanding and promoted student initiative as suggested by Hiebert (1997). I also encouraged a classroom norm of asking questions not only of myself, but of peers. I facilitated these discussions and prompted students to think about the connections throughout the material, giving time for reflection and communication of ideas as suggested by Hiebert (1997). However, since I promoted student initiative, some students took advantage of my role as a facilitator more than others. In particular, I often held interesting mathematical discussions with Alexa, Kristy, Mark, and Ryan because they would initiate such discussion. Alexa and Kristy initiated such discussions more towards the latter half of the course, whereas Mark and Ryan initiated such discussions more towards the beginning half of the course when they were still fully engaged. Lindsay and Vanessa never initiated conceptual discussions, but rather asked me procedural questions when they needed to. Therefore, the capacity to which students experienced my facilitative role depended both on their presence and their initiative. Alexa and Kristy experienced my role to its fullest capacity due to their consistent attendance and learning initiative. Mark and Ryan experienced my facilitative role partially because of their absences. Lindsay experienced my role more as one of content delivery because of her absences and lack of initiation of meaningful mathematical discussion. Similarly, Vanessa did not initiate conceptual discussions, but because she attended regularly and observed me facilitating the learning of others, she experienced my facilitative role partially. Student experiences of my facilitative role were related to their ability to communicate and reflect about mathematics.

5.1.3. *Social Culture of Classroom*

Engagement in the community culture was also experienced in different ways. It was related to students' desires to communicate and reflect about mathematics. The social culture of the classroom was geared towards collaborative learning as detailed by Hiebert (1997). However, some students participated in this culture more than others. Alexa and Kristy became key contributors of the classroom norm for learning through their consistent curiosity in understanding the mathematics rather than performing the mathematics. They were also social leaders in the sense that they wanted to participate in genuine learning tasks and they wanted to facilitate the learning of their peers. Benware & Deci (1984) found that students who learn material in order to teach it exhibit higher intrinsic motivation, greater conceptual understanding, and feel more active in the learning process. Mark, through his insightful questions and interesting examples, was also a contributor of the social culture of learning in the classroom. Even when he did not participate in the activities near the end of the term, he still taught others in the class. Ryan was initially engaged in the social culture of the classroom through his insightful questions, but he soon became more focused on his own individual learning. Lindsay was not interested in the social culture of the classroom whatsoever and only participated when she thought it was necessary. Vanessa also wasn't interested in the general collaborative classroom culture, but was interested in staying in collaboration with a select few classmates who formed a sort of sub-culture. Those who avoided involvement in the collaborative classroom environment were more focused on learning the material individually from the accessible learning tools that were provided online.

5.1.4. Learning Tools

Students were provided with a variety of learning tools during and out of class. The tools that were provided were designed to promote the development of understanding as detailed by Hiebert (1997). As mentioned earlier, students were encouraged to work together in teams, ask each other questions, ask me questions, work on whiteboards, use manipulatives, and even watch the videos on personal computing devices during class. Outside of class, they were provided with videos, online contact with their teacher and classmates, and textbook practice problems. All students used all of these tools at some point in the course, but only some of them continued to use all of these tools throughout the course. All participants made good use of out of class tools. However, only those who participated in group learning activities consistently (represented by Alexa and Kristy) made complete use of all learning tools provided. Mark also made good use of most of the tools because he would use his own initiative to take advantage of tools provided in the classroom to better understand the mathematics he was learning. However, Ryan, Lindsay, and Vanessa, through their individualistic focus, did not make complete use of the tools that were provided for them to construct meaning in mathematics.

5.1.5. Accessibility of Features

That being said, all students in the class had equitable access to all features of the classroom. In fact, they had autonomous choice over which classroom components they wanted to interact most with. Some chose not to pursue particular features, but the invitation was open to them. During class, I facilitated learning in a way that no ideas were diminished. In fact, I at times drew attention to mathematically incorrect comments, framing them as though they provided an excellent learning example, drawing positive attention to the students who offered those comments. In general, student ideas were equitably respected, and this practice was encouraged within the classroom culture. Further, work completed during class time was often accessible online for those who were unable to attend. Resources in the course website were updated frequently to promote accessibility of content. The ultimate goal was to encourage equitable accessibility of tasks, teacher, culture, and tools in the classroom through communication and reflection of ideas as promoted by Hiebert (1997).

5.1.6. Summary of Classroom Features

Overall, through a variety in engagement in classroom features, students experienced the classroom in different ways. The defining characteristic in the differentiation of learning experiences in the class is that of communication in addition to reflection. Those who chose to collaborate with others engaged in communication, a defining element of Hiebert's (1997) framework for a classroom designed for developing understanding. These students made good use of all of the classroom elements provided. Those who avoided collaboration with others lacked this element of comunication, and were left only with individualistic reflection. Hiebert's (1997) five critical features of a classroom designed for understanding are meant to work together as a system. Hiebert (1997) claimed that understanding is compromised when one or more of these features are absent. Since this study is not focused on measuring understanding, but rather describing student experiences of the flipped classroom, it can not be concluded that students who participated in all components of the classroom gained deeper understanding than students who didn't. However, the possibility is there and could leave room for future research in this area. Instead, degrees of student interaction with Hiebert's (1997) essential classroom components serve as an indicators of student experiences in the classroom and can be analyzed in relation to factors such as goals, self-efficacy, and anxiety in the following sections.

Table 2 below shows a summary of student interaction with Hiebert's (1997) five critical classroom features as evidenced by the cases presented earlier. In particular, it shows a bifurcation of experiences in Math 084 into complete and incomplete. Those who experienced the complete flipped classroom made use of all elements provided including the most important element, collaboration. Those who experienced an incomplete version of the flipped classroom lacked this element of collaboration because they focused on learning individually. Again, it should be noted that all six of the cases analyzed successfully completed the course with a grade of B+ or higher as discussed in Chapter 4 Results and Case Analyses. Therefore, this bifurcation is not related to achievement.

Table 2: Summary of Engagement in Classroom Features

	Alexa	Kristy	Mark	Ryan	Lindsay	Vanessa
Genuine Tasks	●	●	⭘	⭘	☾	☾
Teacher Facilitator	●	●	⭘	⭘	☾	⭘
Community Culture	●	●	●	☾	☾	⭘
Learning Tools	●	●	●	⭘	⭘	⭘
Accessibility	●	●	●	●	●	●
	Complete Flipped			Incomplete Self-Paced		

● high
⭘ occasional
☾ low

As noted, those who did not participate in all of the classroom elements tended to work individually, which is also referred to as self-paced learning. The defining characteristics of a self-paced learning environment in mathematics education include learning modules or units that students work through at their own pace, tests taken after each module or unit if the student feels ready, and a teacher who provides feedback on tests and acts as a tutor when needed (Schoen, 1976). In this study, the videos acted as interactive and motion packed learning modules. When students fell behind with the material, they progressed through the videos at their own pace and rescheduled tests according to this pace. Students were provided with detailed feedback on tests as well as teacher facilitation if they attended regularly.

However, the self-paced option did not offer students genuine learning tasks nor the teacher support of developing a collaborative classroom culture geared towards the development of understanding, which is primarily evoked through the presence of communication as mentioned earlier. It did, however, provide students with learning tools, a teacher in the role of a facilitator, and equitable accessibility of these elements. In a sense, one could view a self-paced classroom as a subset of the flipped classroom. Students had the autonomous opportunity to choose between these two learning structures based on how they chose to interact with the elements of the course.

5.2. Role of Autonomy

The aforementioned bifurcation in how the participants experienced the classroom was made possible, in part, by the autonomy provided in the structure of the flipped classroom's learning environment. Students had the autonomous opportunity to elect the ways in which they would interact with the elements of the course. Stefanou et al.'s (2004) framework of autonomy offers a lens for further analysis of this bifurcation. The framework proposes a differentiation between manifestations of cognitive autonomy, procedural autonomy, and organizational autonomy support.

5.2.1. Cognitive Autonomy Support

Students who experienced the complete flipped classroom engaged in a collaborative environment with rich tasks and tools guided by my role as a facilitator who promoted accessibility of information. They were supported in a cognitively autonomous way because they were able to "discuss multiple approaches and strategies, find multiple solutions to problems, justify solutions for the purpose of sharing expertise, be independent problem solvers, debate ideas freely, and ask questions" (Stefanou et al., 2004, p. 101). The opportunity for cognitive autonomy support was present throughout the entire term and the element of communication, as defined by Hiebert (1997), was crucial in the establishment of cognitive autonomy support in the class. The genuine learning tasks, the facilitative role of the teacher, and the social culture of learning were the most conducive to fostering autonomous communication, which contributed to cognitive autonomy support.

Those who engaged completely in group learning activities and were active members of the classroom culture of learning strongly demonstrated cognitive autonomy. Although Alexa and Kristy both initially felt uncomfortable with the freedom they were being given in mathematical reasoning, they quickly discovered the benefits of having the autonomy over the cognitive dimension. Mark also exhibited cognitive autonomy. Even though he did not participate in group learning tasks after a few absences from class due to illness, he still exhibited autonomy over the cognitive dimension by self-initiating his learning process. Mark, however, only exhibited cognitive autonomy during the first part of the course. His focus on efficiency near the end of the course took precedence over his willingness to explore mathematical ideas. Lindsay and Vanessa did not take interest in exploring mathematical reasoning at any point in the term. In fact, they avoided any prompting they received to discover why certain mathematical properties exist. Rather they were interested in clarifying necessary procedures. The tendency to focus on procedures rather than understanding can be seen as a desired for cognitive structure. In general, cognitive autonomy seems to be a defining characteristic of those experiencing the class as a complete flipped classroom that is conducive to building understanding.

5.2.2. Procedural and Organizational Autonomy Support

Unlike cognitive autonomy, procedural and organizational autonomy were not as consistently supported throughout the term. In fact, the organizational and procedural dimensions as defined by Stefanou et al. (2004) were rather structured throughout the first half of the term. With the intent to establish a culture conducive to learning, students were randomly assigned groups, asked to sit in different places, given due dates, provided with a manner in which to hand in work, given an example of which materials to use to handle materials, etc. Once the classroom culture was established, these elements were eventually supported more autonomously. According to Jang et al. (2010), the provision of structure for procedural and organizational elements is necessary as long as it is provided in an autonomously supportive manner. Stefanou et al. (2004) claimed that it is unclear as to how much structure or autonomy these dimensions require. However, Stefanou et al. (2004) did claim that cognitive autonomy support is essential in promoting engagement, and Jang et al. (2010) concurred with this idea based on how they described autonomy support. Therefore, students experiencing the complete flipped classroom were provided with cognitive autonomy support as well as organizational and procedural structure, which over the course of the term became more autonomously supported. *Figure 6* below shows a representation of this shift in procedural and organizational autonomy over the course of the term based on a subjective scale used to express my perceptions on the levels of procedural and organizational autonomy I provided in each month.

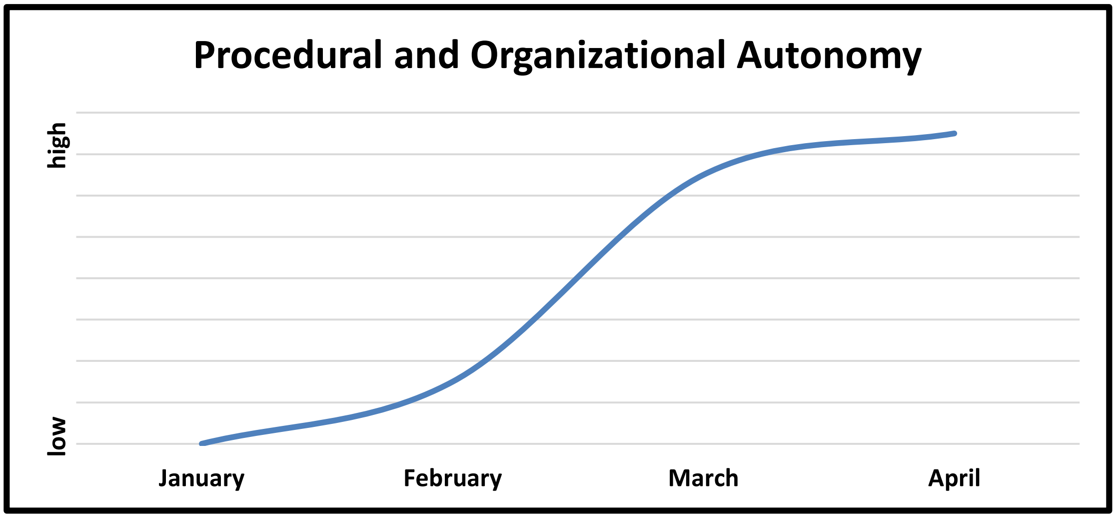

Figure 7: Procedural and Organizational Autonomy

It is interesting to note that the bifurcation of experiences began to occur for students at the same time that organizational and procedural dimensions began to be supported more autonomously. Students noticed the liberty they had and made choices in classroom interaction based on their needs. Those who found the cognitively demanding components of the classroom (Alexa, Kristy, and Mark), such as the genuine learning tasks, interesting, pursued them. However, those who saw that it was more efficient for them to complete course requirements (Ryan, Lindsay, Vanessa) used the liberties of procedural and organizational autonomy to engage in the classroom according to a self-paced model, focusing on the videos and the textbook rather than collaborative classroom tasks.

5.2.3. Summary of the Role of Autonomy

In summary, the presence of cognitive autonomy support was crucial in differentiating between those who experienced the flipped classroom completely from those who experienced the classroom as more of a self-paced learning environment. It also made a difference in the level of engagement in building understanding that students experienced. The procedural and organizational dimensions were initially structured in an autonomous way in order to set classroom norms. However, when these dimensions became more autonomous, students bifurcated into two types of classroom interaction: flipped and self-paced according to their desires for cognitive autonomy. *Figure 8* below serves as a subjective visual representation of students' expressed desires for either high cognitive autonomy, occasional cognitive autonomy, or no cognitive autonomy as coded from the case data in relation to the time in the term.

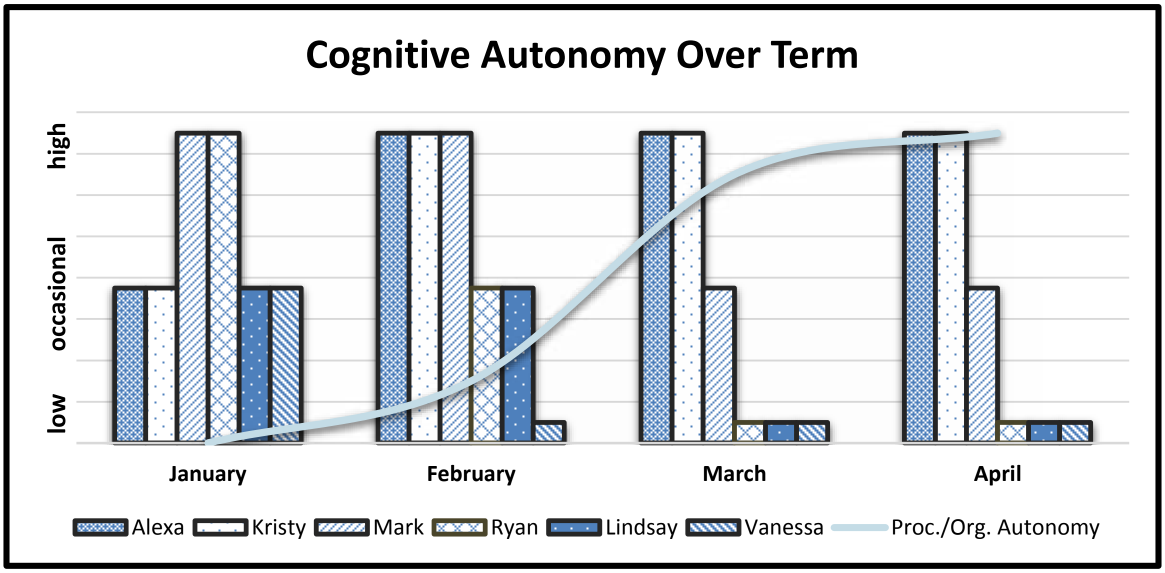

Figure 8: Cognitive Autonomy over Term

More generally, the students who interacted in the complete flipped classroom engaged in high cognitive autonomy support throughout the term, and the students who treated the classroom in a more self-paced manner opted out of cognitive autonomy support as soon as they knew that this was an option for them (See *Table 3* below). Although this result could be due to student acclimatization with the learning environment over the course of the term, it is more likely a manifestation of student goal orientations.

Table 3: Cognitive Autonomy Summary

	Alexa	Kristy	Mark	Ryan	Lindsay	Vanessa
Cognitive Autonomy	●	●	●	☾	☾	☾
● high		Complete			Incomplete	
☾ low		Flipped			Self-Paced	

5.3. Goals

The bifurcation between those choosing to engage in the complete flipped classroom and those choosing to treat the course in a self-paced manner can also be seen as a reflection of student goal orientations. Dweck (1986) defined goal orientations as either learning goal orientations, which are evidenced by the desire to understand, or performance goal orientations, which are evidenced by the need to gain affirmation of competence. Hannula (2006) posited that such orientations are often blended within individuals, with particular goal orientations dominating because of psychological needs and emotions in the learning process.

In the class under study, students with learning goal orientations were more willing to engage in the complete flipped classroom. In particular, Alexa and Kristy indicated high goals of learning. They found the collaborative activities enjoyable because of the opportunities the afforded such as testing out ideas, learning from others, and teaching others. Alexa and Kristy also pursued the learning activities regardless of whether they contributed to their grade of not, indicating low performance goal orientations. Mark also exhibited a strong learning goal orientation throughout most of the term except for after he missed classes due to illness and needed to focus on being able to successfully complete the course. The absences caused him to focus more on performance near the end of the course due to his psychological needs for course completion and advancement. Alexa, Kristy, and Mark all predominantly engaged in the course according to the complete flipped classroom model.

However, students with higher performance goal orientations focused on being efficient in completing the course, and therefore engaged in the classroom in a more self-paced manner. Lindsay and Vanessa both avoided opportunities to create meaning in the material, and therefore had low learning goal orientations. They both often expressed their need to complete the course successfully. Lindsay was shyer about it, but Vanessa was very verbal about her psychological needs for attaining high scores. Lindsay and Vanessa also both expressed appreciation and need for efficiency in learning. They did not want to participate in activities that would not directly lead them to knowing how to perform particular mathematical operations. They did not see value in understanding the concepts more deeply. Ryan, however, initially expressed goals of learning, but as soon as he was behind in the material, a dominant performance goal was observed. It is interesting to note that without observational data, it may not have been particularly evident that Ryan had a strong performance goal orientation because his learning goal orientation was also prominent and he often used learning and performance interchangeably when referring to learning in general. Although he often asked conceptual questions during class, his performance goal was observed when he began to opt out of components of the class that did not contribute to his grade. He also commented on his grades claiming that he wanted to attain a higher grade in the course than he did. Lindsay, Vanessa, and Kyle all predominantly engaged in the course in a self-paced manner by opting out of components of the classroom that did not affect their grades as soon as procedural and organizational autonomy was more prominent in the course.

Overall, goal orientations of learning were associated with engagement in the complete flipped classroom and goal orientations of performance were associated with participation in the self-paced approach of the flipped classroom environment (See *Table 4* below).

Table 4: Goal Orientations

	Alexa	Kristy	Mark	Ryan	Lindsay	Vanessa
Learning Goal Orientation	● high	● high	● high	○ occasional	☾ low	☾ low
Performance Goal Orientation	☾ low	☾ low	○ occasional	● high	● high	● high
		Complete Flipped			Incomplete Self-Paced	

● high
○ occasional
☾ low

5.4. Self-Efficacy

The prominence of past negative experiences in adults returning to study mathematics and the negative influence that past negative experiences have on self-efficacy (Zimmerman, 2000) contribute to the likelihood of low self-efficacy in a mathematics upgrading setting. Five out of the six cases exhibited initially low mathematical self-efficacy based either on self-reports on surveys or observational evidence. Increasing student self-efficacy is instrumental in developing persistence in learning and decreasing the likelihood of adverse emotional reactions (Bandura, 1997). Vieira & Grantham (2011) claimed that self-efficacy can be related to autonomy support in relation to a task. The flipped classroom as implemented in this research positively affected student mathematical self-efficacy, especially for those who invited cognitive autonomy support and experienced the complete flipped classroom.

In particular, Alexa and Kristy self-reported strong increases in self-efficacy. They both noted very negative experiences with mathematics in high school, and in turn, entered the course with low mathematical self-efficacy. It was observed that they pursued cognitive autonomy in the flipped classroom and felt comfortable testing out their ideas and teaching others around them. This was instrumental in boosting their mathematical self-efficacy. By the end of the term, their mathematical self-efficacy was very high. This was evidenced by their statements of confidence in mathematics on their follow-up surveys. In addition, Mark noted that he already had high self-efficacy when he entered the course, and this was maintained if not slightly increased throughout the term based on my observations and his self-reports.

Further, those who engaged in the course in a more self-paced manner (Ryan, Lindsay, and Vanessa) entered the course reporting low mathematical self-efficacy due to past negative experiences and were observed to experience somewhat of an increase in self-efficacy throughout the term as evidenced by some of their self-reports of “feeling better” about doing mathematics. Lindsay and Vanessa in particular entered the course with extremely negative past experiences and exhibited exceptionally low self-efficacy. Ryan also entered reporting low self-efficacy. Over the course of the term, through obtaining positive feedback on their work, Ryan, Lindsay, and Vanessa were able to have a more positive view on their abilities to perform specific mathematical tasks as evidenced by various observations and self-reports. Although they did not seem to attain particularly high levels of mathematical self-efficacy by the end of the term, somewhat of an increase in self-efficacy was observed in each of them based on their statements relating to their abilities to complete mathematical tasks.

Although all cases exhibited positive effects on mathematical self-efficacy, more prominent increases of mathematical self-efficacy were observed in those utilizing all class features compared to those treating it in a self-paced manner (See *Table 5* below). Overall, the flipped classroom as implemented in this study was beneficial with regards to self-efficacy.

Table 5: Changes in Self-Efficacy

	Alexa	Kristy	Mark	Ryan	Lindsay	Vanessa
Self-Efficacy	↑	↑	→	↗	↗	↗
↑ increased ↗ somewhat increased → continued		Complete Flipped			Incomplete Self-Paced	

5.5. Anxiety

The comfortable and unthreatening learning environment was generally conducive to relieving mathematics anxiety, however, it is difficult to make generalisations based on the limited number of participants who entered the course with strong mathematical anxiety. From the cases, the students who indicated high levels of anxiety with mathematics at the beginning of the course were Kristy, Lindsay, and Vanessa. The others (Alexa, Mark, and Ryan) did not indicate high amounts of anxiety at any point during the course. From those who initially felt anxiety, one experienced a complete elimination of anxiety with mathematics, and the others continued to feel anxiety with mathematics to a certain extent. The one whose anxiety was completely eliminated was Kristy, who engaged in the flipped classroom completely and experienced a great positive transformation in all aspects of learning mathematics. She claimed on the follow-up survey that she initially had extremely high anxiety with mathematics based on past experiences, and that she was leaving the course with no anxiety with mathematics whatsoever. This may be related to her increase in self-efficacy and development of a strong learning goal orientation. The other two cases who entered with high anxiety were Lindsay and Vanessa. These students did not participate in all components of the flipped classroom. They did not experience the same anxiety relief as Kristy, however, they did experience slight increases in self-efficacy. Lindsay displayed her anxiety with learning mathematics by avoiding class during times when she felt she would potentially be cognitively challenged. Vanessa did not experience anxiety with learning mathematics, but rather with writing mathematics tests. She battled with this on every test throughout the term and felt that if she did not have the test anxiety, she would have performed better on most of her tests. *Table 6* below provides a summary of changes in anxiety in the case studies according to self-reports and observations.

Table 6: Changes in Anxiety

	Alexa	Kristy	Mark	Ryan	Lindsay	Vanessa
Anxiety	⦸	↓	⦸	⦸	→	→
		Complete Flipped			Incomplete Self-Paced	

↓ decreased
↘ slightly decreased
→ continued
⦸ no anxiety

Overall, it is unclear from this particular study whether the flipped classroom model decreased mathematics anxiety or not. This may be a deeper issue that would require more thorough exploration.

5.6. Emergent Results

In addition to the factors detailed above, the specificity of the data and the process of inductive analysis allowed for unanticipated results to emerge. In particular, student attendance seems to play a strong role in how students experienced the flipped classroom. Further, there seems to be an interrelatedness between the theories used in this research.

5.6.1. Attendance

Although student attendance was certainly not considered as a mediating factor in a flipped classroom at the outset of this study, the analysis of the cases very clearly shows that absences were a strong mediating factor in having students slip into a self-paced experience of the class. Once students slipped into this mode of study, they did not return to engaging in the class completely.

Alexa, Kristy, and Vanessa attended regularly and maintained their interaction with the class throughout the term. Alexa and Kristy engaged completely throughout the term, while Vanessa opted to use the class in a self-paced manner early in the term. These cases had regular attendance and regular interaction.

Mark, Ryan, and Lindsay underwent significant periods of absence in the term. Mark and Ryan encountered uncontrollable challenges in their lives that extrinsically caused them to be absent, altering their interactions in the class, while Lindsay intrinsically chose to be absent because of her choice in interaction with the class. In particular, Mark missed several classes due to illness. After this period of absence, he began to work individually. At first, he used the self-paced mode of study to catch up with material he missed, but then he continued to use it in order to move ahead of schedule. Similarly, Ryan missed several classes due to a funeral, and then an English paper that took more precedence for him. After these periods of absence, Ryan used his class time in a self-paced manner in order to catch up with the material. He did not return to engaging in the complete class experience. Lindsay was also absent a lot. However, unlike the others, there was no significant external reason for her absence. She noted that she was absent when she was too far behind to participate in class activities. Absence seemed more like a coping mechanism for her.

In general, falling behind seems to be highly associated with absence. Some students fell behind because of extrinsically influenced absences, while others intrinsically chose to be absent because they had fallen behind. Falling behind can be extremely frustrating and can lead adults to withdraw from a course (McAlister, 1998). In the flipped classroom, procedural and organizational autonomy allowed self-pacing to be a management skill, a sort of coping mechanism for falling behind. Remarkably, students who fell behind were able to catch up through the use of the video resources that were provided as part of the flipped classroom. Had these students not been able to access content delivery materials out of class time, they may have not been able to complete the course with so many absences, which could have led to withdrawal or failure.

Table 7 below summarizes the students who had higher attendance and those who had lower attendance. It indicates that absence seemed to be generally related to self-pacing.

Table 7: Summary of Attendance

	Alexa	Kristy	Mark	Ryan	Lindsay	Vanessa
Attendance	●	●	☾	☾	☾	●
● high		Complete			Incomplete	
☾ low		Flipped			Self-Paced	

This emergent result indicates that the autonomy provided in the flipped classroom allowed for students who fell behind for either extrinsic or intrinsic reasons to use self-pacing as a management skill in order to catch up with the required material and complete the course. Although it may be a criticism that autonomy allows for self-paced behaviour, it should be seen as a benefitting factor because students who may have had troubles completing the course were able to do so. These students may not have engaged in opportunities for developing deeper understanding that are afforded by the complete flipped classroom, but they were able to successfully complete the course in a self-paced manner if that was all they were able to manage. This speaks to the importance of autonomy as a mediating factor in student experiences of the flipped classroom.

5.6.2. Connecting the Theories

In an effort to better understand student experiences in the flipped classroom, several theories have been used to analyze student interactions and visible qualities within the setting in the preceding sections. *Table 8* below presents a summary of these analyses.

Table 8: Summary of Analyses

	Alexa	Kristy	Mark	Ryan	Lindsay	Vanessa
Genuine Tasks	●	●	⭘	⭘	☾	☾
Teacher Facilitator	●	●	⭘	⭘	☾	⭘
Community Culture	●	●	●	☾	☾	⭘
Learning Tools	●	●	●	⭘	⭘	⭘
Accessibility	●	●	●	●	●	●
Cognitive Autonomy	●	●	●	☾	☾	☾
Learning Goal Orientation	●	●	●	⭘	☾	☾
Performance Goal Orientation	☾	☾	⭘	●	●	●
Self-Efficacy	↑	↑	→	↗	↗	↗
Anxiety	⃠	↓	⃠	⃠	→	→
Attendance	●	●	☾	☾	☾	●

● high
⭘ occasional
☾ low
⃠ no anxiety
↑ increased
↓ decreased
↗ slightly increased
↘ slightly decreased
→ continued

Complete Flipped Classroom	**Incomplete Self-Paced Classroom**
Communication	
Reflection	Reflection
Cognitive Autonomy	Cognitive Structure
Goal of Learning	Goal of Performance
Increased Self-Efficacy	

It may be noticed that there is an interrelatedness among the theories used in the analyses. Students who engaged in the flipped classroom more completely (Alexa, Kristy, and Mark) evidenced stronger learning goal orientations (Dweck, 1986), elected more cognitive autonomy (Stafanou et al., 2004), were less swayed by changes in organizational and procedural structure (Jang et al., 2010), experienced more desirable effects on self-efficacy (Bandura, 1997) and anxiety (McLeod, 1992), and attended class more frequently. In contrast, students who engaged in the flipped classroom less completely (Ryan, Lindsay, and Vanessa) evidenced stronger performance goal orientations (Dweck, 1986), avoided electing cognitive autonomy (Stefanou et al., 2004), were more influenced by changes in organizational and procedural structure (Jang et al., 2010), did not experience as clear desirable effects on self-efficacy (Bandura, 1997) and anxiety (McLeod, 1992), and tended to attend class less frequently. As such, connections may be seen between cognitive autonomy (Stefanou et al., 2004), organizational and procedural structure (Jang et al., 2010), higher self-efficacy (Bandura, 1997), and stronger learning goal orientations (Dweck, 1986). These connections may require further study, but they serve as indicators for the construction of an environment conducive to desirable learning conditions.

5.7. Summary of Analyses

From the above analyses, it is evident that students in the flipped classroom (as implemented in this study and detailed in Section 3.3: Math 084 Flipped) experienced the class in different ways. The class offered students an autonomous opportunity to engage in a variety of components such as learning tasks, teacher facilitation, community culture, learning tools, and accessibility to material. Most importantly, albeit it was not completely supported out of class, cognitive autonomy support was provided during class. At first, when procedural and organizational structure was provided in an autonomously supportive way, all students participated in all components of the class. As the term progressed, more autonomy was provided over procedural and organizational dimensions in the class. Simultaneously, a bifurcation of student experiences occurred. When procedural and organizational autonomy was provided, students split into those engaging in the flipped classroom completely and those interacting with it in a more or less self-paced manner. Some students downgraded to the self-paced option after a series of absences because they had to catch up with the material that they fell behind with. One of these students continued to be absent in an effort to learn the material more efficiently and avoid collaborative opportunities during class. This bifurcation into two types of learning experiences may be more prominently attributed to a variety in student goal orientations. Students who engaged in all components of the flipped classroom tended to exhibit learning goal orientations. Whereas students who treated the class in a self-paced manner (opting out of the more collaborative class components) tended to portray performance goal orientations. Further, those who experienced the complete flipped classroom tended to seek cognitive autonomy whereas those who used it as a self-paced learning environment tended to avoid cognitive autonomy, seeking cognitive structure. The key difference between those students who experienced the complete flipped classroom and those students who used it in a self-paced manner was the presence of communication and collaboration in the interaction with the genuine group learning tasks, the teacher as a facilitator, and the classroom culture of learning. Self-paced students avoided these element of the class, often viewing them as less efficient methods of completing the course. However, classroom opportunities for collaboration such as group learning tasks and a community of learners allowed for cognitive autonomy to be manifested in a more complete manner than if it was only provided by the facilitative role of a teacher.

Further, students also received varying benefits depending on their interaction in the class. Five out of the six cases entered the course with low mathematical self-efficacy. Those who engaged in the complete flipped classroom out of those five tended to exhibit higher increases in mathematical self-efficacy. The self-paced students out of those five also exhibited increases in self-efficacy, but the increases were categorized as slight increases because the students did not attain high levels of mathematical self-efficacy by the end of the term. Changes in anxiety were also observed. However, only three out of the six cases entered the course with reported mathematical anxiety. Only one of those participated in the complete flipped classroom, and her anxiety decreased so much that it was nonexistent by the end of the term. However, the other two anxious cases who engaged in the class in a self-paced manner continued to experience mathematical anxiety at the end of the term. Overall, more prominent benefits were found in students who interacted in the complete flipped classroom as opposed to those students engaging in it in a self-paced manner.

Essentially, the students who engaged in the complete flipped classroom were taking advantage of the collaborative elements of the class which provided them with opportunities to communicate and to be supported in a cognitively autonomous manner. These students held strong learning goal orientations and experienced positive effects on their mathematical self-efficacy. Meanwhile, the students who experienced the classroom in a self-paced manner focused on less collaborative components of the classroom where they could work individually and efficiently in an effort to satisfy their performance goal orientations. These students also tended to appreciate cognitive structure rather than cognitive autonomy. The bifurcation of student experiences in the class occurred half way through the term when procedural and organizational autonomy was more prominently provided. It is also interesting to note that once students downgraded to using the course in a self-paced manner, they did not return to using the elements of the course completely. Most importantly, students were able to successfully complete the course with a final grade of B+ or higher regardless of the manner in which they chose to experience the course. However, those experiencing the complete flipped classroom exhibited stronger benefits with regard to goal orientation and self-efficacy.

6. Conclusions

Current literature on flipped classrooms indicates that liberty over use of class time that the approach affords provides teachers with a platform for educational improvement (Brinkley, 2012; Bull et al., 2012; EDUCASE, 2012; Johnson, 2013; Tucker, 2012). Educators are implementing the flipped classroom in various ways depending on their teaching goals (Johnson, 2013). Tucker (2012) noted that “it's not the instructional videos on their own, but how they are integrated into an overall approach, that makes the difference” (p. 82). Students need to be given room to think and interact in an autonomously supportive way within a flipped classroom (Brinkley, 2012). Most importantly, the flipped classroom is a way for teachers to change their roles from teaching in a teacher-centered manner to teaching in a student-centered manner (EDUCASE, 2012). This was my initial motivation in implementing a flipped classroom. However, it is still unclear in the literature how the freed up class time should be best used when implementing the approach.

> The Flipped Classroom can create more classroom time to provide rich, meaningful learning activities . . . But what is the best use of this time to support student learning? This is a question that requires further research. (Johnson, 2013, p. 81)

Literature on flipped classrooms seems to allude to either using class time in a rich problem solving manner through the use of engaging activities (Brinkley, 2012), or in a self-paced manner through the use of learning resource packages (Johnson, 2013). Recommendations for class time often assume that teachers are in control of student interactions and literature often assumes that student engagement in activities will occur. For example, “instead of students listening passively to a lecture, [students] are engaged in hands on or active learning” (Brinkley, 2012, para. 2).

However, to think that students experience a flipped classroom in the same way is naïve. This research has shown that the flipped classroom, through the autonomy it affords, can create varying environments for students depending on their goal orientations, self-efficacy, anxiety, and attendance.

6.1. Answering the Research Questions

The main intent of this research was to describe how students experience a flipped classroom. After an in-depth analysis of multiple data sources through analytic induction, the research questions as stated in Section 2.6.1 Research Question can now be answered.

6.1.1. Experiences in the Flipped Classroom

How do students experience a flipped classroom?

In this study, the flipped classroom afforded the capacity for a collaborative student-centered classroom learning environment. It also provided students with the autonomous opportunity to choose ways in which they would interact in the class. In summary, students in the adult mathematics upgrading course Math 084 bifurcated into experiencing the flipped classroom in one of two ways: the complete flipped classroom and a self-paced classroom.

Students who experienced the complete flipped classroom engaged themselves autonomously in the collaborative learning tasks provided, the facilitative role of the teacher, and the social culture of learning in the classroom community. They also had the tools and accessibility to materials. For example, Kristy, a student who engaged completely in all components noted the importance of both components in the development of her conceptual understanding.

> Although I want to say that the at home lectures were the most valuable part of the class, the group activities played an equal role in how well I learned the mathematical concepts. (Follow-up survey)

On the contrary, students who experienced the flipped classroom as more of a self-paced classroom did not pursue engagement in such collaborative opportunities. These students often opted out of electing cognitive autonomy that accompanied opportunities for collaboration in order to pursue performance goals rather than learning goals in an effort to efficiently complete the course. Interestingly, this bifurcation of student interaction coincided with the increase in my provision of procedural and organizational autonomy. It also coincided with increasing student absences. Once students fell behind in the material or experienced a series of absences, they typically resorted to treating the course in a self-paced manner, an interaction that they continued until the completion of the term.

Both the complete flipped classroom and the self-paced option that the flipped classroom afforded were highly student-centered and allowed students to pursue their goals in the class. Although it is desirable for students to pursue goals of learning, it is not always what they desire. This speaks to the ever-present tension between student and teacher goals. It is also a good reminder of the fact that a goal cannot be forced onto anyone. Instead, the goal can be encouraged and nurtured through providing opportunities for developing deeper understanding if a student so desires. Although student understanding was not explicitly measured in this study, it is notable to mention that Alexa, a student who engaged completely in all components of the flipped classroom expressed how the flipped classroom helped develop her understanding of mathematics.

> I am walking out of this class with a high mark AND understanding math. I honestly never thought I would be able to understand math. I actually enjoy math now. (Alexa - Week 14 survey)

The flipped classroom in this study provided students with an invitation to pursue goals of learning without forcing it to be the only option. Students could still complete the course and satisfy the prerequisites they needed by interacting in a self-paced manner, but more importantly, those who became interested in developing deeper meaning in mathematics were given the opportunity to do so through the collaborative nature of the classroom learning environment.

6.1.2. Interrelated Factors in the Experiences

How are factors such as goals, self-efficacy, and anxiety interrelated with student experiences in a flipped classroom?

The bifurcation of student experiences in the flipped classroom proved to be interrelated with various factors such as goals, self-efficacy, and anxiety that were identified at the outset of this study. However, additional results emerged through the process of analytic induction. Attendance was also found to be an interrelated factor in the bifurcation of student experiences. Further, all of the abovementioned factors were found to be interrelated not only with the bifurcation of student experiences in the flipped classroom, but also interrelated with each other.

In summary, it was found that students who engaged in all of the elements of the flipped classroom, including opportunities for collaboration and cognitive autonomy (Stefanou et al., 2004), evidenced strong learning goal orientations (Dweck, 1986), high increases in mathematical self-efficacy (Bandura, 1997), indicative results for desirable effects on anxiety (McLeod, 1992), and more frequent and consistent class attendance. They were also less swayed by changes in organizational and procedural structure (Jang et al., 2010) than their self-paced counterparts. Students who chose work through material in a more self-paced manner did so by opting out of opportunities for collaboration and cognitive autonomy (Stefanou et al., 2004). This tendency towards interacting in a self-paced manner was often mediated by absences and coincided with an increase in procedural and organizational autonomy in the class. It should be noted that these students may have easily dropped out of the course had they not been provided with an extensive set of resources to help them complete the course as many adult students do when they fall behind in course material (McAllister, 1998). In general, it was found that students who resorted to treating the class in a self-paced manner tended to exhibit strong performance goal orientations (Dweck, 1986) with a focus on efficiency in completing required tasks. As more organizational and procedural autonomy was provided, these students tended to focus on completing the minimum requirements of the course. They did not experience as clearly desirable effects on self-efficacy (Bandura, 1997) as their complete flipped classroom counterparts did, and no conclusive results were found regarding their anxiety in mathematics.

However, it should be noted that all cases showed evidence of desirable effects of the flipped classroom on student mathematical self-efficacy. For example, Alexa, who engaged in the flipped classroom completely noted that she “enjoyed teaching others during the group activities” because it made her “feel better about [herself]” and made her realize that she “really can do [those] hard questions, most with no problem” (Follow-up survey). Even Lindsay, who treated the course in a completely self-paced manner noted that she had performed “much better in [the] class than [she] originally thought” and felt that the flipped classroom format allowed her to learn things she “didn’t know before” (Week 8 survey). Even though she often opted out of collaborative classroom activities, the opportunity to observe others collaborating around her was helpful in developing her understanding of certain concepts.

The positive effect of the flipped classroom on mathematical self-efficacy is an important result because it is found that self-efficacy is instrumental in academic functioning (Bandura, Barbaranelli, Caprara, & Pastorelli, 1996; Albert Bandura, 1989; Pajares, 1996; Schunk, 1989; Vieira & Grantham, 2011; Zimmerman, 2000). In particular, self-efficacy allows for persistence in learning (Bandura, 1997), more positive emotional reactions (Bandura, 1997), higher intrinsic interest (Zimmerman, 2000), and higher performance in mathematics (Siegel, Galassi, & Ware, 1985). In particular, autonomy is found to help elicit more positive self-efficacy (Vieira & Grantham, 2011), which is evident in this study.

The interrelatedness of relational understanding (Skemp, 1976), cognitive autonomy (Stefanou et al., 2004), learning goal orientations (Dweck, 1986), high self-efficacy (Bandura, 1997), and consistent attendance is also an important result. All of these factors proved to be related to student interaction in all of the elements of a complete flipped classroom that offered opportunities for communication and reflection (Hiebert, 1997) in a desirable manner. Further, this research supports the premise of Jang et al.'s (2010) theory that classrooms conducive to engagement give structure and autonomy. In particular, organizational and procedural dimensions (Stefanou et al., 2004) should be structured, while the cognitive dimension should be provided with autonomy in order to promote student engagement in opportunities for communication and reflection (Hiebert, 1997). This somewhat counters Stefanou et al.'s (2004) claim that organizational and procedural dimensions should be provided with autonomy as well as the cognitive dimension. However, the main result of this research contributes to these theories by affirming that cognitive autonomy is an essential ingredient in promoting student engagement in learning opportunities.

6.2. Limitations and Opportunities for Further Study

Although this study was conducted as a small scale exploration of six case studies in one particular implementation of the flipped classroom, the abovementioned results indicate that further research could be conducted on a larger scale to affirm the correlation between the abovementioned factors of relational understanding, cognitive autonomy, learning goal orientations, high self-efficacy, and consistent attendance. The potential correlation between these factors can have strong implications for guiding best practices of teachers who desire to promote deeper student mathematical understanding within a collaborative learning environment that supports cognitive autonomy such as a flipped classroom.

Now that student experiences in a flipped classroom have been explored in this small scale study, future studies may also want to look at exactly how each of the two ways of interaction in a flipped classroom (complete and self-paced) impact student understanding of the material in comparison to each other and in comparison to a control group that is not taught according to a flipped classroom model.

Further, student achievement in a flipped classroom could also be studied further. All cases in this study represented students who completed the course with a B+ or higher, and therefore were considered to have completed the course successfully. As mentioned in Chapter 4 Results and Case Analyses, these students represented the student population in the class relatively well because there was only one student in the flipped classroom who received a C+. All others received either A or B grades or did not complete the course. This leaves room for investigation of whether the flipped classroom pushes students into either succeeding in the course or dropping out of the course, or if it was just an instance that occurred within this small scale study.

Finally, a flipped classroom is merely a mindset with no clear method of implementation. Further implementation methods could be explored. For example, content delivery videos could be used as content review rather than content preview. Class time could be treated in a more structured manner. Assessment strategies such as standards based grading could be also be explored. There are many opportunities for exploration of various methods of flipped classroom implementation. That is the beauty of the mindset of a flipped classroom: it is completely malleable and can be used to accomplish a teacher's goals without compromising the delivery of the curriculum.

6.3. How have I grown?

Over the course of this research, I have grown as a teacher, as a researcher, and as a learner. The most conducive factor in my growth was my discovery of autonomy.

As a teacher, I learned about the delicate tension between providing my students with autonomy and with structure. Discovering the framework developed by Stefanou et al. (2004) of procedural, organizational and cognitive autonomy was pivotal in my understanding of autonomy provision in mathematics education. Further, the discovery of Jang et al.'s (2010) theory of providing both structure and autonomy to increase student engagement led me to realizing the necessity of procedural and organizational structure paired with cognitive autonomy in developing student engagement. This lens has helped me understand my tensions with providing autonomy in the classroom. I now have a framework with which to think about my future teaching.

I also learned about the unresolvable tension between my goals and my students' goals. I held goals of learning (Dweck, 1986) and development of relational understanding (Skemp, 1976) for my students, but I found no way to enforce these goals in my students. This is because their goals didn't necessarily coincide with my goals that I had for them. I learned that there is no way to guarantee a change in a student's goal orientation. This is just the nature of a student and there are many factors affecting their goals with a course. I can manage this tension by understanding that it is appropriate to live with this tension by developing coping mechanisms such as persistently continuing to adjust and experiment with various teaching strategies (Oesterle, 2011).

In the process of conducting this study, I have also grown as a researcher. Given that this is my first major piece of research in mathematics education, the mere process of reviewing the literature and using it to explain occurrences in my classroom was extremely fruitful. I now have the confidence to pursue further topics in mathematics education.

Finally, in reflecting on my own actions as a teacher in the process of conducting this research, I have been able to reflect on my own actions as a learner in response to the autonomy I have been provided with as a master's student. In the Masters of Secondary Mathematics Education Program at SFU, my supervisor and my professors have provided me with high cognitive autonomy support, moderate procedural and organizational structure, and an environment that is conducive to communication and reflection of ideas. The autonomous and collaborative conditions I have been provided with have afforded several opportunities for me to pursue my personal interests in mathematics education. My personal interests have been encouraged and supported through the provision of necessary resources that I have felt motivated to pursue. Being able to take charge of my learning has been extremely empowering and has led me not only to increase my self-efficacy in conducting research, but has also led me to wholeheartedly enjoy my work in the field of mathematics education. This is a true testament to my findings in this research of the importance of cognitive autonomy within a collaborative setting where communication and reflection are encouraged.

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Appendices

Appendix A.

Week 3 Survey January 24, 2013

Math 084 Flipped Classroom Progress Survey *(Jan. 24, 2013)* **Name:** ______________

Please give as detailed responses to the following questions as possible. All feedback is welcome.

1. Describe your current attitude towards mathematics.

2. Describe what your past mathematics classes looked like.

3. Based on these past experiences, describe what you expected this class to be like before you attended the first class.

4. The structure you are experiencing in this class is called a flipped classroom. What is your understanding of this model and what is it meant to accomplish and why?

5. What concerns did you initially have or do you still have regarding the format of the flipped classroom?

6. What do you find to be most enjoyable about this class? Why?

7. What do you find to be least enjoyable about this class? Why?

8. Do you think you are performing well in the class? How is it indicated to you?

Appendix B.

Week 3 Interview Questions January 24, 2013

These are some guideline questions. Feel free to alter as necessary☺

1. Based on your experiences in previous mathematics classes, describe your past experience with math classes.
2. Would you be happy if your classes this term were conducted in a similar manner to previous mathematics classes? Why or why not?
3. How would you describe your current attitude towards mathematics?
4. What does a flipped classroom mean for you?
5. What benefits do you see in using this instruction model?
6. What do you like and/or dislike about how the in-class time is spent?
7. How do you engage in course material out-of-class time? Do you meet with others? Do you use the forum on the website? Do you contact the instructor?
8. Do you think you are performing well in the class? How do you know?
9. What changes do you think should be made to the flipped model classroom to enhance learning for future students?

My key areas of interest right now are:

- Attitude – What effects does the class have on their attitude towards math or towards learning…
- Assessment System: How are the students **motivated**? Eg. What makes them work on their math and what makes them want to do more practice or corrections . . . is it flexible test dates, no grades given on homework, portfolio feedback/dialogue with instructor, etc.
- Out of class time: What impact do the videos have on their learning? – some students take detailed notes: pause/rewind, etc. while others skim through or just print out screen shots. . . .
- Class time: How does the in class time contribute to their learning – ask for favorite activities, discussions, flex time (where they choose what to do) etc.
- Autonomy: sense of choice

Appendix C.

Week 8 Survey March 7, 2013

Math 084 Flipped Classroom Progress Survey *(March 7th, 2013)* **Name:** ______________

Please give as detailed responses to the following questions as possible.

Describe your experience with mathematics in high school. Was there a point in time when mathematics became your least favorite subject? Why do you think this happened?

Having experienced a flipped mathematics classroom for almost two months, answer the following to describe your experience with it:

- What do you think the flipped classroom model is meant to accomplish and why?

- Describe what you expect to be doing when you come to class.

- If you leave class early, why do you choose to do so?

- Describe what you do to prepare for the class at home. How does your approach change if you are stuck on a concept?

- Underline all options that apply. If none express what you do, make a note of what you do.

  How many of the videos do you watch? All/most/some

  How much of the videos do you watch? Entire video/just beginning/scroll through examples

  What do you do when you watch them? Take notes/just listen

  Do you re-watch videos? Yes/No

  If yes, why do you re-watch and what do you differently the second time around?

  Do you use the textbook examples to learn from? Yes/No

  Do you complete practice problems? All suggested/most/some

- Has there been anything that has motivated you to work harder in this class than you first expected to? Explain.

- What have you learned about mathematics this term that you didn't know before? Which component of the flipped classroom do you think has contributed most to this growth?

- Do you think your experiences with this class have changed the way you would approach learning in other classes? How so?

Appendix D.

Week 8 Interview Questions March 7, 2013

These are some guideline questions. Feel free to alter as necessary ☺ . . . probe as much as you can . . . if they make a statement try to find out why they make the claim and what exactly they mean by it.

1. How would you **describe your current attitude towards mathematics**? What do you consider mathematics to be? *(If they say they hate math, probe further as to why they say so and what their **definition of math** is . . . textbook work, or maybe problems?)*

2. Describe your **past experience with mathematics classes**.
   (work towards painting a picture of their high school math experience including conflicts/ attitudes/environment etc.)

3. **What does a flipped classroom mean** for you?
   What **benefits** do you see in using this instruction model?
   *(probe for **confidence levels, motivation, autonomy** . . .)*

   What do you like/dislike about **in –class time**? (group work etc.)
   How do you engage in course material **out-of-class time**?
   What do you do when you get stuck? *(probe for **motivation to complete work out of class**)*
   Do you meet/talk with others? Do you use the forum on the website? Do you contact the instructor?

4. **Do you think you are performing well** in the class? How do you know?

5. Do you think that your **approach to learning in other classes has changed because of this class**? How so?

6. What **changes** do you think should be made to the flipped model classroom to enhance learning for future students?

Appendix E.

Week 14 Survey April 18, 2013

Math 084 Flipped Classroom Final Survey *(April 18th, 2013)* **Name: ______________**

Please give as detailed responses to the following questions as possible.

What does the flipped classroom model mean to you NOW? What is it meant to accomplish? What did you like/dislike? Which components were most useful for you and which could be changed?

Looking back on the course:

- Did the way you engaged with the videos change over the term? How so? Why do you think?
- Did the way you used class time change over the term? How so? Why do you think?
- If you left early or missed class, explain what caused you to do so?

Would you recommend this class (using the flipped classroom model) to a friend? If yes, would you recommend it to every friend or do you think it is better suited for some more than for others? What advice would you give them?

Please complete this survey once more to see if anything has possibly changed for you☺

Anxiety is defined as a feeling of worry, nervousness, or unease. ***Please indicate the level of your anxiety in the following situations.***	Not at all	A little	A fair amount	Much	Very much
1. Taking the math section of a college entrance exam	1	2	3	4	5
2. Realizing that you have to take a certain number of math classes to fulfill requirements	1	2	3	4	5
3. Signing up for a math course	1	2	3	4	5
4. Buying a math text	1	2	3	4	5
5. Walking into a math class	1	2	3	4	5
6. Looking through the pages in a math text	1	2	3	4	5
7. Reading a formula in a science text	1	2	3	4	5
8. Reading and interpreting graphs or charts	1	2	3	4	5
9. Watching a teacher work on an algebraic equation on a whiteboard	1	2	3	4	5
10. Being given a homework assignment of many difficult problems	1	2	3	4	5
11. Picking up a math text to begin working on a homework assignment	1	2	3	4	5
12. Getting stuck on a math problem	1	2	3	4	5
13. Getting ready to study for a math test	1	2	3	4	5
14. Studying for a math test	1	2	3	4	5
15. Thinking about an upcoming math test one day before	1	2	3	4	5
16. Taking a test or quiz in a math course	1	2	3	4	5
17. Waiting to get a math test returned to you in which you expected to do well	1	2	3	4	5
18. Starting a new chapter in a math text	1	2	3	4	5
19. Listening to another student explain a math formula or problem	1	2	3	4	5
20. Thinking about a math course during your daily activities	1	2	3	4	5

Please respond to each of the following statements.	Strongly agree	Somewhat agree	Neither agree or disagree	Somewhat disagree	Strongly disagree
1. Making mistakes is part of learning mathematics	O	O	O	O	O
2. Group work helps me learn mathematics	O	O	O	O	O
3. I like doing mathematics	O	O	O	O	O
4. There are several ways to find the correct solution of a mathematical problem	O	O	O	O	O
5. Solving a mathematics problem is demanding and requires thinking	O	O	O	O	O
6. Mathematics is continuously evolving; new things are still being discovered	O	O	O	O	O
7. Mathematics is a collection of facts	O	O	O	O	O
8. Mathematics enables people to better understand the world they live in	O	O	O	O	O
9. I get satisfaction from solving mathematical problems	O	O	O	O	O
10. When I get stuck on a mathematical problem I tend to give up quickly	O	O	O	O	O
11. I think I will be able to use what I learn in mathematics in other courses	O	O	O	O	O
12. Mathematics has been my worst subject	O	O	O	O	O
13. I have not worked very hard at learning mathematics	O	O	O	O	O
14. I start mathematical problems and questions confidently	O	O	O	O	O
15. It is a waste of time when a teacher makes students think on their own about how to solve a mathematical problem	O	O	O	O	O
16. I am good at math	O	O	O	O	O
17. Those who are good at mathematics can solve problems in a few minutes	O	O	O	O	O
18. In mathematics the teacher knows the right answer	O	O	O	O	O
19. Mathematics is a mechanical and boring subject	O	O	O	O	O
20. I like mathematics because the teacher shows me how to do it and I do it	O	O	O	O	O
21. I expect to do well on the mathematics tests and assessments we do	O	O	O	O	O
22. I prefer mathematics when I have to work hard to find a solution	O	O	O	O	O
23. I am certain I can learn how to solve difficult mathematics problems	O	O	O	O	O
24. Mathematics has no relevance to my life	O	O	O	O	O
25. Studying mathematics is a waste of time	O	O	O	O	O
26. Mathematics is a worthwhile and necessary subject	O	O	O	O	O

27. I study mathematics because I know how useful it is	O	O	O	O	O
28. Knowing mathematics will help me earn a living	O	O	O	O	O
29. I think mathematics is an important subject	O	O	O	O	O
30. Everyone can learn mathematics	O	O	O	O	O
31. Mathematics is used all the time in people's daily lives	O	O	O	O	O
32. If I try hard enough I can understand the mathematics we are taught	O	O	O	O	O
33. Discussing different solutions to a mathematics problem is a good way of learning mathematics	O	O	O	O	O
34. Time used to understand why a solution works is time well spent	O	O	O	O	O
35. Routine exercises are very important in the learning of mathematics	O	O	O	O	O
36. Only very intelligent students can understand mathematics	O	O	O	O	O
37. Only the mathematics to be tested is worth learning	O	O	O	O	O
38. Learning mathematics is mainly about memorizing	O	O	O	O	O
39. Getting the right answer in mathematics is more important than understanding why the answer works	O	O	O	O	O
40. My only interest in mathematics is getting a good grade	O	O	O	O	O

Appendix F.

Week 14 Interview Questions April 18, 2013

These are some guideline questions. Feel free to alter as necessary ☺ . . . probe as much as you can . . . if they make a statement try to find out <u>why</u> they make the claim and <u>what exactly they mean</u> by it. Probe for changes.

1. How would you **describe your current attitude towards mathematics**? What do you consider mathematics to be? *(If they say they hate math, probe further as to why they say so and what their* ***definition of math*** *is . . . textbook work, or maybe problems?)*

2. Has your **attitude towards mathematics changed** over the course of this term? How so? Why?

3. Describe your **past experience with mathematics classes.**
   (work towards <u>painting a picture of their high school math experience</u> including conflicts/ attitudes/environment etc.)

4. **What does a flipped classroom mean** for you?
   What **benefits** do you see in using this instruction model?
   (probe for ***confidence levels, motivation, autonomy*** *. . .)*

   What do you like/dislike about **in –class time**? (group work etc.)
   How do you engage in course material **out-of-class time**?
   What do you do when you get stuck? *(probe for* ***motivation to complete work out of class)***
   Do you meet/talk with others? Do you use the forum on the website? Do you contact the instructor?

   How did your interaction with the class (videos, class time, practice exercises, etc.) **change over the term?

5. **Do you think you are performing well** in the class? How do you know?

6. Do you think that your **approach to learning in other classes has changed because of this class**? How so?

7. What **changes** do you think should be made to the flipped model classroom to enhance learning for future students?

8. Would you recommend this course to a friend? If so, **what would you tell them** before they started the class?

Appendix G.

Follow-up survey April 26, 2013

Reflections on a Flipped Classroom: Follow-up Survey

Please respond to each of the following by checking **true** or **false** as well as providing a **brief clarification** to your response *(feel free to alter the statement to accurately describe your position – use a different font to show this difference)*:

Name: ________________________

1. I loved the videos because they allowed me to work at my own pace
   ☐ True ☐ False
   Comment:

2. If I fell behind, I was able to use the videos to catch up
   ☐ True ☐ False
   Comment: . . . Did you fall behind? How often?

3. I took detailed notes from the videos because I liked seeing the procedures
   ☐ True ☐ False
   Comment:

4. I focused on memorizing these procedures
   ☐ True ☐ False
   Comment:

5. During group work, I tried to recall procedures from the videos and make them work
   ☐ True ☐ False
   Comment:

6. The videos helped me complete my homework because they provided complete examples
   ☐ True ☐ False
   Comment:

7. I took notes from the videos so that I could ask about certain concepts in class
   ☐ True ☐ False
   Comment:

8. The videos helped me understand the math rather than just helping me complete my homework
   ☐ True ☐ False
   Comment:

9. During class, I just wanted to get my graded problems done
   ☐ True ☐ False
   Comment:

10. During class, I found the activities took too much time
    ☐ True ☐ False
    Comment:

11. I tended to avoid coming to class when I was behind
    ☐ True ☐ False
    Comment: If true, why so?

12. I came to class prepared to ask questions
    ☐ True ☐ False
    Comment:

13. I loved being able to ask questions in class
    ☐ True ☐ False
    Comment:

14. I enjoyed class time because I could clarify concepts with my classmates and/or my teacher
    ☐ True ☐ False
    Comment:

15. I loved the activities because I could build a stronger understanding of the math
    ☐ True ☐ False
    Comment:

16. The activities gave me room to try out my ideas
    ☐ True ☐ False
    Comment:

17. I loved the activities because I got to see other approaches to problems from my peers
    ☐ True ☐ False
    Comment:

18. During the activities, I often became confused by other approaches
    ☐ True ☐ False
    Comment:

19. I enjoyed teaching others during the group activities
    ☐ True ☐ False
    Comment:

20. The flipped classroom model gave me enough time to think through concepts in order to understand them completely thanks to the flexibility and amount of resources available
    ☐ True ☐ False
    Comment:

21. The flipped classroom model allowed me to set my own goals and complete them
    ☐ True ☐ False
    Comment:

22. There was too much leniency in the class and I found it difficult to stay motivated to complete my work, causing me to fall behind at some points during the term
☐ True ☐ False
Comment:

23. I completed my corrections because I could get more marks that way
☐ True ☐ False
Comment:

24. Completing my corrections gave me a better understanding of the material
☐ True ☐ False
Comment:

25. If corrections were not worth marks, I would not have done them
☐ True ☐ False
Comment:

26. Working with others was the best part about this class
☐ True ☐ False
Comment:

27. Being able to work at home through the material at my own pace was the best part about this class
☐ True ☐ False
Comment:

28. My goal with this class was to gain better understanding about math
☐ True ☐ False
Comment:

29. My goal with this class was to attain a good grade
☐ True ☐ False
Comment:

30. I know I can succeed in future mathematics courses
☐ True ☐ False
Comment:

Also, if you could give me some details about:

1. Which math classes did you complete in the past and how were they for you?

2. Did you find that this class helped you with any anxiety you had towards math or not? Explain.

3. Did you find yourself become more confident in completing math problems over the term or not? Explain.

4. Describe your favorite type of activity (one that seemed close to textbook work, one that was open-ended, or one that helped you make connections) and why you enjoyed it. If you didn't like the activities, please explain what you didn't like about them and how they could be better.

If you have any further reflections, please share: