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Experimentation of Humanoid Walking Allowing Immediate Modification of Foot Place Based on Analytical Solution

Mitsuharu Morisawa, Kensuke Harada, Shuuji Kajita, Shinichiro Nakaoka,
Kiyoshi Fujiwara, Fumio Kanehiro, Kenji Kaneko, Hirohisa Hirukawa

***Abstract*— This paper proposes a method of a real-time gait planning for humanoid robots which can change stride immediately at every step. Based on an analytical solution of an inverted pendulum model, the trajectories of the COG (Center of Gravity) and the ZMP (Zero-Moment Point) are parameterized by polynomials. Since their coefficients can be efficiently computed with given boundary conditions, this framework can provide a real-time walking pattern generator for humanoid robots. To handle the unexpected result caused by immediate changes of foot placement, we made single support periods as an additional trajectory parameter and the ZMP fluctuation was suppressed by mixing the oppsite phase of the ZMP error. The effectiveness of our method is shown by experiments of the humanoid robot HRP-2.**

I. INTRODUCTION

Humanoid robots have the advantage in working in an environment which are designed for human being. They are expected to contribute to the aging society with fewer children in the near future as labors. Thus, the humanoid robots that have the same size as a human have actively researched and developed [1]-[4]. It is necessary for a robot to have a functionally equivalent mobility to human in such environments. For examples, the high mobility is requested by some functions such as a dynamic obstacle avoidance and teleoperation.

Generally, to change the landing position, it is necessary by two steps at least. Thus, how to generate a biped gait planning with a high response is an important issue such as shown in Fig.1. To generate a motion pattern in real-time, the inverted pendulum model which is represented as the linear differential equation, is derived by an approximate dynamics of the COG [5]-[8]. Kagami and Nishiwaki *et al.*[5] realize a fast generation of dynamically stable humanoid robot walking pattern by discretizing the differential equation. It is difficult to satisfy the boundary condition of the COG in this method. Kajita *et al.*[6] proposed the generation method of biped walking pattern by using preview control. In this method, it is necessary to delay the preview time to change the landing position. Harada *et al.*[7] generated a real-time biped gait using an analytical solution. The connection of two trajectories is discussed in case of modification of next landing position. The generated biped pattern causes a curvature in case of connecting the trajectories in double support phase. Sugihara *et al.*[8] generated a biped walking

The authors are with Humanoid Robotics Group, Intelligent Systems Reseach Institute, National Institute of Advanced Industrial Science and Technology (AIST), 1-1-1- Umezono, Tsukuba, Japan
m.morisawa@aist.go.jp

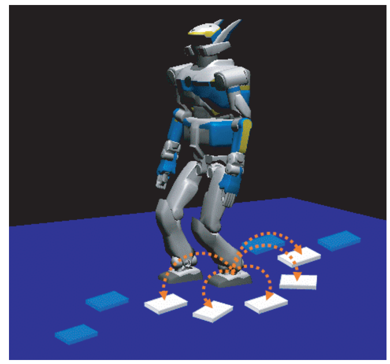

Fig. 1. Real-time gait planning

pattern every one step in real-time by relaxing the boundary condition of the ZMP. A generation system of a dynamically stable humanoid walking pattern which can updates the pattern at a short cycle is proposed by Nishiwaki *et al.*[15]. Wieber [16] generated a walking pattern only from the ZMP constraints by using Linear Model Predictive Control. Authors were achieved immediate modification of landing posigion and proposed the suppression methods of the ZMP fluctuation of transitional response [17].

In this paper, a real-time gait planning method based on analytical solution is generated that can modify next landing position arbitrarily. The modification of landing position can be changed just before lifting up the foot of swing leg. Even if the desired stride is changed at the next step, a stable walking pattern can be generated by adjusting single support period and the shaping ZMP trajectory. The validity of our method was shown by experiments result.

II. SIMULTANEOUS PLANNING OF THE COG AND THE ZMP

The previous on-line modification of gait pattern could change only little landing position or was difficult to connect the ZMP trajectories continuously. In this paper, a biped gait is divided into the m-th intervals every single or double support phase and each interval is represented by applying an analytical solution of a linear inverted pendulum in addition to the ZMP polynomial which is proposed by Harada *at et*[7].

Firstly, $\boldsymbol{x}_G^{(j)} = [x^{(j)} \ y^{(j)} \ z^{(j)}]^T$ and $\boldsymbol{p}^{(j)} = [p_x^{(j)} \ p_y^{(j)} \ p_z^{(j)}]^T$ denote the COG and the ZMP position belonging to the j-th interval. Let us focus on the COG motion on sagittal plane, and an dynamic equation of the

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COG in x-axis can be approximated by an inverted pendulum with a constant height ($z^{(j)} - p_z^{(j)} = \text{const.}$) which is given as follows:

$$\ddot{x}^{(j)} = \frac{g}{z^{(j)} - p_z^{(j)}}(x^{(j)} - p_x^{(j)}), \quad (1)$$

where g is gravity constant. The COG motion on frontal plane can be also formulated as the same equation.

At j-th interval, let us assume that the ZMP $p_x^{(j)}$ can be represented by N_j-th order time polynomial, that is

$$p_x^{(j)}(t) = \sum_{i=0}^{N_j} a_i^{(j)}(\Delta t_j)^i, \quad (2)$$
$$\Delta t_j \equiv t - T_{j-1}.$$

Where, T_{j-1} implies the beginning time of j-th interval. Substituting Eq.(2) into Eq.(1), the COG position $x^{(j)}$ can be solved.

$$x^{(j)} = V^{(j)}\mathrm{c}_j + W^{(j)}\mathrm{s}_j + \sum_{i=0}^{N_j} A_i^{(j)}(\Delta t_j)^i, \quad (3)$$
$$j = 1, \ldots, m,$$
$$A_i^{(j)} = \begin{cases} a_i^{(j)} + \sum_{k=1}^{(N_j-i)/2} b_{i+2k}^{(j)} \, a_{i+2k}^{(j)} \\ \quad (i = 0, \ldots, N_j - 2) \\ a_i^{(j)} \quad (i = N_j - 1, N_j) \end{cases}$$
$$b_{i+2k}^{(j)} = \prod_{l=1}^{k} \frac{(i+2l)(i+2l-1)}{\omega_j^2}$$

Where, $\mathrm{c}_j \equiv \cosh(\omega_j \Delta t_j)$, $\mathrm{s}_j \equiv \sinh(\omega_j \Delta t_j)$, and $\omega_j \equiv \sqrt{g/(z^{(j)} - p_z^{(j)})}$. $V^{(j)}$ and $W^{(j)}$ denote the scalar coefficients. In previous research, the coefficients of the ZMP polynomial were set as unknown constants in the first interval of double support phase. On the other hand, all of those are generalized as unknown constants. Thus, $2m + \sum_{j=1}^{m}(N_j + 1)$ number of unknown constants exist in Eq.(3) during m-th interval. To solve the biped gait planning as two point boundary value problem, the boundary conditions concerned with the COG and the ZMP are given as follows:

(I) Initial condition for the COG position and velocity:

$$x^{(1)}(T_0) = V^{(1)} + A_0^{(1)} \quad (4)$$
$$\dot{x}^{(1)}(T_0) = W^{(1)} + A_1^{(1)} \quad (5)$$

(II) Connection of two intervals for the COG position and velocity

$$V^{(j)}\cosh(\omega_j \Delta T_j) + W^{(j)}\sinh(\omega_j \Delta T_j) + \sum_{i=0}^{N_j} A_i^{(j)}(\Delta T_j)^i = V^{(j+1)} + A_0^{(j+1)} \quad (6)$$
$$V^{(j)}\omega_j \sinh(\omega_j \Delta T_j) + W^{(j)}\omega_j \cosh(\omega_j \Delta T_j) + \sum_{i=1}^{N_j} i A_i^{(j)}(\Delta T_j)^{i-1} = W^{(j+1)}\omega_j + A_1^{(j+1)} \quad (7)$$
$$j = 1, \ldots, m-1$$

(III) Terminal condition for the COG position and velocity:

$$x^{(m)}(T_m) = V^{(m)}\cosh(\omega_m \Delta T_m) + W^{(m)}\sinh(\omega_m \Delta T_m) + \sum_{i=0}^{N_m} A_i^{(m)}(\Delta T_m)^i \quad (8)$$
$$\dot{x}^{(m)}(T_m) = V^{(m)}\omega_m \sinh(\omega_m \Delta T_m) + W^{(m)}\omega_m \cosh(\omega_m \Delta T_m) + \sum_{i=1}^{N_m} i A_i^{(m)}(\Delta T_m)^{i-1} \quad (9)$$

(IV) Initial condition for ZMP position and velocity at each interval:

$$p_x^{(j)}(T_{j-1}) = A_0^{(j)} - \frac{1}{\omega_j^2}A_2^{(j)} \quad (10)$$
$$\dot{p}_x^{(j)}(T_{j-1}) = A_1^{(j)} - \frac{6}{\omega_j^2}A_3^{(j)} \quad (11)$$
$$j = 1, \ldots, m$$

(V) Terminal condition for ZMP position and velocity at each interval:

$$p_x^{(j)}(T_j) = \sum_{i=0}^{N_j}\left\{\left(A_i^{(j)} - \frac{(i+1)(i+2)}{\omega_j^2}A_{i+2}^{(j)}\right)(\Delta T_j)^i\right\} \quad (12)$$
$$\dot{p}_x^{(j)}(T_j) = \sum_{i=1}^{N_j}\left\{i\left(A_i^{(j)} - \frac{(i+1)(i+2)}{\omega_j^2}A_{i+2}^{(j)}\right)(\Delta T_j)^{i-1}\right\} \quad (13)$$
$$j = 1, \ldots, m$$

where, $\Delta T_j \equiv T_j - T_{j-1}$. From the boundary condition (I)-(V), the total number of conditions becomes $6m + 2$. Then, all of unknown constants can be calculated by

$$\boldsymbol{y} = \boldsymbol{Z}^{+}\boldsymbol{w}, \quad (14)$$

Where, matrix $\boldsymbol{Z}$ is composed of each timing of the given conditions and depended on only time (not depended on spacial conditions). The details of elements are shown in [17]. Here, at least, the order of ZMP polynomial N_j must satisfy

$$2m + \sum_{j=1}^{m}(N_j + 1) \geq 6m + 2. \quad (15)$$

Using the fourth order polynomial for the first intervals as the ZMP trajectory, the ZMP can be represented by

$$p_x^{(}1)(t) = a_0^{(1)} + a_1^{(1)}(t - T_0) + a_2^{(1)}(t - T_0)^2 + a_3^{(1)}(t - T_0)^3 + a_4^{(1)}(t - T_0)^4. \quad (16)$$

Setting the boundary condition of the ZMP velocity to 0, the time which becomes the maximum fluctuation of the ZMP can be provided by

$$T_{max} = T_0 - \frac{2a_2^{(1)}(T_1 - T_0)}{2a_2^{(1)} + 3a_3^{(1)}(T_1 - T_0)}. \quad (17)$$

Substituting eq.(17) into (16), the maximum value of the ZMP can be obtained. Then the maximum value can be used to judge whether to be able to continue walking when the next landing position is changed, although the difference between the ZMP which is calculated by the linear inverted pendulum model and that is taken a multi-body dynamics into account becomes large as the walking speed increases.

III. Real-time continuous gait planning

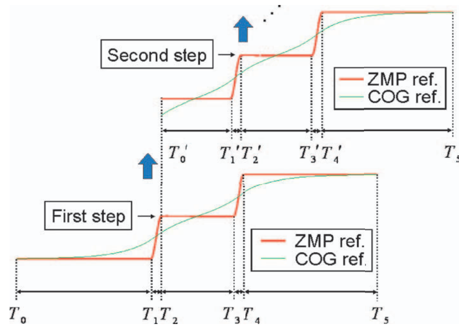
In this section, we explain a method of real-time gait generation by re-planning walking pattern for each step and its problem. Figure 2 (a) illustrates the basic idea. At time T_0, the beginning of walking, we plan a walking pattern of two successive steps for the period of $[T_0 : T_5]$ as shown in the below graph of Fig.2 (a). We use the algorithm explained in the former section, which is fast enough to design two steps within one control cycle of 5[ms]. Since the planned walk will stop after two steps, we re-plan a walking pattern of the second and the third steps at time $T_2(=T_0^{'})$ for the period of $[T_0^{'} : T_5^{'}]$ as shown in the above graph of Fig.2 (a). By repeating this for every step, a continuous walking can be generated.

Although this method seems to allow modification of stride for each step, it does not work as we expect. Let us explain this by using Fig.2 (b). In the below graph of Fig.2 (b), a walking pattern for two steps is planned at time T_0. Then at time $T_2(=T_0^{'})$, we re-plan a pattern whose second step is smaller than previously designed. The resulted trajectory is shown in the above graph of Fig.2 (b). By the sudden change of the stride, the ZMP trajectory at single support period of $[T_0^{'} : T_2^{'}]$ had become convex shape. This is not an appropriate ZMP trajectory at single support phase, because the fluctuation of the ZMP is desired to be as small as possible so that it remains in a support polygon.

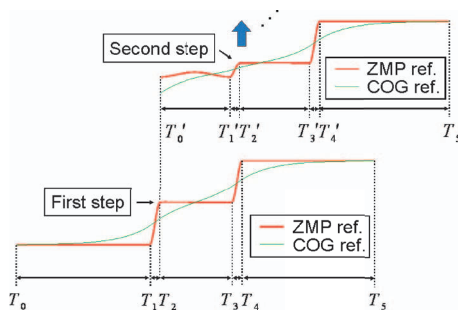
IV. Gait pattern compensation by immediate modification of foot placement

A. Time adjustment in single support phase

As mentioned above, the sudden modification of foot placement will cause the variation of the ZMP for the COG acceleration and deceleration. The variation value of the ZMP becomes large according to the modified length of stride. The COG cannot be traced the desired trajectory unless the ZMP is generated within a support polygon. Thus, let us consider reducing the ZMP variation in single support phase by adjusting a period of single support phase. For example, because a curved ZMP trajectory forward than the desired it implies the deceleration of the COG velocity, an equivalent effect can be obtained by shortening a period of single support phase. On the other hand, in case of a backward ZMP variation, it leads to make a period longer which is equivalent to slow down the COG motion.



(a) Scheduled step



(b) Modified step

Fig. 2. Sequential gait planning

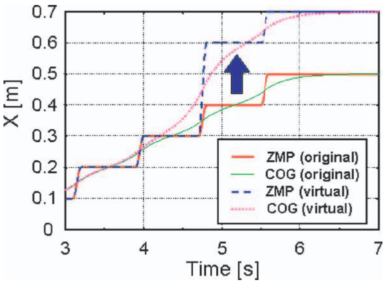
Then, we explain how to determine an adjusting time of single support phase uniquely. Figure 3 (a) shows two preplanned gait patterns. One is that the original COG trajectory is planned by 0.8[s] step cycle and 0.1[s] every step length. At the beginning of single support phase 4[s], a landing position will be modified from 0.4[m] to 0.6[m]. Other COG and ZMP trajectories will be generated on the assumption that the landing position was also preplanned in advance. Figure 3 (b) shows a zoom of Fig.3 (a) at about the time of the modification of foot placement. If it is possible for the COG state to transit from the original pattern to the modified pattern by shifting ΔT, the desired landing position can be realized without unnecessary ZMP variation.

Here, let $x_i, \dot{x}_i$ and $p_i, \dot{p}_i$ be the COG position and velocity and the ZMP position and velocity at the original trajectory, and $x_f, \dot{x}_f$ and $p_f, \dot{p}_f$ be at the modified target one. The transition time can be calculated as

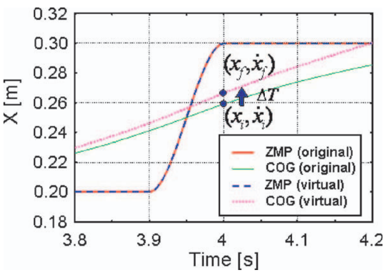
$$\Delta T = \begin{cases} \frac{1}{\omega}\log(r) & (r > 0) \\ \quad \text{s.t. } T_{\min} \leq \Delta T \leq T_{\max} & \\ T_{\max} & (r \leq 0) \end{cases}, \quad (18)$$

$$r \equiv \frac{\omega(x_f^{'} - p_f^{'}) + (\dot{x}_f^{'} - \dot{p}_f^{'})}{\omega(x_i^{'} - p_i^{'}) + (\dot{x}_i^{'} - \dot{p}_i^{'})},$$

where ΔT should be used a value close to integral multiple at the control cycle. If the sum of the transition time and the period of single support phase is positive value, the ZMP trajectory is generated ideally. Since swing leg cannot reach the target landing position in case of too short transition time, we set a lower limit of the transition time. When the transition time becomes too long at the large modification

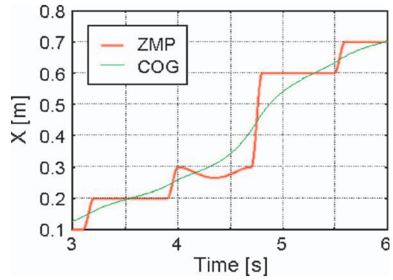


(a) COG and ZMP trajectories

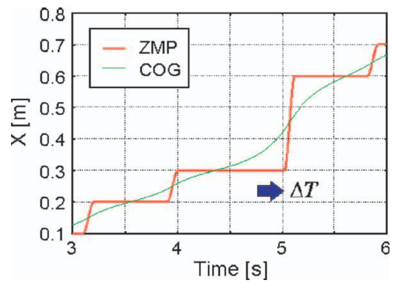


(b) Zoom up of (a)

Fig. 3. Compared with preplanned patterns



(a) Without time adjustment



(b) With time adjustment

Fig. 4. Modification of foot placement

of stride, or cannot be calculated at the change of walking direction, an enough time for single support will be given as maximum value.

Let us consider modifying next step length suddenly from 0.1[m] to 0.3[m] at the beginning of single support phase 4[s] in Fig.4. The generated trajectory without time adjustment is shown in Fig.4 (a). In Fig.4 (a), the ZMP trajectory was fluctuated in the maximum backward by 26[mm]. Adding 0.31[s] to the period of single support phase, the maximum fluctuation of the ZMP was suppressed by up to 1[mm] shown in Fig.4 (b).

In this method, $x_i, \dot{x}_i$ and $x_f, \dot{x}_f$ can be obtained using matrix $\boldsymbol{Z}$ in previous sampling time in Eq.(14). After calculating an adjusting time, matrix $\boldsymbol{Z}$ will be obtained. Since matrix $\boldsymbol{Z}$ does not include any landing position and can be determined by the period of each support phase, the same matrix $\boldsymbol{Z}$ as on sagittal plane also can be utilized for the generation of the COG and the ZMP trajectories on frontal plane. Thus,the COG and the ZMP trajectories can be generated efficiently by calculating a pseudo inverse matrix in Eq.(14) only once at every control cycle.

B. Shaping of ZMP fluctuation

Althogh adjusting a single support period is available to suppress the ZMP fluctuation in the direction of planar motion, the ZMP fluctuation in the orthogonal plane becomes large. Here, this ZMP fluctuation can be compensated not so small by appling a pattern generator using preview control [6], because it has a little bit time to reach the maximum ZMP fluctuation. Here, by setting initial position and velocity of the COG to zeros, the compensating trajectories of the COG and the ZMP are easy to connect the originals smoothly.

The block diagram of the ZMP shaping is shown in Fig.5. Figure 5 (a) implies that the ZMP trajectory had occurred the overshoot by the sudden modification of next landing position at a beginning of single support. Then the COG and the ZMP trajectories are generated so that the output ZMP can follow the unexpected ZMP fluctuation as the desired ZMP by preview control method in (b). Subtracting (b) from (a), newly reference trajectory of the ZMP can be obtained as (c). The reference trajectory of the COG can be also synthesized.

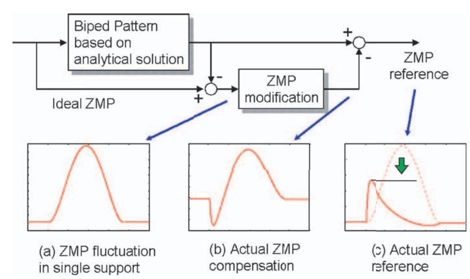


Fig. 5. ZMP shaping

C. Effects of pattern modification

Then the comparison between with and without suppressions are discussed. The maximum ZMP errors which are calcurated by using a multi-body model of humanoid robot HRP-2[4]. In Table.I, although the maximum ZMP errors in all cases without any suppression remain within support polygon as foot size, it was difficult to walk stably in dynamic simulation when the ZMP error is greater than about 0.07 [m] While on the other hand, in Table.II, the maximum ZMP errors becomes a little bit large in case of short modification of foot place applied to proposed method. Howerver, the maximum ZMP errors are enough small for dynamically stable walking at the large modification of foot place. Here, minimum and maximum adjusting time are set to -0.2[s] and 0.35 [s] respectively. These adjusting time should be decided so that the swing leg can reach the desired landing position in consideration of joint limitations such as movable range and angular velocity. Therefore, the humanoid robots are expected to achieve to walk stably as immediate modification of foot place.

TABLE I

WITHOUT ANY ADJUSTMENT OF COG AND ZMP TRAJECTORIES

		before modification [m]						
		-0.3	-0.2	-0.1	0	0.1	0.2	0.3
		Maximum ZMP error[$\times 10^{-2}$ m]						
after modification [m]	-0.3	3.4	3.6	4.6	5.5	6.5	7.5	8.9
	-0.2	3.3	2.1	2.9	3.9	4.9	6.2	7.7
	-0.1	3.7	2.3	1.4	2.4	3.5	5.0	6.7
	0	4.7	3.1	1.7	1.2	2.3	3.9	5.7
	0.1	6.1	4.4	2.8	1.4	1.1	2.6	4.7
	0.2	7.5	5.8	4.1	2.6	2.2	2.6	3.6
	0.3	8.9	7.2	5.6	3.9	3.0	3.3	4.0

TABLE II

PROPOSED SHAPING OF COG AND ZMP TRAJECTORIES

		before modification[m]						
		-0.3	-0.2	-0.1	0	0.1	0.2	0.3
		Maximum ZMP error [$\times 10^{-2}$ m] (upper) Adjustment time [s] (bottom)						
after modification [m]	-0.3	3.4 (0)	3.1 (0.11)	3.2 (0.305)	5.3 (0)	3.3 (0.35)	4.1 (0.35)	4.9 (0.35)
	-0.2	2.9 (-0.11)	2.1 (0)	2.4 (0.19)	3.5 (0)	3.0 (0.35)	3.7 (0.35)	4.6 (0.35)
	-0.1	3.4 (-0.2)	2.8 (-0.2)	1.4 (0)	1.9 (0)	2.7 (0.35)	3.4 (0.35)	4.2 (0.35)
	0	5.6 (-0.2)	4.0 (-0.2)	3.1 (-0.2)	1.2 (0)	2.9 (-0.2)	3.3 (-0.2)	4.2 (-0.2)
	0.1	4.5 (0.35)	3.6 (0.35)	2.8 (0.35)	2.2 (0)	1.1 (0)	3.7 (-0.19)	4.5 (-0.2)
	0.2	4.9 (0.35)	3.9 (0.35)	3.0 (0.35)	3.9 (0)	2.3 (0.19)	2.6 (0)	4.2 (-0.11)
	0.3	5.3 (0.35)	4.2 (0.35)	3.3 (0.305)	5.7 (0)	2.8 (0.305)	2.8 (0.11)	4.0 (0)

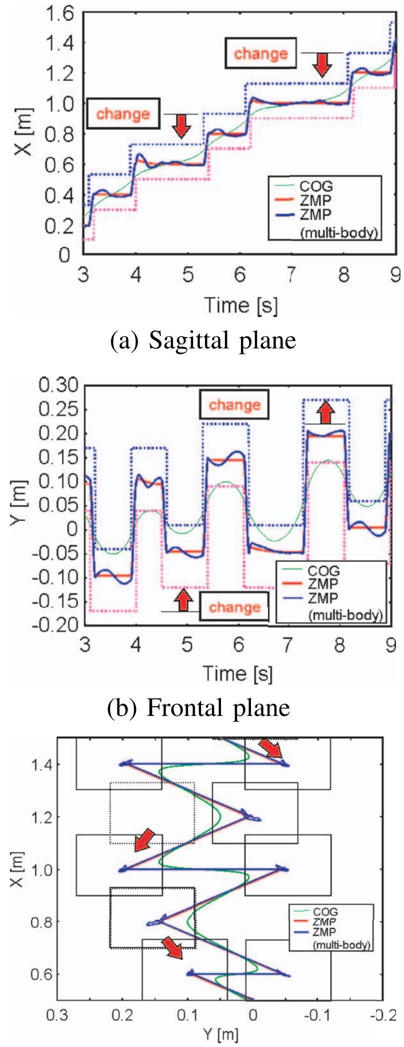


Fig. 6. Reference trajectories of COG and ZMP with the sudden modification of foot place

V. EXPERIMENT

To confirm the effectiveness of the proposed method, experimental result will be shown by using humanoid robot HRP-2. Original walking pattern was generated from foot place straightly. Here, step width is 0.2 [m] and step cycle is 0.8 [s] at all of steps. The command of sudden modification of foot place which moves to just right and left was given. The displacements of left and right was set to 0.05 [m]. The generated trajectories of the COG and the ZMP are shown in Fig.6. In Fig.6 (a) and (b), when the command of change of foot place on the inside, a landing time was adjusted to -0.2 [s] shortly. In contrast, the landing time became +0.28 [s] longer as foot place was changed on the outside. Then stable walking pattern could be generated in Fig.6 (c). The generated walking pattern was enough dynamically stable on the multi-body model.

Here, future three step are connected at the every beginning of single support phase sequentially. If any foot place is not given , step length is set to zero. At the initial and terminal intervals of the biped gait, the fourth order ZMP polynomials are applied ($N_1 = N_7 = 4$).

The third order ZMP polynomials are used at the intermediate interval so that its coefficients can be determined uniquely to reduce the dimension of matrix $\boldsymbol{Z}$ in Eq.(14). This implies that the ZMP coefficients of intermediate trajectories are directly calculated from the boundary condition. The dimension of matrix $\boldsymbol{Z}$ was reduced from 44 to 20. Total computation time became 0.9[ms] (Pentium III 1.26[GHz]).

VI. CONCLUSION

This paper proposed a method of a real-time gait planning which can change stride immediately at every step. By adjusting single support period, the COG and the ZMP trajectories which satisfy geometric boundary condition strictly could be generated. Using an analytical solution of an inverted pendulum, the coefficients of homogeneous solution and the ZMP time polynomial was decided from the boundary conditions of the COG and via points of the ZMP.

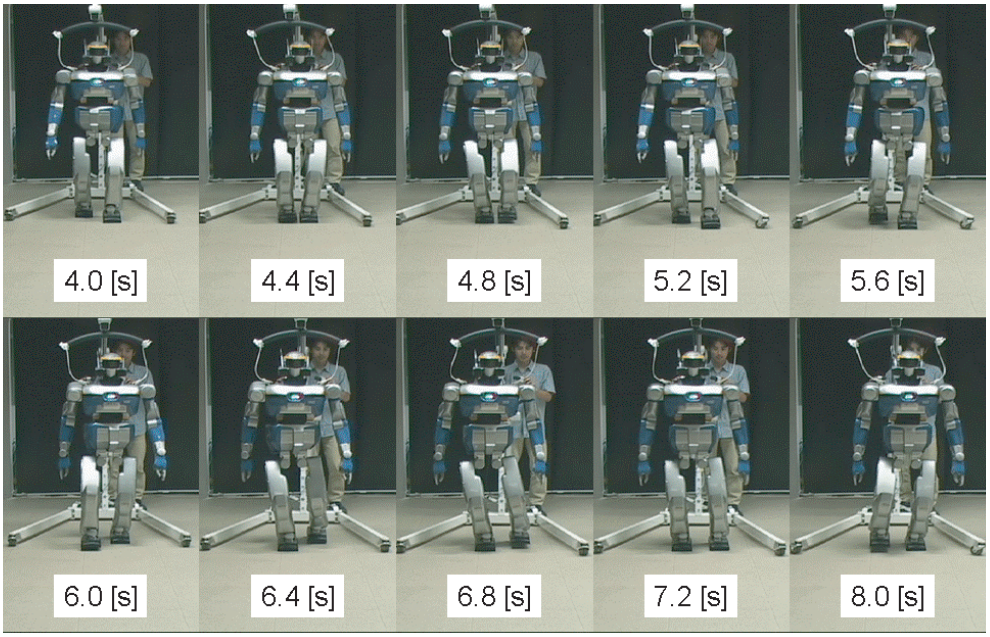


Fig. 7. Snapshot of the playbacked walking pattern with the sudden modification of foot placement

The time that becomes the maximum error margin of the ZMP by using the polynomial can be delayed. Thus, the ZMP error was suppressed by mixing the opposite phase effectively. Even if the next foot placement was longly changed, a stable walking pattern could be generated smoothly by adjusting a period of a single support phase and the shaping ZMP error at single support phase. The effectiveness of the proposed method was confirmed by the sudden change of stride in experiments.

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