Transactions on Modelling and Simulation vol 5, © 1993 WIT Press, www.witpress.com, ISSN 1743-355X

Experimental identification of modal parameters on a full scale structure

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ABSTRACT

With the purpose of measuring vibrations on different types of structures, the Department of Civil Engineering of Porto University has developed a simple and low cost data acquisition system based on a personal computer and on two A/D conversion cards. Several computer codes have been developed in order to either sample, view and store series or to go into a more thorough analysis, allowing the construction of the most well-known spectral estimates and particularly the obtention of FRF estimates for different forms of excitation. For the identification of modal parameters, some single or multi-degree-of-freedom algorithms are also available.

This system has been used to identify modal parameters from a concrete square 6.6x6.6m^2 plate using an impact hammer as a source of excitation and a set of piezoelectric accelerometers as motion transducers.

The great consistency of the identified parameters and the good comparison between those results and the corresponding values obtained from a FEM analysis of the plate evidence the good performance of the system and the quality of the analysis.

1 - INTRODUCTION

The technological development of the last decade made possible the construction of simple, reliable and low cost systems dedicated to the measurement of vibrations on different types of structures. As a consequence, the interest in using experimental techniques for validating mathematical models and characterizing the dynamic behaviour of complex structures has increasingly grown during the last few years. In the civil engineering area, some particular problems, like the effect of underground traffic or explosions on buildings, the influence of flood dischargers on dams

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or the quantification of behaviour coefficients for seismic design have gained importance and contributed to the motivation of developing an excitation and a measurement system at the Department of Civil Engineering of Porto University.

The main requirements for this system were: low cost, flexibility and portability. In fact, the objective was to build a low cost system capable of measuring vibrations on different types of large size structures, which would be robust enough, in order to perform well under rough conditions, and portable, so that two persons would be able to transport, set up all the equipment and carry out the measurements.

Face to those requirements, a simple system based on a personal computer supplied with A/D conversion cards and a set of accelerometers were choosen, what demanded the development of software for data acquisition, signal processing and for the identification of dynamic parameters [1]. This system has proved to constitute a reliable and versatile alternative to traditional equipment in several experimental tests that have been done during the last five years.

In this paper, a brief description of the referred excitation and measurement systems is presented and some special features of the developed software are discussed. Application is made to the identification of modal parameters of a reinforced concrete square 6.6x6.6m^2 plate and a comparison between the identified natural frequencies and mode shapes and the corresponding values obtained from a FEM analysis is made in order to validate the modal model.

2 - EXCITATION AND MEASUREMENT SYSTEMS

Ambient excitation is the simplest form of structural excitation, in terms of equipment and test procedures. Unfortunately, the amount of induced energy is not always sufficient to mobilize the most important mode shapes and, on the other side, the impossibility of measuring the input excitation, usually overcome with the assumption that the corresponding signal is characterized by a band limited white noise, may lead to certain difficulties in the analysis of experimental data. For these reasons, an excitation system has been developed at Porto University, consisting of:

- an impact hammer PCB, model 086B50, equiped with a force sensor, mainly directed to the test of medium size structures, like certain small buildings, small bridges and water reservoirs;
- an electrodynamic shaker APS, model 400 Electro-Seis, also capable of exciting medium size structures;

- an eccentric mass shaker, recently designed and constructed at Porto University, producing a maximum horizontal force of about 25000 N in a frequency range of 0 to 10 HZ, and able to excite large structures, like dams, bridges and tall buildings.

The measurement system is based on a personal computer (Toshiba, model 5200, with a math co-processor) supplied with two A/D data conversion cards (Data Translation, model DT2801A, which include a 12 bit analog to digital converter and two 12 bit digital to analog converters, and are capable of sampling their 8 differential analog input channels at more than 27 kchannel/sec). This system includes a set of twelve piezoelectric high sensitivity accelerometers PCB, model 393C, and four piezoelectric high sensitivity accelerometers Brüel & Kjaer, model 4379S, which are connected to conditioning units Brüel & Kjaer, model 2635, permitting analog filtering and integration to velocity and displacements.

3 - DATA ACQUISITION AND PROCESSING

For the data acquisition, a computer code has been implemented [2], allowing the visualization of data before and immediatly after sampling and storing, as well as the application of certain functions to the measured series, like FFT evaluation and integration. A second computer code can then be used to process all the series obtained in order to estimate Frequency Response Functions (FRF's) and the corresponding coherence functions [1].

The processing of data depends on the type of excitation involved and can be performed according to the following options:

- Ambient excitation - in this case, there is no input measurement and the output series are essencially random in nature. This requires a statistical processing, involving generally very long (usually overlapped) series, whose sequence of operations comprises: a scale conversion, trend removal, decimation, the extraction of overlapped or non-overlapped series, data-window application, raw estimation of auto and cross spectra and, finally, the computation of smoothed estimates of cross and auto spectra and related coherence functions. The approximate evaluation of bias and random errors associated with these estimates is also possible.

- Wide-band forced excitation - this situation includes series obtained either under impact hammer or electrodynamic shaker tests. Basically the sequence of signal processing is identical to the one described above for ambient excitation, except that input series are now available, what allows the evaluation of smoothed FRF estimates using alternatively estimators H1 or H2.

- Stepped-sine excitation - when dealing with this form of excitation, the level of induced energy is usually much higher than the noise level, permitting signals to be treated as deterministic. This leads to a simpler form of processing data, though it should be noted that each output series gives only one spectral line of the FRF estimate. In fact, for each frequency of sine excitation, a short duration series of data points is sampled. After trend removal and, when needed, (i.e., if a significant level of noise is present), a narrow band digital filter centered at the sine frequency is applied. The amplitude of the spectral line of the FRF associated with the excitation frequency is equal to the ratio between amplitudes of response and force series, and the corresponding phase is obtained as the difference between response and force series' phases.

4 - IDENTIFICATION OF MODAL PARAMETERS

A wide variety of methods for the identification of modal parameters are presently available. These methods involve different levels of complexity and are frequently classified according to different criteria, namely their domain of application (time or frequency) and the type of approach accomplished (single or multi-degree of freedom algorithms).

The choice of an algorithm for the identification of modal parameters of a given structure must take into account not only the specific characteristics of the FRF estimates, but also the most suitable conditions of application of the several methods available.

In the special case of large scale structures, whose FRF's usually present a spectral content of interest in a relatively narrow frequency range (less than 100 Hz) and, quite often, a considerable modal interference due to the existance of several rather close natural frequencies, it is usual to consider to be most appropiate the application of frequency domain MDOF algorithms, like the Rational Fraction Polynomial method (RFP).

Although the use of SDOF methods in the frequency domain can origin some difficulties in case of high modal interference, it reveals the remarkable advantage of permitting to obtain, on the basis of relatively simple software, a set of parameters that can be used as initial estimates for MDOF identification methods.

On the other side, and against the philosophy that presently rules the development of algorithms for structural identification in many fields of engineering, which consists of implementing extremely powerful programs where the participation of the user is very reduced, it is the authors' opinion that interactive schemes play a very important role in the problems of modal analysis of civil engineering structures, where the total number of modes of interest is relatively low. In fact, the user's experience and sensibility and the possibility of control of the process of identification can avoid solutions with no physical meaning and improve the quality of the estimates.

The above referred aspects led the Department of Civil Engineering of Porto University to begin developing a new system for data analysis with the implementation of several SDOF techniques of identification in the frequency domain, namely: the peak amplitude method, the circle-fit method, the method of Dobson [3] and a more recent approach developed by Mau and Wang [5]. Several programs for structural identification were thus developed, written in FORTRAN77 and proportioning a certain degree of interactivity and useful graphic output options [1].

As it is well known, the peak amplitude method is the oldest method and also the one that leads to worse results in terms of the identified parameters. It is however an extremely simple and useful approach for practical applications, what justified its implementation.

The programs developed on the basis of the circle-fit and Dobson methods correspond indeed to simplified versions of multi-degree of freedom algorithms, in which the estimation of the parameters corresponding to each mode is preceded by the elimination in the FRF of the residues associated to the modes previously identified. The application of these algorithms, that involve few computer processing time and memory requirement, usually reveals rather efficient and accurate when the modal interference is not too much significative.

With regard to the computer program based on the method developed by Mau and Wang, it is worth referring the great simplicity of application and the better quality of the estimates when compared to those obtained by the peak amplitude method. This algorithm presents the additional advantage of not involving the phase values of the FRF, what is a particularly important aspect when dealing with stepped-sine excitation, as it is not always easy to evaluate this parameter, whose quality of estimation is often rather worse than the corresponding to the estimate of the amplitude.

In order to overcome certain difficulties that can be caused by the very high modal interference that may occur in some structural systems, preventing that the estimates obtained from the application of any of the methods previously referred may be accepted as final estimates, there is still available a program of modal identification developed based on the MDOF algorithm in the frequency domain called Rational Fraction Polynomial method (RFP) [4], which applies the decomposition in orthogonal polynomials of Forsythe to the numerator and denominator of the FRF expression, and the least squares method to the minimization of the square error defined as the square difference between the measured FRF and the theoretical FRF evaluated by a state space formulation. This is a completely general approach, where no restrictions are introduced with regard to the damping characteristics, as it happens with the other formulations previously referred, where proportional damping is currently assumed.

The interactivity and the graphical facilities proportioned by this algorithm permit a high quality analysis of the measured FRF's.

5 - CASE STUDY

5.1 - Definition and numerical modelling

Using the above described measurement and excitation systems it was possible to analyse the dynamic behaviour of a reinforced concrete 6.6x6.6m^2 square plate. Four columns support this varying thickness plate (according to Figure 1), which constitutes the roof of a test tank located at the University site.

In order to make a subsequent comparison between the modal parameters obtained experimentally and the corresponding parameters of the theoretical model, a numerical modelling of the plate was developed based on a convenient discretization of 1/4 of the structure (Figure 1a)) in a finite element mesh composed by 51 elements.

Using a computer program for the dynamic analysis of shells on the basis of a Ahmad FEM algorithm, and simulating different boundary conditions for this part of the plate, respecting the symmetry conditions, it was possible to evaluate the lowest 18 natural frequencies (Table 1) and the corresponding modal shapes for the following hipothesis: symmetry with regard to the medians of the plate (SS); symmetry with regard to one median and anti-symmetry with regard to the other one (SAS); anti-symmetry with regard to both medians of the square (ASAS).

Some of these modal shapes are represented graphically in Figure 2.

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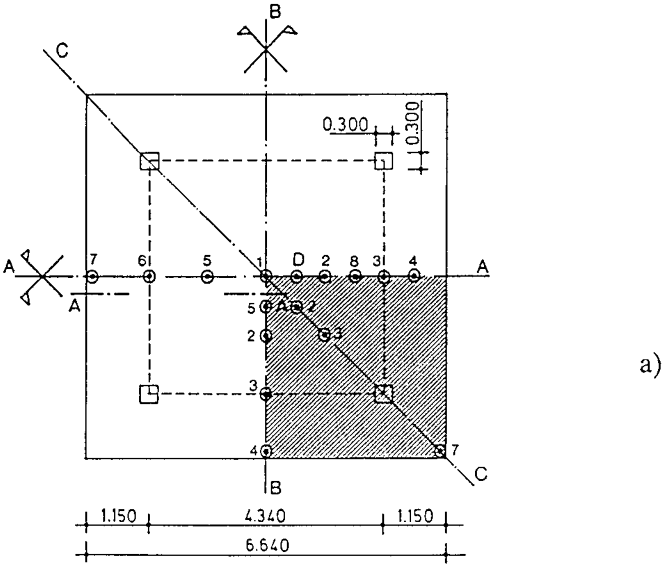

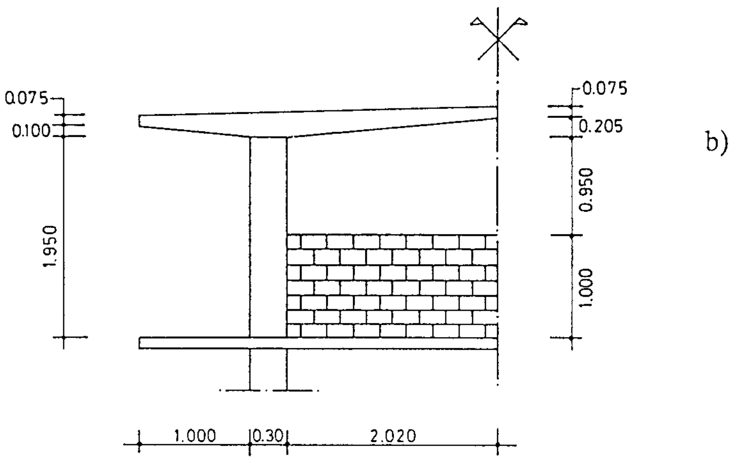

Figure 1 - Schematic representation of the tested structure.

Table 1 - Calculated natural frequencies for the three boundary conditions analysed.

Frequency \ Type of modes	SS	SAS	ASAS
1st	17.97	26.45	53.92
2nd	21.41	39.93	56.18
3rd	46.68	49.98	71.92
4th	62.02	65.23	95.62
5th	89.16	76.02	124.51
6th	111.19	110.23	159.36

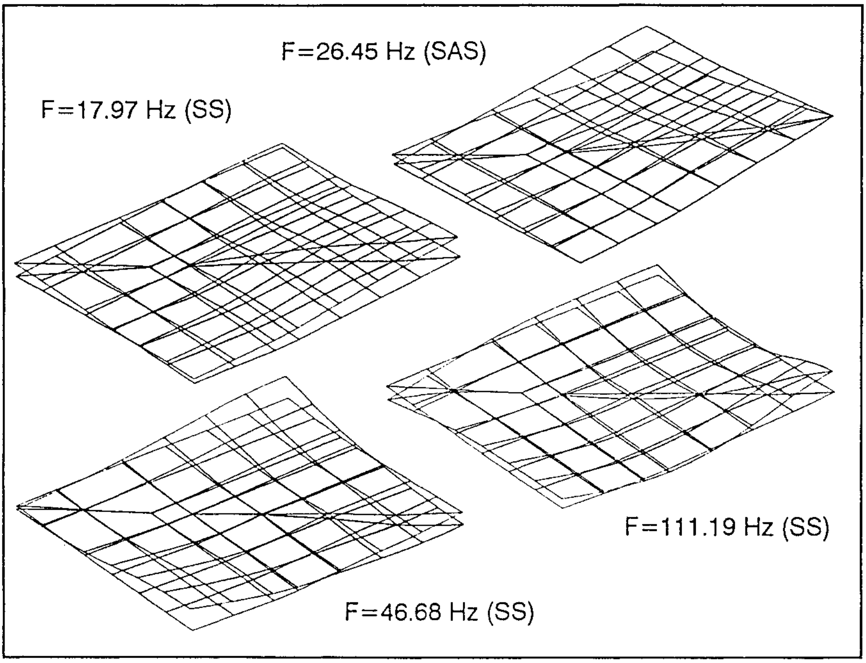

Figure 2 - Illustration of some of the calculated mode shapes.

5.2 - Test procedures

Taking into consideration that the lowest and most important mode shapes are essentially symmetric in relation to one or both of the considered symmetry axes of the plate, a set of measurement points located along those axis and the diagonal axis were choosen (Figure 1a)). Those measurement points correspond to nodes of the defined finite element mesh.

The equipment used during this test consisted of the data acquisition system described in Section 2, of seven accelerometers PCB fastened to metal cubes and pasted to the structure, and of the impact hammer PCB, provided with a soft tip.

As there were only 8 channels available with this setting, the first channel was connected to the impulse hammer and the other 7 were connected to the accelerometers, six of which were placed on positions 1A to 6A of Figure 3 and remained fixed during the tests, while the last one was moved along the other measurement points.

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After testing all the equipment, and since analog filters were not available, several series were sampled at high frequencies (varying from 1000Hz to 2000Hz) under ambient and impact hammer excitations. A brief Fourier analysis showed clearly that the spectral content of the response under both forms of excitation lied in the range 0-100Hz, thus allowing a sampling frequency choice of 400Hz.

The tests performed consisted then of the application of a set of 5 to 10 impulses on each measuring point, storing in each case several time series of 1024 samples, characterizing either the input signal or the corresponding responses at 7 different points of the plate.

At the end, more than 1200 time series with an average duration of 2.5s were recorded, associated to the application of about 170 impulses.

5.3 - Signal Processing

The application of the computer program for data analysis and signal processing, referred in section 3, to the set of sampled series has led to estimates of frequency response functions expressed in terms of the relation acceleration/force. The procedure used comprehended the following operations: scale conversion; trend removal; raw estimate of input and outputs auto spectra and of cross spectra between input and outputs; smoothed estimates of FRF's applying the estimator H_1, and averaging over 5 to 10 raw estimates of auto spectra and cross spectra; estimation of coherence functions.

It should be noted that the application of data windows hasn't revealed necessary as it was observed that the structural responses vanished during the time of observation, leading to a neglectable leackage effect in the sampled series.

An example of one FRF and of the corresponding coherence function is presented in Figure 3.

A brief analysis of all the estimates obtained permitted to extract the following conclusions:

- the estimates of the coherence functions present, in general, rather high values, except in some narrow frequency intervals where the structural response has a very low intensity, thus becoming the signal/noise ratio low. This fact reveals a very good quality of the FRF's obtained, aspect that may still be emphasized when comparing cross FRF estimates (e.g. FRF2-1 and FRF1-2 (Figure 4)) and checking the Maxwell theorem;

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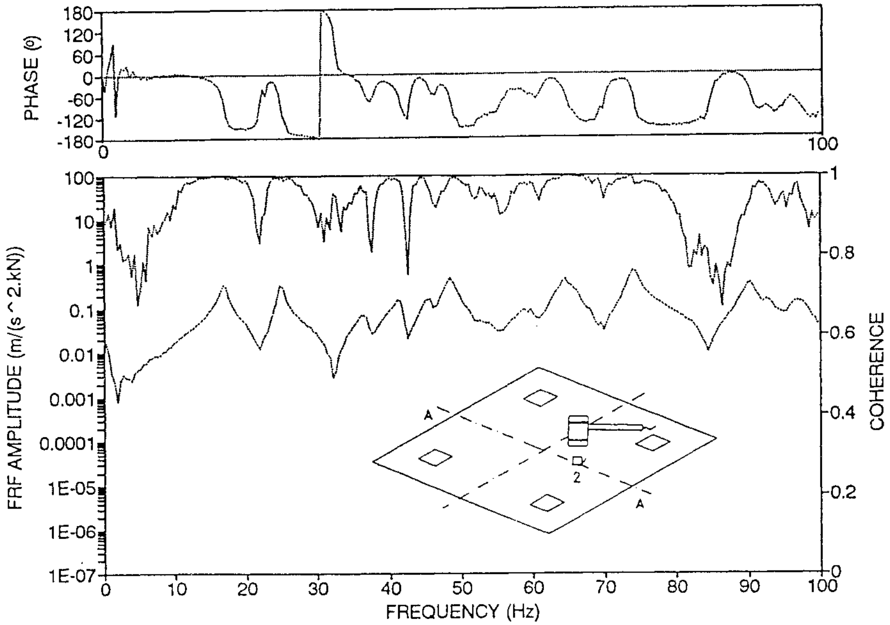

Figure 3 - Example of FRF and coherence function estimate.

- the number of natural frequencies contained in the interval of analysis (0-100Hz) is rather high, what leads to the existance of a significative modal interference and, consequently, to some difficulty in the identification of modal parameters. This fact can be clearly understood observing FRF2-2A, presented in Figure 3, corresponding to a situation in which only (SS) and (SAS) modes were induced;

- taking into consideration that the major part of the response measurements took place in points located along the symmetry axis coincident with the medians of the plate, one may think that essencially, just (SS) and (SAS) mode shapes have been analysed. However, the measured FRF's contain some residues in correspondence with the natural frequencies of the (ASAS) mode shapes. Since the signal/noise ratio is very low along those measurement points, a decrease of coherence may be observed near those natural frequencies;

- Table 1 shows clearly the existance of vibration modes with very close frequencies. The analysis of their shapes, obtained nummerically, reveals significative similarities between them. Moreover, the (SAS) mode shapes are indeed coupled modes, since, as a consequence of the symmetry conditions, each natural frequency corresponds to two different possible deflection configurations. These two aspects justify

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the occurance of a significative complex character in several mode shapes, that can be observed, for instance, in the evaluation of the phase estimate of FRF2-2A presented in Figure 3.

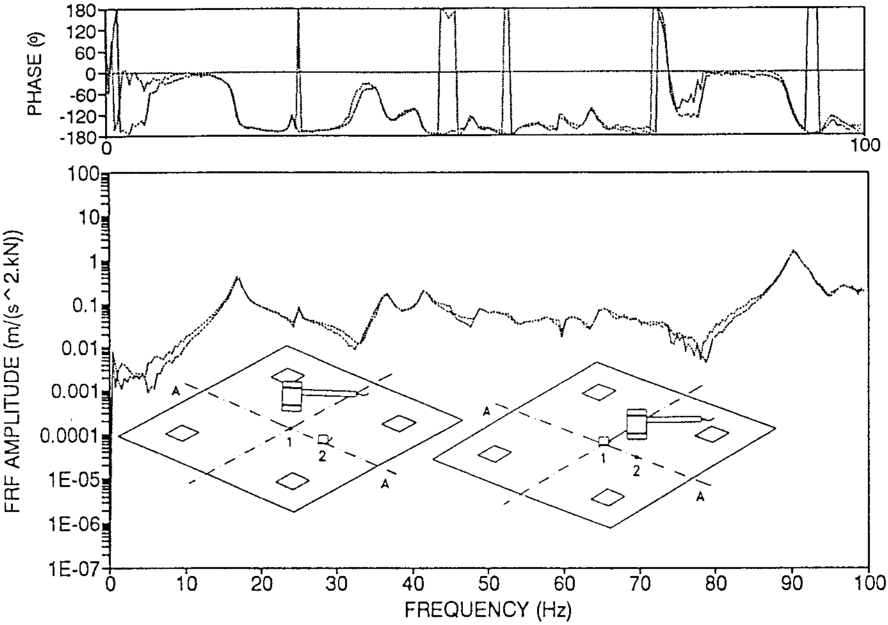

Figure 4 - Superposition of cross FRF estimates (FRF2-1A and FRF1-2A).

5.4 - Identification of modal parameters

The characteristics revealed by the set of FRF estimates obtained suggest that the techniques of one degree-of-freedom are not the most suitable for the identification of the modal parameters of this structure. Therefore, the computer code, previously referred, based on the RFP method, was applied. Figure 5 shows a comparison between one estimated and the corresponding regenerated FRF, obtained on the basis of the identified parameters achieved using this algorithm.

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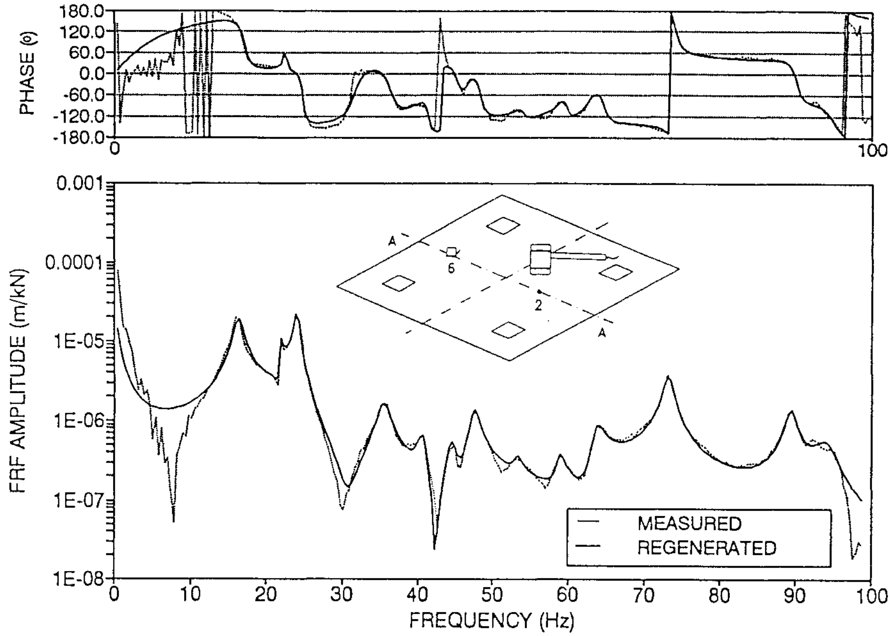

Figure 5 - Example of a measured/regenerated FRF.

The adopted procedure led to an excelent quality of the fittings performed as can be seen on Figure 5. Moreover, the several values identified for each natural frequency present very small variations, what reveals a great consistency in all the dynamic analysis developed. The same conclusion can be drawn based on the values estimated for the modal damping ratios, although greater variations may be observed in this case due to the difficulties presented by the very high modal interference and modal coupling. Table 2 resumes the values of the identified natural frequencies and modal damping ratios.

Table 2 - Identified natural frequencies and damping factors.

Natural Frequency (Hz)	Std-deviation	Damping Ratio (%)	Std-deviation	Type of mode shape
16.96	0.03	3.12	0.24	SS
22:37	0.08	0.60	0.03	SS
24.74	0.04	1.01	0.17	SAS
36.08	0.11	2.40	0.56	SS
40.95	0.26	1.62	0.35	
48.05	0.10	1.26	0.17	SAS
64.81	0.61	1.21	0.41	
74.00	0.17	0.81	0.05	
83.29	1.10	1.48	0.61	
90.22	0.04	0.75	0.08	SS

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It should still be mentioned the important complex nature of the values of the residues identified, aspect that can equally be related to the high modal interference and modal coupling.

5.5 - Validation of the theoretical model

Using the modal parameters identified trough the application of the RFP algorithm to the set of the FRF's analysed, it was possible to obtain estimates of some mode shapes, what permitted a comparison with the theoretical parameters, by analogy with the most similar mode shapes provided by the numerical model.

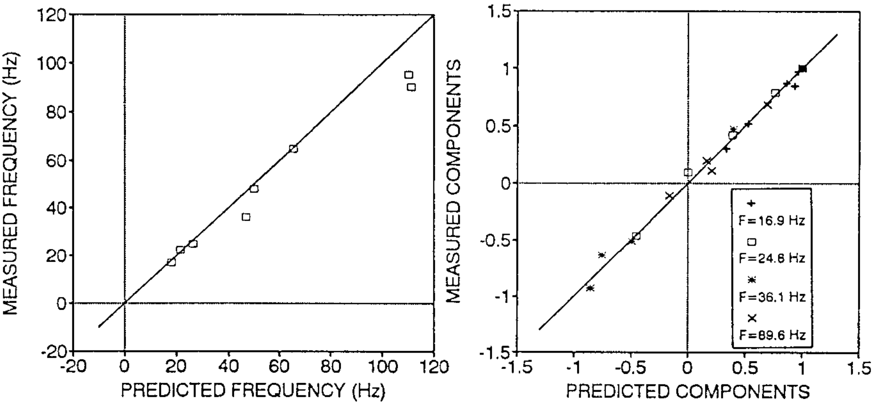

Fig. 6 - Comparison between identified and calculated a) - natural frequencies; b) - components of mode shapes.

The comparison established in terms of natural frequencies is presented in the plot of Figure 6-a), which shows an excelent compatibility between the values of the 1st, 2nd, 3rd, 5th and 6th identified frequencies and the corresponding predicted frequencies, though not so good in the other three cases, whose representative points are located below the straight line of unit slope, suggesting a greater stiffness of the theoretical model. These differences would be particularly expectable for the highest natural frequencies, as the corresponding mode shapes are more dependent on the boundary conditions considered for the plate and on the type of discretization adopted for the constitution of the finite element mesh.

With regard to the modes of vibration, the comparisons were established in terms of the amplitude of the components, despite the strong complex nature of some mode shapes identified. It should be noted that this criterion of comparison is only valid when damping is assumed as

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proportional, what can be considered an acceptable approximation in this case, taking into consideration the very low values of the identified modal damping ratios.

Good correlations between numerical and experimental modal components are also evidenced by MAC (Modal Assurance Criterion) values superior to 0.990 and by Figures 6-b) and 7.

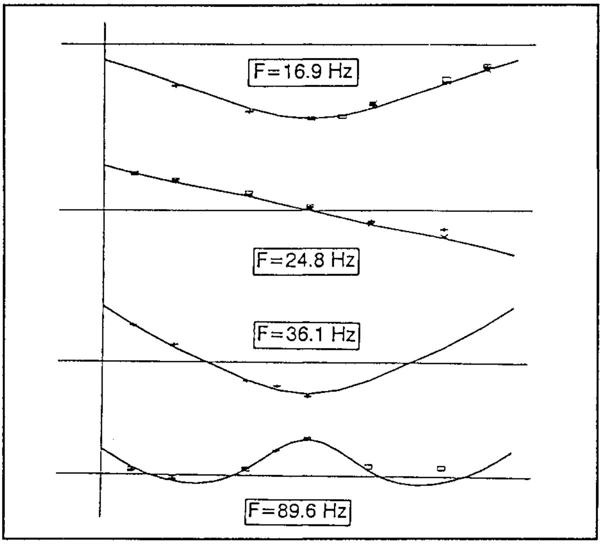

Fig. 7 - Components of the measured/predicted mode shapes along the medians of the plate.

6 - FINAL REMARKS

In this paper an excitation and measurement system developed at the Department of Civil Engineering of Porto University was described, taking into consideration the goal of analysing experimentally the dynamic behaviour of large size Civil Engineering Structures. Though this system presents a great simplicity and low cost, it was necessary to develop appropriate software for the processing of sample series and for the identification of modal parameters, and along this work some features of this task were also discussed.

As a result of this effort, and after the realization of several tests in real structures in order to calibrate the equipment and the software, it was possible to present a detailed study of the dynamic behaviour of a reinforced concrete square plate. This study involved a numerical modelling of the structure and a performance of experimental tests using an impulse hammer as a source of excitation, from which a set of input and response series were recorded and analysed, giving some of the most important natural frequencies, modal damping ratios and the corresponding components of mode shapes for the intrumented points.

Although the hammer excitation is not a traditional source of excitation for medium size structures, due to the low signal/noise ratio, that can lead to some difficulties in the identification of modal parameters and, despite the high complexity of the structure tested as a consequence of the significative modal interference and modal coupling, a great consistency of the identified modal parameters was achieved.

The comparison between predicted and measured frequencies and mode shapes suggested rather good agreement between the theoretical and the experimental models.

This proves the good performance of the developed analysis system and the good quality of the experimental tests.

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