CHAPTER 18

EXPERIMENTAL STUDY ON THE VALIDITY RANGE OF VARIOUS WAVE THEORIES

by

Masataro Hattori*

INTRODUCTION

In the design of coastal structures and the study of nearshore dynamics, rational predictions of the wave kinematics are very important but difficult procedures. Although a large number of nonlinear wave theories have been proposed and used for computing the wave kinematics, there are no theories applicable from the deep water to very shallow water. It is, therefore, very important for coastal researchers and engineers to know which of theories describe well a wave field specified by the wave characteristics and water depth, and to select a particular wave theory for a problem of interest.

Many intensive efforts have been made to examine the validity as well as the applicability of various wave theories. However, there are still no well-accepted guidlines for the application range of the wave theories. The validity evaluation of a particular wave theory has been basically made by means of the following two versions: the analytical (mathematical) validity and the experimental (physical) validity. The analytical validity study has been conducted by various researchers (Dean, 1970; Komar, 1976; Horikawa et al., 1977; Swart, 1978) and revealed the degree of mathematical satisification to the governing equations and boundary conditions for each wave theory. The analytical validity study probably tends to show the relative applicability for various wave theories. It does not ensure that the theory describe well laboratory or field phenomena. Based on the analytical validity of various wave theories by Horikawa et al., Isobe (1985) proposed application ranges for the finite amplitude wave theories in terms of the relative water depth and relative wave height.

The experimental validity refers to how well the prediction of various wave theories agrees with actual measurements (Dean & Dalrymple, 1984). As the wave shoals, wave form becomes more asymmetrical, especially under high wave conditions of interest to design. Such nonlinearity influences greatly the wave kinematics and it makes difficult to predict readily the wave kinematics by several theories. From a practical viewpoint, it is , therefore, requested to establish the application ranges of available wave theories for shoaling waves.

In this study, laboratory data of simultaneous measurements of the

* Professor, Department of Civil Engineering, Chuo University, Kasuga 1-13-27, Bunkyo-ku, Tokyo, 112, Japan.

wave surface elevation and water particle velocities of the shoaling wave were compared with the predictions from various wave theories. On basis of degree of the overall agreement between the measured and predicted time-varying quantities, the validity range of the individual wave theory was specified.

The purposes of the present study are: (1) To propose parameters and criteria for the evaluation of wave theory validity, (2) To evaluate the validity limits for various wave theories, and (3) To determine the application ranges of available wave theories for important ranges of wave conditions of practical interest.

The wave theories included in the evaluation are listed in Table 1. The linear wave theory is the best known and widely used theory. From a practical viewpoint, the Stream Function wave theory of Dean (1965) was adopted as a representative numerical wave theory. The prediction of time-varying quantities by the ninth order irregular Stream Function wave theory, SFM9A, was performed using the measured wave profile, whereas that by the other theories was based on symmetrical profiles with the measured wave height and period.

Table 1 Wave Theories Involved in the Evaluation.

WAVE THEORY		REFERENCE
Linear wave theory	ST 1	Ippen (1966)
Fifth order Stokes	ST 5	Isobe et al. (1978) Fenton (1985)
Third order Cnoidal	CN 3	Isobe et al. (1978)
Stream Function wave theory		Dean (1965)
Fifth order of permanent waves	SFM 5B	
Ninth order of irregular waves	SFM 9A	

EXPERIMENTAL EQUIPMENT AND PROCEDURES

Experiments were performed in a glass-walled wave flume, 0.30 m wide, 0.55 m high and 20 m long. Waves were produced by a flap-type wave generator installed at one end of the flume. A sloping bottom of 1/20 was installed in the flume. Experimental conditions, given by Table 2, covered a wide range of the wave steepness. The last column of Table 2 gives the Ursell parameter, Ur_0, at the wave generator.

Simultaneous measurements of the wave surface elevation and water particle velocities were made at various locations over the slope. Range of the shallow water Ursell parameter, $Ur = gHT^2/h^2$ (H is the wave height, T is the wave period, h is the water depth, and g is the gravitational acceleration), at measuring section was from about 1 to 110. The wave surface elevation, η , was measured by resistance-type wave gages having excellent linearity and stability. The manufacture's stated frequency response of the wave gage is approximately 20 Hz, higher than the harmonic frequency range of experimental waves due to the nonlinearity. Measurements of the horizontal and vertical water particle velocities, u and w, were made using a laser doppler velocimeter of two

Table 2 Experimental Conditions.

Exp.	Run	T(s)	h_i(cm)	H_i(cm)	h_b(cm)	H_b(cm)	H_o'/L_o	Ur_0
1	1	0.80	35.0	6.5	9.0	6.0	0.068	1.14
	2	0.85	35.0	4.4	5.5	5.4	0.041	0.87
	3	1.00	35.0	3.1	5.0	4.3	0.021	0.85
	4	1.40	35.0	2.6	5.3	4.7	0.009	1.41
	5	1.20	40.0	0.9	–	–	0.005	0.30
2	1	1.00	37.5	4.7	7.0	6.1	0.032	1.19
3	1	0.80	33.0	6.5	8.5	6.8	0.068	1.22
	2	0.85	33.0	5.1	6.5	6.1	0.048	1.09
	3	1.00	33.0	3.8	5.5	5.4	0.026	1.12
	4	1.40	33.0	2.5	4.5	4.9	0.009	1.45
	5	1.20	33.0	1.1	–	–	0.005	0.47
4	1	0.80	33.0	6.5	8.5	6.9	0.068	1.22
	2	0.84	43.0	5.6	7.0	5.7	0.049	0.84
	3	0.99	43.0	4.0	5.5	5.0	0.026	0.84
	4	1.40	43.0	2.7	6.0	4.6	0.009	1.14

H_i : Wave height at the uniform water depth h_i.
H_b & h_b : Wave height and water depth at breaking point.
Ur_0 : Ursell parameter at the wave generator.

components (DISA, Model 55 X) in a vertical plane beneath the wave surface measurement. Outputs of the wave surface elevation and water particle velocities were recorded on a 7-channel analogue recorder over 70 wave periods minimally. The data were digitized by an A-D converter at a sampling frequency of 167 Hz for computer processing. After individual waves were determined from the wave gage records by means of the zero-upcrossing method, the time series data were averaged over 50 waves with respect to the phase.

DETERMINATION OF THE EVALUATION PARAMETERS

In the most previous studies (Swart, 1978), the validity of wave theory has been discussed on basis of visual comparisons between the theoretical prediction and measured values. In order to determine rationally the application range for a particular wave theory, the wave theory validity should be evaluated with appropriate criteria of non-dimensional parameters representing degree of the agreement of the theory with the measured values. We adopted the four following parameters:

(1) Ratio of the maximum values: $M_R^+ = Y_{max}/X_{max}$,

(2) Ratio of the minimum values: $M_R^- = Y_{min}/X_{min}$,

(3) Coherence between the measured and predicted time histories:

$$C_o = \sum_{i=1}^{m} X_i Y_i / \left(\sum_{i=1}^{m} X_i \sum_{i=1}^{m} Y_i \right)^{1/2}, \text{ and}$$

(4) Overall root-mean-square (rms) between the time histories:

$$E = [\sum_{i=1}^{m}(X_i - Y_i)^2 / \sum_{i=1}^{m} X_i^2]^{1/2}.$$

X_i and Y_i, as shown in Fig. 1, are the measured and predicted values sampled at various and evenly spaced phases over one wave period of the time history. X_{max} and Y_{max}, and X_{min} and Y_{min} are the maximum and minimum values of X_i and Y_i, respectively.

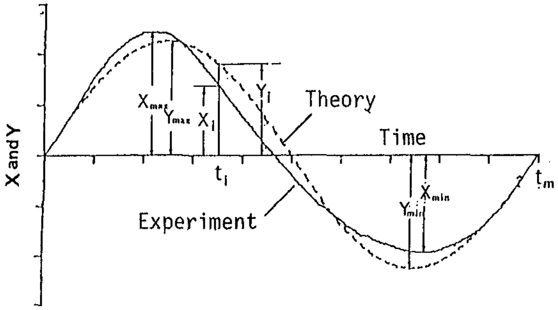

Fig. 1 Definition Sketch for Evaluation Parameters.

From a practical viewpoint, the parameter set of M_R^+ and M_R^- seems to, be of a very useful in the validity evaluation. Since the coherence function C_o indicates the degree of similarity in overall profiles of time variations, the parameter set of C_o and M_R^+ is considered as a favorable measure for the validity evaluation. As the overall rms error E indicates the deviation degree of the theory from the experiment throughout the time history of one wave period, E is considered as the principal parameter for the validity evaluation.

EXPERIMENTAL ERRORS AND CRITERIA FOR THE VALIDITY EVALUATION

If experiments could be made under sufficiently well-controlled conditions, measured data are most likely contaminated with a variety kind of errors associated with the experiments. We should determine criterion for evaluation of the wave theory validity with consideration that disagreement between the theory and experiment is at least of the same order as the estimated experimental error, because the experimental validity is based on the comparison between the theory and experiment.

Prior to the determination of the evaluation criterion, it is necessary to discuss potential errors associated with experiments. The experimental errors are attributed to the following causes: experimental equipment and procedures, measuring instruments and techniques, and data processing.

Ursell parameter at the wave generator for each test run, given in the last column of Table 2, is much less than 13, and indicates that experimental waves are completely free from the secondary wave generation (Swart, 1978). Wave reflection from the sloping bottom was one of the systematic error sources. From measurements of the wave height distribution in a part of uniform water depth, the error due to the reflection was estimated at most ± 5 % under high wave conditions.

In Fig. 2, the experimental results are shown for the variation of wave height for a series of Exps. 1 through 3. The solid line along data plots represents the theory of Shuto (1974), which is the first order Cnoidal wave theory with some second order terms retained in order to take account of the bottom slope effect. For a reference, breaking limits of Yamada and Shiotani (1968) and of Goda (1970), and the isolines of the shallow water Ursell parameter are shown. It is noticed from Fig. 2 that the experiments surprisingly agree well with the theory except for the experiments of low steepness conditions. The small deviation between the measurements and the theory is probably due to capillary effect of the water surface on thin-wire wave gages. The wave measurement error is estimated to be at most ± 2 %.

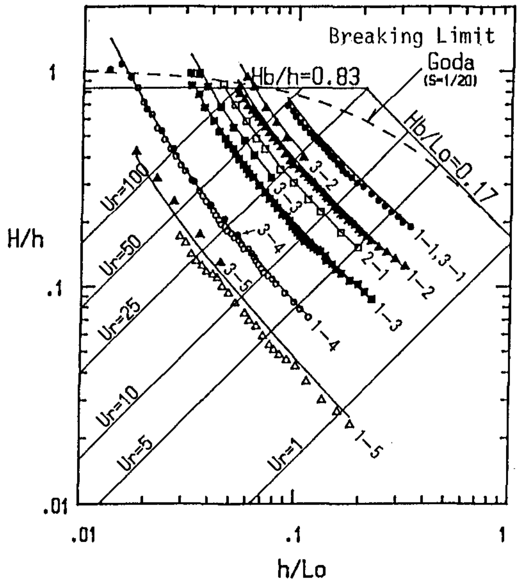

Fig. 2 Wave Height Variation.
——: Theory of Shuto (1974)

The manufacture's stated accuracy of the instruments used was much greater than that required for this class of experiments. Another potential error is due to averaging processes of the measured data, and is estimated ±0.5 % for the wave surface elevation and ± 1 % for the water particle velocity data.

Taking into consideration of the potential errors in the experiment, we determined the overall rms error of $E \leq 0.10$ as the principal criterion of the validity evaluation. Evaluation criteria of the other parameters were determined so as to be equivalent to the principal criterion and were $0.90 \leq M_R^+$ and $M_R^- \leq 1.10$ and $C_o \geq 0.995$. To substantiate the adequancy of the criteria, Fig. 3 presents an example comparison between the measured and predicted time variations of the wave surface elevation and water particle velocities. The theoretical predictions were made by using the measured wave height, wave period, and water depth. E value for the ST 5 theory is equal to the criterion of E = 0.10. Visual comparisons provide fairly well agreements between the experiment and the predictions from the ST 5 theory.

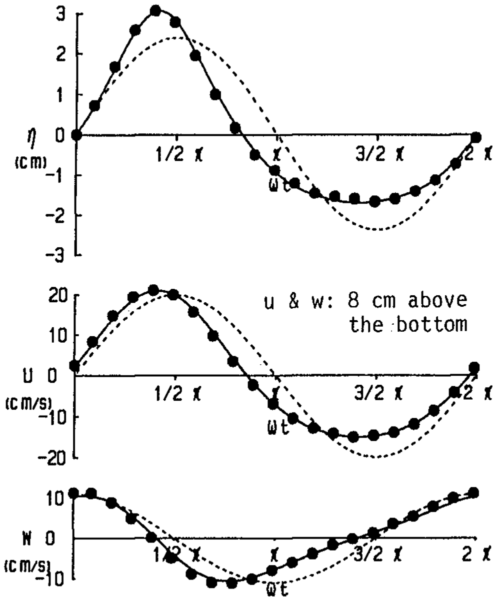

Fig. 3 Comparisons between Measured and Predicted Wave Form and Velocities for ST 1 and ST 5 Theories.
● : measured, -----: ST 1, ——— : ST 5
(Exp. 4-2: H=4.76 cm, h=12.0 cm, T=0.84 s, Ur=22.9)

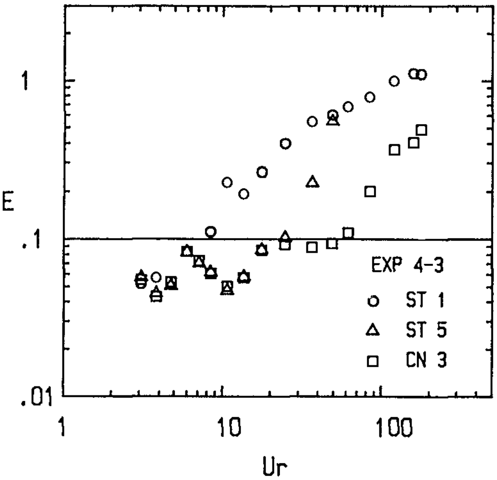

Fig. 4 Validity Limits for Wave Form of ST 1, ST 5 and CN 3 Theories in terms of E and Ur.

VALIDITY EVALUATION FOR THE WAVE SURFACE ELEVATION

Since the prediction by a wave theory is based on the measured wave height and period, the use of M_R^+ and M_R^- is not suitable for the validity evaluation of the wave surface elevation. Thus, validity examinations were performed with respect to the C_o and E parameters.

The validity limits of the ST 1, ST 5, and CN 3 theories for the wave form is shown in Fig. 4, in terms of the overall rms error E and the shallow water Ursell parameter Ur. It was found that the linear theory broke up the validity due to the wave form asymmetry about the mean water level, while the nonlinear wave theory became unable to maintain the validity due to the wave form asymmetry about the wave crest. Nonlinear effects of the shoaling wave on the wave form were discussed with the use of two types of the skewness factor for the wave form (Sekine & Hattori, 1985).

VALIDITY EVALUATION FOR THE WATER PARTICLE VELOCITIES

Since within a region of small Ursell parameter at measuring locations, the wave form is approximately symmetrical, the water particle velocities at the crest and trough phase position are predicted relatively well by either the linear or the Stokes wave theory. As the wave shoals further, the wave form becomes more asymmetrical about the crest as well as the mean water level. Due to such nonlinearity associated with the shoaling, degree of the agreement between the measurement and the prediction of the water particle velocities as well as the wave form depends on the class of wave theory applied to the computation of the wave kinematics.

Figure 5 shows an example comparison of the vertical distributions with depth for the horizontal and vertical water particle velocities under the crest and trough at a section of Ur = 22, where the wave form asymmetry becomes apparent. Under such a circumstance, the nonlinear wave theories, ST 5, CN 3 and SFM 9A, predict fairly well the measured values of the horizontal water particle velocity as seen in Fig. 5. Although the vertical water particle velocity under the crest and trough

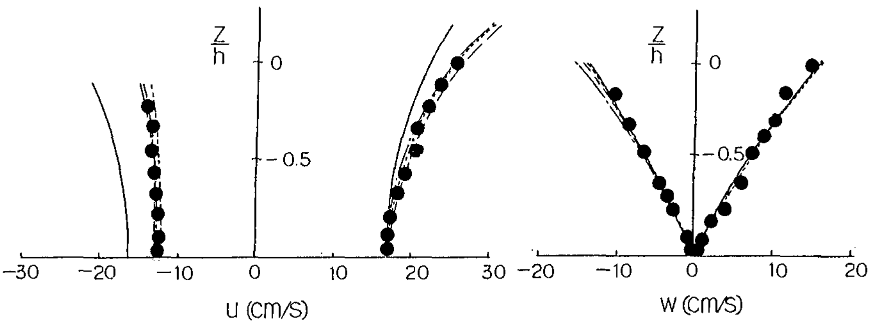

Fig. 5 Vertical Distributions of Water Particle Velocities under Crest and Trough (Exp. 2-1, Ur = 22).
———: ST 1, — — —: ST 5, —·—·— : CN 3, - - - - - -: SFM 9A

is represented excellently well by every wave theory as in Fig. 5, it is considered that degree of the agreement between the theory and measurements for the horizontal water particle velocity clearly depends upon the theoretical modeling for the nonlinear effect.

Based on comparisons of the vertical distributions between the measured and predicted water particle velocities, we decided to examine the wave theory validity for the water particle velocity through time history data of the horizontal water particle velocity. The validity evaluation was based on spatial distribution of the isoline of each evaluation parameter. Figure 6 is an example showing the validity limit of the ST 1 theory. The top figure is for the E value, the middle figure for the parameter set of M_R^+ and M_R^-, and the bottom figure for the parameter set of M_R^+ and C_o. The validity area is illustrated by the shaded part. The horizontal coordinate is represented by the water depth, instead of the horizontal distance from a reference point. The figure by the sloping bottom is the water depth and that in parentheses is the shallow water Ursell parameter at the measuring section. $_{max}$ and $_{min}$ are the envelop of wave height. B.P. denotes the breaking point.

As waves propagate from the intermediate to very shallow water region, the wave form is gradually translated due to nonlinear effects of the wave motion. As a consequence, the linear wave theory loses the validity for the horizontal water particle velocity as well as for the wave form. An interesting evidence is found that the validity limit for the water particle velocity broadens at the lower portions of the water column, because the bottom boundary restricts the orbital motion of water particles due to waves, especially in the vertical direction. This results degradation of the nonlinearity in the velocity time-variation. From a comparison between the top and middle figures in Fig. 6, we notice that the validity limit determined with the parameter set of M_R^+ and M_R^-, very useful in a practical use, is almost the same as that by E. In contrast, a remarkable disagreement is found between the validity limit determined by E and the parameter set of M_R^+ and C_o. Thus the parameter set of M_R^+ and C_o is not suitable for the validity evaluation. As many previous studies have pointed out, the ST 1 theory provides very good predictions of the wave kinematics even in very shallow water region, up to the breaking point.

Figures 7 and 8 show the validity limit of the nonlinear wave theories, ST 5 and CN 3. The validity limit for the ST 5 theory coincides with the nonconvergence or inapplicability of the wave theory. Although the CN 3 theory is recognized to predict reasonably well the water particle velocity in the very shallow water region, the theory becomes unable to maintain the validity due to the nonlinearity associated with the asymmetry about the maximum velocity under the crest. The CN 3 theory provides a favorable agreement with the measured wave surface elevation in the intermediate region (see Fig. 4). On the other hand, the theory tends to overpredict the horizontal water particle velocity at the crest phase position in the same region. Consequently, another validity limit appears in the intermediate water depth region, and this is expected from the assumption employed in the derivation of the Cnoidal wave theory. According to the validity evaluation for the CN 1 theory, the theory represents very poorly the

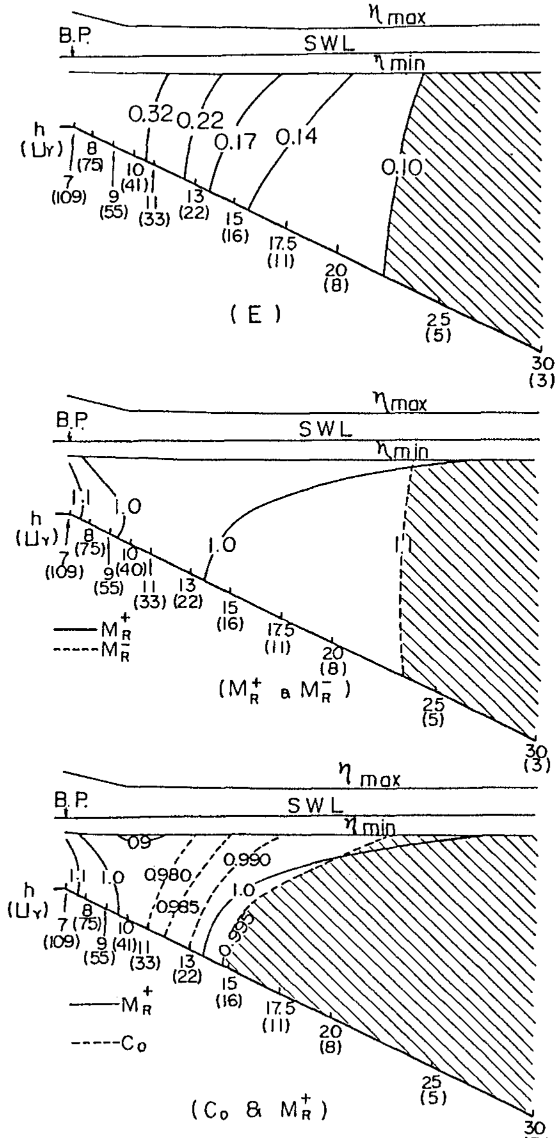

Fig. 6 Validity Limit for Horizontal Velocity of ST 1 Theory (Exp.2-1).

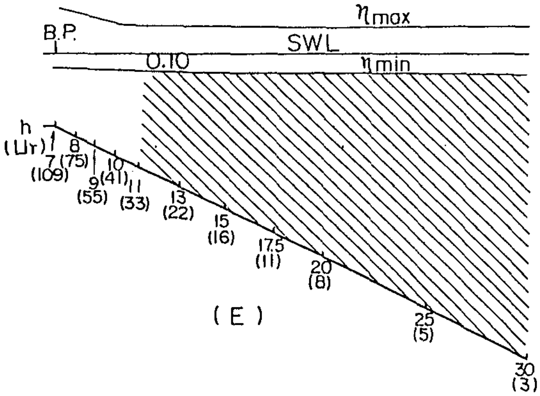

Fig. 7 Validity Limit for Horizontal Velocity of ST 5 Theory (Exp. 2-1)

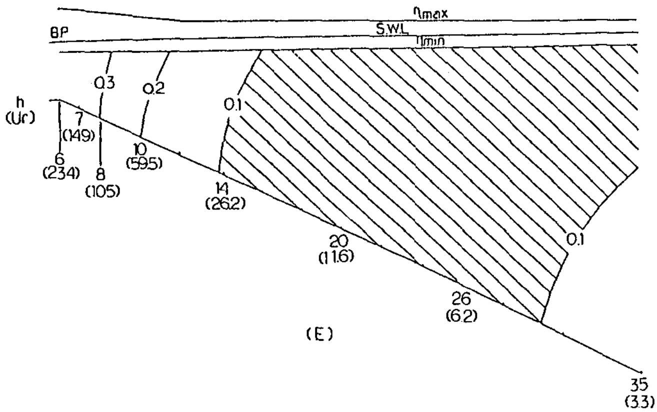

Fig. 8 Validity Limit for Horizontal Velocity of CN 3 Theory (Exp. 4-4).

measures water particle velocity. Thus the rational validity limit of the CN 1 theory could not be determine.

The irregular Stream Function wave theory, SFM 9A, provides the best agreement with the measured water particle velocity over a very wide range from the deep water to very shallow water near the breaking point. This is most probably due to the use of the measured wave surface elevation for the calculation of the SFM 9A theory.

APPLICATION RANGE FOR THE WAVE THEORIES

In the previous sections, we mainly discussed the validity limit for the wave form and the water particle velocity with the aid of the criterion of the overall rms error E = 0.10. The application range of the wave theories can be determined from intercomparisons of the validity limits for the wave form and water particle velocity, in terms of the relative water depth, h/L_0, and relative wave height, H/h, with the auxiliary parameter of the shallow water Ursell parameter, Ur. The results are presented by Figs. 9 through 11.

Application ranges of the ST 1 and ST 5 theories are presented in Fig. 9, in which the broken line is the isoline of the rms error E_T = 0.10 in the free surface boundary conditions for the wave theories, determined from the analytical validity study of Horikawa et al. (Isobe, 1986). Breaking limit by Yamada and Shotani (1968) is also shown. Circle, triangle, and square symbols denote the validity limits for the wave form and the horizontal water particle velocities near the bottom and just below the trough, respectively. According to the validity evaluation results, the validity limit for the wave form coincides with that for the water particle velocities at the upper portions of the water column. Therefore, this determines the application ranges of the ST 1 and ST 5 theories, as shown in Fig. 9.

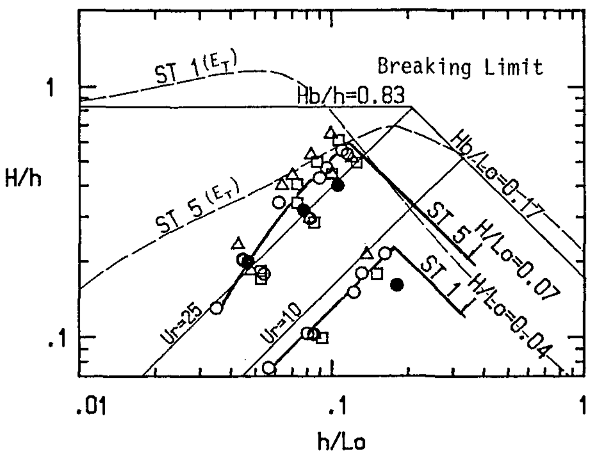

Fig. 9 Application Ranges of ST 1 and ST 5 Theories.
○ ● : the wave surface elevation, △ ▲ : near-bottom horizontal velocity, □ ■ : horizontal velocity below the trough.
— —: isoline of E_T = 0.10 by Isobe (1986)

A special feature found in the application range of the CN 3 theory is that the application range is confined within a region bounded by the application limits in very shallow water and in the intermediate water depth region as in Fig. 10, and provides some limitation in application of the Cnoidal wave theory. The CN 3 theory as well as the ST 5 can not represent favorably the measured values in a region of large values of

Ur, in which the asymmetry in the wave form becomes more remarkable (Iwagaki et al., 1972; Flick et al., 1981). Taking into account that there is an application limit of the CN 3 theory of Ur = 6, and that the application range of the ST 5 theory in a region of Ur > 25 is narrow, the isoline of Ur = 25 is considered as a reasonable demarcation between the applications of the ST 5 and CN 3 theories (Le Mehaute, 1976; Isobe, 1985). From Figs. 9 and 10, it is found that the application ranges determined by the experimental validity indicate a similar trend of the isoline of E_T for each wave theory, especially in the intermediate water depth.

Application range of the irregular Stream Function wave theory, SFM 9A, coincides exactly with the validity limit for the water particle velocities (Fig. 11). The SFM 5B theory, assumed the symmetrical wave form can not predict well the wave kinematics in very shallow water region because of the nonlinear effect. From a comparison between Figs. 10 and 11, we find that the application limit of the SFM5B theory is almost the same as that of the CN 3 theory in a region of large values of Ur.

To conclude the discussions in the present study, we propose a diagram representing the application ranges of the wave theories for waves propagating in the shoaling water. Figure 12 presents the application ranges in terms of h/L_0, h/H, and Ur.

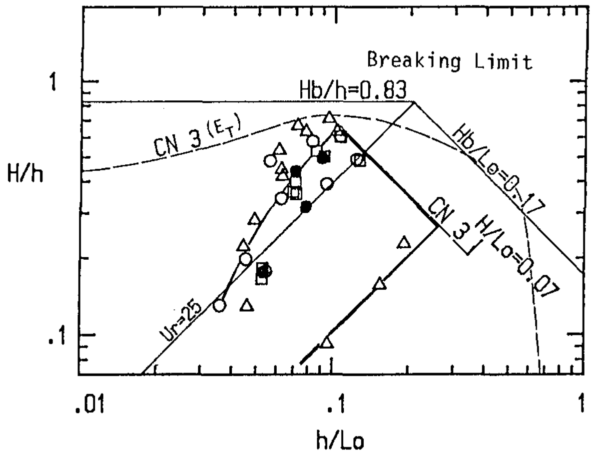

Fig. 10 Application Range of the CN 3 Theory.

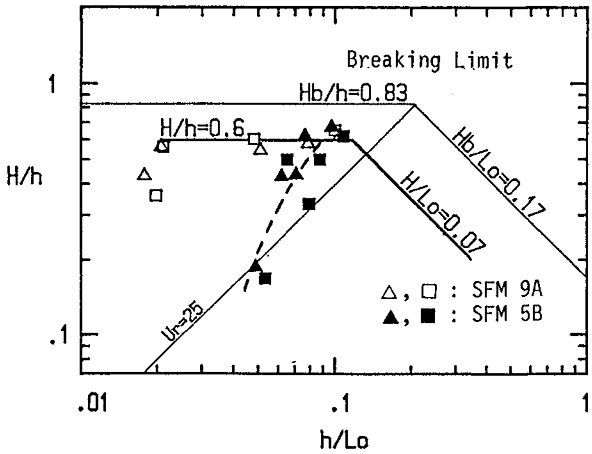

Fig. 11 Application Ranges of the SFM 9A and SFM 5B Theories.

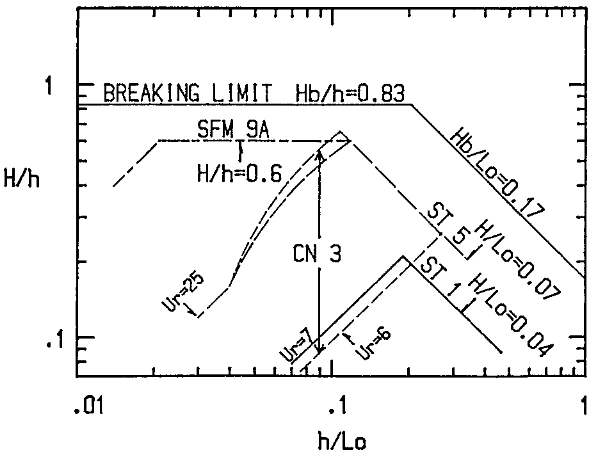

Fig. 12 Application Ranges of the Wave Theories Included in the Present Study.

CONCLUSIONS

Simultaneous measurements of the wave surface elevation and water particle velocities of shoaling waves have been performed at various locations in a wave flume with a slope of 1/20. The validity of wave theories has been evaluated by comparisons between the measured values and predictions from the wave theories. The wave theory validity has been discussed using four parameters, representing the degree of the agreement between the measurement and theory. Based on the validity evaluation, the application ranges for the wave theories have been determined in terms of H/h, h/L_0, and Ur.

The main findings of the present study are as follows:

(1) Amongthe non-dimensional parameters for the validity evaluation, the overall rms error E plays a principal role in the determination of the validity limit of various wave theories. In addition, the parameter set of M_R^+ and R_R^- is favorable to the validity evaluation in a practical problem specified by the wave characteristics and water depth.
(2) For wave surface elevation, the validity range of analytical theory for permanent type waves is limited by the nonlinearity attributed to the wave form asymmery about the mean water level for the linear wave theory and about the crest for the nonlinear theory.
(3) The degree of agreement between the measured and predicted horizontal water particle velocity depends on the elevation above the bottom and on the value of Ursell parameter.
(4) Application ranges of the wave theories included in the evaluation are determined by the validity limits for the wave form and horizontal water particle velocity at the upper portions of water column just below the trough. Within the limit of the present study, the application ranges are determined as in Fig. 12 and Table 3.

Table 3 Application Ranges of the Wave Theories.

ST 1	$0.2 \leqq h/L_0 < 0.4$:	$H/L_0 \leqq 0.04$
	$0.06 < h/L_0 \leqq 0.2$:	$U_r \leqq 7$
ST 5	$0.1 \leqq h/L_0 < 0.4$:	$H/L_0 \leqq 0.07$
	$0.06 \leqq h/L_0 \leqq 0.1$:	$U_r \leqq 32$
	$0.04 < h/L_0 \leqq 0.06$:	$U_r \leqq 25$
CN 3	$0.12 \leqq h/L_0 < 0.4$:	$H/L_0 \leqq 0.07$
	:	$6 < U_r \leqq 25$
	$0.07 \leqq h/L_0 \leqq 0.12$:	$H/h \leqq 0.5$
	:	$6 < U_r \leqq 35$
	$0.04 \leqq h/L_0 \leqq 0.07$:	$6 < U_r \leqq 35$
	$0.02 < h/L_0 \leqq 0.04$:	$6 < U_r \leqq 25$
SFM9A	$0.1 \leqq h/L_0 < 0.4$:	$H/L_0 < 0.07$
	$0.02 < h/L_0 \leqq 0.1$:	$H/h \leqq 0.6$

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