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Experimental Investigation of Kinetic Energy and Momentum Coefficients in Regular Channels with Stiff and Flexible Elements Simulating Submerged Vegetation

Elżbieta KUBRAK, Janusz KUBRAK, and Adam KICZKO

Faculty of Civil and Environmental Engineering,
Warsaw University of Life Sciences, Warszawa, Poland;
e-mails: Elzbieta_Kubrak@sggw.pl, Janusz_Kubrak@sggw.pl,
Adam_Kiczko@sggw.pl (corresponding author)

Abstract

The paper addresses the problem of determination of the energy and momentum coefficients for flows through a partly vegetated channel. These coefficients are applied to express the fluid kinetic energy and momentum equations as functions of a mean velocity. The study is based on laboratory measurements of water velocity distributions in a straight rectangular flume with stiff and flexible stems and plastic imitations of the Canadian waterweed. The coefficients were established for the vegetation layer, surface layer and the whole flow area. The results indicate that the energy and momentum coefficients increase significantly with water depth and the number of stems per unit channel area. New regression relationships for both coefficients are given.

Key words: submerged flexible and rigid vegetation, velocity profile, energy and momentum coefficients.

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1. INTRODUCTION

Understanding the impact of vegetation on water flow conditions has become important in river restoration projects. In the last decades a turbulent flow structure in rivers and channels with vegetation have been extensively studied by Nepf and Vivoni (2000), Nepf and Ghisalberti (2008), Nepf (2012), Cameron *et al.* (2013), Poggi *et al.* (2004), Righetti (2008), and others. In the same time, studies to find an appropriate way of description of the vegetation roughness (see, *e.g.*, Kubrak *et al.* 2008, 2012, 2013) were evaluated. River flows can be described in detail by the Navier–Stokes equation. However, in many practical applications it is possible to simplify the model, reducing computational and data costs. In engineering applications an assumption of a one-dimensional character of a flow is often used, where all state variables refer to averaged quantities. For a channel flow, variables in a wide cross-section are averaged over the water depth and width. The common practical problem is the determination of water profiles in rivers and channels. The solution is based on a one-dimensional form of the energy equation, or the momentum equation, if the water surface profile varies rapidly and is established under a mixed flow regime. Both these equations use different velocity-distributions coefficients, whose values are nearly equal, but involve different meanings of the frictional losses (Chow 1959, Yen 2002).

The expression for the fluid kinetic energy equation as a function of mean velocity requires the energy coefficient α (also termed as the Coriolis coefficient or the Saint Venant coefficient) to be introduced. In the case of the momentum equation it is the momentum coefficient β (also termed as the Boussinesq coefficient). These coefficients might be elaborated on account of velocity measurements or an assumed velocity profile with depth. Fenton (2005) stated that neglecting these coefficients in hydraulic calculations of an open channel flow might introduce an error of 5-10%. The presence of vegetation within a channel causes a distortion of a flow field, which is shaped by plant heights, spacing geometrical and mechanical properties. If a plant height is lower than the water depth, vegetation is considered as short and within the flow field at least two layers can be distinguished, namely: vegetated and surface ones. This simplified classification is an introduction into assessing the vegetation impact on analyzed coefficient values.

To express the kinetic energy in the channel cross-section using the mean velocity, it is necessary to adopt the correction coefficient, defined as:

$$\alpha = \frac{\int v^3 dA}{v_m^3 A} \ . \tag{1}$$

Similarly, the stream momentum in the channel cross-section, expressed as a function of the mean velocity, requires the momentum coefficient β:

$$\beta = \frac{\int v^2 dA}{v_m^2 A} , \tag{2}$$

where v_m is the mean water velocity in the cross-section calculated as $v_m = Q/A$, Q is the discharge, and A is the cross-sectional area.

For a uniform velocity field, the energy and momentum coefficients are equal to one. On the contrary, the coefficients take values higher than one. Analyzing Eqs. 1 and 2 it can be seen that the momentum coefficient β should be smaller than the energy coefficient α. Chow (1959) presented values of the energy and momentum coefficients for natural and artificial channels of uniform cross-sections. In this study, the reported energy coefficient α is within the range of $\alpha \in (1.10 \div 2.00)$, and the momentum coefficient satisfies the following: $\beta \in (1.03 \div 1.33)$. Strauss (1967) stated that the energy and momentum coefficients are functions of a velocity profile over the depth and a cross-sectional shape, parameterized with lengths of the bottom and slope. For wide, rectangular channels, the coefficient values are mostly determined only by the velocity profile over the depth (Rehbock 1922). In the literature there is a lack of information concerning values of these coefficients for uniform channels with vegetation.

This article accounts for the determination of the energy and momentum coefficients in a rectangular, wide channel with a short vegetation simulated with stiff and flexible stems, as well as with plastic imitations of the Canadian waterweed (*Elodea Canadensis sp.*). The outcome and the main novelty of the research are methodological grounds to improve accuracy of one-dimensional flow routing models of partly vegetated channels, used to solve practical problems like calculations of water depths in drainage-irrigation systems.

The application of the rectangular channel allowed to exclude the effect of the channel cross-section shape on the velocity profile and as a result on the computed coefficients. The values of energy and momentum coefficient were determined in three sets: for vegetation and surface layers, and for the total cross-section. As a result it was possible to assess the energy and momentum coefficients for a velocity field affected by the presence of vegetation, also in a free region when values were computed for a surface layer. The common approach in practical studies, involving a one-dimensional modeling of water profiles, is an assumption of a zero flow in a vegetation layer. The article is based on laboratory experiments whose results were already presented by Kubrak *et al.* (2008, 2012, 2013), wherein they were used to verify the model of the velocity profile for a channel with different elements imitating vegetation. The methodology and outcomes of laboratory experiments are given here briefly.

2. EXPERIMENTAL INVESTIGATIONS

The hydraulic experiments were settled in the Hydraulics Laboratory of the Warsaw University of Life Sciences – SGGW. The laboratory channel is characterized by a straight run with an adaptable slope, a rectangular cross-section, and the following dimensions: the length of 16 m, width of 0.58 m, and height of 0.60 m (Fig. 1). The measurements were conducted for steady flow conditions, in the channel with elements simulating the stiffness and flexibility, as a result of the pressure force of a stream on vegetation.

The vegetation was simulated by:

- Stiff wooden stems of a cylindrical shape (Fig. 2a), diameter $d = 0.0022$ m and a height of 0.11 m (Kubrak 2007).
- Flexible stems, made from the PVC, of the elliptical shape, diameters $d_1 = 0.00095$ m, $d_2 = 0.0007$ m, and a height of 0.165 m (Fig. 2b); with a mean value of elasticity modulus $E = 3630$ MPa, determined on the basis of measurements performed with the strength testing machine – INSTRON 5582 (Kubrak 2007).
- Artificial plants imitating the Canadian waterweed (*Elodea Canadensis sp.*), made of a PVC with a height of 0.11 m, presented in Fig. 2c (Wójtowicz *et al.* 2010, Kubrak *et al.* 2013).

The vegetation models were installed on a plate of the $k_s = 0.0001$ m roughness height. The mounting nodes were organized in squares of the side length s (Fig. 3). Two horizontal components of mean velocities (longitudinal and transversal) were measured with the use of a programmable electromagnetic liquid velocity meter (PEMS) manufactured by Delft Hydraulics. The velocity measurements were made for two channel slopes: $i = 0.0087$ and $i = 0.0174$. The experimental variants are set up in Table 1.

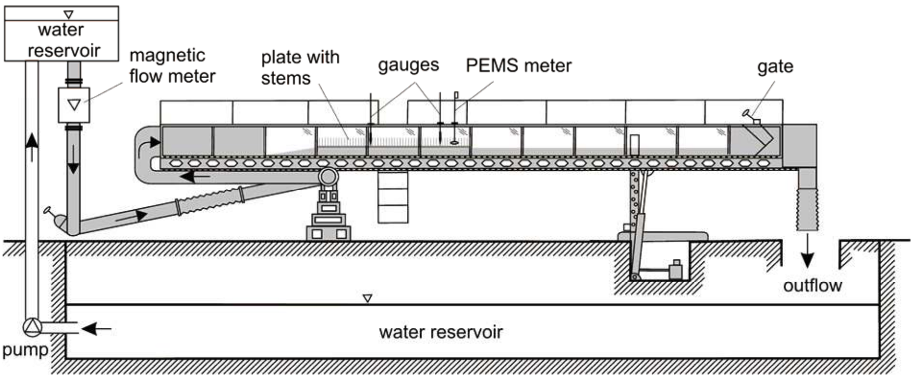
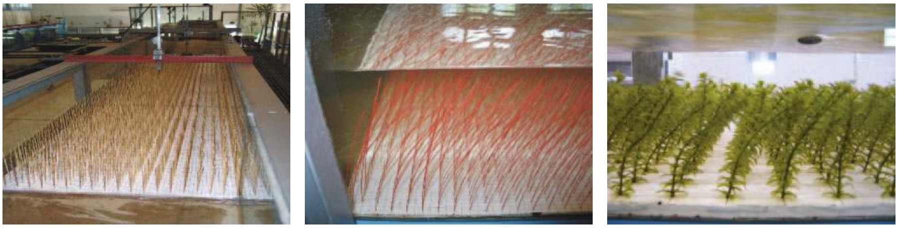


Fig. 1. The layout of the experimental flume.



(a) (b) (c)

Fig. 2. Setup of plant models in the laboratory channel: (a) stiff elements, (b) flexible elements, and (c) submerged elastic elements of artificial plants simulating Canadian waterweed (*Elodea Canadensis sp.*). Photos acquired from Kubrak (2007) and Wójtowicz *et al.* (2010).

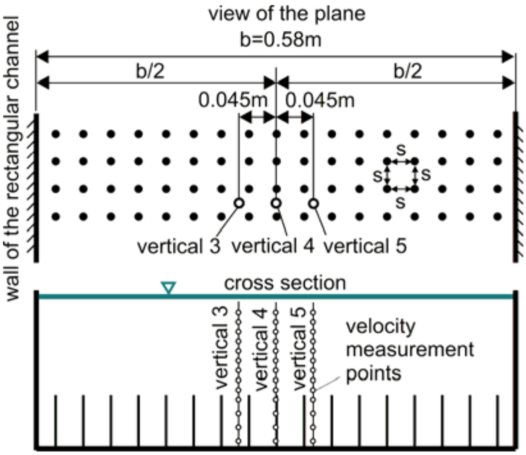


Fig. 3. The artificial vegetation pattern and the placement of measure cross-sections and velocity measure points.

The water discharge was measured with the electromagnetic flow meter, situated at the inflow pipe. The water depth was captured with 0.1 mm accuracy. The water level was controlled with a gate, placed at the channel end (Fig. 1).

Examples of measured and calculated velocity profiles, taken above the stiff and flexible stems, are presented in Fig. 4. Profiles were modeled, basing on the mixing length concept (Kubrak 2007, Kubrak *et al.* 2008, 2013). Because of the high agreement between calculated and measured velocity profiles, resulting in high values of the Linear Correlation Coefficient *R* and small differences between calculated and measured values (MRE), the energy and momentum coefficients were determined taking the calculated profiles.

Table 1

Basic set-up of the experiments of velocity measurements

Element types	Spacing stems s [m]	Number of stems per unit channel area m [stems/m^2]	Slope i [–]	Case
Stiff stems	0.015	4032	0.0087 0.0174	S.1.1 S.1.2
	0.030	1008	0.0087 0.0174	S.2.1 S.2.2
Flexible stems	0.010	10000	0.0087 0.0174	E.1.1 E.1.2
	0.020	2500	0.0087 0.0174	E.2.1 E.2.2
Elements simulating Canadian waterweed	0.045	507	0.0087 0.0174	C.1.1 C.1.2

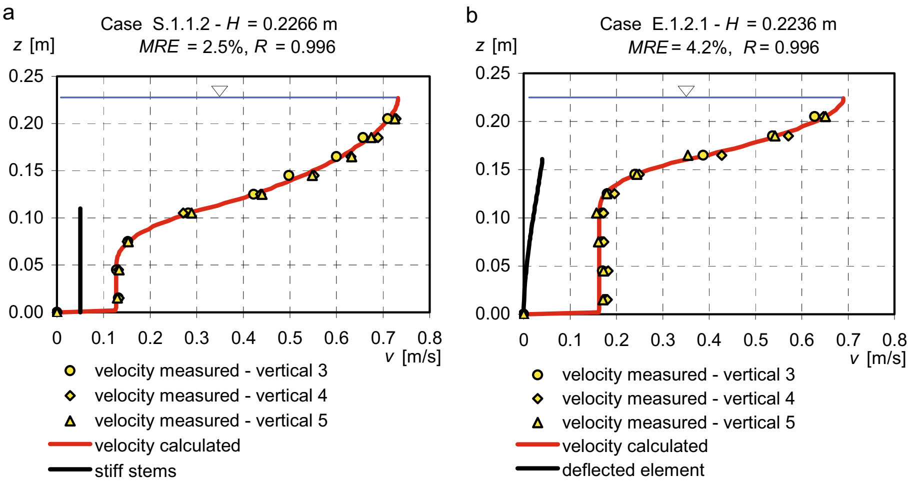


Fig. 4. The calculated and measured velocity profiles: (a) in the channel with stiff stems, and (b) in the channel with flexible stems.

Integration of velocity over the cross-sectional surface A was done using a grid method (Hulsing *et al.* 1966), where the flow field is discretized with the Cartesian grid. For greater accuracy, the size of the grid should be chosen as small as possible and here it always was below 0.0025 m. Assuming that the effective velocity through each grid is equal to that at the center of gravity of the grid, the quantities $\Sigma v dA$, $\Sigma v^2 dA$, and $\Sigma v^3 dA$ are computed. The val-

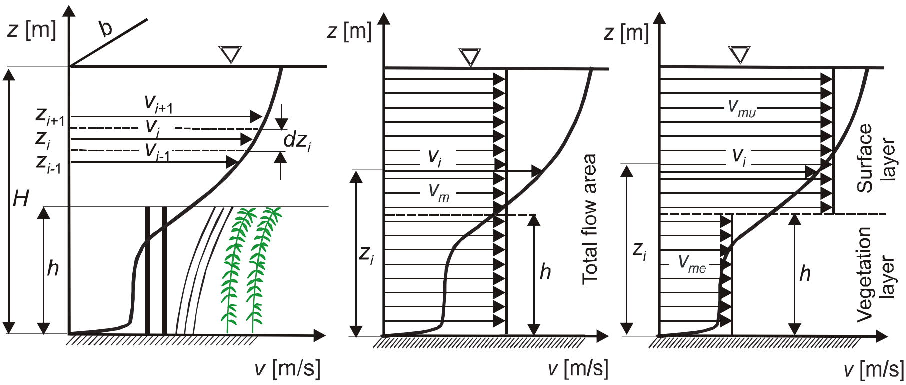


Fig. 5. Vertical profiles of the velocity in the channel used in energy α and momentum β coefficients calculations; v_i is the velocity at the center of gravity of the grid, h – height of the submerged stiff or deflected elements, v_{me} – mean velocity in layer with elements, v_{mu} – mean velocity in layer over elements, and v_m – mean velocity in profile.

ues of α and β are computed according to Eqs. 1 and 2. As the flow field significantly differs in the layer of vegetation and above, the calculations were performed separately for each layer and also for the whole profile (Fig. 5). The coefficients determined for the surface layer reflect the values used in practical applications of the one-dimensional water depth modeling where a flow through vegetation layer is neglected.

3. RESULTS OF CALCULATION AND DISCUSSION

Examples of velocity profiles shaped by stiff stems for which, *i.e.*, coefficients were determined are presented in Fig. 6. Computed energy and momentum coefficients for vegetation layer, upper layer over elements and a total flow area as a function of the ratio of mean velocities of surface and vegetation layers, v_{mu}/v_{me}, are presented in Figs. 7 and 8. The numerical values used in the plots (Figs. 7 and 8) are set together in Table 2.

Figures 6-8 show that the energy α and the momentum β coefficients increase with a variation of the velocity profile. The velocity distribution in the vegetated layer for elements simulating the Canadian waterweed have a similar shape as those for stiff stems: just above a bottom the velocity significantly increases to become uniform for the major part of the vegetated layer and again increases while getting closer to a top of elements. The height of the constant velocity zone decreases with reduction of elements' density and the ratio of water depth and the element height H/h. For smaller elements' densities, the velocity in a constant zone is higher, which results in a more even profile (Fig. 6). That explains why energy α and momentum β coeffi-

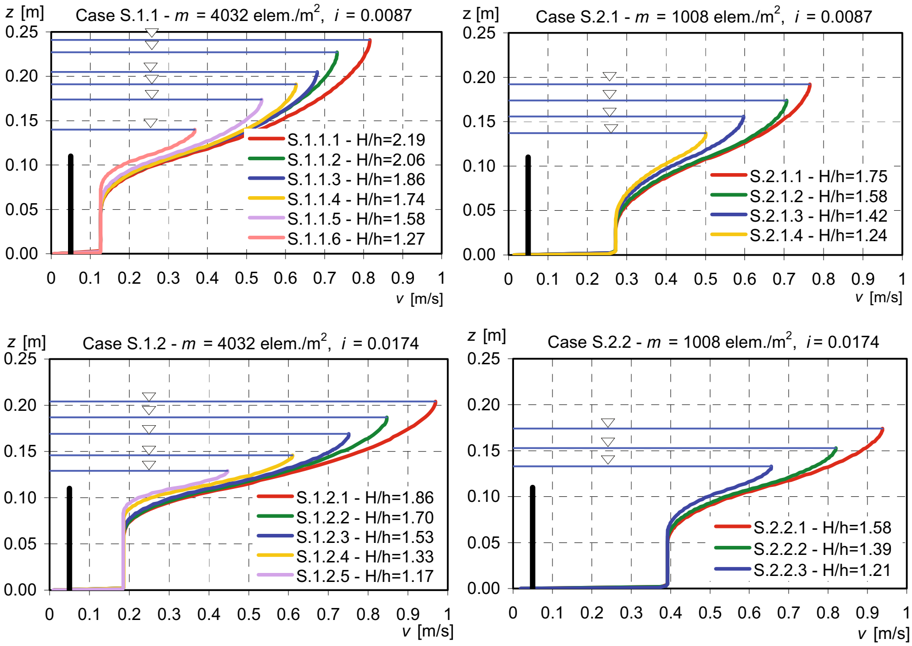
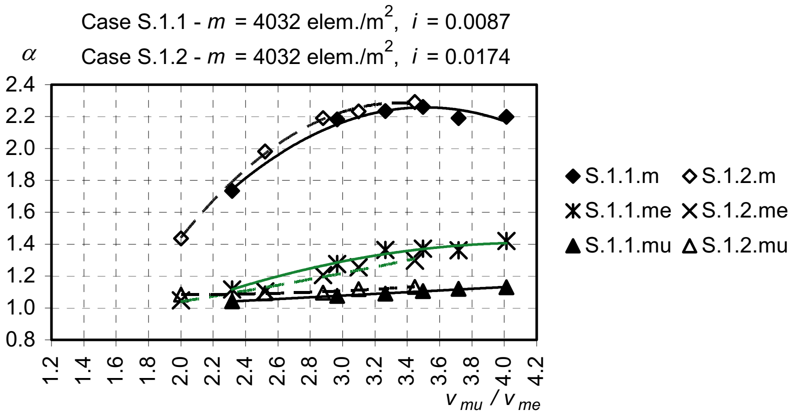


Fig. 6. Computed velocity profiles for flume with stiff elements (cases S.1.1, S.1.2, S.2.1, S.2.2) used for calculation of energy α and momentum β coefficients.

cients for dense elements spacing reach higher values and are characterized by stronger variation than for the surface layer.

The highest variation of the velocity occurs when the total flow area is considered and increases with the flow depth. This is seen in the energy and momentum coefficients values, which follow this schema: they are highest for the total flow area and increase with a flow depth.

The variation of the flow profile decreases with an increase of plant spacing. For spacing of $m = 4032$ stems/m^2 with a flow depth and the stiff stems height ratio of $H/h \in (1.17 \div 2.19)$, the relationship of a mean velocity in the surface and vegetation layer is in the range of $v_{mu}/v_{me} \in (2.00 \div 4.01)$. The corresponding values of the energy coefficient are in the range $\alpha \in (1.44 \div 2.29)$. In the surface layer, where the velocity variation is the lowest, the energy coefficient is within the range of $\alpha \in (1.04 \div 1.13)$. In the vegetation layer, where usually an even velocity distribution is presupposed, values of α coefficients appeared to be higher than in the surface layer and fall within $\alpha \in (1.05 \div 1.42)$. An increase in the spacing of stiff stems from $m = 4032$ stems/m^2 to $m = 1008$ stems/m^2 causes a reduction of the velocity profile variation, characterized by the velocity ratio $v_{mu}/v_{me} \in (1.42 \div 2.04)$.



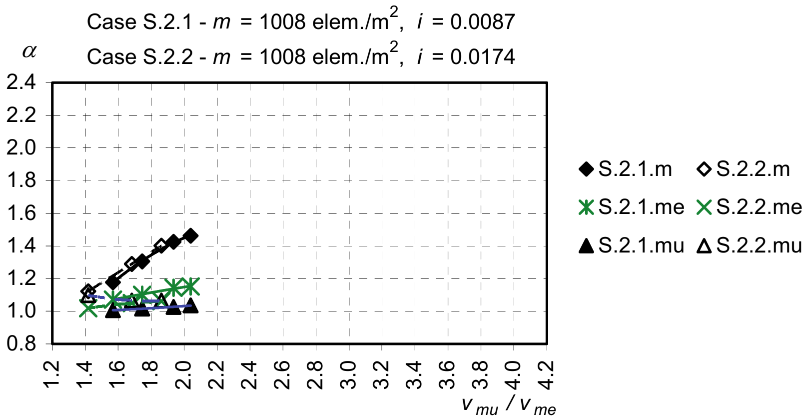


Fig. 7. Computed energy α coefficients in relation to the quotient of mean velocities in layer with stiff elements, over elements and the mean velocity in profile (cases S.1.1, S.1.2, S.2.1, S.2.2), where *me* is the mean velocity in layer with elements, *mu* – mean velocity in layer over elements, and *m* – mean velocity in profile.

Consequently, the values of the energy coefficient are reduced to $\alpha \in (1.12 \div 1.46)$ for the total flow area and $\alpha \in (1.01 \div 1.10)$ for the surface layer. In a vegetation layer, the energy coefficient is in the scope of $\alpha \in (1.02 \div 1.15)$ and slightly surpasses values reported in the surface layer.

The highest values of the momentum coefficient, $\beta \in (1.12 \div 1.40)$, are recorded for the total flow area in a channel with stiff stems of $m =$ 4032 stems/m^2 spacing. The increase of the spacing to $m = 1008$ stems/m^2 reduces the momentum coefficient to the range of $\beta \in (1.04 \div 1.15)$. In the vegetation and surface layers the values of the momentum coefficient are close to unity: $\beta \in (1.00 \div 1.05)$.

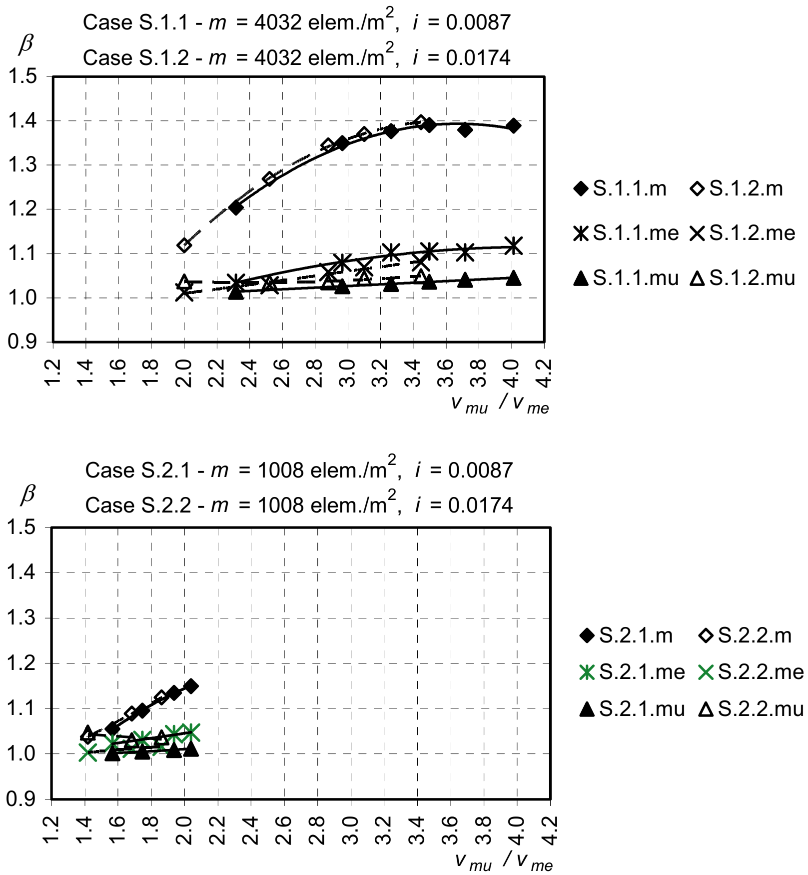


Fig. 8. Computed momentum β coefficients in relation to quotient of mean velocities in layer with stiff elements, over elements and mean velocity in profile (cases S.1.1, S.1.2, S.2.1, S.2.2), where *me* is the mean velocity in layer with elements, *mu* – mean velocity in layer over elements, and *m* – mean velocity in profile.

The results presented in Figs. 7 and 8 suggest that the energy and momentum coefficients depend on the water height above the vegetation and they are not influenced by the channel bottom slope.

A similar dependence was constructed for flexible stems (m = 10000 stems/m^2, m = 2500 stems/m^2) and for plastic imitations of the Canadian waterweed (m = 507 stems/m^2). The calculated ranges of the energy and momentum coefficients as a function of water depth H and the plant height ratio h are presented in Table 2. The results for all investigated plant models are shown in Figs. 9 and 10.

Table 2

The energy and momentum coefficients computed for the vegetation, surface layers, and the total flow area

Case		H [m]	h [m]	Total flow area				Vegetation layer				Surface layer			
				Q [m^3/s]	v_m [m/s]	α [–]	β [–]	Q_{me} [m^3/s]	v_{me} [m/s]	α [–]	β [–]	Q_{mu} [m^3/s]	v_{mu} [m/s]	α [–]	β [–]
S.1.1	S.1.1.1	0.2414	0.11	0.0602	0.430	2.20	1.39	0.0104	0.163	1.42	1.12	0.0498	0.654	1.13	1.05
	S.1.1.2	0.2266	0.11	0.0.501	0.381	2.19	1.38	0.0102	0.159	1.36	1.10	0.0400	0.591	1.12	1.04
	S.1.1.3	0.2047	0.11	0.0406	0.342	2.26	1.39	0.0101	0.159	1.37	1.11	0.0305	0.556	1.11	1.04
	S.1.1.4	0.1911	0.11	0.0346	0.312	2.23	1.38	0.0102	0.159	1.36	1.10	0.0244	0.519	1.09	1.03
	S.1.1.5	0.1736	0.11	0.0263	0.261	2.18	1.35	0.0097	0.152	1.28	1.08	0.0166	0.451	1.08	1.03
	S.1.1.6	0.1399	0.11	0.0144	0.178	1.73	1.20	0.0089	0.139	1.12	1.03	0.0056	0.322	1.04	1.01
S.1.2	S.1.2.1	0.2043	0.11	0.0570	0.481	2.29	1.40	0.0144	0.226	1.30	1.08	0.0426	0.779	1.13	1.05
	S.1.2.2	0.1869	0.11	0.0450	0.415	2.23	1.37	0.0142	0.223	1.25	1.07	0.0308	0.691	1.12	1.04
	S.1.2.3	0.1686	0.11	0.0349	0.357	2.19	1.35	0.0138	0.216	1.21	1.06	0.0212	0.622	1.10	1.04
	S.1.2.4	0.1464	0.11	0.0237	0.279	1.98	1.27	0.0129	0.203	1.10	1.03	0.0108	0.512	1.09	1.04
	S.1.2.5	0.1292	0.11	0.0168	0.224	1.44	1.12	0.0125	0.195	1.05	1.01	0.0043	0.390	1.08	1.04
S.2.1	S.2.1.1	0.1920	0.11	0.0534	0.480	1.46	1.15	0.0212	0.332	1.15	1.05	0.0322	0.677	1.04	1.01
	S.2.1.2	0.1735	0.11	0.0443	0.440	1.43	1.14	0.0209	0.328	1.14	1.04	0.0234	0.635	1.03	1.01
	S.2.1.3	0.1559	0.11	0.0346	0.383	1.31	1.10	0.0200	0.314	1.10	1.03	0.0146	0.548	1,02	1.01
	S.2.1.4	0.1365	0.11	0.0268	0.338	1.18	1.06	0.0195	0.305	1.07	1.02	0.0073	0.476	1.01	1.00
S.2.2	S.2.2.1	0.1742	0.11	0.0587	0.581	1.40	1.13	0.0281	0.441	1.06	1.02	0.0306	0.821	1.07	1.04
	S.2.2.2	0.1531	0.11	0.0460	0.518	1.29	1.09	0.0277	0.435	1.05	1.01	0.0183	0.732	1.07	1.03
	S.2.2.3	0.1331	0.11	0.0349	0.542	1.12	1.04	0.0269	0.421	1.02	1.00	0.0080	0.597	1.10	1.05
E.1.1	E.1.1.1	0.2661	0.165	0.0405	0.262	2.85	1.52	0.0122	0.127	1.21	1.05	0.0283	0.482	1.17	1.06
	E.1.1.2	0.2576	0.165	0.0354	0.237	2.74	1.48	0.0118	0.124	1.15	1.04	0.0235	0.438	1.16	1.06

to be continued

Table 2 (continuation)

Case		H [m]	h [m]	Total flow area				Vegetation layer				Surface layer			
				Q [m^3/s]	v_m [m/s]	α [–]	β [–]	Q_{me} [m^3/s]	v_{me} [m/s]	α [–]	β [–]	Q_{mu} [m^3/s]	v_{mu} [m/s]	α [–]	β [–]
E.1.1	E.1.1.3	0.2475	0.165	0.0308	0.215	2.65	1.45	0.0117	0.123	1.10	1.02	0.0191	0.398	1.18	1.07
	E.1.1.4	0.2275	0.164	0.0245	0.186	2.42	1.38	0.0115	0.120	1.08	1.02	0.130	0.354	1.14	1.05
E.1.2	E.1.2.1	0.2236	0.161	0.0376	0.290	2.51	1.41	0.0172	0.184	1.19	1.05	0.0204	0.568	1.12	1.05
	E.1.2.2	0.2184	0.162	0.0348	0.275	2.46	1.39	0.0171	0.182	1.18	1.05	0.0177	0.541	1.12	1.05
	E.1.2.3	0.2068	0.161	0.0292	0.243	2.15	1.30	0.0166	0.178	1.12	1.03	0.0126	0.473	1.11	1.05
	E.1.2.4	0.1951	0.162	0.0242	0.214	1.79	1.21	0.0163	0.174	1.07	1.02	0.0078	0.407	1.12	1.05
E.2.1	E.2.1.1	0.2386	0.153	0.0509	0.368	1.49	1.15	0.0241	0.271	1.09	1.03	0.0286	0.540	1.06	1.02
	E.2.1.2	0.2136	0.154	0.0400	0.323	1.35	1.11	0.0237	0.265	1.07	1.02	0.0164	0.473	1.06	1.02
	E.2.1.3	0.1935	0.155	0.0322	0.287	1.20	1.06	0.0231	0.257	1.03	1.01	0.0091	0.407	1.07	1.03
E.2.2	E.2.2.1	0.2131	0.132	0.0742	0.600	1.44	1.14	0.0340	0.444	1.17	1.05	0.0402	0.854	1.05	1.02
	E.2.2.1	0.1925	0.131	0.0642	0.575	1.45	1.14	0.0340	0.448	1.18	1.05	0.0301	0.845	1.05	1.02
	E.2.2.1	0.1799	0.133	0.0536	0.513	1.35	1.11	0.0322	0.431	1.14	1.05	0.0203	0.747	1.06	1.03
C.1.1	C.1.1.1	0.1514	0.050	0.0366	0.417	1.68	1.23	0.0055	0.184	1.04	1.02	0.0311	0.539	1.16	1.06
	C.1.1.2	0.1557	0.053	0.0377	0.417	1.74	1.24	0.0057	0.179	1.04	1.02	0.0318	0.544	1.19	1.07
	C.1.1.3	0.1604	0.051	0.0382	0.410	1.72	1.24	0.0052	0.169	1.04	1.02	0.0330	0.530	1.18	1.06
	C.1.1.4	0.1618	0.047	0.0407	0.434	1.66	1.22	0.0050	0.175	1.04	1.02	0.0358	0.547	1.18	1.06
	C.1.1.5	0.1481	0.053	0.0337	0.393	1.65	1.21	0.0061	0.190	1.04	1.02	0.0277	0.512	1.15	1.05
	C.1.1.6	0.1432	0.054	0.0318	0.383	1.62	1.20	0.0064	0.197	1.04	1.02	0.0254	0.502	1.14	1.05
	C.1.1.7	0.1400	0.055	0.0305	0.375	1.62	1.20	0.0065	0.199	1.02	1.01	0.0240	0.493	1.15	1.05

to be continued

Table 2 (continuation)

Case		H [m]	h [m]	Total flow area				Vegetation layer				Surface layer			
				Q [m^3/s]	v_m [m/s]	α [–]	β [–]	Q_{me} [m^3/s]	v_{me} [m/s]	α [–]	β [–]	Q_{mu} [m^3/s]	v_{mu} [m/s]	α [–]	β [–]
C.1.1	C.1.1.8	0.1371	0.056	0.0279	0.350	1.62	1.20	0.0064	0.190	1.04	1.02	0.0215	0.468	1.13	1.04
	C.1.1.9	0.1291	0.058	0.0251	0.335	1.55	1.18	0.0068	0.199	1.02	1.01	0.0182	0.448	1.11	1.04
	C.1.1.10	0.1330	0.055	0.0273	0.354	1.61	1.19	0.0064	0.196	1.02	1.01	0.0210	0.470	1.14	1.05
	C.1.1.11	0.1207	0.062	0.0210	0.300	1.64	1.20	0.0068	0.186	1.02	1.01	0.0142	0.424	1.12	1.04
	C.1.1.12	0.1198	0.062	0.0201	0.290	1.65	1.20	0.0067	0.180	1.03	1.02	0.0134	0.416	1.10	1.03
	C.1.1.13	0.1146	0.062	0.0184	0.277	1.59	1.18	0.0066	0.181	1.02	1.01	0.0118	0.394	1.11	1.04
	C.1.1.14	0.1080	0.061	0.0166	0.265	1.47	1.14	0.0067	0.186	1.02	1.01	0.0099	0.371	1.08	1.03
	C.1.1.15	0.1053	0.065	0.0150	0.245	1.40	1.12	0.0070	0.184	1.02	1.01	0.0079	0.348	1.06	1.02
C.1.2	C.2.1.1	0.1432	0.050	0.0411	0.495	1.54	1.18	0.0076	0.254	1.04	1.02	0.0335	0.633	1.12	1.04
	C.2.1.2	0.1400	0.050	0.0437	0.538	1.54	1.18	0.0083	0.281	1.00	1.00	0.0354	0.685	1.13	1.05
	C.2.1.3	0.1355	0.049	0.0381	0.485	1.07	1.19	0.0071	0.246	1.02	1.01	0.0309	0.624	1.13	1.04
	C.2.1.4	0.1331	0.049	0.0369	0.478	1.64	1.21	0.0070	0.235	1.04	1.02	0.0300	0.629	1.13	1.04
	C.2.1.5	0.1301	0.051	0.0362	0.479	1.59	1.19	0.0075	0.253	1.02	1.01	0.0287	0.625	1.13	1.04
	C.2.1.6	0.1224	0.053	0.0321	0.452	1.44	1.14	0.0088	0.282	1.02	1.01	0.0232	0.585	1.09	1.03
	C.2.1.7	0.1198	0.056	0.0280	0.403	1.56	1.18	0.0081	0.246	1.00	1.00	0.0199	0.546	1.12	1.04
	C.2.1.8	0.1166	0.058	0.0285	0.421	1.44	1.14	0.0098	0.281	1.04	1.02	0.0187	0.570	1.06	1.02
	C.2.1.9	0.1131	0.062	0.0270	0.411	1.45	1.14	0.0104	0.284	1.02	1.01	0.0166	0.571	1.07	1.02
	C.2.1.10	0.1093	0.061	0.0224	0.353	1.53	1.16	0.0086	0.238	1.02	1.01	0.0138	0.503	1.08	1.02

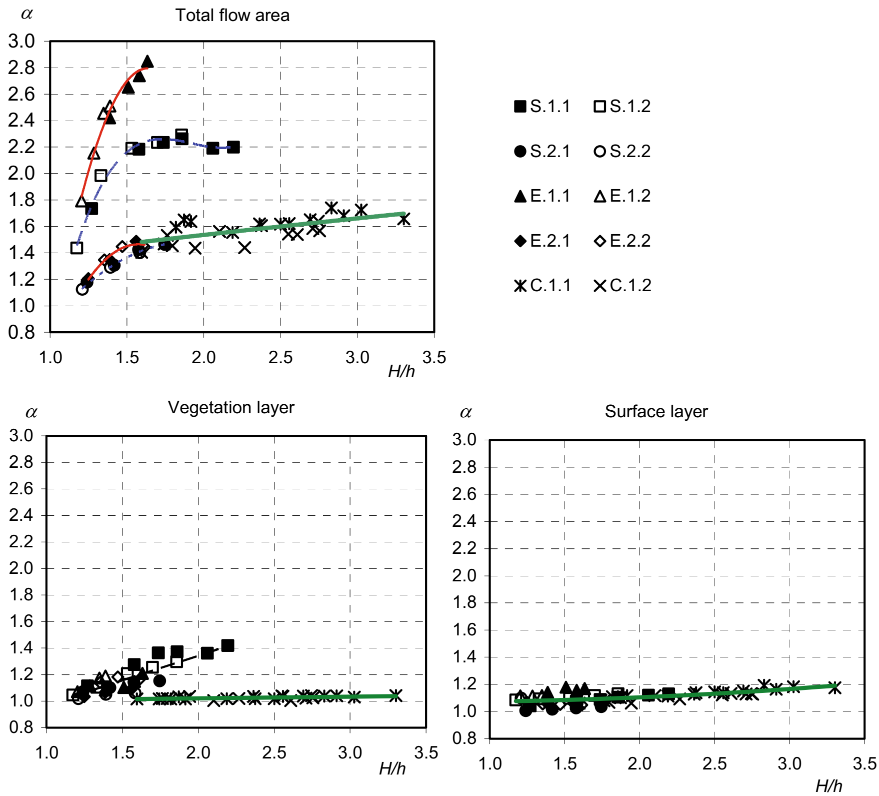


Fig. 9. Calculated energy α coefficients for the velocity profile in the channel and relation between coefficients for different types of artificial vegetation: stiff, flexible stems, and plastic imitations of the Canadian waterweed (*Elodea Canadensis sp.*).

Total flow area. The highest values of the energy coefficient, falling in the range of $\alpha \in (1.79 \div 2.85)$, were obtained for a total flow area with flexible stems of $m = 10000$ stems/m^2 spacing (E.1.1 and E.1.2). Meanwhile, the lowest values of the energy coefficient for the total flow area, in the range of $\alpha \in (1.12 \div 1.49)$, are reported for high spacing of both stiff and flexible stems, respectively, $m = 1008$ stems/m^2 and $m = 2500$ stems/m^2 (S.2.1, S.2.2 and E.2.1, E.2.2). The energy coefficient for the artificial Canadian waterweed was within the range of $\alpha \in (1.40 \div 1.74)$.

As the energy coefficient, the momentum coefficient β reached the highest values of $\beta \in (1.21 \div 1.52)$ for flexible stems of $m = 10000$ stems/m^2 spacing. The lowest values of the momentum coefficient, in the range of $\beta \in (1.04 \div 1.15)$, correspond to the total flow area for high spacing of both stiff and flexible stems. The momentum coefficient for the artificial Canadian waterweed did not exceed the value of $\beta = 1.25$.

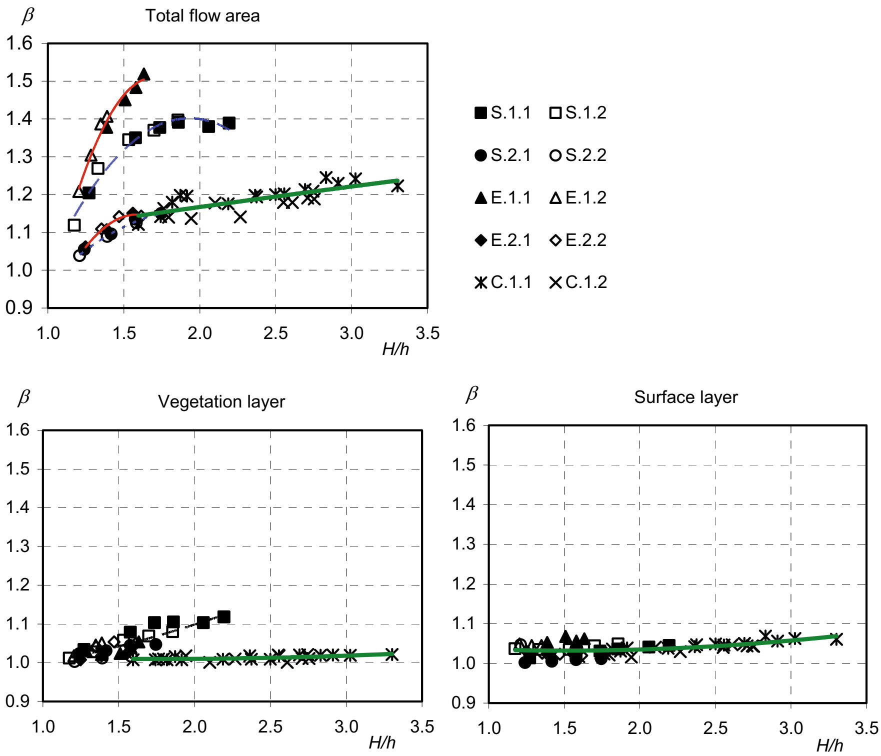


Fig. 10. Calculated momentum β coefficients for the velocity profile in the channel and relation between coefficients for different types of artificial vegetation: stiff, flexible stems, and plastic imitations of the Canadian waterweed (*Elodea Canadensis sp.*).

Vegetation layer. Values of the energy coefficient calculated only for the mean velocity in a vegetation layer, were significantly lower from those for the total velocity profile. There were no strong differences between the energy coefficient determined for stiff or flexible stems and it did not exceed the value of 1.42. Values of the energy coefficient α calculated for the Canadian waterweed were close to unity. A similar behavior is shown by the momentum coefficient β for the vegetation layer, for all types of plant models.

Surface layer. Values of the energy coefficient for the surface layer were close together in all experimental variants and did not exceed the value of 1.19. The corresponding value of the momentum coefficient did not exceed 1.05.

As it is shown in Figs. 7-9, the highest values of the energy and momentum coefficients for the total flow are found for densely spaced flexible stems. These values are higher than those given by Chow (1959) for regular channels without vegetation. Only coefficient values for the total flow with stiff stems were found slightly smaller. Decrease of elements spacing density reduces the coefficients values to the point where they are equal for all analyzed element types.

The calculated values of the energy and momentum coefficients for the total flow area were related to each other (Fig. 11), which allowed to establish a linear relationship between the coefficients:

$$\beta = 0.27\alpha + 0.75 \ , \quad (R^2 = 0.98) \ , \tag{3}$$

or

$$\alpha = 3.70\beta - 2.78 \ . \tag{4}$$

By analogy, the similar relationship was identified for the vegetation layer (Fig. 11).

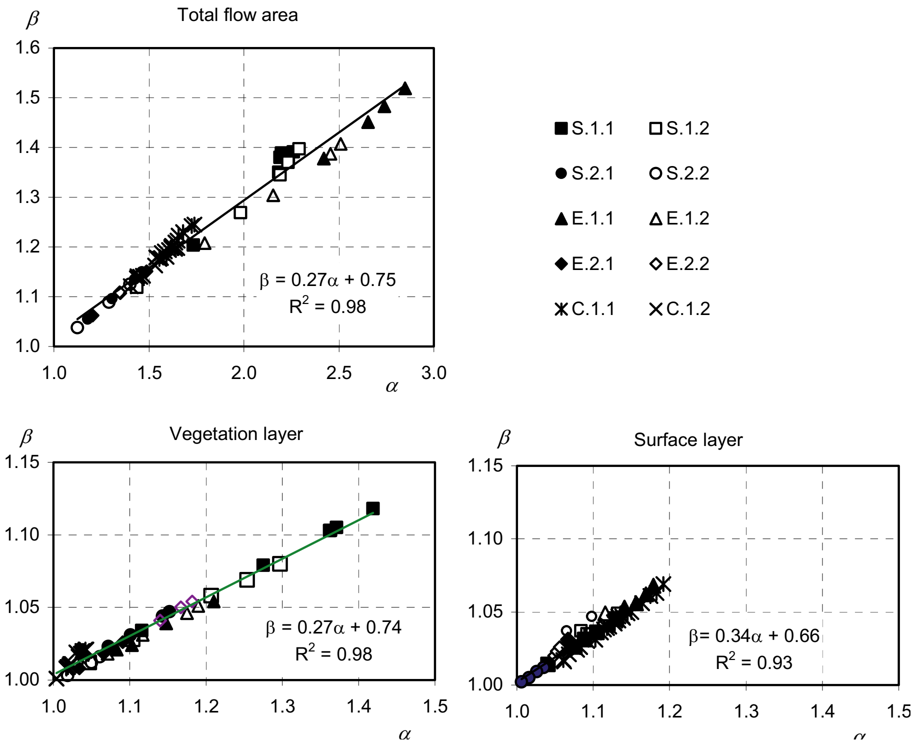


Fig. 11. The relationship between the energy α and momentum β coefficients calculated for the vegetation layer for stiff, flexible stems, and plastic imitations of the Canadian waterweed.

4. CONCLUDING REMARKS

The analysis of calculated values of the energy α and momentum β coefficients for the rectangular channel with stiff, flexible stems, and plastic imitations of the Canadian waterweed, allows to state that:

- The energy and momentum coefficients depend on the velocity variations. The highest velocity variability is obtained in the total profile, smaller in vegetated layer, and the smallest in the surface layer. Therefore, the highest values of the energy α and momentum β coefficients characterize the total flow area, whilst the smallest a surface layer above stiff and flexible elements.
- For the plastic imitations of the Canadian waterweed, the highest velocity variability is found in the surface layer.
- With an increase of plant spacing, the shape of the velocity profile in the vegetation layer gets closer to that in the surface layer; as a result of a velocity variation reduction, also values of the energy and momentum coefficients are reduced.
- The energy and momentum coefficients do not depend on the channel slope.
- The energy and momentum coefficients were related to linear regression relationships 3 or 4. Analogical relationships were elaborated for the vegetation and surface layer.
- In models of one-dimensional channel flow for partly vegetated flow area a higher energy and momentum coefficient values than those given by the present literature should be used. For fully vegetated flow areas, these coefficients should be close to a unity.

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