# Experimental Study of the Rooster Tail Jump and End Sill in Horseshoe Spillways

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## Abstract

In a horseshoe spillways, due to the collision of the falling nappes from their surround walls, in the center of spillway's trough, a spatial hydraulic jump is formed that named "rooster tail". This study by using the physical model of horseshoe spillway, investigates the form, height and length of rooster tail jump. Based on the analytical methods, the effective parameters on rooster tail jump's height and height were determined and their interaction was investigated and linear relationships were proposed to predict jump's length and height. By increasing the amount of water on the spillway's crest and thereby increasing the velocity of flow nappe at the point of contact with the spillway's bed, length and high of rooster tail jump, linearly increased. The result also shows that by increasing number of Froude, the length and height of jump increases and by increasing the spillway's length, the height and length of the rooster tail jump decreases. To control of rooster tail jump in spillway's model, two different size of end sills Inserted at downstream of spillway and result shows that by employing a sill with height of 3.8 cm and 7.6 cm, the flow depth, in average, respectively 122% and 272% increase compared to no sill conditions, also flow state change from super-critical to sub-critical. At the sill of 3.8 cm it was observed that the rooster tail jump did not submerged, but at the height of 7.6 cm the jump submerged and static pressure increased more. The results revealed that by placing the sill of 3.8 and 7.6 cm, respectively 45% and 35% of the maximum pressure entering the bed of the spillway at the collision site is reduced.

*Keywords:* Rooster Tail Jump; End Sill; Horseshoe Spillway; Velocity; Pressure.

## 1. Introduction

The study of the hydraulic jump is a challenging issue because of its intricate nature, where the supercritical flow changes to subcritical flow with a rapid rise of flow depth. The flow in the hydraulic jump was regarded as a turbulent shear layer having an air–water interaction on the upper surface forming the roller in the mixing layer, the extent of which depends on the magnitude of upstream Froude number [1]. Dissipation of the water flow's kinetic energy which is caused by a hydraulic jump inside and downstream of the spillways is requirement. This is essential, not only to protect the banks from erosion, but also to secure that the dam itself and adjoining structures like powerhouse, canal, etc. are not sabotaged by the high velocity turbulent flow [2]. One of the hydraulic structures that hydraulic jump occur inside it, is horseshoe spillway (Figure 1). In a horseshoe spillway, due to the collision of the falling nappes from their surround walls in the center of spillway's trough, a hydraulic jump is formed that named "rooster tail". The rooster tail jump collapses to bed in the downstream of the spillway [3]. Given that the hydraulic jump causes pressure fluctuations,
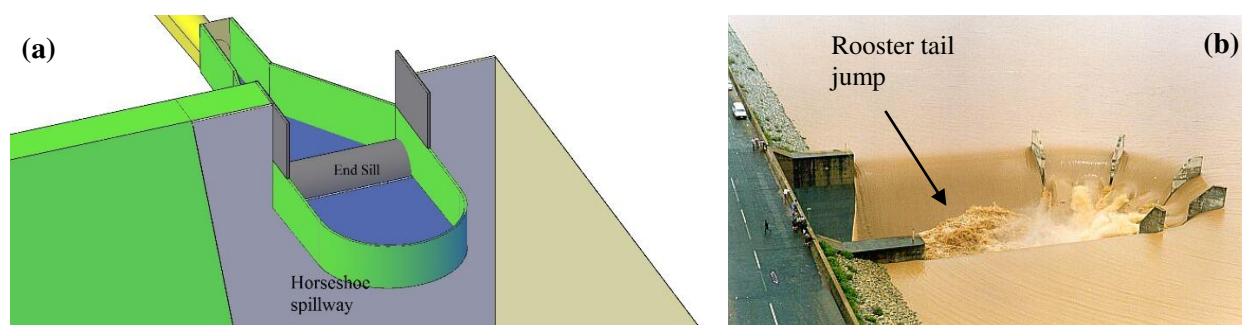
it is a destructive factor for the bed of spillway hence, it is necessary to reduce the severity of this destructive factor. Creating the end sill in the downstream of the Horseshoe spillway (upstream of the chute), can reduce or eliminate the damage caused by increasing the depth of flow in the trouth (Figure 1b and spillway of Nacala Dam in Mozambique.).



**Figure 1. a) rooster tail jump in horseshoe spillway  b) horseshoe spillway constructed in South Africa and its end sill in downstream**

The hydraulic jump and their control have been widely studied, but only few investigators considered rooster tail jump, and little attention was focused on dissipating this jump in horseshoe spillways. Early studies conducted in rectangular horizontal smooth walled channels [4] and application of the momentum principles led to the well-known Bélanger's equation, assuming uniform velocity and hydrostatic pressure distributions upstream and downstream of the hydraulic jump and negligible boundary flow resistance. Herbrand [5] studied the spatial hydraulic jumps such as jump in enlarged stilling basin. Herbrand recommended for preliminary sizing the length of the spatial hydraulic jump as shown in Equation 1.

$$L_B = 0.8L_b = 4(y_2 - y_1) \tag{1}$$

In which $L_B$ is length of jump and $L_b$ is length of broken Jump. Bushra and Noor Afzal [1] investigate the hydraulic jump in circular and U-shaped channels. Their results shows that the hydraulic jump length $L_j/h_2$ is independent of $Fr_1$ at large upstream Froude number. Pagliara and Palermo [6] studied hydraulic jumps on rough and smooth beds. They proposed a semi-theoretical predictive relationship to estimate jump characteristics for a wide range of hydraulic and geometric conditions covering both rough and smooth beds. The problem of energy dissipation below hydraulic structures has attracted the attention of hydraulic engineers for a long time. Khatsuria [2] classified the energy dissipators for spillways as follows: 1. Hydraulic jump stilling basins 2. Free jets and trajectory buckets 3. Roller buckets 4. Dissipation by spatial hydraulic jump 5. Impact type energy dissipators. A transverse sill is a basic element of a hydraulic jump dissipator. A sill is able to control the jump, and to dissipate more energy than a classical hydraulic jump (CHJ). Depending on the tail water depth, the sill may or may not be submerged. Non-submerged sill flow is characterized by supercritical flow beyond the sill that is the energy dissipation is incomplete. The preliminary study of Shukry [7] pointed to an optimum position of sill. In the same year, Bradley and Peterka published their series of papers on jump dissipators. Basin III contains as a main element baffle blocks. The sill was not considered as baffle blocks were found superior in making the flow downstream from the sill uniform [8].

Hager & Damei Li [9] based on numerical method studied the Sill-controlled energy dissipator and investigated the reduction of tail water level (ΔYS) due to the presence of sill. Their results shows that depending on the quality of tail water bed the sill-controlled stilling basin may be much more efficient, requires less tail water and needs only a reduced length of basin when compared to a classical jump basin. Also, the length of basin is defined as the end of bottom roller. Larry W. Mays. [10], illustrates a side-channel spillway in operation the hydraulic features are defined as follows. The chute crest is proportioned to produce subcritical flow in the trough for all discharges to dissipate the overflow energy and produce uniform flow into the chute. Montazar and Salehi Neyshabouri [11], experimentally investigated effects approaching channel' bed slope and elevation and position of end-sill on performance of a U-shape side spillway for different inflow rates. Their results showed that the sill's elevation had the highest impact on both pressure fluctuations and hydraulic performance of spillways. Their results revealed that 3% increase in the slope of channel's bed, it reduces pressure fluctuations. Using a numerical model, Taghizadeh et al. [12], investigated stepping effects on dynamic pressure fluctuations in a three-side spillway. They used finite volume method (FVM) and RNG turbulence in their model. They reported that that their proposed ogee profile form resulted in reduced turbulence in the side channel. They suggested that stepping of the Ogee profile of three-side spillway, would simplify both the construction and operation of these spillways.

Recently McGhin et al. [13] experimentally tested the effect of width deflector that design for the downstream side of the Low-Head dams on eliminating the submerged hydraulic jump. The results demonstrate that a dramatic reduction

in upstream-directed surface velocities over a wide range of headwater and tail water conditions is possible. Felder and Chanson [14] investigated the basic air-water flow properties of Hydraulic Jumps on Uniform Beds Macro-roughness. Results highlighted some distinctive effects of the bed roughness including an upwards shift of the hydraulic jump and an increase in bubble count rate and void fractions in the region close to the jump toe. Jesudhas et al. [15] by using the detached-eddy simulation (DES), studied the three-dimensional classical hydraulic jump with an inlet Froude number of 8.5. From the results, it was inferred that the flow is retarded in the inner layer of the wall jet due to the presence of the bed and is accelerated in the outer region of the wall jet due to the circulatory flow in the roller region. Parsamehr et al. [16] experimentally investigated the characteristics of hydraulic jump on rough bed with discontinuous roughness elements of lozenge shape over adverse slope. They found that by increasing the height of roughness elements and steeping the adverse slope, the sequent depth ratio, and relative length of the jump decreased while the energy loss increased. Valero et al. [17] by using published experimental data and computational fluid dynamics (CFD) models, compared the performance of energy dissipation of a Type III Basin on smooth chute and stepped-chute. The results shows that relative to a smooth chute, the turbulence generated by a stepped chute causes a higher [18-20].

The current study for the first time investigates the rooster tail jump in the classic horseshoe spillways by using the physical model. Based on the dimensional analysis, the effective parameters and dimensionless numbers in rooster tail jump are specified and relationships have been proposed to predict the length and height of the jump. Also, in this research, the End Sill has been used to control the rooster tail jump in the downstream of spillway and its effects on reducing the pressure on the spillway's floor have been investigated.

## 2. Material and Methods

### 2.1. Experimental Procedure

Experiments of this study were carried out in the hydraulics laboratory of the Water Engineering Department, University of Tabriz. The water supply system used in this study consisted of an underground pool equipped with a pump (with a maximum flow rate of 60 L/s) to transfer water from the underground pool to a free surface cylindrical tank with a volume of 2.5 m$^3$. The cylindrical tank was equipped with an internal weir to keep water at a constant level of 3.7 m. A rectangular cube with dimensions of 180 cm (length) × 100 cm (width) × 120 cm (height) was used as the main reservoir. The crest height in spillway were 95.0 cm higher than the reservoir's bed. In order to simulate quiescent conditions in the reservoir, a flow tranquilizer was employed at the 10 cm upstream of the spillways (Figure 2). The flow tranquilizer was made of two meshed-metallic frames (100 cm width ×120 cm height) with a 10 cm distance in between. Distance between the frames was filled with wool fabrics and gravel with a grain size ranging between 18.0 - 32.0 mm.
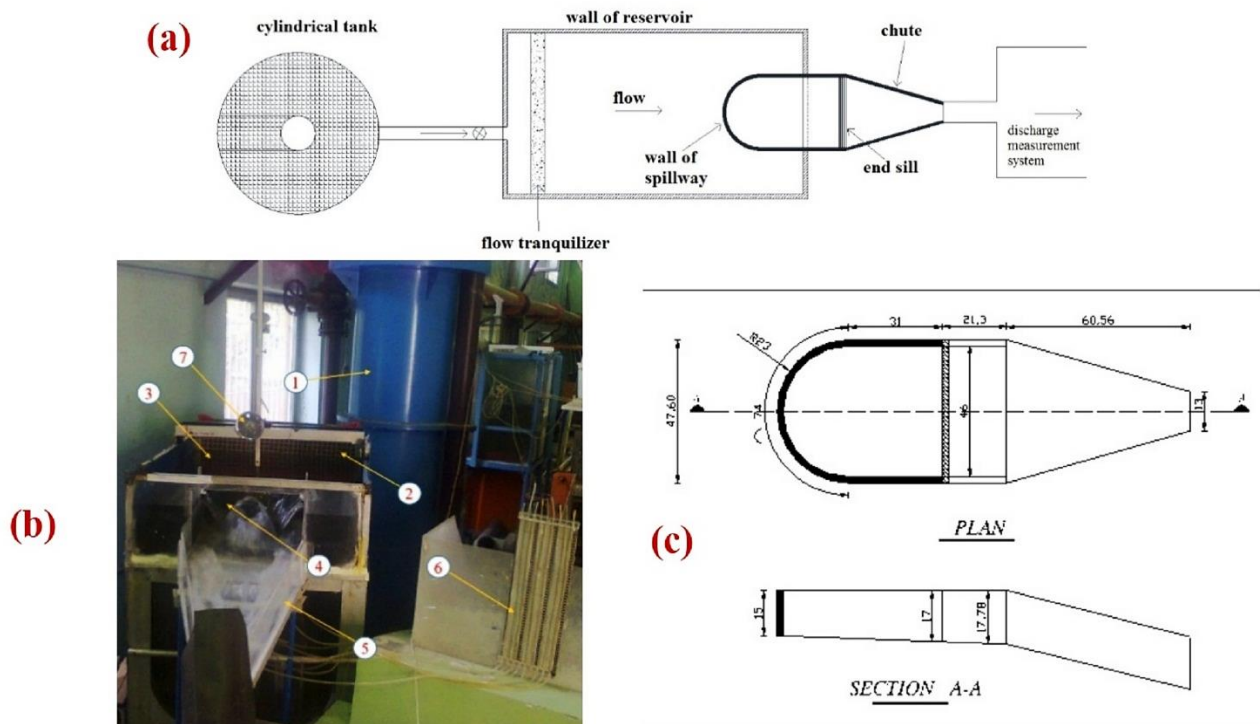


**Figure 2.  a) Schematics plan-view of experimental setup b) picture of the experimental setup: (1) cylindrical tank, (2) flow tranquilizer, (3) reservoir, and (4) spillway (5) chute (6) piezometer (7) point gage  C) dimensions of model (dimensions in cm)**

First set of experiments, six models of classic horseshoe spillways with different crest lengths were made from

plexiglass and tested under different flow conditions. Each model of classic horseshoe spillway was comprised of a curved segment, two straight parallel walls, and with a floor with a slope of 0.036. The length and radius of the curved walls in all spillways were 74 and 23 cm, respectively. The heights of the walls at the beginning of the spillways were 15 cm. the length of the two parallel walls from 0 to 30 cm by step-wise (6 cm) increased. Six different models of classic horseshoe spillways with external crest lengths of 76, 88, 100, 112, 124, and 136 cm were constructed and used for experiments. Fig.3(c). Hydraulic characteristics of spillway at each model was investigated under seven different flow rates of 7.5, 8.5, 9.5, 10.5, 11.5 12.5, and 13.5 L/s.

## 2.2. The Rooster Tail Jump and End Sill

Due to the geometry of the horseshoe spillway, during the passing of flow from the spillway to downstream, rooster tail jump was formed inside the spillway (trough) as shown in Figure 3. To study the rooster tail jump, all 42 experiments (with different length of spillway and discharges) were repeated. The characteristics of rooster tail jump that studied are length and height of rooster tail.
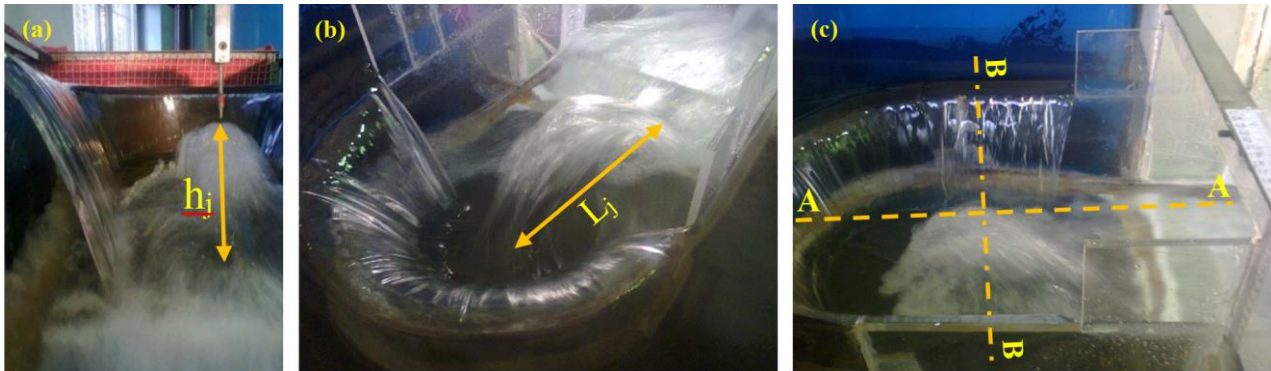


**Figure 3. rooster tail jump in the horseshoe spillways a) downstream view b) upstream view c) side view**

In order to investigate the: 1)effect of creating End Sill (at downstream of horseshoe spillway) on the flow conditions and 2) checking the Ability to control of rooster tail jump, the physical model of two semicircular sills with a radius of 3.8 and 7.6 cm at the downstream of spillway was built (Figure 4). Then, the flow height, static pressure and flow velocity at different flow rates (before and after the insertion of the sills) were measured, and the maximum pressure at the point of flow jet collision to the bottom floor, was calculated by using Equation 5. The placement of the sill is immediately upstream of the junction of spillway to chute that is located out of the reservoir's wall (Figures 2 and 3).
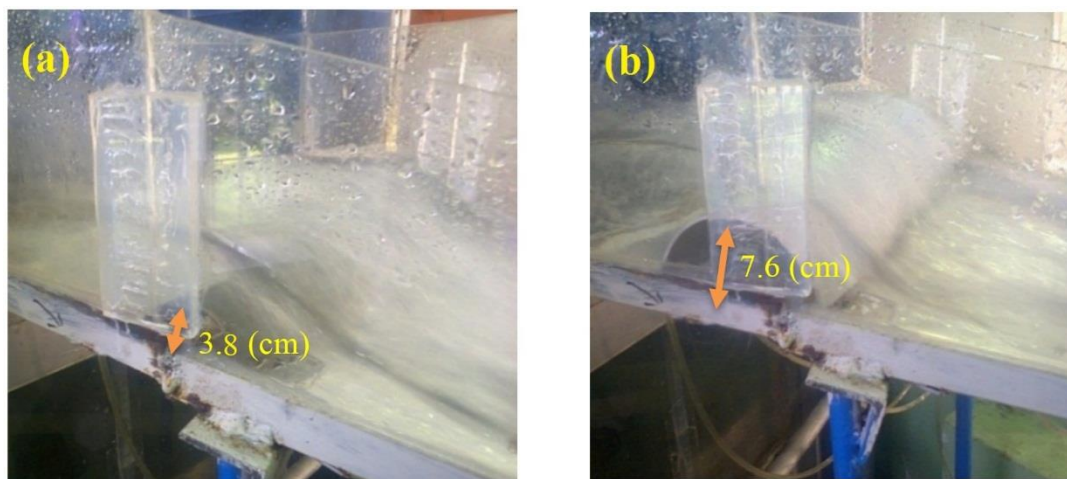


**Figure 4. Placing the sill against the flow in the downstream of horseshoe spillway a)sill height: 3.8 (cm)**
**b) sill height:7.6 (cm)**

## 2.3. Pressure in the Stilling Basin with Flow Jets

In the Stilling basin with flow jets, a jet with a vertical and relatively flat surface impacted to the floor, and by deviations and spreads of flow, its energy is depreciated. Usually on this Stilling basins, there is a pillow of water with a depth of $t_L$ that the jet flow with the thickness $t_j$ and velocity of $V_1$ is collision to it. Vischer and Hager [13] proposed maximum pressure for PM in this Stilling basin by Equation 2.

$$\frac{P_M}{\rho V_1^2/2} = 7.4\,\frac{t_j}{t_L} \tag{2}$$

In order to obtain a relationship between upstream discharge of unit width ($q_u$) and the length and height of the jump, $q_u$ calculated from Equation 3.

$$q_u = Q/L_w \tag{3}$$

In order to compare the flow depth in the downstream of spillway with the critical depth, Equation 4 (that used for the rectangular channels) has been used. Where $y_c$ is the critical depth, $q_d = Q/B$ is the flow discharge in the unit wide ($B$ is wide of downstream channel that equal to 46 cm) and g is gravity acceleration.

$$y_c = (\frac{q_d^2}{g})^{1/3} \tag{4}$$

### 2.4. Experimental Measurements

The inflow water head and depth of flow in downstream of rooster tail jump was measured at 60.0 cm upstream of the spillways using a point gauge. Current velocity was measured with a micro-current meter. In order to measure the static pressure at the downstream of horseshoe spillway (before connecting to the chute), five piezometers were installed and their values were read in various experiments (Figure 2). Using a point gauge with an accuracy of ± 0.1 mm, flow profiles over and inside the spillways were measured every 2.0 cm along the A-A and B-B axes shown in Figure 3b. The total outflow discharge from the spillways was measured using a stilling basin equipped with a calibrated triangular spillway, with an apex angle of 53º 8´ at the downstream of stilling basin. The relationship between the discharge flow rate and the water head overflowing the triangular weir was calculated as [14]:

$$Q = \frac{8}{15}\sqrt{2g}\,C_{de}\,\tan\frac{\theta}{2}\,H_d^{2.5} \tag{5}$$

Where $Q$ (m³ s⁻¹) is discharge flow, $g$ (m s⁻²) is gravitational acceleration, $C_{de}$ is the discharge coefficient, $\theta$ is the apex angle of triangular weir, and $H_d$ is the water head overflowing the triangular weir. Rating curve of the triangular weir downstream of the model is presented in Figure 5.
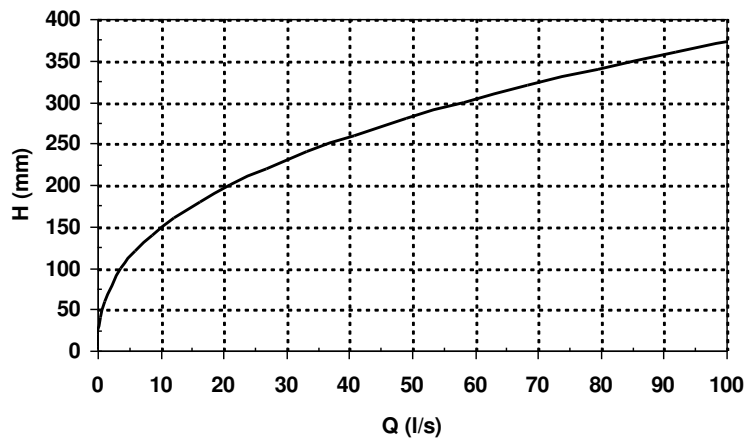


**Figure 5. Rating curve of the triangular weir**

### 2.5. Dimensional Analysis of Length and Height of Rooster Tail Jump

A general function of effective variables on the length and height of rooster tile jump, can be defined follows:

$$L_j, h_j = f(L_w, h, P, S_0, Q, D, \rho, \mu, g,) \tag{6}$$

Where $L_w$ is the spillway's crest length, $h$ is water head on the spillway, $P$ is spillway's height from reservoir's bed, $S_0$ is spillway's bed slop, Q is flow discharge, D is diameter of the semi-circular crest, $\mu$ is water viscosity, $\rho$ is water density, $\sigma$ is surface tension, and $g$ is acceleration of gravity. By choosing three parameters $Q$, $L_w$ and $\rho$ as repeating variables, length and height of rooster tile jump in classic horseshoe spillways ($L_j$) could be a function of dimensionless parameters derived from the π-Buckingham theorem:

$$L_j, h_j = f(\frac{h}{L_w},\ \frac{P}{L_w},\frac{D}{L_w},\frac{L_w^5 g}{Q^2},\frac{L_w}{Q*\vartheta}) \tag{7}$$

Since experimental parameters of P, and D were constant during the experiments, effects of their dimensionless terms were also excluded from the model. Therefore Equation 7 can be simplified to Equation 8:

$$L_j , h_j = f(\frac{h}{L_w}, \frac{L_w^5 g}{Q^2}, \frac{L_w}{Q * \vartheta}) \tag{8}$$

In order to avoid the effect of surface tension on the crest of spillway, the minimum water heights on the crest, 2 cm was considered.

## 3. Result and Discussion

### 3.1. Rooster Tail Jump

The cross sectional and the longitudinal profiles for rooster tail jump in the horseshoe spillways with different crest lengths for instance at a discharge of 11.5 L/s is showed in Figure 6. Also the cross sectional and the longitudinal profiles for the classic spillways with different flow discharges at a crest lengths L$_w$=88 cm is showed in Figure 7.
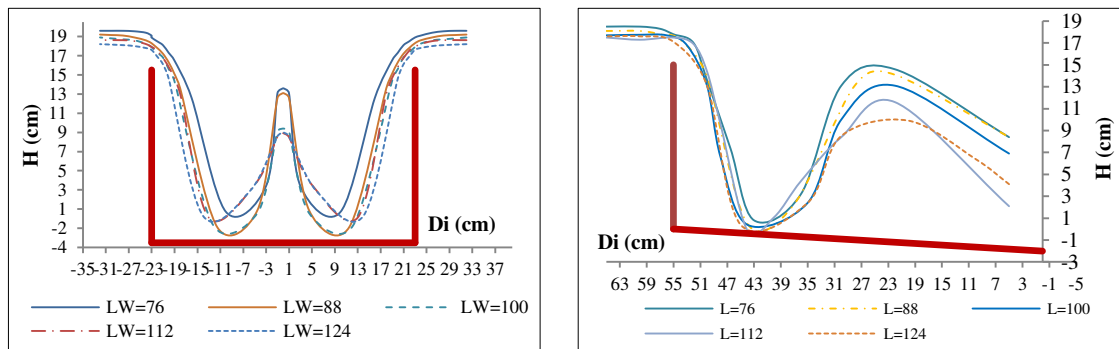


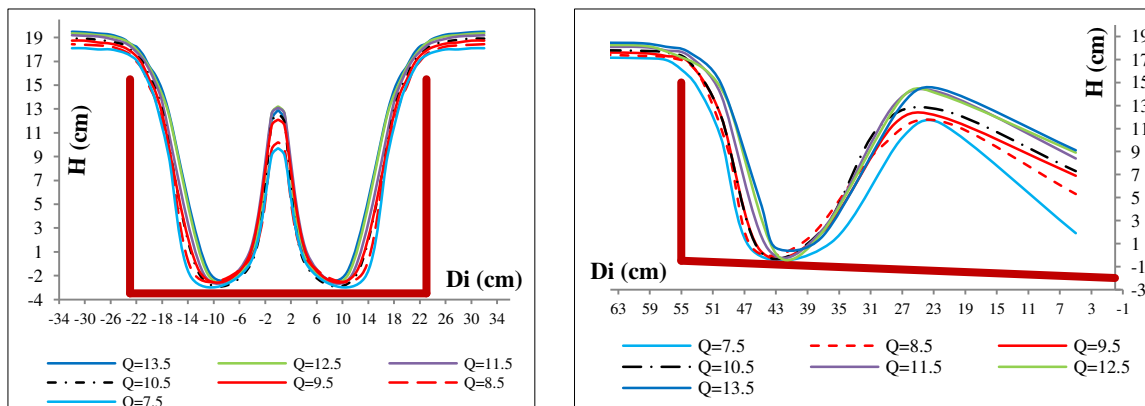**Figure 6. Cross sectional and the longitudinal profiles for rooster tail jump (Q=11.5 L/s)**



**Figure 7. Cross sectional and longitudinal profiles for rooster tail jump in deferent discharge (lw=88 cm)**

The cross-sectional profiles of the flow in the classical spillways, show that for a constant discharge, by the reducing the crest length, the water head over the spillway increases and the distance of the location of jump formation from the spillway's wall increases. In addition, by increasing the spillway's length, the height and length of the rooster tail jump occurring inside the spillway decreases. Figure 6 show that for a constant crest length, by the increasing the flow discharge, the water head over the spillway increases and the distance of the location of jump formation from the spillway's wall increases. In addition, by increasing the spillway's flow discharge, height and length of the rooster tail jump occurring inside the spillway increases. In the profiles of Figures 6 and 7, it is determined that height of rooster tail hydraulic jump, is approximately equal to the level of chute walls in downstream of spillway and therefore requires an increase in wall height or control the jump height. Based on the analytical methods, diagrams of dimensionless numbers against the length and high of jump plotted. Figure 8 shows the length (L$_j$) and height (h$_j$) of rooster tail jump against the ratio of water head to crest length (h/L$_w$).
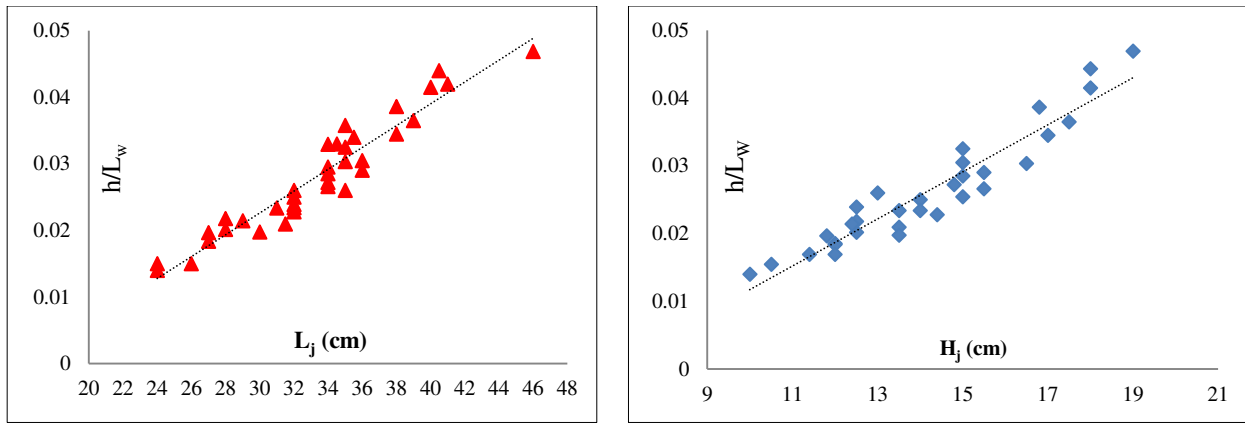
**Figure 8. Variations of the height and length of jump to the ratio of water head to crest length (h/Lw)**

Figure 8 shows that by increasing the ratio of water head to crest length ($h/Lw$), length and high of rooster tail jump, linearly increasing. The reason for this phenomenon, is increasing the amount of water on the spillway's crest and thereby increasing the velocity of flow nappe at the point of contact with the spillway's bed. Linear relationships between $h/Lw$ and length and high of rooster tail jump are shown in Equations 9 and 10 (dimensions are in cm).

$$L_j = 16.63 + 625 \frac{h}{Lw} \tag{9}$$

$$h_j = 6.6 + 285 \frac{h}{Lw} \tag{10}$$

Another dimensionless parameter that the efficiency of length and high of rooster tail jump is the ratio of $L_w^5 g/Q^2$ that shown in Figure 9.
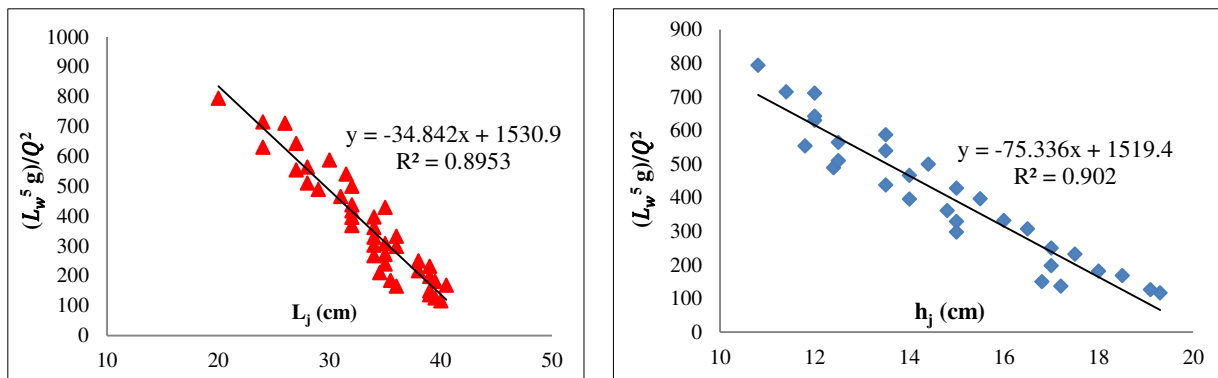


**Figure 9. Variations of the height and length of jump to the discharge of classic horseshoe spillway**

Figure 8 shows that by decreasing the ratio of $L_w^5 g/Q^2$, length and high of rooster tail jump is linearly increasing. Also Figure 8 shows that increasing the flow discharge through the spillway (at a given crest length) increases the length and height of the jump. Dimensionless number of $L_w^5 g/Q^2$ somehow represents the inverse of the Froude number, so in this jump, with increasing number of Froude, the length and height of jump increases. Figure 10 shows diagram of the length (Lj) and height (hj) of rooster tail jump against the ratio of $L_w/Q \times \vartheta$.
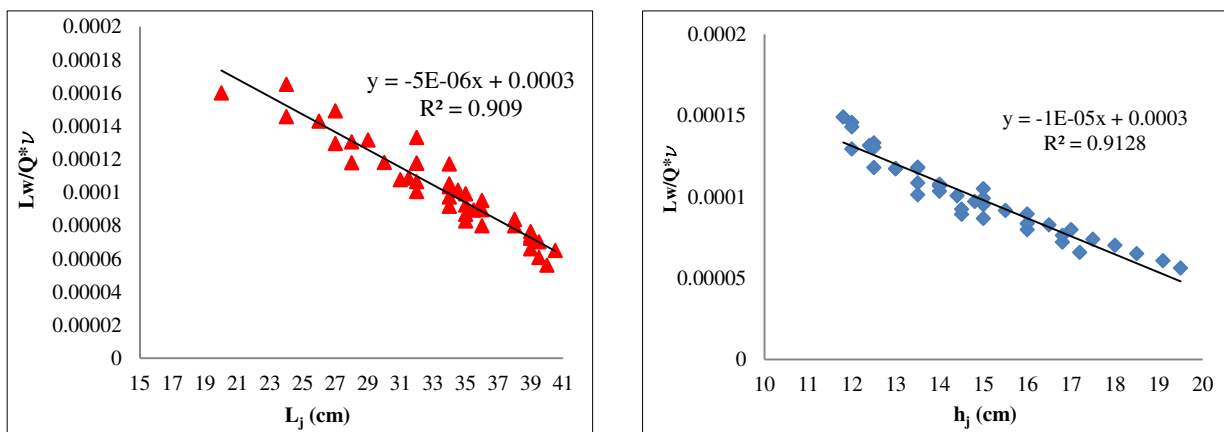


**Figure 10. Variations of the height and length of jump to the discharge of classic horseshoe spillway**

Figure 10 shows that by increasing the ratio of $L_w/Q \times \vartheta$, length and high of rooster tail jump are decreasing. The diagrams of upstream discharge of unit width ($q_u$) and the length and height of the jump are plotted in Figure 11. Also regression relationships between $q_u$ and length- height of the rooster tail jump are shown in Figure 11.
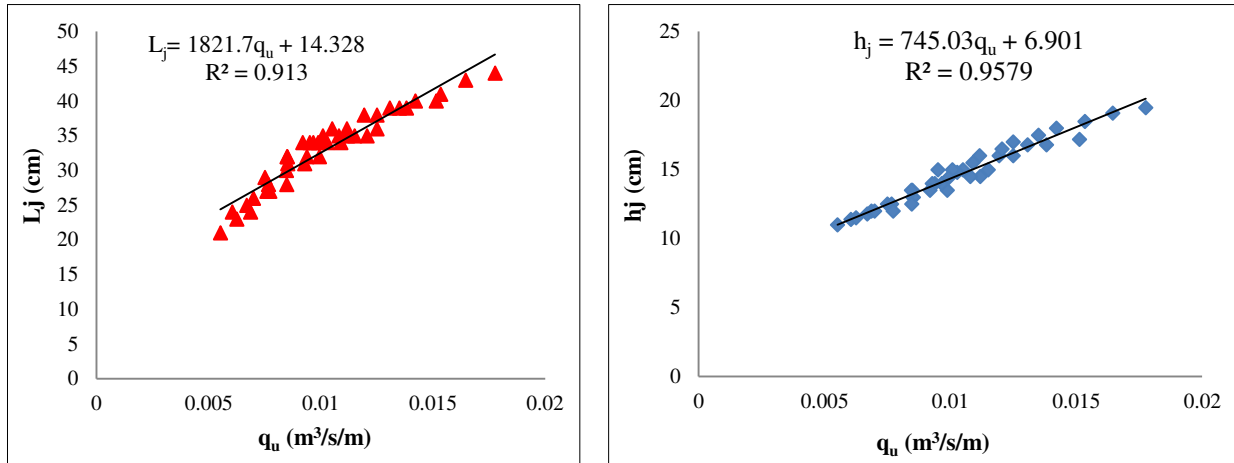


**Figure 11. variations of the height and length of jump to the discharge of classic horseshoe spillway**

As seen in Figure 11, by increasing the $q_u$, both length and height of the rooster tail jump linearly increased.

## 3.2. The Effect of End Sill

The average static pressure of flow at the aforementioned location in the different discharge and $L_w$=100 cm is shown in Figure 12.
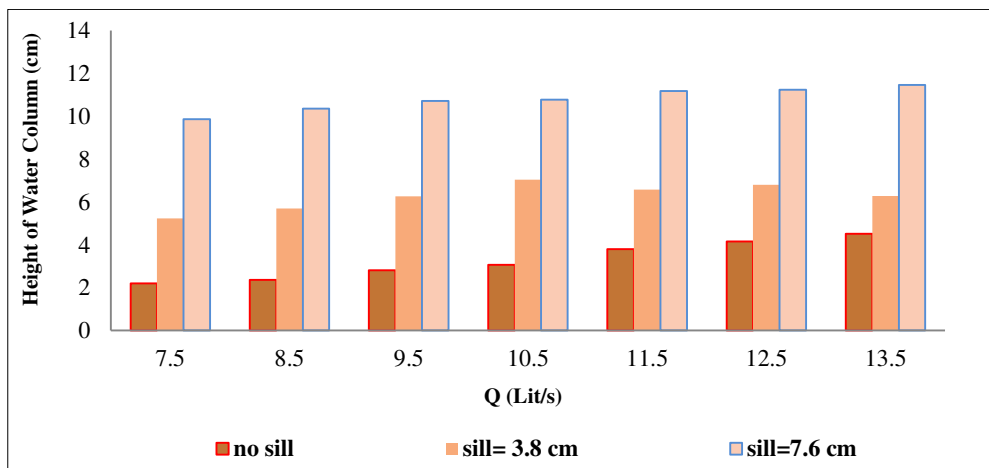


**Figure 12. comparative bar diagram of static presser in downstream of spillway**

Based on the measurement of the depth of flow in downstream of hydraulic jump with crest length of 112 and 100 cm, and its control with the static pressure, it was determined that by employing a sill with height of of 3.8 cm, the flow depth, in average, 122% increase compared to no sill conditions. By employing a sill with height of 7.6 cm, flow depth, in average, 272% increase compared to no sill conditions. The flow before the applied of sill, in all of discharges are super-critical, but by applying the sill, it change to the sub-critical state. Also, the comparison of results for spillway with two different crest lengths, shows that by decreasing the $L_W$, the flow depth at the downstream of spillway in the state of no sill, increased in discharge of less than 10.5 lit/s and decreased for discharge values greater than 10.5 lit/s. With applying the sill by height of 3.8 cm, decreasing the $L_W$ causes that flow depth is increases. Comparison of the flow depth on the sills and the calculated critical depth from Eq.4 indicates that the flow on the sill with height of 7.6 cm is approximately equal to the critical depth, but in the sill with height of 3.8 cm higher flow depth Creates than the critical depth. The results of measuring the flow velocity in the downstream of the spillway in the three mentioned states indicate that, by applying a small sill (3.8 cm), the flow velocity in the downstream of the horseshoe spillway decreases by 60% relative to the no-sill state. It is also observed that by operating a sill of 7.6 cm, the flow rate is reduced by 80% compared to the no-sill state. During the experiments, it was observed that the depth of flow increased with the insertion of the sill, and it was also observed that the rooster tail jump did not submerged at a sill of 3.8 cm, but by placing the sill with a height of 7.6 cm the rooster tail jump submerged and static pressure increased more. Also it was observed that sill with a height of 7.6 cm further reduces the flow turbulence (Figure 13).
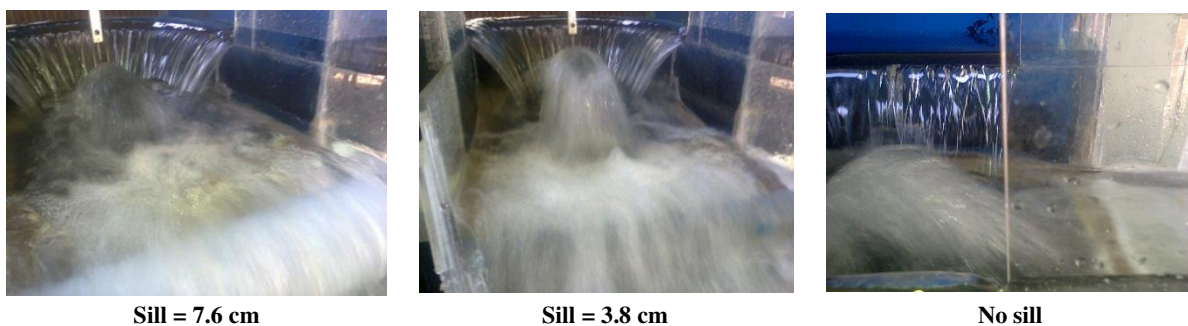
| Sill = 7.6 cm | Sill = 3.8 cm | No sill |

**Figure 13. Compare the flow conditions in the bottom of the horseshoe spillway**

Based on the results of the experiment, the Eq.5 was calculated in different discharges for two crest lengths of 100 and 112 cm, as shown in Figure 14 and in contrast to the flow depth of the jump's downstream.
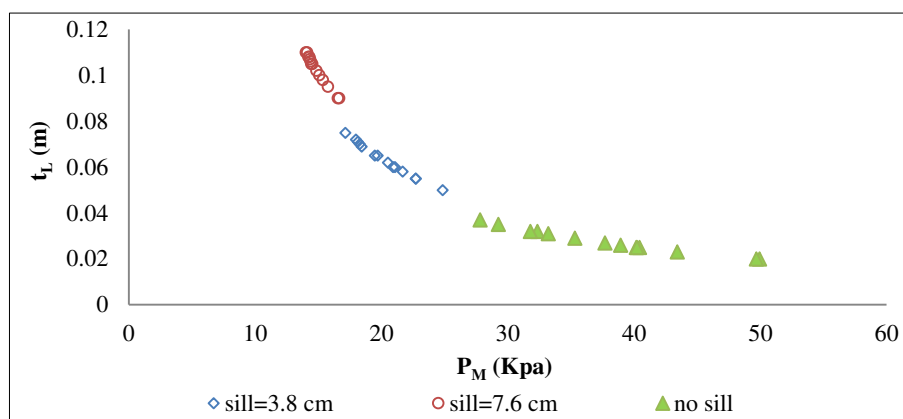


**Figure 14. Comparison of the pressure at the position of impacting jet to bed with and without end sill**

With reference to Figure 14, it is shown that with the increase of the depth of downstream ($t_L$), which is directly related to the sill height, the pressure caused by rooster tail jump collisions is reduced. By placing the sill of 3.8 cm, 45%, and by placing the sill of 7.6 cm, 35% of the maximum pressure entering the bed of the spillway at the collision site is reduced. The power relationship of pressure at the point of jumps collision to the bed of spillway with a depth of downstream ($t_L$) showed by Equation 11. The regression coefficient of the Equation 11 is 0.99.

$$P_M \ = \ 2.8837 * t_L^{-0.71} \tag{11}$$

## 4. Conclusion

The hydraulic jump and their control have been widely studied, but only few investigators considered rooster tail jump so in this study by using the physical model of horseshoe spillway, the form, height and length of rooster tail jump investigates. In a horseshoe spillways, due to the collision of the falling nappes from their surround walls, in the center of spillway's trough, a spatial hydraulic jump is formed that named "rooster tail". The results of this study shows that by reducing the crest length of horseshoe spillway, the water head over the spillway increases and the distance of the location of jump formation from the spillway's wall increases Also by increasing the amount of water on the spillway's crest and thereby increasing the velocity of flow nappe at the point of contact with the spillway's bed, length and high of rooster tail jump, linearly increased. Therefore, the choice of longer spillway crest is recommended to reduce the rooster tail jump effects. Based on the analytical methods, the effective parameters on rooster tail jump's height and height were determined. The result also shows that by increasing the ratio of water head to crest length ($h/Lw$), length and high of rooster tail jump, linearly increasing (Linear relationships were proposed to predict jump's length and height). Dimensionless number of $L_w^5 g/Q^2$ somehow represents the inverse of the Froude number. The results shows that by increasing the Froude number, the high of rooster tail jump is linearly increasing. By increasing the Dimensionless number of $L_w/Q \times \vartheta$, length and high of rooster tail jump are decreasing. By increasing the upstream discharge of unit width (qu) both length and height of the rooster tail jump linearly increased.

To control of rooster tail jump in spillway's model, two different size of end sills Inserted at downstream of spillway and result shows that by employing the sills with height of 3.8 cm and 7.6 cm, the flow depth, in average, 122% and 272% respectively increase compared to no sill conditions, also flow state change from super-critical to sub-critical. Also, the comparison of results for spillway with two different crest lengths, shows that by decreasing the $L_W$, the flow depth at the downstream of spillway in the state of no sill, increased in discharge of less than 10.5 lit/s and decreased

for discharge values greater than 10.5 lit/s. by applying the sills with height of 3.8 cm and 7.6 cm, flow velocity in the downstream of the horseshoe spillway, in average, respectively 60% and 80% decreases compared to no sill conditions. The results revealed that by placing the sill of 3.8 and 7.6 cm, respectively 45% and 35% of the maximum pressure entering the bed of the spillway at the collision site is reduced. In the experiments it was observed that sill with a height of 7.6 cm further reduces the flow turbulence.

## 5. Conflicts of Interest

The authors declare no conflict of interest.

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