Experimental design in determining the parametric errors of CMMS

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Abstract

Procedures to determine the parametric error behaviour of co-ordinate measuring machines (CMMs) from measurements of calibrated artefacts such as ball plates are well established in principle. However, since their practical implementation requires significant resources, both in terms of measurement time and computation, it is important that these procedures provide reliable information. In recent modelling and simulation work at NPL, measurement experiments involving ball plates, ring gauges, step gauges and spheres have been analysed. The results show that the design of the experiment has a critical influence on the accuracy of the results obtained and that experiments, at first sight quite similar, can have markedly different degrees of effectiveness. This paper discusses how modelling and numerical simulation can be used to assess and improve effectiveness of different strategies involving a range of artefacts.

1 Introduction

In a (conventional) co-ordinate measuring machine (CMM) with three mutually orthogonal linear axes, the position of the probe tip centre is inferred from scale readings on each of the three machine axes. For a CMM with perfect geometry, the scale readings alone are sufficient to provide accurate co-ordinate measurements. However, in practice, CMMs will have imperfect geometry with respect to the straightness of the axes, the squareness of pairs of axes and rotations describing roll, pitch and yaw. These systematic

errors – so-called *parametric errors* – have to be taken into account if the accuracy potential of the CMM is to be more fully realised.

Procedures have been developed to determine these parametric errors from the repeated measurements of calibrated artefacts such as ball or hole plates, e.g., Cresto[2], Kruth, Vanherk & de Jonge[6], Kunzmann, Trapet & Waldele[7] and Zhang *et al.*[8]. However, their practical implementation requires significant resources, both in terms of measurement time and computation, and it is therefore important that these procedures provide reliable and sufficiently comprehensive information. To arrive at effective procedures, it is necessary to consider the mathematical model of CMM behaviour, the type of calibrated artefact, and the way the measurement and calibration information is analysed, i.e., the parameter estimation method. This paper considers these issues.

2 Mathematical models of CMM behaviour

The location $\mathbf{x}^* = (x^*, y^*, z^*)^T$ of the (centre of the) CMM probe tip can be modelled as $\mathbf{x}^* = \mathbf{x} + \mathbf{e}(\mathbf{x}, \mathbf{a}, \mathbf{p})$, where $\mathbf{x} = (x, y, z)^T$ are the scale values,

$$\mathbf{e}(\mathbf{x}, \mathbf{a}, \mathbf{p}) = \mathbf{e}_0(\mathbf{x}, \mathbf{a}) + R(\mathbf{x}, \mathbf{a})\mathbf{p} \tag{1}$$

describes the systematic error in terms of a positional term $\mathbf{e}_0$, a rotation matrix R specified by three rotation angles and a probe offset $\mathbf{p}$. The components $\mathbf{e}_0$ and R are each a function of the scale values $\mathbf{x}$ and model parameters $\mathbf{a}$, and different types of behaviour can be modelled by choosing different functions for $\mathbf{e}_0$ and R.

In a completely general model, the six component functions are represented by empirical functions of the scale readings $\mathbf{x}$, by multivariate polynomials or tensor product splines, for instance. We term this an *empirical model*. In the *kinematic model* of CMM behaviour, (e.g. Zhang[8] *et al.*) it is assumed that the probe location is built up from the behaviour along each axis. For example, if $\mathbf{x}_x = (x, 0, 0)^T + \mathbf{e}_{0,x}(x, \mathbf{a})$ and $R_x(x, \mathbf{a})$ describe the three positional and three rotational terms of the error behaviour along the x-axis as a function of the x-scale value, with y- and z-motion described similarly, the overall motion can be described by

$$\mathbf{x}^* = \mathbf{x}_x(x, \mathbf{a}) + R_x(x, \mathbf{a})\left\{\mathbf{x}_y(y, \mathbf{a}) + R_y(y, \mathbf{a})\left[\mathbf{x}_z(z, \mathbf{a}) + R_z(z, \mathbf{a})\mathbf{p}\right]\right\}, \tag{2}$$

or similar, the exact representation depending on the CMM architecture. The 18 error terms are each a function of a single variable and can be represented using polynomials or splines, for example. Often the kinematic model is used in a linearised form with all higher order terms ignored.

3 Measurement of calibrated artefacts

To determine the kinematic errors of a CMM, one can design experiments to measure each of the 18 functions individually and then amalgamate the results to determine the composite error. A second approach is to use the CMM to measure one or more calibrated, dimensionally stable, artefacts in different locations and orientations and determine the errors from these measurements and calibration information.

As an example of the second approach, suppose a ball plate is placed in a number of positions within the working volume of a CMM and the scale readings corresponding to measurements of the spheres are recorded. Let $\mathbf{x}_i$ be the scale readings corresponding to an estimate of the centre of the jth sphere in the kth position of the plate using the lth probe. This measurement information is described by three model equations of the form

$$\mathbf{x}_i + \mathbf{e}(\mathbf{x}_i, \mathbf{a}, \mathbf{p}_l) = T(\mathbf{b}_j, \mathbf{t}_k) + \mathbf{f}_i, \tag{3}$$

where $\mathbf{e}$ describes the kinematic errors, $\mathbf{p}_l$ is the lth probe offset, $\mathbf{b}_j$ is the location of the jth sphere for the ball plate in a fixed frame of reference, $\mathbf{t}_k$ are parameters describing the kth transformation $\hat{\mathbf{x}} = T(\mathbf{x}, \mathbf{t})$ and $\mathbf{f}_i = \mathbf{f}_i(\mathbf{a})$ are parameter-dependent deviations of the model values from the data. If the model validly represents the data, the values of $\mathbf{f}_i$ corresponding to the best estimates (below) of $\mathbf{a}$ are estimates of the measurement errors corresponding to the transformed sphere centre co-ordinates. These errors can be expected to behave as random variables. Calibration information associated with the ball plate can be in the form of estimates d_{jl} of the inter-sphere distances and modelled as

$$||\mathbf{b}_j - \mathbf{b}_l|| = d_{jl} + f_{jl}, \tag{4}$$

where the f_{jl} again represents deviations of the model values from the data. In theory, best estimates of the parametric errors defined by $\mathbf{a}$ can be determined by minimising

$$\sum \alpha_i^2 \mathbf{f}_i^{\mathrm{T}} \mathbf{f}_i + \sum \beta_{jl}^2 f_{jl}^2 \tag{5}$$

with respect to the parameters $\mathbf{a}$, $\{\mathbf{b}_j\}$ and $\{\mathbf{t}_k\}$ subject to the constraints (3–4). Here α_i and β_{jl} are weights chosen to reflect the relative uncertainty in the measurement and calibration information.

The same approach can be adopted for the multiple measurement of other calibrated artefacts such as step gauges, ring gauges (cylinders) and

spheres. Let **b** be the parameters defining the artefact surface in a fixed frame of reference. The model equations associated with measurements of the artefact can be written as

$$m_i(\mathbf{x}_i + \mathbf{e}(\mathbf{x}_i, \mathbf{a}, \mathbf{p}) + \mathbf{f}_i, \mathbf{b}, \mathbf{t}_k) = 0, \tag{6}$$

stating that $\mathbf{x}_i+\mathbf{e}(\mathbf{x}_i, \mathbf{a}, \mathbf{p})+\mathbf{f}_i$ lies on the surface of the artefact in the kth position specified by transformation parameters $\mathbf{t}_k$. The calibration information associated with the artefact can be encoded in equations of the form

$$c_j(\mathbf{d}_j + \mathbf{f}_j, \mathbf{b}) = 0, \tag{7}$$

where $\mathbf{d}_j$ are calibration data and $\mathbf{f}_j$ represents the error in $\mathbf{d}_j$. Estimates of the parametric errors specified by **a** can be found by minimising

$$\sum_i \mathbf{f}_i^{\mathrm{T}} A_i \mathbf{f}_i + \sum_j \mathbf{f}_j^{\mathrm{T}} B_j \mathbf{f}_j \tag{8}$$

with respect to **a**, **b** and $\{\mathbf{t}_k\}$ subject to the constraints (6) and (7). Here, the matrices A_i and B_j are estimates of the inverses of the covariance matrices of $\mathbf{f}_i$ and $\mathbf{f}_j$, respectively. This type of optimisation problem can usually be converted into an *unconstrained* nonlinear least squares problem and solved iteratively using some variant of the Gauss-Newton algorithm (see, for example, Gill, Murray and Wright[4]), the basic steps of which we now briefly describe.

Suppose we wish to minimise $\sum_{i=1}^{m} f_i^2(\mathbf{c})$ with respect to the parameters $\mathbf{c} = (c_1, \ldots, c_n)^{\mathrm{T}}$ and that **c** is the current estimate of the solution. Let $J = \left[\frac{\partial f_i}{\partial c_j}\right]$ be the *Jacobian matrix* of partial derivatives and let $\mathbf{p}_{GN}$ be the least squares solution of the matix equation $J\mathbf{p}_{GN} + \mathbf{f} = \mathbf{0}$, (see, e.g., Golub and Van Loan[5]). An updated estimate of the solution is given by $\mathbf{c} = \mathbf{c} + \mathbf{p}_{GN}$.

The Jacobian matrix at the solution can be used to estimate the covariance matrix $V_{\mathbf{c}}$ of the fitted parameters through

$$V_{\mathbf{c}} = \hat{\sigma}^2 \left(J^{\mathrm{T}} J\right)^{-1}, \tag{9}$$

where $\hat{\sigma} = \left(\mathbf{f}^{\mathrm{T}}\mathbf{f}/(m-n)\right)^{1/2}$ estimates the standard deviation of the residual errors. If $h(\mathbf{c})$ is a function of the fitted parameters then the standard uncertainty of h is given by

$$u(h) = \left(\left(\nabla_{\mathbf{c}} h\right)^{\mathrm{T}} V_{\mathbf{c}} \nabla_{\mathbf{c}} h\right)^{1/2}. \tag{10}$$

The advantage of the approach indicated by eqns (6-8) is that uncertainties associated with the measurement and calibration information are properly taken into account so that, firstly, the resulting estimates of the parametric errors are (relatively) unbiased and, secondly, the estimates of uncertainties associated with the fitted parameters (following eqns (9-10)) are valid (assuming the underlying model is appropriate) and can be used directly to determine the effectiveness of a particular measurement/calibration strategy.

4 Simulation of measurement experiments

Using numerical simulation, it is possible to analyse a vast number of measurement experiments to develop an understanding of the measurement system and to design effective measurement strategies. The simulations reported on here involve a CMM of working volume $0 \leq x, y, z \leq 1$ m with a kinematic error model and a repeatability error of 0.2 μm in each coordinate. It is assumed that these errors are uncorrelated and drawn from a normal distribution. The accuracy of the parameters estimates can be gauged by u_{max}, the maximum, and $\bar{u}$, the mean, uncertainty for distances between pairs of randomly chosen error-corrected points within the working volume of the CMM. This repeatability error gives a natural baseline with which to compare these statistics.

The estimation of uncertainty requires that all the system parameters can be determined from the measurement and calibration information. If the system is not identifiable, i.e., the associated Jacobian J is rank deficient, it is possible to use the singular value decomposition (SVD, Golub and Van Loan[5]) to analyse the modes of behaviour that remain free. If $\mathbf{v}$ is a right singular vector corresponding to a zero singular value of J then a perturbation of the parameters $\mathbf{c}$ in the direction of $\mathbf{v}$ is not detected from the data. (The right singular vectors of J correspond to the eigenvectors of $J^{\mathrm{T}}J$.)

Example: isotropic axis roll. The following example shows the type of rank deficiency that can occur. Suppose a CMM has an error behaviour approximated by

$$\mathbf{x}^* = \mathbf{x} + R(\lambda\mathbf{x})\mathbf{p},$$

where R is the linearised rotation matrix corresponding to roll about each

axis given by

$$R(\mathbf{x}) = \begin{bmatrix} 1 & -z & y \\ z & 1 & -x \\ -y & x & 1 \end{bmatrix},$$

and λ is a small perturbation parameter. Then

$$\mathbf{x}^* = \mathbf{x} + R(\lambda\mathbf{x})\mathbf{p} = \mathbf{x} + \mathbf{p} + \lambda\mathbf{x} \times \mathbf{p} = R(-\lambda\mathbf{p})\mathbf{x} + \mathbf{p},$$

showing that measurements of an artefact using a CMM with isotropic axis roll are the same as those of a rotated artefact by a CMM with no axis roll. This means that, irrespective of the measurement strategy and calibration information, this type of behaviour is not detectable from the multiple measurement of an artefact *unless* measurements are taken using more than one probe offset with the artefact in a fixed position. In general, at least three probe offsets are required.

4.1 Ball plates experiments

The results of extensive simulations of measurement experiments using ball plates is presented in Cox *et al.*[1]. One suite of experiments involves the measurement of a 7×7 square ball plate positioned in the horizontal planes $z = 0.1$ and $z = 0.9$ using a single vertical probe offset $(0, 0, -L)^T$, and twice in each of the planes $x = 0.5$ and $y = 0.5$ with probe offsets $(\pm L, 0, 0)^T$ and $(0, \pm L, 0)$, respectively. Two sets of calibration information are considered: A) the Euclidean distances between the centres of neighbouring balls on all rows and columns of the grid are prescribed, giving 84 items of calibration data, and B) the Euclidean distances between the centres of the four balls at the vertices of the grid measured along the rows and columns of the grid are prescribed, giving 4 items of calibration data.

It is found that for a symmetrical configuration of ball plate positions, calibration information A leads to a full rank system and moderately accurate estimates $u_{max} = 0.84$ μm, $\bar{u} = 0.36$ μm, but case B is rank deficient with a symmetrical change of the location of the balls compensated for by a symmetrical arrangement of scale errors. If the configuration is modified to break the symmetry, through translation and rotation of the ball plate, both types of calibration information given comparable results with u_{max} and $\bar{u}$ better than 0.63 and 0.23 μm, respectively. These results show that variations of a measurement strategy can have markedly different behaviour in terms of calibration requirements and accuracy.

4.2 Other calibrated artefacts

In this section, we present the results of some numerical simulations involving a step gauge, (plain setting) ring gauge and a sphere (realised by a fixed length arm pivoting about a fixed point, for example).

Step gauge experiment. In this experiment, a 0.8 m long step gauge was placed in seven positions, three parallel to the CMM axes and four parallel to the space diagonals of the CMM's working volume. The step gauge had faces at every 0.025 m along its length which were measured using one or more probes. The calibration information was given in terms of the calibrated distances of each face from a reference face at one end of the gauge.

Ring gauge experiment. In this experiment a 0.5 m diameter ring gauge was measured in six positions, two positions parallel to each of the planes defined by the CMM axes. The positions were chosen so that as much of the working volume was sampled as possible. Measurements lying approximately in two or more circles on the gauge's cylindrical surface were simulated for each of the six positions. The calibration information was given in terms of the calibrated diameter.

Sphere experiment. A sphere of diameter 0.5 m was placed in five positions centred at (0.25,0.25,0.25), (0.75,0.75,0.25), (0.25,0.75,0.75), (0.75,0.25,0.75) and (0.50,0.50,0.50). Measurements using one or more probes along a number of circles on each sphere position were simulated. The calibration information was given in terms of the calibrated diameter.

Tables 1 and 2 demonstrate the effectiveness of the measurement experiments to determine quartic and quardratic error behaviour, respectively. Table 1 shows the numerical results for the case where the 18 parametric errors are represented by 5th order (quartic) polynomials. The uncertainty values indicate that the step gauge data is quite inadequate in determining accurate parametric errors. This poor performance can be partially explained by the design of the experiment in which the seven sets of measurements are relatively isolated from each other, with little or no overlap in the measurements of the gauge in one position with the measurements for another position. It is intuitively clear that such a strategy would not be good for detecting squareness errors, for example. On the other hand the ring gauge and the sphere data allow for a determination of the parametric errors to a reasonable accuracy. The sphere data indicates that well-designed experiments could offer an alternative to ball plate methods.

Table 2 shows the corresponding numerical results for the case where the 18 parametric errors are represented by 3rd order (quadratic) polynomi-

Artefact	m	m_c	n	u_{max} (μm)	$\bar{u}$ (μm)
Step gauge	469	33	102	37.9	7.3
Ring gauge	242	1	94	2.9	1.2
Ring gauge	627	1	94	1.8	0.8
Sphere	275	1	85	1.8	0.7
Sphere	605	1	85	0.9	0.3

Table 1: Results of numerical simulations using calibrated artefacts for determining the parametric errors modelled as quartic polynomials. Notation: m is the number of measurements simulated, m_c the number of items of calibration information, n the number of model parameters, u_{max}, the maximum, and $\bar{u}$, the mean, uncertainty for distances between pairs of error-corrected points.

Artefact	m	m_c	n	u_{max} (μm)	$\bar{u}$ (μm)
Step gauge	469	33	66	2.2	0.8
Ring gauge	242	1	58	1.3	0.6
Ring gauge	627	1	58	0.8	0.4
Sphere	275	1	49	0.7	0.3
Sphere	605	1	49	0.5	0.3

Table 2: As Table 1 but with parametric errors modelled as quadratic polynomials.

als. The uncertainty values indicate that all the experiments give acceptable information with the ring gauge and sphere data producing average uncertainties not too far from the repeatability of 0.2 μm. These results indicate that these types of artefacts can be used to detect low order changes in the parametric errors and are therefore appropriate for periodic interim checking and as a means of providing additional information to be incorporated with information obtained as part of a periodic reverification of the CMM.

4.3 Ball gauge experiments

We have also carried out comprehensive numerical simulations involving a ball gauge – essentially a one dimensional ball plate where all the balls are positioned along a single axis. They are potentially important in that

Artefact	m	m_c	n	u_{max} (μm)	$\bar{u}$ (μm)
7×7 ball plate	294	4	234	0.63	0.23
9-ball gauge	363	1	222	0.69	0.25

Table 3: Comparison of a ball plate experiment and a ball gauge experiment designed to mimic a ball plate experiment. Notation as in Table 1.

they are easier to construct, position and calibrate than a 2-D ball plate. The large reference artefact designed and constructed by NPL for use in the verification of large CMMs is a type of ball gauge (Forbes and Peggs[3]). One important question is whether or not a ball gauge can be used for the accurate determination of the parametric errors of a CMM. We have found that such a gauge can indeed be so used if the measurement strategy has sufficient in-built rigidity. We place the ball gauge along the four sides and two diagonals of a square. This arrangement of six positions of the ball gauge simulates one ball plate, which we can use as we would an actual ball plate. Table 3 shows the results of a numerical experiment mimicking the experiment described in section 4.1 using a "ball plate" synthesised using a ball gauge with 9 balls. This pseudo-ball plate has effectively $4+6\times8 = 52$ balls and can be compared with experiments with a 7×7 ball plate. Table 3 shows that the ball gauge results are comparable to those for the ball plate.

5 Summary and concluding comments

In determining CMM behaviour, there are a large number of potential approaches involving choices of artefact, measurement strategy, calibration information, etc. It is impractical to test more than a few such approaches and numerical simulation is an obvious tool to guide their development. However, for the numerical simulations to give valid results it is essential that they based on an adequate model of the measurement experiment which properly takes into account CMM behaviour, measurement and calibration information.

Results of simulations at NPL have shown that a number of calibrated artefacts can be used in effective strategies, under the assumption that the artefacts are sufficiently stable. Moreover, many of these strategies require minimal calibration information, often little beyond scale-setting data. However, the relationship between measurement configuration and the ac-

curacy of the estimates is complex and any proposed method should be simulated in order to confirm its suitability. In this, the calculation of valid uncertainties associated with the parameter estimates is crucial. Without some such validation it is quite possible to implement methods that give unreliable results or fail to detect significant types of error behaviour.

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