

Experimental validation of a coprime linear microphone array for high-resolution direction-of-arrival measurements^{a)}

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Abstract: Coprime linear microphone arrays allow for narrower beams with fewer sensors. A coprime microphone array consists of two staggered uniform linear subarrays with M and N microphones, where M and N are coprime with each other. By applying spatial filtering to both subarrays and combining their outputs, $M + N - 1$ microphones yield $M \cdot N$ directional bands. In this work, the coprime sampling theory is implemented in the form of a linear microphone array of 16 elements with coprime numbers of 9 and 8. This coprime microphone array is experimentally tested to validate the coprime array theory. Both predicted and measured results are discussed. Experimental results confirm that narrow beampatterns as predicted by the coprime sampling theory can be obtained by the coprime microphone array.

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Date Received: August 13, 2014 **Date Accepted:** February 4, 2015

1. Introduction

Microphone arrays are used in a wide variety of situations for many acoustic applications, including passive-sonar localization, spatial-audio recording, and isolation of a desired signal from noise.¹⁻⁴ In particular, acoustic source localization based on microphone array signal processing can be used to locate gunfire or aircraft in defense and homeland-security applications, to localize noise sources in the design and manufacturing process of machines in the aerospace industry,⁵ or to localize an unknown number of spatially distributed sound sources in spatially distributed noise.⁶ All of these applications benefit from array designs and beamforming techniques that produce narrow beams. However, conventional array theory limits resolution by the spatial Nyquist theorem. In this work we report on experimental validation of an array design and beamforming technique based on coprime number theory,⁷ which allows for array designs with a drastically reduced number of sensors having spacing exceeding the separation limit governed by the spatial Nyquist theorem.

To address this fundamental limitation, alternative array-processing methods and sensing schemes, such as compressive sensing and random-array sensing, have been explored.^{2,8} Lang *et al.*⁹ did theoretical work on maximum-entropy methods for

^{a)}Aspects of this work have been presented at the 165th ASA Meeting in San Francisco, California in 2013.

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array design computing array layouts with uneven sampling that had more degrees of freedom. To achieve higher degrees of freedom becomes a technical challenge in the array beamforming technique.

Vaidyanathan and Pal⁷ recently presented theoretical models for sparse sensing using what they called a “coprime array.” A coprime array comprises two uniform linear subarrays that are coincident (the same starting point and both continue in the same direction) with M and N components, where M and N are a coprime pair. This particular array arrangement has a number of advantages over randomly distributed sparse arrays⁸ because the microphone locations are easily determined, requiring no exhaustive searches, and the outputs of the two subarrays can be combined in a unique way due to the co-primality of the subarrays. These coprime arrays with appropriate signal processing can achieve higher resolution than is classically possible by the Rayleigh limit, allowing for accurate direction-of-arrival (DOA) estimation with a drastically reduced number of sensors.

Following Vaidyanathan and Pal,⁷ coprime numbers have drawn increasing attention in a wide range of acoustics and radar applications.^{10–12} This letter reports on the experimental validation conducted using a linear coprime array consisting of two subarrays of nine and eight microphones in a well-controlled quasi-anechoic environment. This letter discusses preliminary results. More systematic experiments and sophisticated data analysis are subjects of future research.

2. Basic theory

In order to describe how coprime numbers can be exploited to achieve more than $M \cdot N$ degrees of freedom using only $M + N - 1$ sensor elements for beamforming and DOA estimation, this section begins with the basics of beamforming a conventional uniform linear array (ULA).¹³ The following description also relies heavily on the recent work by Vaidyanathan and Pal.⁷

The beampattern of a conventional ULA of K microphones as illustrated in Fig. 1(a), with each receiving a signal $h(k)$, can be expressed as

$$H_K = \sum_{k=0}^{K-1} h(k)e^{-jzk}, \quad (1)$$

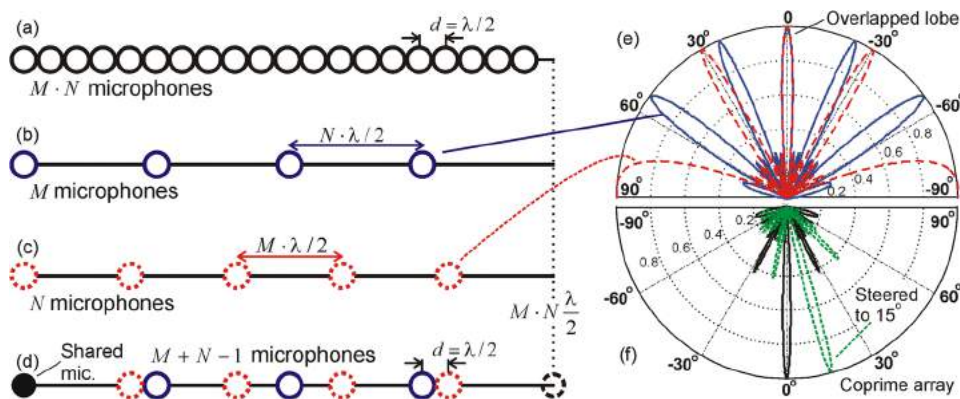


Fig. 1. (Color online) Linear coprime microphone array with coprime numbers $M = 4$, $N = 5$ and its beam patterns for normal incident waves. (a) Conventional linear microphone array requiring $M \cdot N$ microphones in order to achieve a comparable beam width. (b) Subarray with $M = 4$ microphones. (c) Subarray with $N = 5$ microphones. (d) Staggered coprime microphone array consisting of $M + N - 1$ microphones, the first one (far left) is shared by two subarrays. (e) Beam patterns of subarray (solid line), consisting of M microphones as shown in (b) and subarray (dashed line), consisting of N microphones as shown in (c). Grating lobes at the frontal direction overlap completely. (f) Beam patterns of the combined coprime microphone array for the normal incidence and the one steered to 15° .

where $\alpha = 2\pi d \sin \theta / \lambda$ with θ being the azimuthal angle from normal, $j = \sqrt{-1}$, $d = \lambda/2$ being the inter-microphone spacing, and λ being the wavelength of interest. When the inter-microphone spacing is $d \leq \lambda/2$, the beampattern of this ULA of length $L = K \cdot d$ contains one main lobe. At the limit, $d = \lambda/2$, governed by the spatial Nyquist theorem, the beampattern retains a single main lobe.¹³ The inter-microphone spacing $d = \lambda/2$ is termed *elementary spacing* in the following. The frequency associated with the wavelength is termed *design (or critical) frequency*. Given the aperture length L and elementary spacing d , resulting in K microphones, the design frequency is determined by

$$f_0 = \frac{c}{2d} = \frac{c \cdot K}{2L}, \quad (2)$$

with c being the sound speed.

If M and N are a pair of coprime numbers, in other words, M and N have no common dividers other than 1, the uniform linear array can be constructed consisting of two virtual uniform line subarrays with drastically sparse spacing of the microphones, but still achieve similar degrees of freedom. For a sparse uniform linear array, as illustrated in Fig. 1(b), consisting of M microphones with inter-microphone spacing much larger than the elementary spacing $d_M = N\lambda/2 = N \cdot d$ (with N being a positive integer), the sparse ULA of the given length L will contain fewer microphones ($M < K$), resulting in a beampattern

$$H_M(z^N) = \sum_{m=0}^{M-1} h(m)z^{-Nm}, \quad (3)$$

with $z = \exp(j\pi \sin \theta)$ for simplicity. This simplified notation clarifies the coprime relationship between M and N as noted by $H_M(z^N)$. Figure 1(b) illustrates the physical arrangement of this sparse ULA; note that this sparse ULA consists of M microphones with a microphone spacing of N times the half-wavelength, and its far-field beampattern function can be expressed¹³ as

$$P_M(\theta) = \frac{\sin\left(\frac{kMd_M}{2} \sin \theta\right)}{\sin\left(\frac{kd_M}{2} \sin \theta\right)} e^{j[k(M-1)d_M/2] \sin \theta} = \frac{\sin\left(\frac{MN\pi}{2} \sin \theta\right)}{\sin\left(\frac{N\pi}{2} \sin \theta\right)} e^{j[(M-1)N\pi/2] \sin \theta}, \quad (4)$$

since $kd_M = N\pi$ with $k = 2\pi/\lambda$. Figure 1(e) illustrates its beampattern (solid-line) assuming the same values of $h(m)$ sensed by all the microphones, containing a number (N) of equally sensitive grating lobes (Ref. 4), also governed by the spatial Nyquist Theorem. The interval between these grating lobes is $\lambda/d_M = 2/N$, and the null-to-null beam width of the grating lobes (in α -domain) is $2\lambda/Md_M = 4/(MN)$. In a similar fashion, the same length of the sparse ULA with an inter-microphone spacing $d_N = M\lambda/2$ will contain N microphones,

$$H_N(z^M) = \sum_{n=0}^{N-1} h(n)z^{-Mn}. \quad (5)$$

Figure 1(c) illustrates the subarray with N elements, its beampattern (dashed line) assuming the same values of $h(n)$ sensed by all the microphones, is shown in Fig. 1(e), containing M grating lobes with their interval $\lambda/d_N = 2/M$, the null-to-null beam width (in α -domain) is $2\lambda/Nd_N = 4/(MN)$. Two staggered ULA subarrays as also shown in Fig. 1(d) contain $M + N - 1$ microphones, with one microphone shared by the two subarrays. Note that the beampattern of the subarray consisting of M uniformly spacing microphones (subarray M in short) contains N equally sensitive grating lobes, while the subarray N contains M grating lobes. As shown in Fig. 1(e) only one

(main) lobe of each subarray exactly overlaps, the rest of the grating lobes are largely separated from each other so that a straightforward cross-correlation processing results in the beampattern of the staggered coprime arrays,

$$H_C(z) = H_M(z^N)H_N^*(z^M). \quad (6)$$

The width of the beampattern's main lobe, as illustrated in Fig. 1(f), has the null-to-null beam width of $4/(MN)$ (in α -domain), which is similar to that of a conventional ULA with $M \cdot N$ elements. From one of the subarrays, say, the subarray N , containing N microphones with the inter-microphone spacing $d_N = M\lambda/2 = L/N$ for given aperture length L , the design frequency that satisfies the constraints of both subarrays is

$$f_c = \frac{cMN}{2L}. \quad (7)$$

This is akin to replacing K in Eq. (2) with $M \cdot N$.

Two distinct sidelobes, being significantly smaller than the main lobe, result from partially overlapped grating lobes of the two subarrays. The other smaller sidelobes are similar to those of conventional ULAs. The beampattern can also be steered to an arbitrary looking angle, as shown in Fig. 1(f), which illustrates the beampattern steered to a 15° looking angle.

3. Experimental study

This work has conducted a number of experimental tests for validating the coprime array theory discussed in Sec. 2. Section 3.1 first describes the design of a coprime microphone array and then discusses the experimental results in Sec. 3.2 in comparison with those simulated using the theory briefly described in Sec. 2.

3.1 Coprime array design

A total of 16 electret microphones are flush-mounted on a flat carriage 1 m long ($L = 1$ m), 0.03 m wide, and 4 mm thick. The flat carriage is designed for easy flush-mounting on a surface so to be easily deployable in a semi-anechoic chamber on the rigid floor for experimental investigations, as described below. The coprime numbers are chosen to be $M = 9$ and $N = 8$ for a design frequency $f_c = (cMN)/(2L) \approx 12.35$ kHz according to Eq. (7). The elementary microphone spacing governed by the spatial Nyquist theorem amounts to $d = \lambda/2 \approx 1.39$ cm. Figures 2(a) and 2(b) illustrate the coprime microphone array consisting of two staggered linear subarrays across a 1 m aperture and a photograph of the coprime line array, respectively. For the given aperture length, it has to contain 72 microphones in order for the conventional linear microphone array to fulfill the spatial Nyquist theorem. At this design frequency such

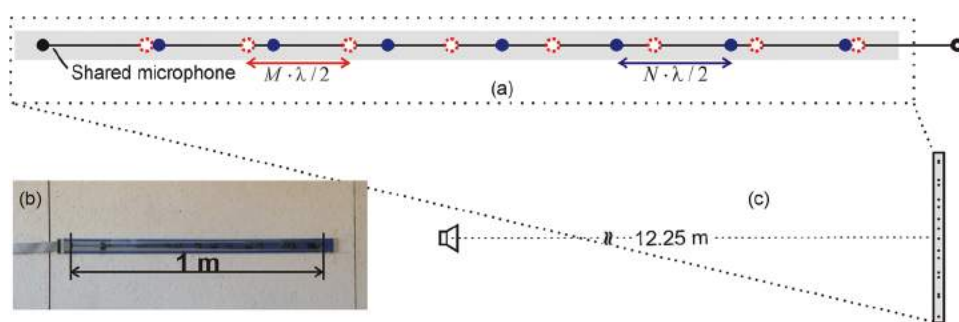


Fig. 2. (Color online) Design, photograph, and the experimental setup of a coprime microphone array. (a) Design of the coprime microphone array of 1 m long containing 16 microphones for a coprime pair of $M = 9$ and $N = 8$. (b) Photograph of the coprime microphone array. (c) Experimental setup in an anechoic environment with the coprime microphone array as shown in (a) and (b).

a conventional linear microphone array will have a predicted beampattern given by Eq. (1). The following experimental results using the coprime microphone array containing only 16 microphones demonstrate that the drastically reduced number of microphones arranged in a coprime array achieve comparable narrowness of the beampattern.

In order to validate the coprime microphone array beampattern as given by Eq. (6) and shown in Fig. 1(f) the experimental setup and the preliminary results are discussed in Sec. 3.2.

3.2 Experimental results

The coprime microphone array in the form of a flat carriage is placed on the ground of a semi-anechoic chamber. A sound source radiating frequency contents up to 16 kHz is placed on the floor 12.25 m away from the center of the microphone array as shown in Fig. 2(c) so that the sound source can be considered to be in the far field. Impulse responses between the sound source and the 16 microphones are measured at each angle as the coprime microphone array is turned between -90° and $+90^\circ$ with a 1° angular step. To validate the performance of the coprime microphone array in terms of its beampattern at the design frequency, a narrow bandpass filter (12th-octave) centered at 12.35 kHz is applied to all measured impulse responses. The beampatterns of two subarrays are calculated using Eqs. (3) and (5) with experimentally measured, bandpass-filtered impulse responses $h(n)$, yet without electronic steering (interelement delay), but rather via a simple summation for every angular orientation of the coprime array at for a 1° step.

After calculating the total signal energy for each angular step, the normalized beampatterns are plotted in Fig. 3(a). Figure 3(b) illustrates the beampattern of the combined two subarrays using Eq. (6). For ease of comparison, the predicted beampatterns with the same values for all $h(n)$ in Eqs. (3) and (5) [$h(n) = 1$, in particular] are plotted in Fig. 3(c). Each grating lobe in the experimentally measured beampatterns of the subarrays, as shown in Fig. 3(a), corresponds to the predicted ones, as shown in Fig. 3(c). The subarray $M = 9$ contains 8 grating lobes, while the subarray $N = 8$ contains 9 grating lobes. Two sidelobes in the combined beampattern from the

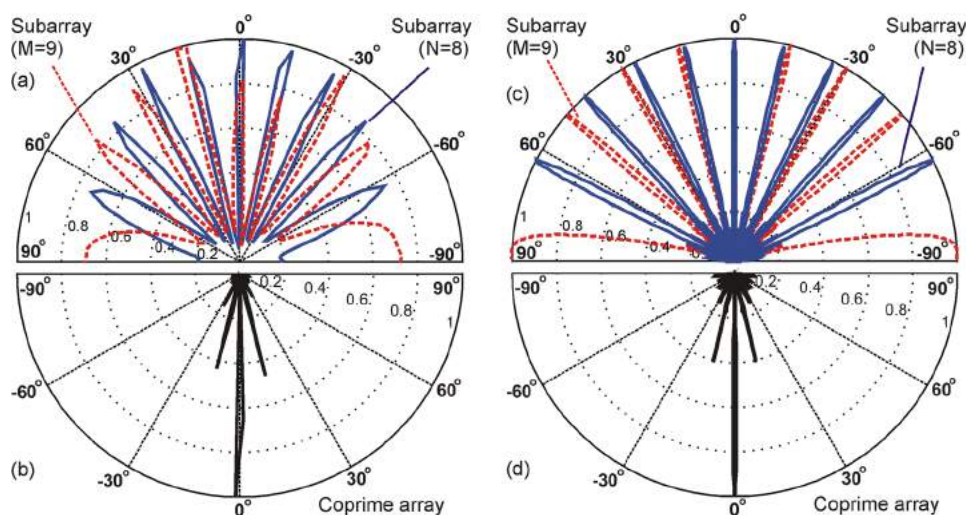


Fig. 3. (Color online) Comparison between predicted and experimentally measured beampatterns of a coprime microphone array with coprime numbers $M = 9$, $N = 8$ for normal incident waves. (a) Experimentally measured beampatterns of the two subarrays. (b) Experimentally measured beampattern of the coprime array of 16 microphones. (c) Predicted beampattern of the two subarrays. (d) Predicted beampattern of the coprime microphone array.

experimental data, as shown in Fig. 3(b), are also similar to those illustrated in the predicted beampattern, as shown Fig. 3(d).

4. Conclusion

Exploiting coprime number theory, microphones can be arranged sparsely with fewer total elements, exceeding the separation limit governed by the spatial Nyquist theorem. Two sparse uniform line subarrays, each with a coprime number of microphones, are staggered into one combined line array such that the subarrays produce narrow grating lobes that overlap with one another exactly in just one direction. By combining the subarray beampatterns it is demonstrated by experimental measurements that the shared beam is retained while mostly canceling the other superfluous grating lobes—as predicted by theory. This work has implemented a coprime microphone array with a coprime pair of 9 and 8 forming a coprime microphone array of a total of 16 microphones. At the design frequency, experimental beampatterns derived from narrowband (12th-octave) measurements are shown to correspond with predicted results. The narrow beampattern obtained from only 16 microphones across a 1 m aperture linear coprime array can be used for estimating directions of arrivals with high angular resolution exceeding that predicted by conventional theory of a uniform linear array. Future analysis of experimental investigations should compare sidelobe-levels of the coprime and the conventional arrays. More systematic experiments and sophisticated data analysis are subjects of future research.

Acknowledgments

The authors are thankful to Professor MingSian Bai for many insightful discussions and to John Paul Nichols for preliminary experimental results in the scope of his MS degree thesis at the Graduate Program in Architectural Acoustics, Rensselaer Polytechnic Institute.

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