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EXPERIMENTAL MEASUREMENT METHOD TO DETERMINE THE PERMITTIVITY OF EXTRA THIN MATERIALS USING RESONANT METAMATERIALS

S. Xu[1,2], L. Yang[1,2], L. Huang[1,2], and H. S. Chen[1,2,*]

[1]The Electromagnetics Academy at Zhejiang University, Zhejiang University, Hangzhou 310027, China

[2]Department of Information Science & Electronic Engineering, Zhejiang University, Hangzhou 310027, China

Abstract—The permittivity of extra thin silk cloth is usually measured through some complex methods in the past. Here we propose a convenient and flexible method to measure the permittivity of extra thin silk cloth using resonant metamaterial structures. The metamaterial structures used here are symmetric split ring resonators (SRRs). The principle is that the resonant frequency of the SRRs is very sensitive to the permittivity of the surrounding medium. Therefore, the relative permittivity of an extra thin medium as silk cloth can be determined. Our experimental measurement shows that the relative permittivity of the silk cloth is 4.5. A piece of printing paper is also measured with a relative permeability of 1.4. The effectiveness of the method in determining the permittivity of a solid medium is very useful in future applications.

1. INTRODUCTION

Silk has been widely used as natural textile since it was discovered in ancient China [1, 2]. Besides the cloth applications, silk can also be treated as a highly robust and biocompatible material substrate. For example, in a recent work, silk is used as the substrate of metamaterials that can work as absorbers in terahertz frequencies [3]. Compared with natural material, metamaterials are man-made structures that exhibit unnatural electromagnetic properties. The physical properties of the negative index material with negative permittivity and permeability,

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* Corresponding author: Hong Sheng Chen (hansomchen@zju.edu.cn).

or left handed materials (LHMs), such as reversed Doppler effect, Cerenkov radiation, and Snell's law, were predicted by Veselago [4]. Shelby et al. fabricated a metamaterial sample composed of rods and split-ring resonators (SRR) and observed negative refraction with prism experiments for the first time [5]. Various types of structures have been designed and verified to be LHMs [6, 7]. Besides negative refraction property, many other properties of LHMs have now been extensively investigated [8, 9]. Currently, many novel applications have been achieved with metamaterials, such as hyperlens [10, 11], invisibility cloak [12–17], antennas [18–21] and absorbers [22, 23]. In particular, the inclusion of the silk in metamaterials [3] broadens the application of metamaterials. As the electromagnetic properties of the metamaterials on silk are very closely related to the permittivity of the silk, it is very important to accurately measure the electromagnetic parameters of the silk. However, the permittivity of a given kind of silk cloth is not easy to be measured because the silk cloth is too thin, lithe and ductile. As a kind of isotropic nonmagnetic material, the permeability of silk cloth is μ_0 in Giga Hertz or higher frequencies. Thus, the permittivity is the only parameter that needs to be determined. Previous work shows the permittivity of silk cloth can be measured through some complex and inconvenient methods. One method is to use dielectric rod resonators [24]. However, in this method, the length of rod resonators need be changed many times. The accuracy is also considerably restricted by the uncertainty in the actual value of the surface resistance of the conducting plates. Terahertz time-domain spectroscopy (THz TDS) [25, 26] was also used for measuring the refractive index of materials from a few tens of GHz to several THz but is a complex and inconvenient method because two THz pulses need be propagated through a sample and because the isolation of the first pulse may not be possible in the cases of low refractive indexes and/or thin samples, and the accuracy of the method is restricted by the uncertainty of the measured signals.

In this paper, resonant metamaterials are used to measure the permittivity of an extra thin silk cloth. The resonant metamaterial used in the method is symmetric split-ring resonators (SRR) [27, 28]. The resonant frequency of symmetric SRR is sensitive to the real part of permittivity of the substrate, and the quality factor depends on the image part of permittivity of the substrate. Thus, it is reliable to use resonant metamaterials for measuring the permittivity of the silk cloth.

2. METHOD

The resonant metamaterial structures can be equivalent to LC resonant circuits. They have shown strong frequency dispersion. The electromagnetic properties of resonant metamaterials are sensitive to the shape of structures, the dimensions of the unit cell, and the materials parameters of metal and the surrounding medium. Among resonant metamaterial structures, the resonant frequency of symmetric SRR [27, 28] is very sensitive to the real part of permittivity of the substrate, and the qualify factor depends on the loss of substrate in microwave band. The gap between two SRRs can be equivalent to a capacitance, and the gap of each SRR can also be treated as a capacitance. However, the distance between two SRRs is relatively large, so the impact of the capacitance between two SRRs can be neglected in our case. The effective capacitance and inductance can be calculated and approximated in a single SRR. The geometrical inductance per unit length of the structure, L_g, is given by the area enclosed by the resonator: $L_g = \mu_0 w S$. Because of the difference on the distance between two SRRs, the calculation method of the effective capacitance shown in Figure 1 is different from that in [27]. An approximation method of microstrip gap capacitance shown in [29] and [30] can be used here. It has already been shown that the capacitance of SRR is related to the properties of the substrate (Figure 4 in [29]). The strong analogy with inductor — capacitor (LC) circuits has motivated substantial efforts in establishing simple circuit models for an estimate of the resonance frequency $\omega_0 = (L_g C_g)^{-1/2}$, in terms of geometrical parameters of the SRR structure and the surrounding materials. We can see that the resonant frequency of the SRR is very sensitive to the permittivity of the substrate close to the SRR, therefore can be used to measure the permittivity of a thin material. In experiment we choose symmetric SRR to measure the permittivity of the silk cloth.

The symmetric SRR is printed on one side of the substrate with a relative permittivity of ε_r. We use simulation to get the relation of the resonant frequency of the SRR and the permittivity of the substrate. All the simulations are performed with CST. The simulated environment is the same as the experimental environment, i.e., a rectangular waveguide, with the space of $300\,\text{mm} * 54\,\text{mm} * 10.8\,\text{mm}$. The setup of the boundaries is as follows: the boundaries in z and y directions are PEC (perfect electric conducting), and the boundaries in x direction are PML (perfect matched layer). The symmetric SRR based on the substrate (Figure 1) is placed in the middle of the simulated space. S_{11} and S_{21} parameters can be measured in

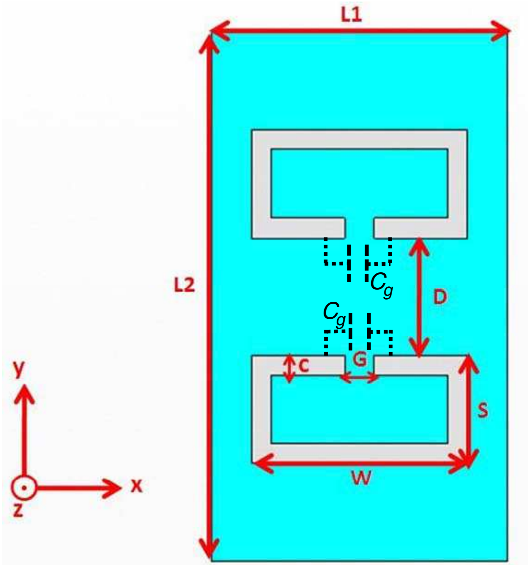

Figure 1. The setup of the symmetric unit cell. The dimensions of symmetric SRR structure are: $C = 2\,\text{mm}$, $D = 12\,\text{mm}$, $G = 3\,\text{mm}$, $W = 22\,\text{mm}$, $S = 11\,\text{mm}$. The length and wide of the substrate are: $L1 = 30\,\text{mm}$, $L2 = 54\,\text{mm}$.

simulation. The dimensions of symmetric SRR structure that we use here are: $C = 2\,\text{mm}$, $D = 12\,\text{mm}$, $G = 3\,\text{mm}$, $W = 22\,\text{mm}$, $S = 11\,\text{mm}$, as shown in Figure 1. The height and width of the substrate are: $L1 = 30\,\text{mm}$, $L2 = 54\,\text{mm}$. The permittivity ε_r and thicknesses of the substrate are changed in the simulation. The resonant frequency decreases when the substrate with a given permeability becomes thicker in the numerical simulation. The resonant frequency also decreases when the permeability of substrate turns to a large value. The curves of resonant frequency of SRR structure versus the relative permittivity of substrate with different thicknesses are shown in Figure 2. Here, only three curves (thicknesses are 0.2 mm, 0.4 mm and 0.6 mm) are plotted in Figure 2. Actually, curves with any thickness of substrate can be easily obtained with numerical simulation. We therefore can find a unique permittivity at a given thickness of substrate to match each resonant frequency of SRR structure.

In order to verify the accuracy of our method, we simulated and measured a material whose permittivity has been known. The picked material is a kind of foam whose relative permittivity is 1.05, which has been used as substrates of metamaterials and antenna in some

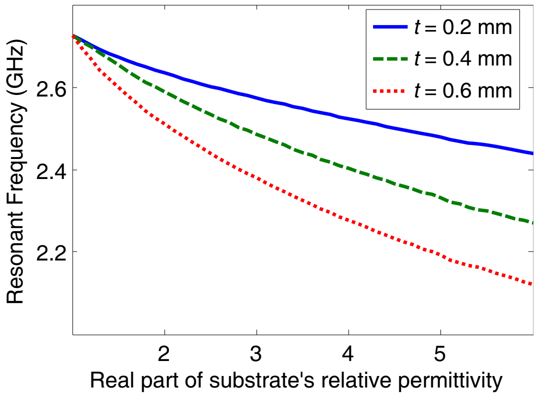

Figure 2. The relation between the resonant frequency of the SRR and the relative permittivity of the substrate with different thickness.

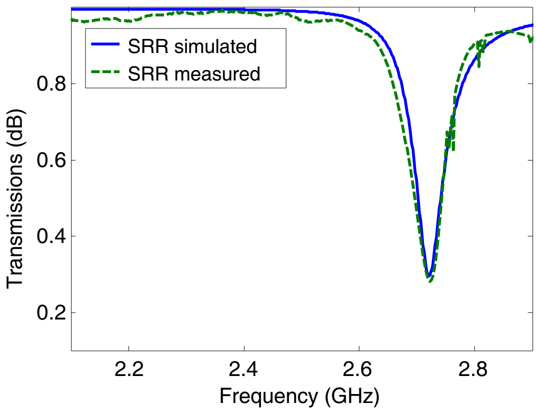

Figure 3. The simulated (dash) and measured (solid) transmissions of symmetric SRR with foam. The relative permittivity of foam in the simulation is set as 1.05.

previous research works [31, 32]. A symmetric SRR unit cell with the dimensions given above is fabricated, and an 8-mm thick foam is used as the substrate. It is measured in a rectangle waveguide. The waveguide works on 1.72 GHz to 2.71 GHz. The experimental result of this case has been shown in Figure 3, and the resonant frequency of symmetric SRR with foam substrate is 2.723 GHz. It should be noticed that the measured resonant frequency is greater than the working frequency band of the waveguide. However, the experimental result is still believable because the waveguide can also work on a higher frequency above the cutoff frequency. There are only some tiny oscillations at some frequency points above the working frequency band of the waveguide in the experimental measurement, but without affecting the resonant frequency. In Figure 3, the measured curve is mainly smooth except two little shakes, so the experimental curve here is reliable. From the relation between the resonant frequency and the permittivity of the substrate (the curve for substrate with 8-mm thick are not shown here), we can see that a resonant frequency of 2.723 GHz corresponds to a permittivity of 1.05, in agreement with the known value. The simulated transmission curve of the SRR on the foam with a permittivity of 1.05 is also shown in Figure 3. We see the transmission curves measured from the simulation and experiments are in good agreement. This test shows the effectiveness of our method in determining the permittivity of a solid medium.

3. RESULTS

To measure the permittivity of the silk cloth, we put the silk cloth on the top of the metamaterial composed of a symmetric SRR unit cell. The symmetric SRR is supported by an 8-mm thick foam. The thickness of the silk cloth is 0.2 mm. The metamaterial plus the silk sample is measured in the same rectangle waveguide as shown above. The experimental transmission coefficient is shown in Figure 4, from

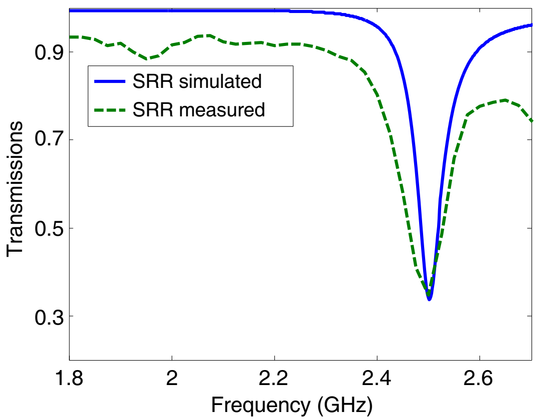

Figure 4. Measured (solid) transmission of symmetric SRR with 0.2 mm-thick silk cloth substrate and the simulated (dash) transmission of symmetric SRR with 0.2 mm-thick substrate whose relative permittivity is 4.5.

which we see that the resonant frequency is 2.5 GHz. Then, we find the point of interaction in the curves of Figure 2 and get that the real part of relative permittivity of the silk is 4.5. We also show in Figure 4 the simulated transmission coefficients of the SRR with the silk medium using a relative permittivity of 4.5. We find that the simulated results are in good agreement with the experimental ones, as shown in Figure 4.

In order to further confirm that the measured permittivity is correct, we use the silk cloth with a different thickness for measurement. We put the silk cloth with a thickness of 0.4 mm on the same metamaterial composed of SRR. We measured its transmission in the same rectangle waveguide. The measured transmission coefficient is plotted in Figure 5. The measured resonant frequency is 2.371 GHz. From the relation of the resonant frequency and the permittivity shown in Figure 2, we can see that the relative permittivity of silk cloth is 4.5, which is in good agreement with the previous measurement in a different thickness of the cloth. The simulated transmission coefficient is also shown in Figure 5, from which we can see that both the simulation and experiment results are in good agreement. From the above two measurements, we can find that the relative permittivity of the silk cloth is equal to 4.5.

To further show the effectiveness of this method in determining the electromagnetic parameters of an extra thin material, we use a

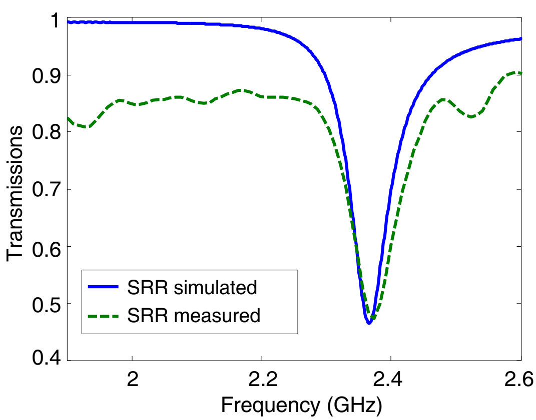

Figure 5. Measured (solid) transmission of symmetric SRR with 0.4 mm-thick silk cloth substrate and the simulated (dash) transmission of symmetric SRR with 0.4 mm-thick substrate whose relative permittivity is 4.5.

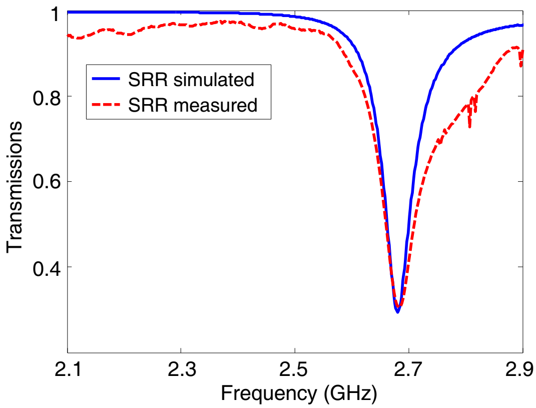

Figure 6. Measured (dash) transmission of symmetric SRR with a 0.2 mm-thick A4 printing paper and the simulated (solid) transmission of symmetric SRR with 0.2 mm-thick substrate whose relative permittivity is 1.4.

piece of printing paper as another example. The paper is of 70 g/m^2 A4 printing paper with a thickness 0.2 mm. We put the printing paper on the top of the metamtaerial composed of symmetric SRRs with the same dimensions shown above. The measured resonant frequency of metamaterial cell is 2.683 GHz. We search this frequency point on the 0.2-mm curve in Fig. 2 and find that the measured resonant frequency point in experiment matches the simulated one when the relative permittivity is 1.4. Both the simulated and measured transmission coefficients are shown in Figure 6. In the simulation, we set the relative permittivity of the paper to be 1.4, and we can find from Figure 6 that the curve match well with the experimental result.

4. CONCLUSION

In conclusion, we measure the relative permittivity of a silk cloth through waveguide measurement based on metamaterials. The principle of this method is that the resonant frequency of the metamaterial composed of SRR is very sensitive to the material parameters of the surrounding medium. Compared with traditional measurements, our method is much easier to carry out. Firstly, the experimental instruments we used here are a rectangular waveguide and a R&S vector network analyzer, which are quite simple. Secondly, the resonant frequency of metamaterial is very sensitive to the permeability of substrate so that it provides a high sensitivity to pick out the material property. Finally, only one metamaterial sample needs to be fabricated and measured once. Thus, it can save the time of measurement. Measuring the electromagnetic properties of a silk cloth is very important because metamaterials can be realized on silk cloth with accurate functions, and some novel applications can be achieved on silk or cloth substrate.

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