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Experimental verification of RV-MTS model for fracture in soda–lime glass weakened by a V-notch †

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Abstract

Fracture phenomenon was experimentally investigated in a soda–lime glass weakened by a V-notch under tensile-shear loading. Fracture tests were conducted using a new test sample called the V-notched Brazilian disc (V-BD) specimen. The fracture resistance and fracture initiation angle were obtained initially from the test results. Afterward, a fracture model was utilized to estimate the experimental results. Very good correlation was found between the experimental and theoretical results both for the fracture resistance and the fracture initiation angle in notches having different notch angles and various notch tip radii. Experimental results revealed that for a constant notch tip radius, the failure load under pure tensile loading conditions decreases as the notch angle increases. For a constant notch angle, as the notch tip radius increases, the fracture load in the soda–lime glass V-BD specimens enhances in the whole domain from pure tensile to pure shear loading. Moreover, for a constant notch tip radius, the notch angle has almost no effect on the fracture initiation angle when the specimen is predominantly under tensile loading conditions.

Keywords: Fracture; Notch; Soda–lime glass; Toughness

1. Introduction

Soda–lime glass is a brittle material often utilized in engineering components and structures because of its transparency. Decorative products such as crystal dishes, vases, and so forth are made of different types of glasses. Some are valuable and relatively expensive while some are used extensively in daily life. Such products may have V-shaped notches, which are used in the design because of their elegance. Although glass components are not commonly designed for load-bearing purposes, they can be indirectly subjected to mechanical loads applied to other parts of the structure or the loads imposed during transportation or accidental falling. Under such mechanical loads, notched glass products are prone to sudden fracture because of the initiation and rapid propagation of cracks.

V-notches in glass products play the role of a stress raiser and dramatically decrease their load-bearing capacity because of the stress concentration in the notch tip area. Therefore, it is important to investigate the fracture resistance of soda–lime glass components under different loading conditions, especially in the presence of stress concentrators like cracks and notches.

Cracked and V-notched glass components can be subjected to three different types of in-plane loading: pure mode I, pure mode II, and mixed mode I/II loading. Under pure mode I loading conditions, any two respective points along the crack or notch face open relative to the bisector line without any sliding. In pure mode II, the two respective points slide relative to the bisector line without any opening. Any combination of mode I and mode II deformation is called mixed mode I/II or in-plane tensile shear loading.

The fracture resistance of soda–lime glass in the presence of a sharp crack under different loading conditions has been investigated in past research. Gong et al. [1] statistically determined the mode I fracture resistance (or fracture toughness) of soda–lime glass using the indentation technique. Koike et al. [2] studied the subcritical crack propagation of glass by double cleavage drilled compression test specimen under pure mode I. The fracture resistance of common soda–lime glass was experimentally investigated by Abrams et al. [3] using a four-point bend configuration. Fett et al. [4] investigated the fatigue crack growth in soda–lime–silicate glass at different environmental conditions. Park et al. [5] studied the loading rate effects on mode I fracture resistance of soda–lime glass

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during flexural bending. Deriano et al. [6] determined the fracture resistance of soda–lime glass under mode I loading conditions using a single-edge notched beam and indentation methods. Geandier et al. [7] also used this method to determine the mode I fracture resistance of soda–lime glass. Researchers have investigated, both experimentally and theoretically, the mixed mode fracture resistance of soda–lime glass in the presence of a sharp crack (see for example [8-14]). Shetty and his colleagues [15-17] and Awaji and colleagues [18, 19] have reported extensive experimental results for mixed mode fracture in soda–lime glass obtained through experiments conducted on a well-known disc-type specimen with a central crack called the centrally cracked Brazilian disc (CCBD) specimen.

There are several well-known failure criteria in the study of mixed mode I/II (i.e., combined tensile shear) brittle fracture in cracked components made of different materials like soda–lime glass. Investigators often used the maximum tangential stress (MTS) criterion [20], the minimum strain energy density (SED) criterion [21], and the maximum energy release rate or Griffith criterion [22]. Ayatollahi and Aliha [23] employed the generalized MTS (GMTS) criterion to predict the tensile-shear fracture in soda–lime glass. Aliha and Ayatollahi [24] used the same criterion to provide very good estimates for the results related to dental resin materials in the presence of sharp cracks. Despite extensive studies on brittle fracture in cracked glass specimens, a review of literature revealed that fracture in soda–lime glass components containing a V-shaped notch has not been investigated either experimentally or theoretically.

The main objective of the present study is to determine experimentally the fracture resistance for a commercial grade soda–lime glass in the presence of V-notches. In addition, it assesses if a recently developed brittle fracture model can be used for predicting the test results. For this purpose, a series of fracture tests was conducted on soda–lime glass using a Brazilian disc specimen with a rhombic central slit, called the V-notched Brazilian disc (V-BD). Fracture resistance and fracture initiation angle were obtained under tension-shear loading for the V-BD specimens of various notch angles and different notch tip radii. A fracture criterion called the round-tip V-notched MTS (RV-MTS) [25] was employed to predict the notch fracture resistance and the fracture initiation angle in the soda–lime glass V-BD specimens. Fracture resistance and fracture initiation angle of the soda–lime glass samples can be estimated more accurately using the results of the RV-MTS criterion.

2. Experiments

One of the cracked specimens frequently used in the past for conducting mixed mode I/II fracture experiments on brittle materials like glass is the CCBD specimen (see for example Refs. [15-19]). A modified version of the CCBD specimen, called V-BD, was utilized in this research to perform mixed mode fracture tests for V-notches. Fig. 1 displays the V-BD specimen schematically. In the figure, β is the angle between the loading direction and the notch bisector line, and parameters 2α, D, $d/2$, and P are the notch angle, disc diameter, notch depth, and applied compressive load, respectively. When the direction of the applied load P is along the notch bisector line (i.e., $\beta = 0$), the upper and lower corners of the rhombic slit are subjected to pure mode I deformation (i.e., pure opening mode). When the angle β enhances gradually from zero, the loading condition varies from pure mode I to pure mode II (i.e., pure shear notch deformation). For a specific angle, called β_{II}, pure mode II deformation is achieved. The mode II loading angle β_{II} is always less than 90°; it also depends on the notch depth, its opening angle 2α, and the notch tip radius. The angles β_{II} can be determined using the finite element (FE) method as elaborated in Ref. [25].

Table 1. Mechanical properties of the tested soda–lime glass.

Property	Value
Bulk density [kg/m^3]	2450
Mean tensile strength (σ_t) [MPa]	14
Young's modulus (E) [GPa]	72
Plane-strain fracture toughness (K_{Ic}) [MPa m$^{0.5}$]	0.6

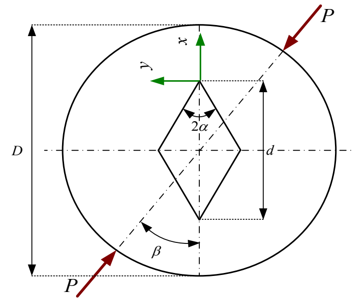
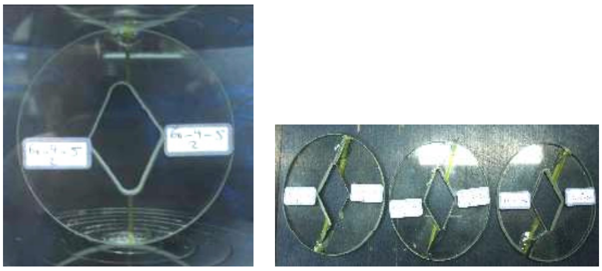


Fig. 1. The V-BD specimen.

The material used for fabricating V-BD specimens was a common soda–lime glass containing (in mol %) 72% SiO_2, 14% Na_2O, 6.8% CaO, 6.2% MgO, 0.5% Al_2O_3, 0.2% K_2O, 0.3% SO_3, and 0.03% Fe_2O_3 with the mechanical properties presented in Table 1. The bulk density was determined using the buoyancy method. The experimental procedures for determining the other properties listed in Table 1 are found in Ref. [26].

For all the soda–lime glass V-BD specimens, the disc diameter (D), notch depth ($d/2$), and thickness were 80, 20, and 6 mm, respectively. To study the effects of the notch angle and the notch tip radius on the fracture behavior of the soda–lime glass specimens, three values of notch angle 2α = 30, 60, 90 (deg.) and three values of notch tip radius ρ = 1, 2, 4 mm were considered for fabricating the specimens. The specimens were cut precisely from a soda–lime glass plate of 6 mm thick using a high-precision 2-D CNC water-jet cutting machine. A total number of 162 mixed mode I/II fracture tests were performed for various notch geometry parameters and different



(a) (b)

Fig. 2. The soda–lime glass specimens: (a) Under mixed mode loading; (b) Broken after mixed mode fracture tests.

loading angles β from 0 (pure mode I) to β_{II} (pure mode II). Tests were conducted on different angles β of 0, 5, 10, 15, 20, 25 (deg.); 0, 5, 15, 20, 25, 30 (deg.); and 0, 5, 15, 20, 30, 35 (deg.) for 2α = 30, 60, and 90, respectively. For each geometry shape and loading angle, three fracture tests were performed using a universal tension-compression test machine under displacement control condition with a loading rate of 0.1 mm/min. Fig. 2 shows the sample soda–lime glass V-BD specimens before and after the fracture test. Experimental observations showed that the load-displacement curves for the tested soda–lime glass specimens were all linear and the fracture occurred abruptly (Fig. 3). Moreover, in all specimens, fracture was initiated from the notch border and extended toward the external border of the specimen. Mean values of the experimental fracture loads (P_f) recorded by the test machine for each specimen are presented in Table 2. The first, second, and third figures denoted for each specimen correspond to the notch angle, the notch tip radius, and the loading angle, respectively.

Table 2. The mean values of the experimental fracture loads for the soda–lime glass V-BD specimens.

Specimen	P_f (N)	Specimen	P_f (N)	Specimen	P_f (N)
30-1-0	1730	60-1-0	1280	90-1-0	850
30-1-5	1680	60-1-5	1200	90-1-5	901
30-1-10	1780	60-1-15	1280	90-1-15	1072
30-1-15	1600	60-1-20	1550	90-1-20	1220
30-1-20	1580	60-1-25	1654	90-1-30	1700
30-1-25	1980	60-1-30	1930	90-1-35	2233
30-2-0	2100	60-2-0	1680	90-2-0	1095
30-2-5	1810	60-2-5	1653	90-2-5	1116
30-2-10	2003	60-2-15	1680	90-2-15	1080
30-2-15	1810	60-2-20	1730	90-2-20	1280
30-2-20	1910	60-2-25	1860	90-2-30	1910
30-2-25	1890	60-2-30	1935	90-2-35	2280
30-4-0	2460	60-4-0	2125	90-4-0	1365
30-4-5	2005	60-4-5	1860	90-4-5	1210
30-4-10	2080	60-4-15	1730	90-4-15	1320
30-4-15	2150	60-4-20	1900	90-4-20	1485
30-4-20	2001	60-4-25	1910	90-4-30	1930
30-4-25	2111	60-4-30	2030	90-4-35	2250

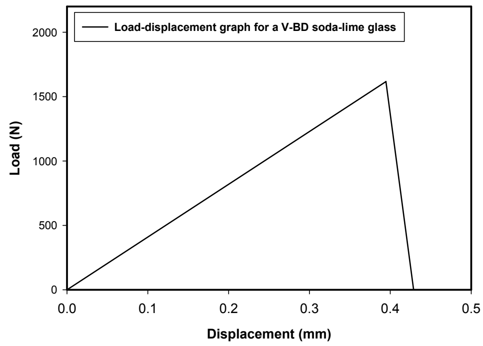


Fig. 3. A sample load-displacement curve obtained for soda–lime glass V-BD specimen.

3. Failure theory

As shown in Fig. 3, the recorded load-displacement diagrams of the soda–lime glass V-BD specimens were linear from the beginning up to the final fracture. Therefore, the use of a brittle fracture theory based on linear elastic fracture mechanics is applicable for estimating the test results. To estimate the experimental results theoretically, a failure criterion proposed by the authors [25] is used herein. The proposed criterion, called RV-MTS, a modified version of the maximum tangential stress (MTS) criterion, was presented to estimate the onset of mixed mode brittle fracture for V-notches [25]. The first hypothesis of the RV-MTS criterion suggests that brittle fracture initiates radially from a point on the notch border along a direction for which the tangential stress at a critical distance $r_{c,V}$ is a maximum [25]. The direction θ corresponding to this point is called the fracture initiation angle θ_0. The second hypothesis of RV-MTS criterion proposes that brittle fracture in a V-notched component takes place when the tangential stress $\sigma_{\theta\theta}$ along θ_0 and at a critical distance $r_{c,V}$ attains a critical value $(\sigma_{\theta\theta})_c$.

The parameter $r_{c,V}$ is the critical distance for round-tip V-notches and the parameter $(\sigma_{\theta\theta})_c$ is a material property commonly considered to be the ultimate tensile strength σ_u for brittle and quasi-brittle materials [25]. Details of the RV-MTS criterion and its mathematical derivation (which can be found in Ref. [25]) are not repeated here for brevity. Using this criterion, a set of fracture curves can be plotted for V-notched components made of a brittle material (such as soda–lime glass) with an arbitrary notch angle and notch tip radius. These curves can be utilized to predict the fracture resistance and fracture initiation angle in V-notched soda–lime glass components under any combination of shear and tension (or mode I and mode II) loading. The main parameters needed for using the fracture curves are the mode I and mode II notch stress intensity factors (NSIFs) $K_I^{V,\rho}$ and $K_{II}^{V,\rho}$, the mode I notch fracture toughness $K_{Ic}^{V,\rho}$, the notch mode mixity parameter M^e_V, and the fracture initiation angle θ_0. These parameters are defined and described in detail in Ref. [25]. In the next section, the theoretical curves of the RV-MTS criterion are plotted for the tested V-notched soda–lime glass specimens and are employed to estimate the experimental results.

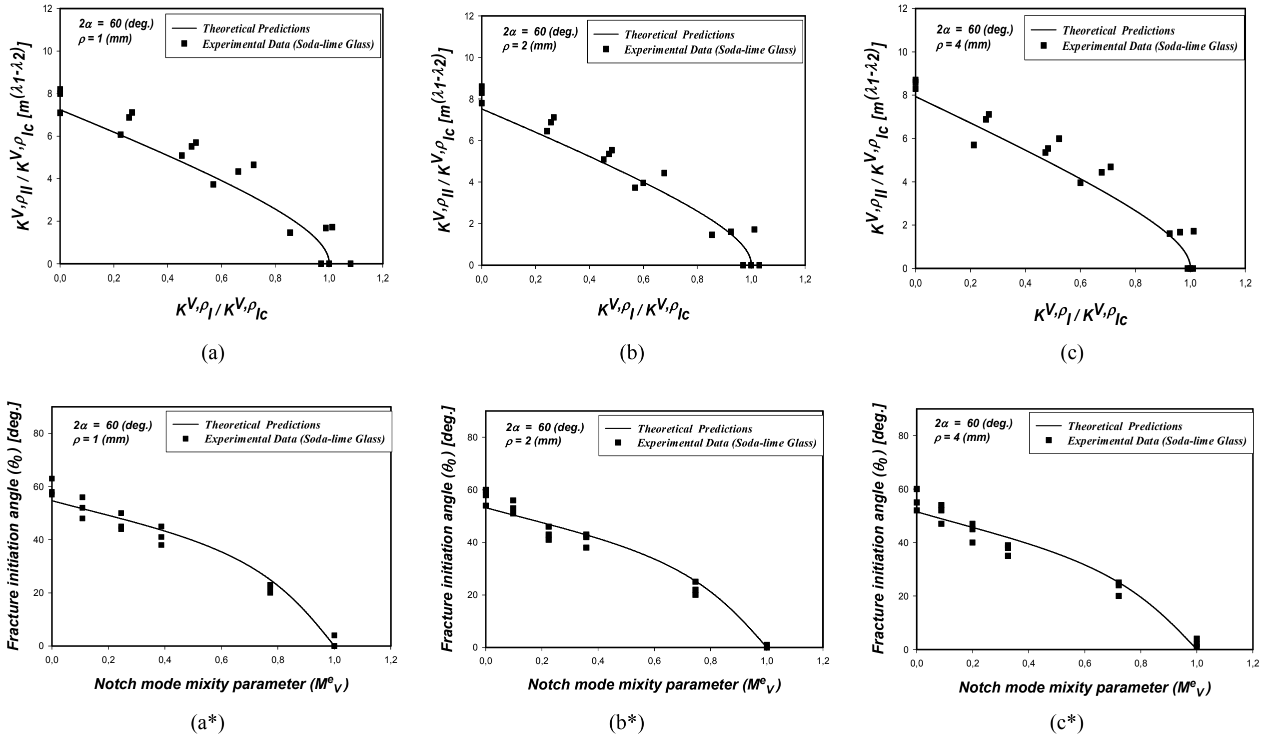


Fig. 4. Fracture curves for V-notched soda-lime glass with notch angle of 60 (deg.) and various notch tip radii together with the experimental results.

4. Results and discussion

To compare the experimental results with the theoretical fracture curves, it is essential to convert the fracture loads obtained for the tested soda–lime glass specimens presented in Table 2 to the corresponding critical NSIFs. For this purpose, one can use the FE method as described in Ref. [25]. Here, the results of the RV-MTS criterion in predicting the mixed mode I/II fracture resistance and the fracture initiation angle of V-BD specimens are compared with the experimental results obtained for the soda–lime glass described in Section 2. Fig. 4 shows the fracture curves and the curves of fracture initiation angle for the soda–lime glass V-BD specimens with the notch angle of $2\alpha = 60$ (deg.) and various notch tip radii (i.e., $\rho = 1$, 2, 4 mm) together with the experimental results. Figs. 4(a), (b), and (c) indicate that as the notch tip radius increases, the notch fracture resistance enhances because of lower stress concentrations near the notch tip. Moreover, as presented in Figs. 4(d) to 4(f), as the parameter M_V^e decreases from 1 (pure mode I) to 0 (pure mode II), the fracture initiation angle (θ_0) grows from 0^o up to a mode II fracture initiation angle. These graphs also show that by increasing the notch tip radius, the fracture initiation angle slightly reduces. Although not shown here, for a constant notch tip radius, the notch angle has almost no effect on the fracture initiation angle when mode I is dominant (i.e., $M_V^e > 0.6$). However, as the contribution of mode II enhances, the effect of the notch angle on the fracture initiation angle increases slightly such that its maximum influence takes place in pure mode II loading conditions. Note that the fracture loads presented in Table 2 are the mean values of the test results. However, all the test results are shown in Fig. 4 in order to present the scatter in the experimental results. A very good correlation is seen in Fig. 4 between the theoretical and experimental results obtained for the onset of mixed mode fracture and for the fracture initiation angle in the soda–lime glass V-notched specimens. Although not shown here, very good agreement was also found between theoretical and experimental results for the other notch angles (i.e., 30^o and 90^o). A comparison between the theoretical and mean experimental results showed that the test results could be estimated using the RV-MTS criterion with the mean discrepancies of about 6% and 8% for the fracture resistance and the fracture initiation angle, respectively. One major reason suggested for the deviations between experimental and theoretical results is that the stress field used in the RV-MTS model is an approximate expression that only satisfies the boundary conditions in limited points on the notch edge and not on the whole edge [25].

V-notches are sometimes used in the design and fabrication of decorative glass products to make them elegant. However, the notches make these products and components vulnerable to mechanical failure because of the fragility of the soda–lime glass. To estimate the load that a V-notched soda–lime glass component can sustain, the type of loading (i.e., mode I, mode II or mixed mode) from the calculated NSIFs should be determined first. Then, the RV-MTS fracture curves can be employed conveniently to estimate the fracture resistance of the component particularly for complex loading conditions, where both tensile and shear deformations are present in the notched component.

5. Conclusions

(1) Fractures in V-notched soda–lime glass specimens were examined both experimentally and theoretically under combined tensile-shear loading conditions. For the experiments, the V-BD specimen was used, which provided the entire loading domain from pure mode I to pure mode II.

(2) A brittle fracture criterion, RV-MTS, was employed to estimate the extensive test results related to the mixed mode fracture resistance and the fracture initiation angle for soda–lime glass specimens weakened by the V-notch.

(3) The experimental results could be estimated effectively using the RV-MTS criterion.

(4) The designers of glass products can utilize the RV-MTS failure model to estimate the fracture resistance of glass components containing a V-notch. Moreover, the RV-MTS model can be used to predict the fracture initiation angle, which sometimes plays a vital role in the assessment of overall damage in glass components and structures containing V-shaped notches.

References

[1] J. Gong, Y. Chen and C. Li, Statistical analysis of fracture toughness of soda–lime glass determined by indentation, *Journal of Non-crystalline Solids*, 279 (2001) 219-223.

[2] A. Koike, M. Tomosawa and S. Ito, Sub-critical crack growth rate of soda–lime–silicate glass and less brittle glass as a function of fictive temperature, *Journal of Non-crystalline Solids*, 353 (2007) 2675-2680.

[3] M. B. Abrams, D. J. Green and S. Jill Glass, Fracture behavior of engineered stress profile soda lime silicate glass, *Journal of Non-crystalline Solids*, 321 (2003) 10-19.

[4] T. Fett, J. P. Guin and S. M. Wiederhorn, Interpretation of effects at the static fatigue limit of soda–lime–silicate glass, *Engineering Fracture Mechanics*, 72 (2005) 2774-2791.

[5] J. K. Park, K. Yasuda and Y. Matsuo, Effect of crosshead speed on the fracture toughness of soda–lime glass, Al_2O_3 and Si_3N_4 ceramics determined by the surface crack in flexure (SCF) method, *Journal of Materials Science*, 36 (2001) 2335-2342.

[6] S. Deriano, A. Jarry, T. Rouxel, J. C. Sangleboeuf and S. Hampshire, The indentation fracture toughness (K_C) and its parameters: the case of silica-rich glasses, *Journal of Non-crystalline Solids*, 344 (2004) 44-50.

[7] G. Geandier, S. Denis and A. Mocellin, Float glass fracture toughness determination by Hertzian contact: experiments and analysis, *Journal of Non-crystalline Solids*, 318 (2003) 284-295.

[8] M. Y. He, H. C. Cao and A. G. Evans, Mixed mode fracture: the four-point shear specimen, *Acta Metallic Materialia*, 38 (1990) 839-846.

[9] M. Li and M. Sakai, Mixed-mode fracture of ceramics in asymmetric four-point bending, *Journal of Americans Ceramic Society*, 79 (1996) 2718-2726.

[10] J. E. Ritter, A. Huseinovic, S. S. Chacravarthy and T. J. Lardner, Subcritical crack growth in soda–lime glass under mixed-mode loading, *Journal of Americans Ceramic Society*, 83 (2000) 2109-2111.

[11] V. M. Sglavo, T. Micheletti and S. Di Suonzo, Influence of crack geometry and loading mode on fracture and fatigue of soda–lime–silica glass, *European Journal of Glass Science and Technology*, 44 (5) (2003) 337-344.

[12] G. S. Glaesemann, J. E. Ritter and K. Jaku, Mixed-mode fracture in soda–lime glass using indentation flaws, *Journal of Americans Ceramic Society*, 70 (9) (1987) 630-636.

[13] S. W. Freiman, A. C. Gonzalez and J. J. Mecholsky, Mixed-mode fracture in soda–lime glass, *Journal of Americans Ceramic Society*, 62 (1979) 206–208.

[14] K. Ikeda and H. Igaki, Mixed-mode fracture criterion in soda–lime glass, *Japanese Society of Mechanical Engineers*, 33 (1990) 84-88.

[15] D. K. Shetty, A. R. Rosenfield and W. H. Duckworth, Mixed mode fracture of ceramics in diametral compression, *Journal of Americans Ceramic Society*, 69 (1986) 437-443.

[16] D. K. Shetty, A. R. Rosenfield and W. H. Duckworth, Mixed-mode fracture in biaxial stress state: application of diametral-compression (Brazilian disk) test, *Engineering Fracture Mechanics*, 26 (6) (1987) 825-840.

[17] D. K. Shetty and A. R. Rosenfield, Slow crack growth in glass in combined mode I and mode II loading, *Scripta Metallic Materialia*, 25 (5) (1991) 997-1002.

[18] H. Awaji and T. Kato, Griffith criterion for mode II fracture of ceramics, *Alison, Balkema, Rotterdam*, (1998) 1199-1204.

[19] H. Awaji, T. Kato, S. Honda and T. Nishikawa, Criterion for combined mode I-II of brittle fracture, *Journal of Ceramics Society of Japan*, 107 (1999) 918-924.

[20] F. Erdogan and G. C. Sih, On the crack extension in plates under plane loading and transverse shear, *Journal of Basic Engineering Transaction ASME*, 85 (1963) 519-525.

[21] G. C. Sih, Strain–energy–density factor applied to mixed mode I/II crack problems, *International Journal of Fracture*, 10 (1974) 305-321.

[22] M. A., S. L. Pu and J. Underwood, ASTM STP 560. Philadelphia: *American Society for Testing and Materials* (1974) 2-28.

[23] M. R. Ayatollahi and M. R. M. Aliha, Mixed-mode fracture in soda–lime glass analyzed by using the generalized MTS criterion, *International Journal of Solids and Structures*, 46 (2009) 311-321.

[24] M. R. M. Aliha and M. R. Ayatollahi, On mixed mode I/II crack growth in dental resin materials, *Scripta Materialia*, 59 (2008) 258-261.

[25] M. R. Ayatollahi and A. R. Torabi, Investigation of mixed mode brittle fracture in rounded-tip V-notched components, *Engineering Fracture Mechanics*, 77 (2010) 3087-3104.

[26] M. R. Ayatollahi and A. R. Torabi, Tensile fracture in notched polycrystalline graphite specimens, *Carbon*, 48 (2010) 2255-2265.

Nomenclature

K_{Ic} : Plane-strain fracture toughness
$K^{V,\rho}_{I}$: Notch stress intensity factor-mode I
$K^{V,\rho}_{II}$: Notch stress intensity factor-mode II
$r_{c,V}$: Critical distance for rounded tip V-notch
2α : Notch opening angle
ρ : Notch tip radius
σ_{υ} : Ultimate tensile strength
$\sigma_{\theta\theta}$: Tangential stress
θ_0 : Fracture initiation angle

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