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EXPERIMENTS IN OPTIMAL CONTROL OF A FLEXIBLE ARM  
WITH PASSIVE DAMPING

by

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Abstract

This paper presents a hybrid active and passive control scheme for controlling the motion of a lightweight flexible arm. A straightforward development of LaGrange's equations using a series expansion of assumed flexible modes provides a time domain model for controller design. The active controller design was approached as a steady state linear quadratic continuous regulator. A constrained viscoelastic layer treatment was employed to achieve passive damping. The passive damping treatment serves to enhance the system's stability while providing sound justification for the use of a highly truncated dynamic model and reduced order controller. Initial experimental results comparing controller performance with and without passive damping demonstrate the merit of the proposed combined active/passive approach.

Introduction

Recently a considerable volume of literature has been devoted to the problem of controlling the motions of structures having flexible structural members. While much of this research is performed in the interest of controlling large spacecraft, several investigators [1-6] have considered applying similar principles to the control of industrial

manipulators in the interest of improving manipulator performance and relaxing the structural stiffness requirement imposed by the more conventional rigid body control techniques. Such is the motivation for the work presented in this paper.

Although manipulators are somewhat complicated structures having several flexible links and joints that move independently, many problems associated with controlling such devices can be approached, without loss of generality, by considering a simple single link, single axis configuration. The present investigation concentrates on a single link arm which rotates in the horizontal plane about a pinned end. The authors apply established methods for developing a time domain dynamic model and active controller. Active control is applied to only the first two flexible modes and the rigid body mode. In order to reduce the effect of the ignored flexible modes a constrained viscoelastic layer damping treatment is applied to the surface of the flexible beam. This approach involves sandwiching a thin layer of viscoelastic material between the flexible member's surface and a stiff elastic constraining layer. When elastic deformation of the structure occurs, shear induced plastic deformation imposed in the viscoelastic layer provides the desired mechanical damping effect. Lane [7] has shown, analytically, that the incorporation of the prescribed passive damping treatment can result in faster settling times and considerable improvements in system stability. This paper presents the results of initial experiments performed with the aim of verifying this assertion.

### Experimental Facility

The experimental facility is a complete laboratory for examining the control of flexible arms with frequencies as high as 100 Hz. The system consists of a flexible arm with payload, DC torque motor with servo-amp, A/D and D/A conversion for measurement sampling, signal conditioning and 16 bit computer system for implementation of control algorithms. The control computer is equipped with floating point hardware, 64 megabyte hard storage, 24 channels of A/D conversion and 2 channels of D/A conversion. A typical value for 32 bit floating point multiplication is 17 microseconds. The physical configuration of the flexible arm, torque motor and sensors is illustrated in figure 1. Figure 2 is a block diagram of the system components. The arm is a four foot long rectangular aluminum beam with cross sectional dimensions of  $3/4 \times 3/16$  inches. With the active feedback controller operating, the first two natural frequencies of the beam approach its clamped-free modes, accordingly these are the modes assumed in modeling the arm's dynamics. The clamped-free frequencies of the arm with the payload in place are 2.0 and 13.5 Hz.

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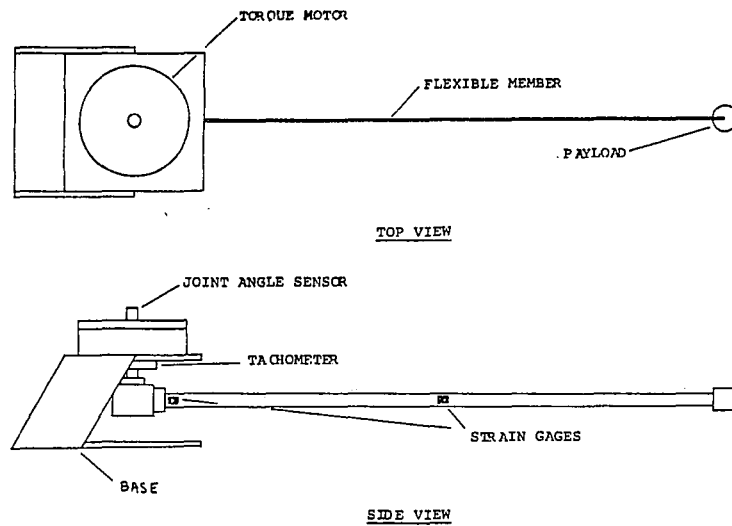


Figure 1. Experimental Arm

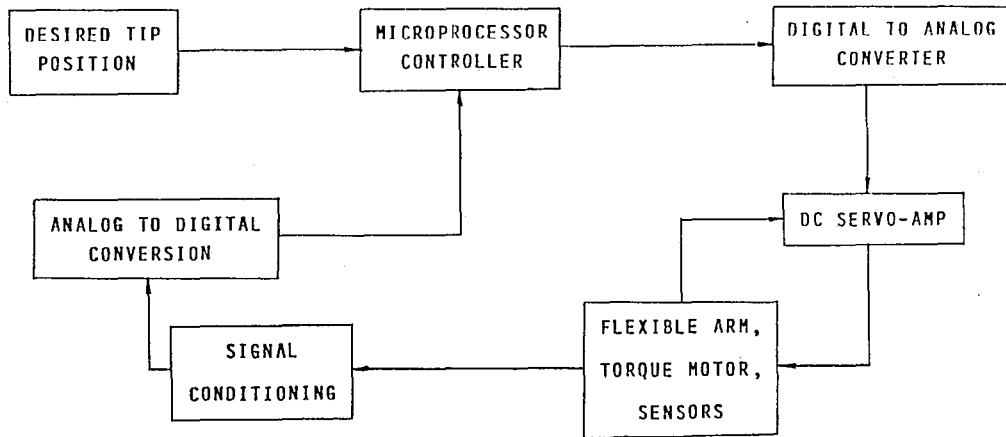


Figure 2. System Block Diagram

## Dynamic Modeling

The first step in controller design is to construct an analytical model of the physical system. The model must include the major features of the real system, yet still lend itself to available analysis tools. A truncated series of assumed modes was selected, with the first mode being a rigid body rotation. Two additional flexible modes corresponding to clamped-free beam vibrations complete the series. LaGrange's

equations are formulated for the three mode series after normalizing the flexible modes. The resulting dynamic equations are then linearized by assuming small motions and neglecting terms of higher order than one. The equations can then be organized into a sixth order state space model of the following form:

$$\begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{z}} \end{bmatrix} = \begin{bmatrix} \phi_{aa} & \phi_{ab} \\ \phi_{ba} & \phi_{bb} \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{z} \end{bmatrix} + \begin{bmatrix} \tilde{\Gamma}_a \\ \tilde{\Gamma}_b \end{bmatrix} u \quad (1)$$

$$\tilde{x} = \begin{bmatrix} \theta \\ q_1 \\ q_2 \\ \dot{\theta} \end{bmatrix} \quad \tilde{z} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \quad (2)$$

$\tilde{x}$  - measured state vector       $\theta$  - joint angle  
 $\tilde{z}$  - unmeasured state vector       $\dot{\cdot}$  - time derivative  
 $\tilde{\Gamma}$  - input vector       $u$  - control torque  
 $q$  - flexible mode

A detailed description of the modeling procedure may be found in [8,9].

### Modal Reconstruction

The control system employed requires the entire state vector be identified for the control law. Direct measurement of the modal quantities is not possible, however, modal displacements can be calculated as linear combinations of strain measurements. Equation 3 is the basic relationship between the flexible modes and the strain. Since we are interested in reconstructing two separate modes, two strain measurements are made, one from the base of the beam and one from the midpoint. Four active gages are used in a full bridge at each measuring point. This implementation compensates for torsional, axial and transverse strains that would otherwise reduce disturbance rejection. Equation 4 is the form of the reconstruction relation used to obtain the modal amplitudes from the strain measurements.

The coefficients for the reconstruction can be determined by inserting the assumed mode functions into equation 4. Experiments agreed well with the analytical model, resulting in a nearly orthogonal relationship between the modes providing 6 to 8 decibels of rejection between the recon-

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structured modes. A reduced order Luenberger observer was employed to estimate the modal velocities from the modal amplitudes.

$$\begin{bmatrix} \epsilon(y_1) \\ \epsilon(y_2) \end{bmatrix} = \begin{bmatrix} c_{11} \frac{\partial^2 \phi_1(y_1)}{\partial y^2} & c_{12} \frac{\partial^2 \phi_2(y_1)}{\partial y^2} \\ c_{21} \frac{\partial^2 \phi_1(y_2)}{\partial y^2} & c_{22} \frac{\partial^2 \phi_2(y_2)}{\partial y^2} \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} \quad (3)$$

$$\tilde{\xi} = A^{-1} \tilde{\epsilon} \quad (4)$$

$\epsilon(y)$  - indicates strain at position  $y$

$\phi(y)$  - spatial mode functions

$q(t)$  - time dependent modal amplitude

### Control System Design

The objective of fast response of the flexible arm's payload to commanded positions, is in opposition to minimizing excitation of the flexible modes. The first requires high rates and torques, while the latter favors smooth application of smaller torques. This problem is an excellent candidate for optimal control, which provides a solution with relative weighting on the various states.

Equation 5 is the standard formulation of a linear quadratic continuous transient regulator problem where  $X$  is the full state vector. The steady state solution of the control law was computed using subroutines in the ORACLS [10] software package. The basic problem was modified [11] so that the closed loop poles of the system could be specified with an arbitrary degree of stability.

$$P = \int (X^T Q X + R u^2) dt \quad (5)$$

$$u = K^T X$$

where

$Q, R$  = weighting matrices

and

$K^T$  = control law

The selection of the elements of the weighting matrices remains to a large degree trial and error. In this system it is noted that the second flexible mode is very energetic and large penalties on the second mode velocity or states coupled to the second mode result in high gains on the second state. These high gains result in excessive control action. Additionally the second mode is not a static deflection mode and high state gain causes problems due to measurement errors. The control objective in the second mode is therefore damping rather than steady state error reduction.

### Passive Damping

Lane [7] introduced the concept of controlling a flexible manipulator arm using a hybrid active/passive control strategy. Passive control involves moving the flexible system's poles to the left by physically adding mechanical damping to the system. This has the effect of improving stability and response of the overall system while reducing the detrimental effects of both control and observation spillover. The application of constrained viscoelastic damping layers was proposed as a passive control measure. The approach involves sandwiching a thin film of viscoelastic material between the flexible member's surface and an elastic constraining layer. Materials having high elastic moduli provide the most effective elastic constraining layers. When elastic deflection of the structure occurs, shear induced plastic deformation is imposed in the viscoelastic layer. The energy dissipation associated with the plastic deformation provides the desired mechanical damping. This concept is illustrated in Figure 3. For further details regarding the

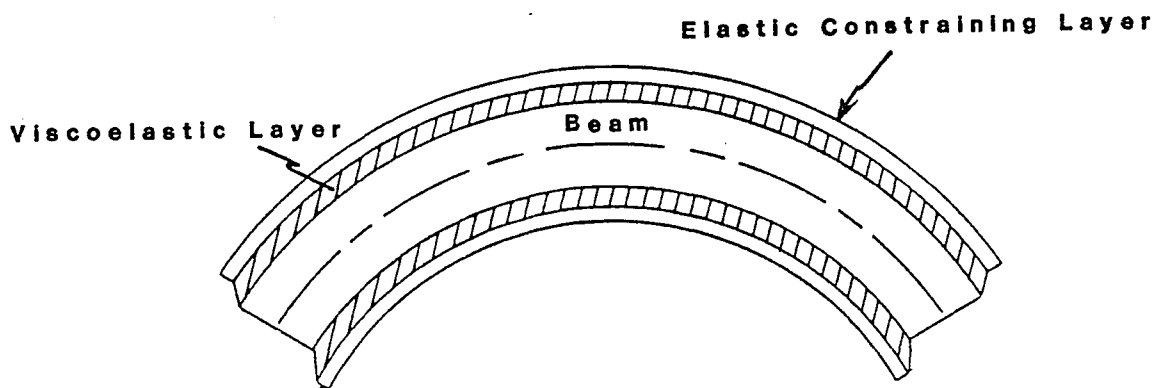


Figure 3. Treated Beam Element Under Flexure

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theory associated with constrained viscoelastic layer damping the reader is referred to references [12-15].

Plunkett and Lee [12] have observed that a relationship exists between the length of the elastic constraining layer and the damping ratio. For example, if the constraining layer is very long, relatively little shear is induced in the viscoelastic layer at locations remote to the endpoints. Conversely, if the constraining layer is very short, a more uniform shear distribution results, however the plastic deformation is of small magnitude, even at the endpoints. This suggests the existence of some optimal length, to which constraining layer sections could be cut to provide the optimal damping for a given configuration. Plunkett and Lee have developed a method for calculating this optimal section length. The damping obtained through application of constrained viscoelastic layers is frequency dependent and accordingly section length optimization is performed with respect to a prescribed frequency. With regard to the present application the section length has been selected so as to optimize the damping in the vicinity of the lowest frequency uncontrolled modes. It is important to note that when the constraining layer is not sectioned, the damping is optimal for very low frequencies and consequently, non-sectioned treatments generally will not enhance the control of flexible structures significantly. Figure 4 compares theoretically calculated damping ratios for sectioned and non-sectioned treatments. The data represents the aluminum beam discussed above, with a .002 inch viscoelastic layer and .010 inch thick steel constraining layer partitioned into 1.72 inch sections. The treatment is applied to both sides of the beam. It is evident from these curves that the damping for the frequency range of interest is substantially increased by simply cutting the constraining layer into sections. Figure 4 also includes the experimentally measured damping ratios for the treated beam described above. Figure 5 presents a comparison between the frequency response for an untreated beam and one incorporating the sectioned constrained layer treatment.

Upon examination of the sectioned constraining layer data presented in figure 4, one finds that while the shapes of the experimental and theoretical curves are in reasonable agreement, the damping ratios are significantly overestimated by Plunkett and Lee's method. The authors find that for very thin constraining (.0015 inch steel) layers the experimental and theoretical values agree within about 10%, however when the constraining layer thickness is increased, in the interest of increasing the damping provided, the agreement tends to be poor. The authors believe that this trend may be attributed to a number of simplifying assumptions made by Plunkett and Lee, which provide adequate results for thin constraining layers, but may be violated as constraining layer thickness

is increased. Probably foremost among these is the assumption that the constraining layer experiences only axial stress that is uniform throughout its thickness. Alberts is presently investigating the extension of Plunkett and Lee's model to accommodate thicker constraining layers.

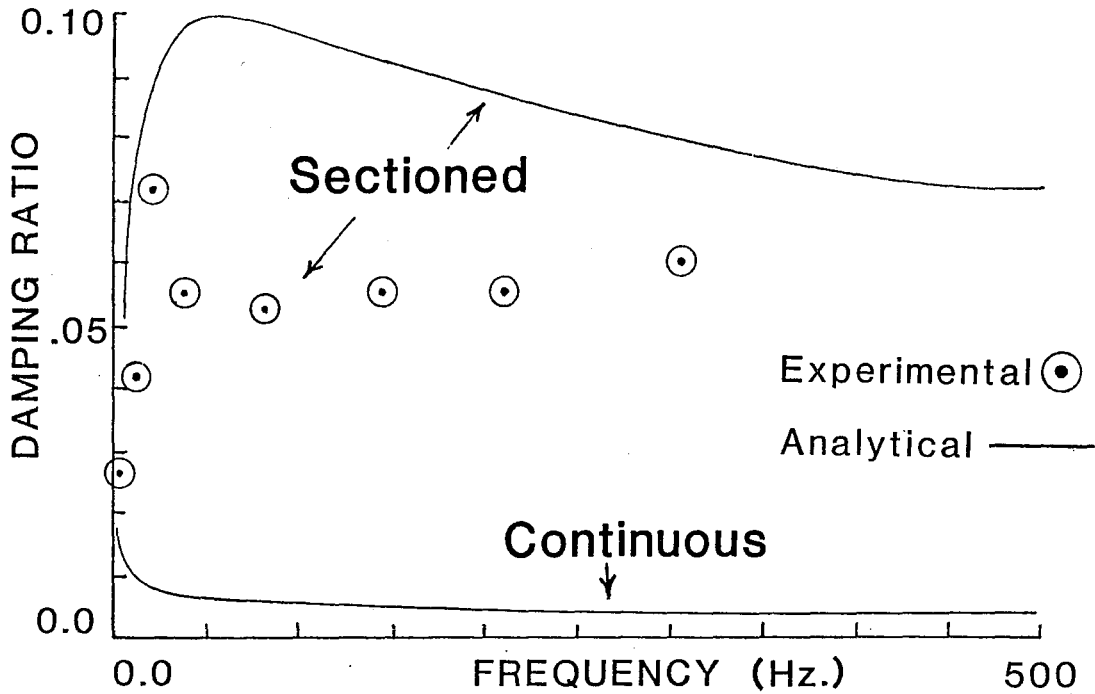


Figure 4. Damping Ratio "vs" Frequency

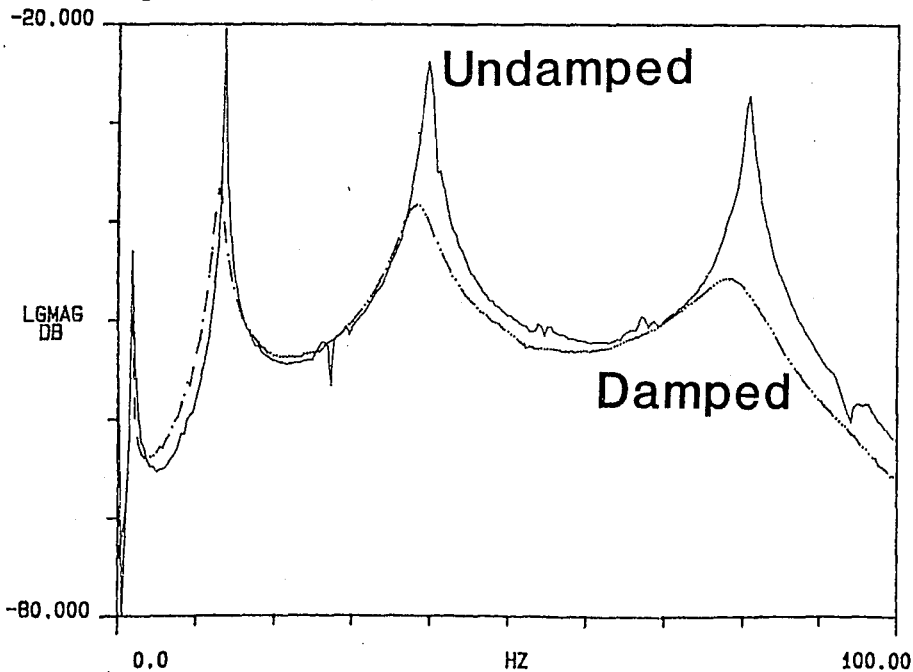


Figure 5. Frequency Response of Damped and Undamped Beams



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Although the performance of the damping treatment falls short of the theoretical predictions, the constrained layer damping technique remains a lightweight, unobstructive, inexpensive and highly effective means of reducing the vibrations of the higher modes. The treatment described adds approximately 0.024 inches to the thickness of the beam and 0.42 pounds per square foot of treated surface to its weight. In applications where very light weight is desired, treatments utilizing carbon fiber composites as the constraining layer material provide performance similar to steel but weigh only 0.094 lbs. per square foot of treated surface.

### Experimental Results

The results of employing the active controller described in previous sections in connection with controlling a beam with no passive damping treatment are represented by the time response curves of figure 6. From these results the effectiveness of the modal feedback for settling the first mode of vibration is readily apparent. The second flexible mode's amplitude is too small to view in this plot however the effect of the active controller on this mode is evident from frequency response curves (figure 9). It was observed that

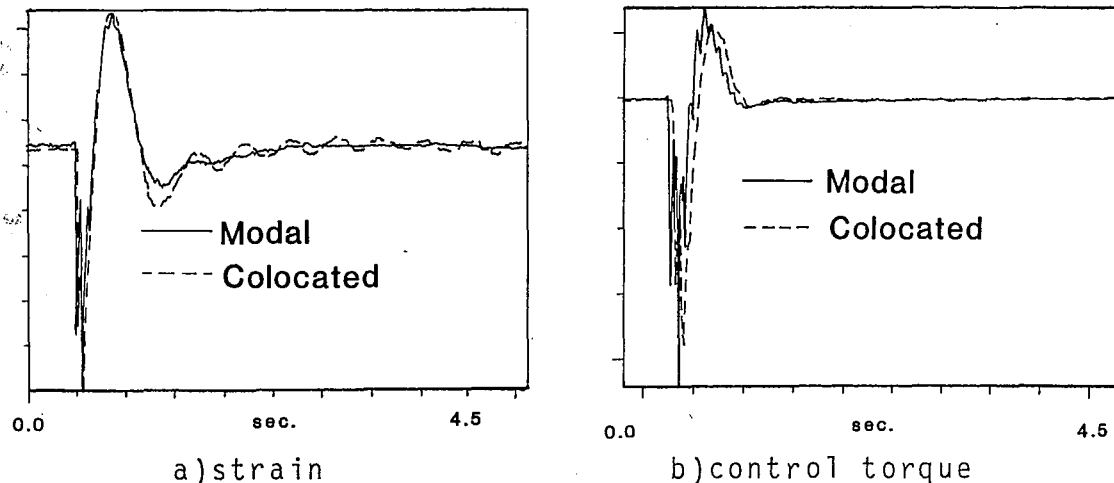


Figure 6. Time Response of Stable System Without Passive Damping

by increasing the gain on the rigid body mode, instability could be induced in the uncontrolled modes. This proved to be a good opportunity to demonstrate the stabilizing effect of the proposed passive damping method. Starting with the gain matrix used to generate the results of figure 6, the rigid body gains were progressively increased until instability was induced in the lower uncontrolled modes. In this condition the arm was initially quiescent, but the application of a step position command resulted in growing oscillations in the

uncontrolled flexible modes. Figure 7a represents the response of the untreated beam with rigid body position and rate feedback only. In this case instability occurs at about 22 Hz. which apparently corresponds to the system's third closed loop pole as shown in figure 9. When modal feedback is included (figure 7b) the instability occurs at 41 Hz. which corresponds with the fourth closed loop pole.

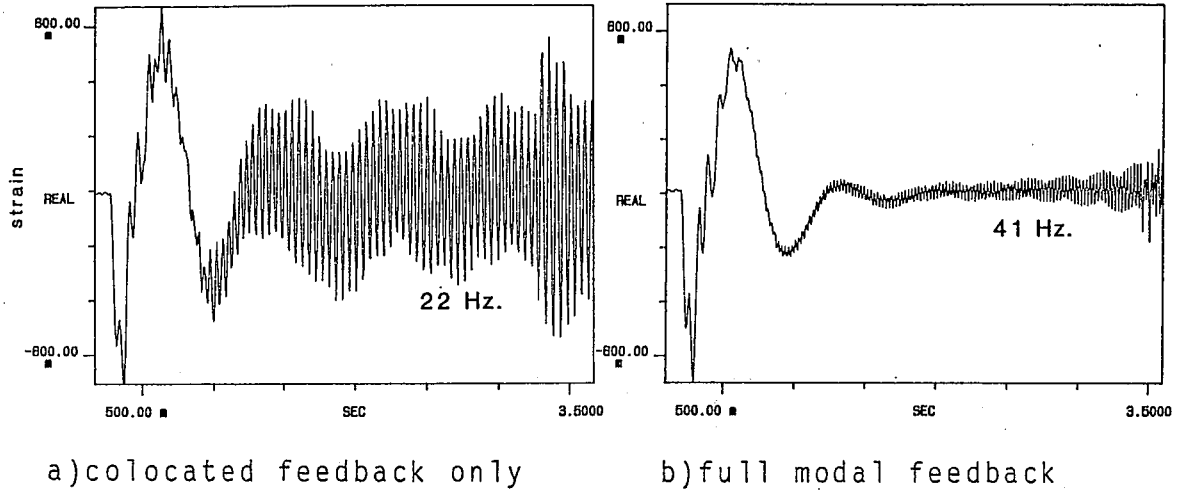


Figure 7. Time Response of the Unstable System Without Passive Damping

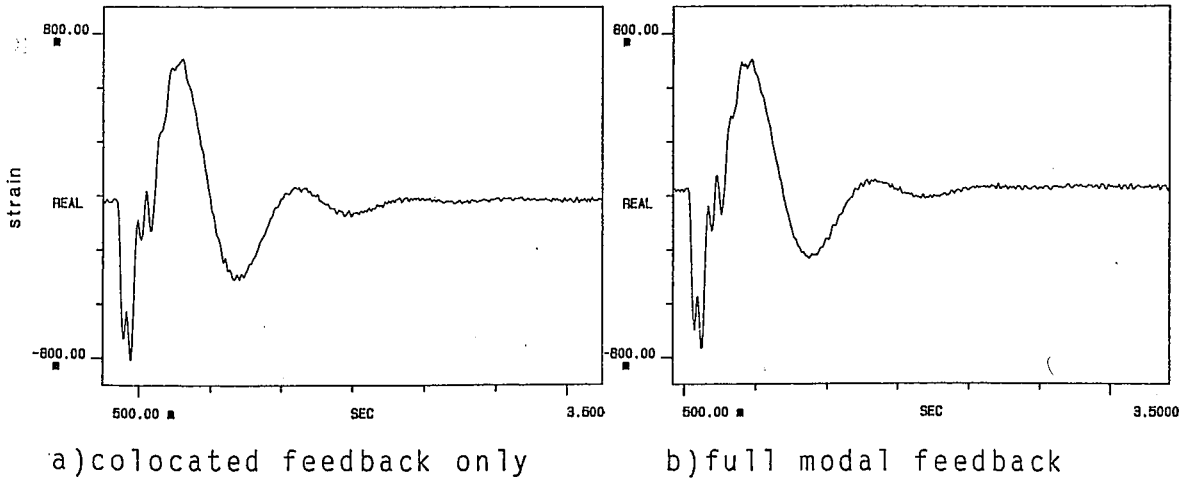


Figure 8. Time Response of the Same System (figure 7) Stabilized with Passive Damping

Without changing the gain matrix, the arm in the experimental system was replaced by an identical arm incorporating the constrained layer damping treatment. As shown in figures 8a and 8b the treatment has eliminated the instability. Figure 9 represents the transfer functions between input torque and payload acceleration for the open-loop system and

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the colocated rate and position feedback system, both with passive damping. The frequency response of this system with full modal feedback is very similar to the colocated feedback case with the lower frequency poles slightly attenuated and broadened, as might be expected.

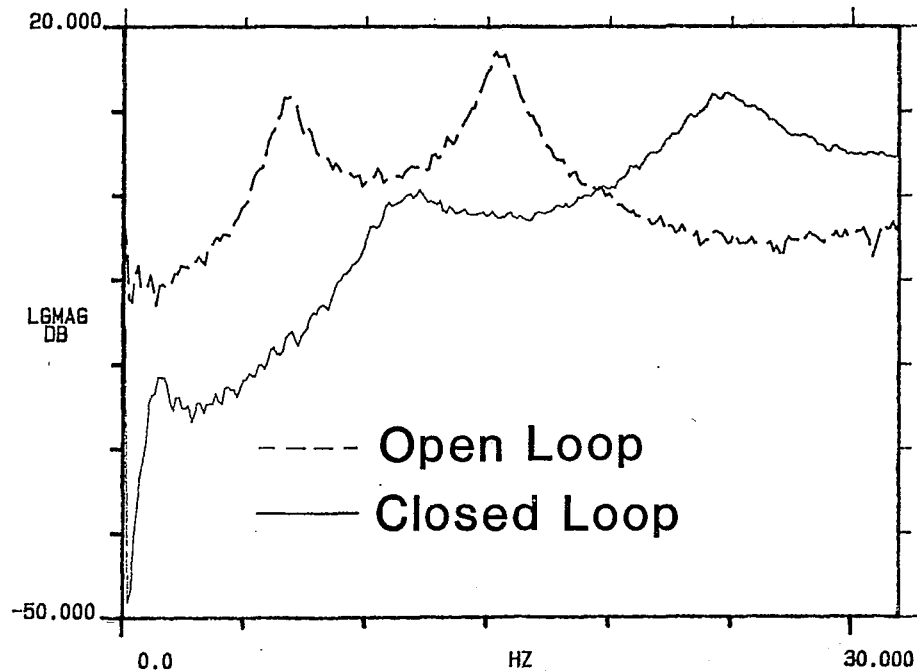


Figure 9. Frequency Response of Open Loop System and Closed Loop, Colocated Feedback System With Passive Damping

### Conclusions

The control of the rigid body mode and the first two flexible modes of a lightweight arm has been demonstrated using a standard steady state linear quadratic regulator. Increasing the rigid body mode feedback gains was found to lead to instability in the low frequency uncontrolled modes. A constrained viscoelastic layer damping treatment, incorporating the notion of length optimized, sectioned constraining layers has been shown to provide an easy to apply and inexpensive method of stabilizing these uncontrolled modes.

In obtaining the initial results presented, the weighting matrices, degree of stability and the estimator dynamics were chosen rather arbitrarily. The authors acknowledge that the values selected for the initial tests may not be the most appropriate selections. The authors are presently working towards "tightening up" the control loop such that the full performance capabilities of the system may be realized.

Finally, it may be appropriate to note that utilizing the proposed hybrid control scheme in space may pose some problems not experienced in earthly applications insofar as the physical properties of viscoelastic materials are somewhat dependent upon temperature and apparently certain viscoelastic materials are subject to degradation and outgassing [16] when exposed to the space environment. Nonetheless, the application of viscoelastics to damping problems in space is an area being actively pursued. Trudell, et.al. [16] suggest that the adoption of passive damping measures will play a crucial role in the successful solution of in large space structure vibration control problems.

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#### References

1. Book, W.J., "Analysis of Massless Elastic Chains With Servo Controlled Joints", Journal of Dynamic Systems, Measurement and Control, Vol.101, September 1979, pp.187-192
2. Book, W.J., "Recursive Lagrangian Dynamics of Flexible Manipulator Arms Via Transformation Matrices", Carnegie-Mellon University Robotics Institute Technical Report, CMU-RI-TR-8323, Dec. 1983
3. Truckenbrodt, A., "Modelling and Control of Flexible Manipulator Structures", 4th CISM-IFTOMM Symp., Warszawa, 1981
4. Cannon, R.H. and Schmitz, E., "Initial Experiments on the End-Point Control of a Flexible One Link Robot", International Journal for Robotics Research, MIT Press, Cambridge Mass., Vol.3, No.3, Fall 1984
5. Cannon, R.H. and Schmitz, E., "Further Experiments on the End-Point Control of a Flexible One Link Robot", submitted for publication in Journal of Dynamic Systems, Measurement and Control, 1984
6. Sangveraphunsiri, V., The Optimal Control and Design of a Flexible Manipulator Arm, Ph.D Dissertation, Dept. of Mech. Eng., Georgia Inst. of Tech., 1984

## EXPERIMENTS IN CONTROL WITH PASSIVE DAMPING

7. Lane, J.S., Design and Control Principles for Flexible Arms Using Active and Passive Control, M.S. Thesis, Dept. of Mech. Eng., Georgia Inst. of Tech., Mar. 1984
8. V. Sangveraphunsiri, "The Optimal Control and Design of a Flexible Manipulator Arm", PhD Dissertation, Dept. of Mech. Eng., Georgia Institute of Technology, Atlanta, Ga.
9. T. Alberts, S. Dickerson, W. J. Book, "Modeling and Control of a Flexible Manipulator Arm: Volume I", MHRC-TR-85-06, Georgia Tech., Feb. 1985, Release Date July 1985
10. E. S. Armstrong, "ORACLS-A System for Linear-Quadratic-Gaussian Control Law Design", NASA TP-1106, April 1978.
11. Anderson, B. O. D., and Moore, J. B.: "Linear System Optimization With Prescribed Degree of Stability", Proc. IEEE, Vol. 116, No. 12, December 1969
12. Plunkett, R. and Lee, C.T., "Length Optimization for Constrained Viscoelastic Layer Damping", J. Acoust. Soc. of Amer., Vol. 48, No. 1, 1970, pp. 150-161
13. Kerwin, E.M. Jr., "Damping of Flexural Waves by a Constrained Viscoelastic Layer", J. Acoust. Soc. Amer., Vol. 31, No. 7, July 1959, pp. 952-962
14. Nashif, A.D. and Nicholas, T., "Vibration Control by Multiple-Layered Damping Treatment", Shock and Vib. Bull., No. 41, Part 2, Dec. 1970, pp. 121-131
15. Torvik, P., "The Analysis and Design of Constrained Layer Damping Treatments", Damping Applications for Vibration Control, Ed. by P. Torvik, ASME, N.Y., 1980
16. Trudell, R.W., Curley, R.C. and Rogers, L.C., Passive Damping in Large Space Structures, AIAA-80-0677, 13 pgs.

