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Experimental determination of the intensity distribution across a source with a grating interferometer

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Abstract. An interferometric technique based on the use of a diffraction grating is used to recover the intensity distribution across a spatially incoherent planar source. Information about the source profile is obtained through measurements of visibility and position of the fringe pattern at transverse planes beyond the grating. Such quantities are shown to be independent of the wavelength of the radiation, so that broadband sources can be analysed without any spectral filtering. Experimental results are given.

1. Introduction

A look at the numerous papers published concerning coherence theory shows that an exciting subject is the possibility of obtaining information about a source starting with knowledge of the spatial correlation of the light field at pairs of points in the space surrounding the source itself [1]. Starting points for these investigations are often the Young interference experiment and the van Cittert–Zernike theorem. The connection between the features of the interference fringe pattern and the complex degree of coherence of the light at the two apertures in the Young scheme is well known [2]. On the other hand, if the field illuminating the interferometer originates from an incoherent source, due to the van Cittert–Zernike theorem, the correlation between fields at the Young apertures is simply related to the intensity distribution across the source [2]. These concepts are at the basis of techniques used to determine angular diameters of sources (as in Michelson's stellar interferometry [2, 3]) and, more generally, the intensity distributions across them (as in multiple-element interferometry, employed in radio astronomy [4]). In both cases it is necessary to filter the signal to make it quasi-monochromatic. Alternatively, by making use of the so-called space–frequency equivalence principle [5], the same information can be obtained from spectral measurements, provided that the bandwidth of the radiation is broad enough [5–13].

In this paper we use a different method to recover the intensity distribution across a one-dimensional spatially incoherent source, whose power spectrum is assumed to be space independent. The double aperture characterizing the Young interferometer is replaced by a grating with a sinusoidal transmission function. Information about the source intensity distribution is obtained by measuring the visibility and the position of the fringes in the field propagated at different planes beyond the grating.

One of the advantages of this modified interference scheme is that white fringes are produced on the observation plane, so that measurements on the fringe pattern may be performed without any spectral filtering of the radiation. This means that the amount of power available for the measurements is much higher than in a traditional Young scheme. Of course, the use of achromatic fringes in grating interferometry is rather well known [14, 15] and has been put to use in high brightness interferometry and imaging devices [16–18]. The role of the mutual coherence function in these devices has also been noted [17].

Our method recalls the achromatic Michelson interferometer proposed by Cutter and Lohmann in 1974 [19]. Here, experimental results pertaining to the case of an incoherent source consisting of two parallel bright strips are reported. We show that, by making use of a cooled CCD camera to record the fringe patterns and an algorithm based on the fast Fourier transform (FFT) to evaluate the visibility pertaining to each image, a very easy and accurate reconstruction of the intensity distribution across the source can be obtained.

Similar procedures were also presented in [20, 21], where a rotating grating was used to determine the degree of coherence of a partially coherent light field, but limited to the quasi-monochromatic case.

2. Theoretical analysis

We shall consider the scheme illustrated in figure 1. A spatially incoherent planar source lies in the plane $\xi\eta$. In a plane xy, whose distance from the source is denoted by D, a one-dimensional grating is placed. The latter is considered as characterized by the transmission function

$$\tau(x) = \cos\left(\frac{2\pi}{P}x\right) \tag{1}$$

where P is the period. Actually, in a practical implementation of the method, the transmission function of the grating may be arbitrary, provided that a suitable $4f$ optical system is used beyond the grating to retain only two symmetric diffraction orders (see figure 2). We shall study the intensity distribution at a distance z from the grating (uv plane).

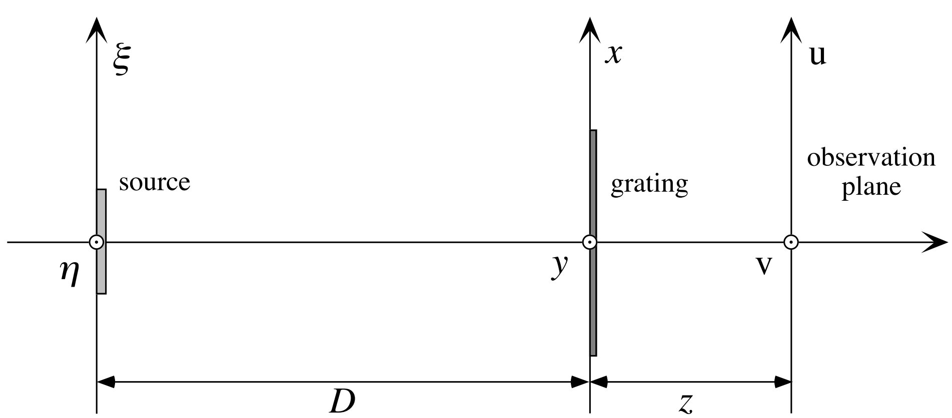

Figure 1. Geometry and notations of the scheme used.

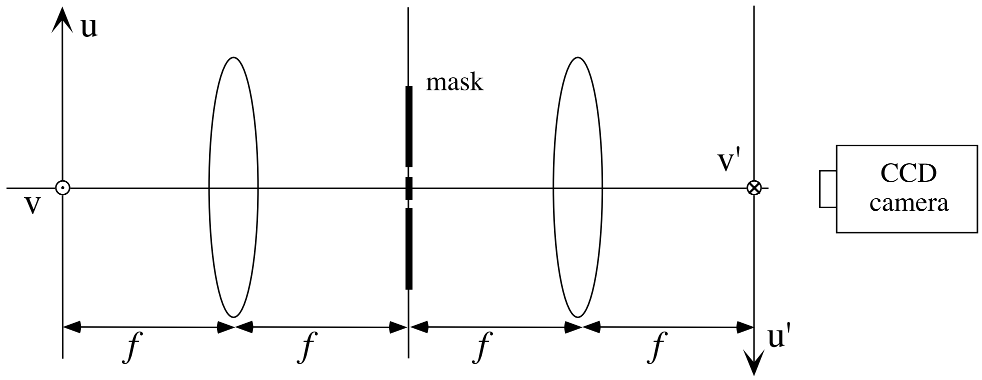

Figure 2. Detection apparatus using a $4f$ optical processor.

In the present analysis a one-dimensional source will be considered, i.e. a source which is very long in the η direction and whose power spectrum [2] is independent of the coordinate η. Let us denote by $S_0(\xi;\nu)$ the power spectrum at the source plane and assume that the normalized power spectrum [2], s_0, does not depend on the ξ coordinate within the source domain, so that we can write

$$S_0(\xi;\nu) = I_0(\xi)\, s_0(\nu) \tag{2}$$

where I_0 is the optical intensity, defined as

$$I_0(\xi) = \int_0^\infty S_0(\xi;\nu)\,\mathrm{d}\nu. \tag{3}$$

Let $W_G(x_1, x_2; \nu)$ be the cross spectral density [2] at frequency ν between two points x_1 and x_2 on the grating plane . It can be written in the general form

$$W_G(x_1, x_2;\nu) = \sqrt{S_G(x_1;\nu)\, S_G(x_2;\nu)}\ \mu_G(x_1, x_2;\nu) \tag{4}$$

where S_G and μ_G are the power spectrum and the spectral degree of coherence, respectively, of the field illuminating the grating. Since the source is assumed to be completely spatially incoherent, these quantities can be evaluated through the van Cittert–Zernike theorem, yielding

$$\mu_G(x_1, x_2;\nu) = \exp\left[\mathrm{i}\frac{2\pi\nu}{cD}\left(x_2^2 - x_1^2\right)\right]\frac{\tilde{I}_0[\nu(x_2 - x_1)/cD]}{\tilde{I}_0(0)} \tag{5}$$

and

$$S_G(x;\nu) = \frac{1}{D} s_0(\nu)\,\tilde{I}_0(0) \tag{6}$$

where the tilde denotes Fourier transform and c is the speed of light. Note that the power spectrum at the grating plane is actually independent of the coordinate x.

The quadratic phase factor in equation (5), which arises when the cross spectral density is propagated under paraxial approximation, will be neglected in the following. This is allowed if the plane xy is in the far zone with respect to the source plane. In general, we can always consider using a lens of suitable focal length to counterbalance the spherical curvature.

Hence the spectral degree of coherence becomes a function, say h_G, of $\nu(x_2 - x_1)/c$ and equation (5) can be written as

$$\mu_G(x_1, x_2; \nu) = h_G\left[\frac{\nu}{c}(x_2 - x_1)\right] = \frac{\tilde{I}_0\left[\nu(x_2 - x_1)/cD\right]}{\tilde{I}_0(0)}. \tag{7}$$

On using the propagation laws for the cross spectral density [22], the power spectrum at the observation plane turns out to be

$$S_z(u; \nu) = \iint \tau^*(x_1)\,\tau(x_2)\,W_G(x_1, x_2; \nu)\,K_z^*(u, x_1; \nu)\,K_z(u, x_2; \nu)\,\mathrm{d}x_1\,\mathrm{d}x_2 \tag{8}$$

where the asterisk denotes the complex conjugate and K_z is the paraxial propagation kernel, given by

$$K_z(u, x; \nu) = \sqrt{\frac{-\mathrm{i}\nu}{cz}}\exp\left[\pi\mathrm{i}\frac{\nu}{cz}(u - x)^2\right]. \tag{9}$$

After simple calculations we find, for the power spectrum at the output plane, the following expression:

$$S_z(u; \nu) = s_0(\nu)\frac{\tilde{I}_0(0)}{2D}\left[1 + \left|h_G\left(\frac{2z}{P}\right)\right|\cos\left(\frac{4\pi}{P}u - \alpha\right)\right] \tag{10}$$

where equation (7) has been used, and

$$\alpha = \arg\left\{h_G\left(\frac{2z}{P}\right)\right\} \tag{11}$$

arg denoting the argument.

Equation (10) has the typical structure of an interference pattern. Note, however, that fringe period and position are independent of the frequency ν. As a consequence, if we use a source emitting broadband radiation, white fringes are formed on the observation plane instead of the coloured ones that would be seen by making use of a Young scheme. On integrating equation (10) with respect to ν, the following intensity distribution, detectable at the output plane, is obtained:

$$I_z(u) = \frac{\tilde{I}_0(0)}{2D}\left[1 + \left|h_G\left(\frac{2z}{P}\right)\right|\cos\left(\frac{4\pi}{P}u - \alpha\right)\right]. \tag{12}$$

From equation (12) we see that the modulus and phase of the spectral degree of coherence of the light impinging on the grating may be determined by measuring the visibility and position of the fringes for different distances (z) of the observation plane. It follows from equation (7) that the source intensity distribution can be recovered, up to a constant factor, by means of the inverse Fourier transform of h_G. Since I_0 is a real function, analysis of the fringe patterns at planes $z \geqslant 0$ is sufficient to completely retrieve the intensity profile of the source, because in this case h_G is Hermitian, i.e. $h_G(-t) = h_G^*(t)$ [23].

From an experimental point of view, the function h_G has to be sampled at a discrete set of values of z. From Shannon's theorem [23] it follows that the sampling interval, say Δz, depends on the spatial extent of the source. One generally has an *a priori* estimate of such an extension, say $\Delta\xi$. It then follows from the sampling theorem that $\Delta z \leqslant PD/2\Delta\xi$.

On the other hand, the resolution of the system is related to the maximum value of the distance, say z_M, where measurements are performed, the maximum spatial frequency on the source plane being given by $2z_M/PD$.

3. Experimental results

The experimental set-up basically reproduces the scheme sketched in figure 1. The source consists of a pair of slits uniformly illuminated by an incandescent lamp, which is placed fairly close to the slits, in order to ensure that they are illuminated incoherently. The slits are 1.55 mm wide, their centres are 5.1 mm apart and the distance D between the source and the grating is 175 cm. The latter is a 80 lines/mm phase grating, and diffraction orders other than $+1$ and -1 are filtered by means of a $4f$ optical system (see figure 2) consisting of two lenses and a suitable transmission mask at their common focal plane. The mask consists of an opaque screen with two holes, wide enough to allow the chromatically dispersed spectral components of the source to pass through.

The fringe pattern at the output plane is acquired by a CCD camera. Images are sent to a personal computer to evaluate the parameters (position and visibility) of the fringe pattern. Since the incoherent source is chosen to be symmetrical, the phase α (related to the position of the fringes) only assumes values 0 or π and is then simply determined by observing modulation inversions in the fringe pattern. As far as visibility determination is concerned, an FFT-based algorithm has been implemented which is able to extract information even from very low-contrast images.

Experimental values (dots) of $h_G(2z/PD)$ versus z are shown in figure 3, where a fitted curve (broken) has been added to improve the data presentation. The recovered intensity distribution on the source plane is reported in arbitrary units in figure 4 (dots), together with the actual profile of the source (full curve). It is seen that the agreement is quite good, although the number of samples ($N = 32$) is not very high. The symmetry of the resulting

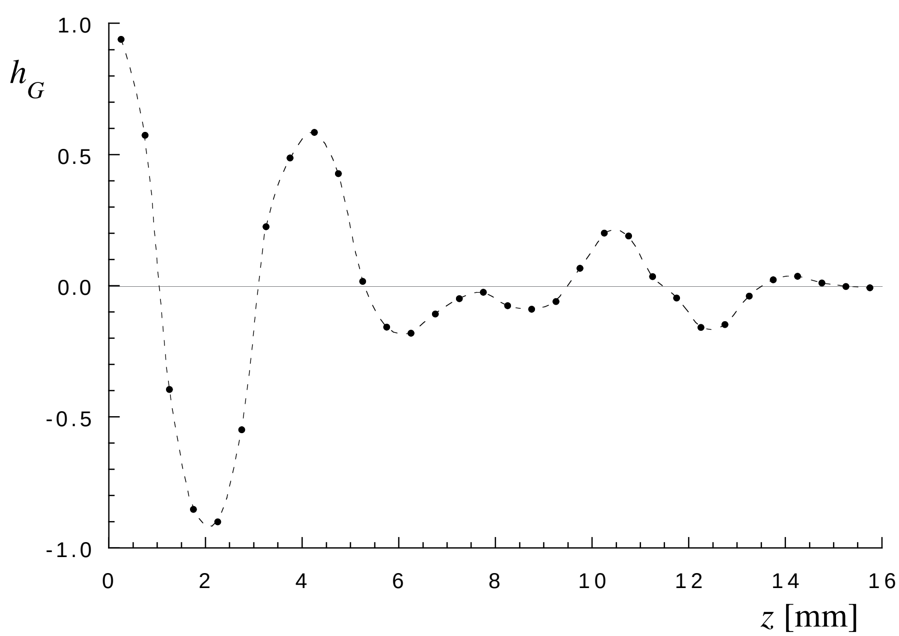

Figure 3. Experimental values of h_G (dots) versus z with an interpolating curve (broken).

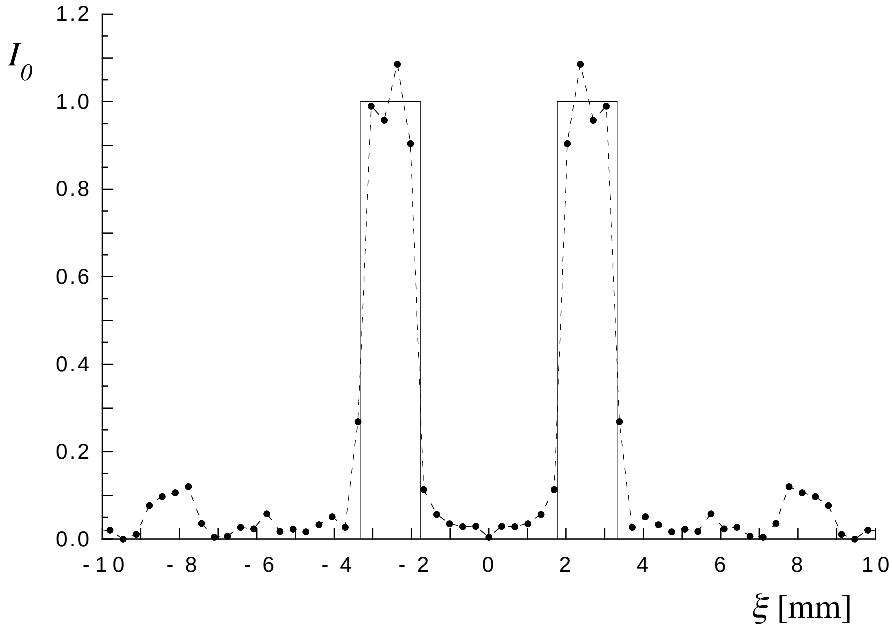

Figure 4. Recovered (dots) and actual (full curve) intensity profile of the source (arbitrary units).

curve with respect to the origin of the ξ-axis is due the fact that in the present case the Hermitian function h_G is real, so that it is also symmetric.

4. Conclusions

The determination of the intensity profile of a spatially incoherent broadband source has been obtained by means of a modified Michelson stellar interferometer. It is accomplished through measurements of the spectral degree of coherence of the field radiated from the source at a transverse plane, where a diffraction grating is placed. Measurements are performed by determining the position and visibility of fringe patterns produced beyond the grating. In contrast to techniques based on Young interferometers, this method gives higher light throughput and leads to white light fringe patterns.

The greatest distance from the grating where measurements are performed fixes the resolution limit of the system. Since the maximum value of such a distance depends on the transverse size of the grating, the maximum available resolution turns out to be comparable to that pertinent to a conventional telescope with the same aperture as the grating. Nonetheless, our approach presents some advantages with respect to conventional image-forming optical systems. For example, it does not suffer from chromatic aberrations and the use of lenses is not necessary. Hints for further improvements of resolution and performance of systems of this kind can be found in [21].

The case of a one-dimensional source has been studied, but the proposed technique can be extended to the case of a general planar source.

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