TECHNICAL MEMORANDUM

ANL-CT-77-16

EXPERIMENTS ON FLUIDELASTIC VIBRATIONS OF TUBE ARRAYS

by

S. S. Chen, J. A. Jendrzejczyk, and M. W. Wambsganss

Components Technology Division

ARGONNE NATIONAL LABORATORY

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MASTER

Base Technology

April 1977

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LMFBR Components
(UC-79k)

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SUMMARY

The work reported herein was performed as part of the base technology activity under Flow Induced Vibration Program (189a No. CA054) sponsored by ERDA/RDD. The overall objective of the activity is to develop new and/or improved, experimentally-validated analytical methods and guidelines for designing LMFBR components to avoid detrimental flow induced vibration.

Many reactor system components, such as heat exchanger tubes and reactor fuel pins are long, slender, beam-like components typically arranged in bundles and immersed in a flowing liquid. As such, they are susceptible to flow induced vibration. Due to fluid coupling, the tubes will respond in one or several of coupled modes. Therefore, understanding the coupled modes is essential in design to avoid detrimental flow-induced vibrations.

A general method of analysis was developed recently for predicting the natural frequencies, mode shapes, and tube responses of tube arrays in liquid; the method can be applied to tube arrays arranged in any pattern. The objective of this report is to present the results of several series of experiments designed to verify the theory.

Four series of tube arrays are tested. The arrangements of the tubes are as follows: (a) a row of five tubes with the gap to tube radius ratio (G/R) equal to 2.0, 1.0, and 0.25; (b) three-tube arrays in the staggered arrangement with G/R equal to 2.0, 1.0 and 0.5; (c) seven-tube arrays in the staggered arrangement with G/R equal to 1.5, 1.0 and 0.4; and (d) four-tube array in a square pattern with G/R = 0.5. The fourth series is tested under five different conditions: (1) fully submerged in unconfined water; (2) partially submerged in water; (3) near a flat wall; (4) contained in a circular cylinder; and (5) fully submerged in a liquid of high viscosity (mineral oil). A means to excite the tubes is provided by an electromagnetic exciter assembly. A servo system can be used to control input. Response in the form of tube acceleration is measured using two accelerometers mounted on each tube. The data is processed in a fast Fourier Transform Analyzer.

An analysis is made for each test case. In the analysis, the equations of motion and boundary conditions for each tube are derived including fluid coupling. Using the uncoupled modal function, one can reduce the partial differential equations to a system of second order ordinary differential equations, from which coupled natural frequencies and tube responses can be calculated.

The detailed information for uncoupled and coupled natural frequencies, mode shapes, damping, and tube response are presented in the report. The experimental data and analytical results are found to be in good agreement. Therefore, the analytical method developed earlier can be applied to different situations. The results of this investigation have applications to heat exchanger tube banks, fuel assemblies, and other structural components with multiple circular cylindrical elements.

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NOMENCLATURE

a_{ij}	Coupling coefficient given in Eq. (8)
c_i	Viscous damping coefficient
c_{ij}	$c_i \delta_{ij}$
e_i	Total external force per unit length
E_i	Young's modulus of tube i
f_i	Generalized force given in Eq. (8)
G	Gap between tubes
G_e	Gap between the center of outside container and the center of tube array
G_w	Gap between tube and flat wall
h	Water depth
I_i	Moment of inertia of tube i
k	Number of tubes
k_i	Spring constant at tube support
k_{ij}	Stiffness matrix given in Eq. (8)
ℓ	Tube length
m_i	Tube mass per unit length
M_i	Concentrated mass at tube end
p_i	$\frac{k_i \ell}{E_i I_i}$
q_i	Generalized coordinate
R	Tube radius
R_c	Radius of outside container
t	Time
u_i	Tube displacement
x,y,z	Cartesian coordinates
$\gamma_{ij}, \bar{\gamma}_{mn}$	Added mass matrix

NOMENCLATURE (Contd.)

δ_{ij}	Kronecker's delta
ϕ_i	Modal function of uncoupled mode for tube i
Ω_i	Circular frequency of uncoupled mode of tube i
ω	Natural frequency of coupled mode

I. INTRODUCTION

When a tube bundle vibrates in a liquid, due to fluid coupling, the tubes will oscillate as a group with definite phase relations among the tubes; this type of oscillation is called coupled vibration. On the other hand, if only one of the tubes is oscillating while all others are held stationary, it is called uncoupled vibration. Coupled and uncoupled vibrations of tube bundles are important in many practical system components, such as heat exchanger tubes, nuclear fuel assemblies, transmission lines, piles, and other similar components. Understanding the vibrational characteristics of tube bundles is essential in design to avoid detrimental flow-induced vibrations.

Many investigations of tube bundles vibrating in a liquid have been made to obtain a better understanding of tube vibration phenomena; a list of those investigations can be found in Reference 1. However, those studies are limited to some special tube arrangements. A general method of analysis was developed recently for predicting the natural frequencies, mode shapes, and tube responses of a group of tubes in a liquid [1-3]; the method can be applied to tube bundles arranged in any pattern. The objective of this report is to present the results of four series of experiments designed to obtain the information on coupled tube/fluid vibrations. Experimental data are found to be in good agreement with the analytical results; thus, the theory developed previously is sound and can be used with confidence.

II. EXPERIMENTAL DESCRIPTION

The test rig is shown in Fig. 1.* It is a tank fabricated from four lucite plates fastened to a base plate. The tank is 35.56 cm (14 in.) long, 35.56 cm (14 in.) wide, and 40.64 cm (16 in.) deep. A brass block bolted to the tank base is provided for installing the test assembly.

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*Figures begin on page 15.

Four sets of tube assembly are tested. The arrangements of the tubes are shown in Fig. 2: (1) a row of five tubes with the gap to the radius ratio (G/R) equal to 2.0, 1.0, and 0.25; (2) a group of three tubes in the staggered arrangement with G/R equal to 2.0, 1.0, and 0.5; (3) a group of seven tubes in the staggered arrangement with G/R equal to 1.5, 1.0, and 0.4; and (4) a group of four tubes arranged in a square pattern with G/R = 0.5. The fourth series is tested under five different conditions: (a) fully submerged in unconfined water; (b) partially submerged in water; (c) near a flat wall; (d) contained in a circular cylinder; (e) fully submerged in a liquid of high viscosity (mineral oil). The tube assembly consists of a group of brass tubes with 1.27 cm (0.5 in.) outside diameter, 0.159 cm (1/32 in.) wall thickness, and 30.48 cm (12 in.) long. Each tube element is soldered to a brass plate as shown in Fig. 3. The assembly is then mounted on the brass block at the center of the tank base.

A means to excite the tubes is provided by an electromagnetic exciter assembly, which is represented by the arrows in Fig. 2. This exciter assembly consists of a permanent magnet embedded in an aluminum mounting block which is screwed to the free end of a tube in the assembly, and a coil supported by a post fixed to the tank base. An alternating current applied to the coil from a signal generator induces an alternating magnetic field which, in turn, produces a force on the tube through the embedded magnet. In each assembly, only one tube is provided with the exciter assembly; those tubes with magnet are tube 5 in case 1, tube 3 in case 2, tube 6 in case 3 and tube 4 in case 4. A servo system can be used to control the current input or acceleration output, as desired.

Response, in the form of tube acceleration, is measured using two accelerometers mounted on each tube. The accelerometers are cemented to a small aluminum block with their sensitive axes orthogonal to each other.

The mounting aluminum block is then mounted to the free end of the tube by a screw such that the sensitivity axes are parallel to the x and y directions. These accelerometers are provided by the manufacturer with leads attached. The wire leads from the accelerometers are passed through the tube, tube base plate, brass block, and tank base, and then attached to a charge amplifier. A fast Fourier transform analyzer is used to process the data.

III. TEST PROCEDURES

A. Air Environment

Testing in air was performed to determine the natural frequency and damping for each tube and to observe the effect of coupling in air. The test for each tube assembly consists of two phases.

(1) Uncoupled Vibration - Additional supports to the free ends of the tubes are provided using wooden wedges and two posts fixed to the tank base for all tubes except the one to be tested. Those tubes with an additional support have much higher natural frequencies, and can be considered as fixed. The tube is excited by plucking it at the free end. The transient response of the tube is recorded. Then, the damping is obtained from the log decrement of the acceleration trace and the natural frequency is determined from the power spectrum of the acceleration. Each tube is tested in two orthogonal directions. Figure 4 shows a typical example of the results from the tests.

(2) Coupled Vibration - The coupling effect of the air is small. However, there is some tube interaction caused by mechanical coupling through the base end plate. After the uncoupled natural frequency and damping are found for each tube, the additional supports at the top of the tubes are removed. The tube assembly is then excited by plucking any one of the tubes. From the power spectrum of the response of any one of the

tubes, one can identify the coupled natural frequencies in air. The coupling in air is found to be small; thus, coupled vibration test in air was made only for a few cases. Figure 5 shows a typical example of the power spectral density of tube acceleration.

B. Liquid Environment

All tests were conducted in water except the last test in the fourth series in mineral oil. Testing in liquid was performed to determine the uncoupled natural frequency and damping for each tube, coupled natural frequencies, mode shapes, steady-state response, and transient response. These tests consist of five phases:

(1) Uncoupled Vibration - The same procedure as in air environment was used to determine the natural frequency and damping for each tube in two orthogonal directions. In this test, all other tubes except the one being tested are jammed tight by wedges to eliminate responses of the surrounding tubes. Figure 6 shows the acceleration-time trace and power spectral density for a typical case.

(2) Coupled Natural Frequencies - Coupled natural frequencies were determined from the power spectrum of accelerometer signals in response to a plucking of the tubes. Since there are many modes in a frequency band and some of the modes are not easily excited, different methods of plucking have to be used to find all coupled modes. For example, two tubes may be plucked simultaneously. Figure 7 shows the power spectrum for tubes 4 and 5 when tubes 1 and 5 are excited at the same time. The test is repeated several times to obtain all natural frequencies of coupled modes. The coupled natural frequencies obtained from each test are then averaged.

(3) Mode Shapes - Once all coupled natural frequencies are determined from plucking tests, a sinusoidal current with its frequency equal to one

of the coupled natural frequencies is applied to the coils. Thus, the tubes are set in resonance. After the tubes reach the steady state, the two accelerometer signals from each tube are recorded on an FM magnetic tape for all tubes simultaneously. From these steady-state acceleration-time traces, one can study the relative motions (mode shapes) of the tubes. This procedure is carried out for all coupled natural frequencies.

(4) Steady-state Responses - For each tube assembly, a sinusoidal current with a constant amplitude controlled by a servo system is applied to the coils and slowly swept through the frequency band of interest. The responses of the tubes as functions of the excitation frequency are recorded. From the response curves, coupled natural frequencies can be identified and the damping of some coupled modes can be estimated using the bandwidth method.

(5) Transient Responses - The transient responses of the tubes are also recorded for some tube assemblies. This is done by plucking one tube and recording the acceleration for each tube simultaneously.

IV. ANALYTICAL METHOD

One of the objectives of the experiments is to verify the theory developed earlier [1,3]. Therefore, analytical studies are made for each tube assembly for comparison. The mathematical model is given in Fig. 8. The method of analysis is the same as that presented in Refs. 1 and 4.

In a group of k tubes, each tube can move in the x and y directions. Let u_i ($i = 1$ to k) and u_i ($i = k+1$ and 2k) represent the displacement components of tube i in the x and y directions respectively. The equation of motion for tube i is [1]

$$E_i I_i \frac{\partial^4 u_i}{\partial z^4} + c_i \frac{\partial u_i}{\partial t} + m_i \frac{\partial^2 u_i}{\partial t^2} + \sum_{j=1}^{2k} \gamma_{ij} \frac{\partial^2 u_i}{\partial t^2} = e_i \quad , \qquad (1)$$

where t is time, z is axial coordinate, m_i is tube mass per unit length,

E_iI_i is flexural rigidity, c_i is damping coefficient, e_i is excitation force, and γ_{ij} is added mass matrix. For partially submerged tubes, Eq. (1) is applied to the portion submerged in liquid, while the equation of motion for the portion in air is corresponding to Eq. (1) by setting $\gamma_{ij} = 0$. Note that the variables with the index i (or j) from 1 to k are associated with the motion in the x direction while from k+1 to 2k in the y direction.

The method for calculating the added mass matrix for tube arrays in an infinite fluid and contained in a circular cylinder have been discussed previously [1,3]. For tube arrays near a flat wall, the following procedure can be employed. A group of k tubes near a flat wall is mathematically equivalent to the problem of 2k tubes which are located and move symmetrically about the flat wall as shown in Fig. 9. Therefore, if the added mass matrix for 2k tubes in an infinite fluid is known, the corresponding added mass matrix for k tubes near a flat wall can be calculated. Let the added mass matrix for 2k tubes in an infinite fluid be $[\bar{\gamma}_{mn}]$, m,n = 1,2,3,...4k, and the added mass matrix for k tubes near a wall be $[\gamma_{mn}]$, m,n = 1,2,3,...2k. γ_{mn} is given by

$$\begin{aligned}
\gamma_{mn} &= \bar{\gamma}_{mn} + \bar{\gamma}_{m,k+n} \;, \\
\gamma_{m,k+n} &= \bar{\gamma}_{m,2k+n} - \bar{\gamma}_{m,3k+n} \;, \\
\gamma_{k+m,n} &= \bar{\gamma}_{2k+m,n} + \bar{\gamma}_{2k+m,k+n} \;, \qquad (2) \\
\gamma_{k+m,k+n} &= \bar{\gamma}_{2k+m,2k+n} - \bar{\gamma}_{2k+m,3k+n} \;, \\
m,n &= 1,2,3\ldots k.
\end{aligned}$$

Each tube is elastically restrained against rotation by a torsional spring with spring constant k_i, and a concentrated mass M_i attached to the free end; the mass M_i represents the masses of the accelerometers, permanent magnet, and aluminum mounting block. Therefore the boundary conditions are:

At zero:

$$u_i = 0 \quad \text{and} \quad E_i I_i \frac{\partial^2 u_i}{\partial z^2} = k_i \frac{\partial u_i}{\partial z} \quad ;$$

and at z = ℓ:

$$\frac{\partial^2 u_i}{\partial z^2} = 0 \quad \text{and} \quad E_i I_i \frac{\partial^3 u_i}{\partial z^3} = M_i \frac{\partial^2 u_i}{\partial t^2} \quad . \tag{3}$$

where ℓ is the length of the tubes. Without loss of generality, the initial state of the tubes may be assumed as follows:

$$u_i(z,t)\Big|_{t=0} = 0 \quad \text{and} \quad \frac{\partial u_i(z,t)}{\partial t}\Big|_{t=0} = 0 \quad . \tag{4}$$

It is assumed that only the vibrations of the mode with no nodal point along the tube axis are significant and all others are negligible. Let

$$u_i(z,t) = q_i(t)\phi_i(z) \quad , \tag{5}$$

where ϕ_i is the modal function for uncoupled vibration of tube i such that

$$\frac{1}{\ell}\int_o^\ell \phi_i^2(z)dz = 1 \quad . \tag{6}$$

Using Eqs. (1), (3), (4), and (5) yields

$$[M]\{\ddot{Q}\} + [C]\{\dot{Q}\} + [K]\{Q\} = \{F\} \quad ,$$

and

$$\{Q\}_{t=0} = \{0\} \quad , \tag{7}$$

$$\{Q\}_{t=0} = \{0\} \quad ,$$

where [M], [C], and [K] are symmetric matrices with elements m_{ij}, c_{ij} and k_{ij}, and [Q] and [F] are general coordinate and generalized force with elements q_i and f_i, in which

$$m_{ij} = m_i\delta_{ij} + \gamma_{ij}a_{ij} \quad ,$$

$$c_{ij} = c_i\delta_{ij} \quad ,$$

$$k_{ij} = (m_i + \gamma_{ii})\Omega_i^2\delta_{ij} \quad ,$$

$$f_i = \frac{1}{\ell}\int_0^\ell e_i\phi_i dz \quad , \tag{8}$$

$$a_{ij} = \frac{1}{\ell}\int_0^\ell \phi_i\phi_j dz \quad ,$$

and Ω_i is the circular frequency of the uncoupled mode of tube i in water.

For free vibration, neglect damping and forcing terms and let

$$\{Q\} = \{\bar{Q}\}\exp(j\omega t) \quad . \tag{9}$$

Natural frequencies and mode shapes of coupled tube/fluid vibration are computed from the undamped homogeneous equations

$$[K]\{\bar{Q}\} = \omega^2[M]\{\bar{Q}\} \quad . \tag{10}$$

Equation (10) will give 2k natural frequencies for a group of k tubes.

Using the normal modes, Eq. (7) can be reduced to a set of 2k uncoupled modal equations. Then the response to an excitation can easily be calculated.

V. COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS

In the analysis, uncoupled natural frequencies measured in air are used to establish the torsional spring constant k_i. Then the uncoupled natural frequency Ω_i and modal function ϕ_i in water for each tube in the x and y directions are calculated. Coupled natural frequencies and mode shapes are determined from Eq. (10). Once the normal modes of coupled motion are determined, tube responses to any excitation can be calculated rather easily.

A. Uncoupled Natural Frequencies

Uncoupled natural frequencies, modal damping ratio, and related parameters are given in Tables 1 to 6,* where the dimensionless spring constant p_i is defined by $\frac{k_i \ell}{E_i I_i}$ (ℓ = tube length, E_i = Young's modulus, and I_i = moment of inertia). The nominal gap-to-radius ratios are given in Fig. 2. However, in the tested assemblies, the average gap-to-radius ratio at the free end is different from that at the base. In computations, the average value is used; those values are given in the tables in parentheses.

It can be seen from Tables 1-6 that the calculated values are in reasonable agreement with the experimental data. In general, the measured frequencies are somewhat lower than the calculated values; this is believed to be due to the fact that wedges are employed in experiments to hold the tubes such that the tubes will be closer to the measured tube and the added mass effect will be larger than calculated values. It is noted that the modal damping in water is much larger than that in air. Since the tubes are soldered to the base, damping is small.

B. Coupled Natural Frequencies

For a group of k tubes, there are 2k coupled natural frequencies. These coupled natural frequencies from experiments and analyses are shown in Tables 7-13.

Theoretically, the motions for a row of tubes in the in-plane and out-of-plane are uncoupled [2]. Experimental results show that they indeed are independent of each other. Therefore, the results for the cases of tube rows are presented in two groups in the x and y directions.

The calculated and measured values of coupled natural frequencies agree very well. For partially submerged cases, as the fluid depth

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*Tables begin on page 36.

becomes small, fluid coupling decreases and the theory and experiment do not agree very well. This is attributed to the base coupling which is not accounted for in the theory.

The damping ratio shown in Tables 7-12 is calculated from Eq. (6) assuming that the damping coefficient $c_i = 2\Omega_i(m_i + \gamma_{ii})\zeta_i$, where ζ_i is the uncoupled modal damping ratio in water. In general, the damping ratio for coupled modes increases as the frequency increases and its magnitude is approximately the same as those for uncoupled modes.

C. Mode Shapes

The theoretical mode shapes for coupled modes are obtained from Eq. (10), while the experimental mode shapes are obtained using the acceleration-time traces recorded from steady-state excitation at a resonance frequency. For simplicity, only three typical cases are presented in Figs. 10-12, where the experimental frequencies are the driving frequencies.

The arrows on the figures indicate the relative position for each tube, and the lengths of the lines represent the magnitudes of tube displacements. Comparing the theoretical and experimental results, one can see that the agreement is very good. There are a few exceptions. For example in the case of 5 tubes, tube 1 has a relatively large component in the in-plane direction. This may be associated with the fact that the tubes may not be precisely parallel or the principal axes of tube 1 may not be in the x and y directions. The other examples are those of modes 3 and 5 for 3 tubes. In those modes, the theoretical and experimental results do not agree well. This can be explained easily. The tube assembly is excited at tube 3 in the y direction. However, corresponding to those modes, tube 3 has a very small displacement component; i.e., the tube assembly is not easily excited by applying force to tube 3. At those frequencies, not only those natural modes are excited, but other modes also are excited. Therefore, the experimental results cannot represent the actual modes but are combinations of several natural modes. If the assembly were excited at the other tubes, correct experimental results would have been obtained for those two modes. Modes 4 and 6 for four tubes are also not in agreement; this is attributed to the same reason.

D. Steady-State Responses

The excitation force is proportional to the input current. Its magnitude is small and not easily calibrated. Therefore, the excitation force is not measured directly in experiments. In order to compare the theoretical and experimental results, the magnitude of the force used in the analysis is obtained using the experimental response values for the tube excited at a frequency equal to 50 Hz; i.e., the analytical and experimental accelerations at 50 Hz are made equal for the tube excited. The results are presented in Figs. 13 and 14 for two typical cases. It can be seen that the agreement is very good. Because of fluid coupling, those tubes not directly excited have large response in the frequency band containing coupled natural frequencies of the system.

E. Transient Responses

An example of the transient responses for 5 tubes is presented in Fig. 15. Similar results are obtained for other cases. For conciseness, those results are not presented here. The transient responses are obtained by plucking. Since the excitation force is not well defined, it is difficult to compare with analytical calculations. Therefore, no analytical results are presented for transient responses.

Based on the experimental results we note that there exist beating phenomena for all tubes. This occurs because there are five natural frequencies which are relatively close to one another. The disturbance was given to tube 1, it has a large response instantly. On the other hand, the peak amplitudes occur later for other tubes; it takes time to develop large tube oscillations.

VI. EFFECTS OF VARIOUS PARAMETERS ON TUBE RESPONSES

A. Gap-to-Radius Ratio

For a group of tubes vibrating in a liquid, the frequency band of coupled modes [2] becomes wider as the gap-to-radius ratio (G/R) decreases. This can be seen clearly from Table 8 for 3-tube arrays. Based on the experimental data, the limits of the frequency band are 64.02 Hz to 71.52 Hz for G/R = 2.0, and 57.42 Hz to 74.47 Hz for G/R = 0.5. Other tube arrays also show similar effect. This is attributed to fluid inertia coupling; as the gap decreases, the fluid coupling becomes larger.

B. Flat Wall

Figure 16 shows the experimental data for the natural frequencies of coupled modes as a function of the ratio of the gap to tube radius (G_w/R). As G_w/R decreases, the frequency band becomes wider; the wall effect is similar to that of reducing the tube pitch (G/R).

C. Eccentricity of Outside Container

Figure 17 shows the effect of the eccentricity (G_e/R) on natural frequencies (experimental data). Most of the natural frequencies decrease with increasing G_e/R, but the fourth mode increases. However, the effect is small.

D. Water Depth

The variations of coupled natural frequencies (experimental data) with fluid depth are shown in Fig. 18. Two effects are observed as the fluid depth increases: (1) the frequency band becomes wider, (2) the coupled natural frequencies become lower.

E. Fluid Viscosity

The natural frequencies of uncoupled and coupled vibrations and damping are shown in Table 14 and 15 for the 4-tube array vibrating in water and mineral oil (specific gravity = 0.935, and viscosity = 41 cp). The effect of fluid viscosity on damping is significant, but its effect on natural frequencies is small. The role of fluid viscosity on coupled vibration of multiple tubes is similar to that of two coaxial shells [5].

F. Uncoupled Vibration

Figures 19 and 20 show the steady-state responses of two-tube arrays under two different conditions: (a) all other tubes are held rigid except the one being excited; and (b) all tubes are free to vibrate. It can be seen that the tube responds differently under two different conditions. This illustrates that using uncoupled modes to study multiple-tube problem may result in error.

VII. CONCLUSIONS

Four series of tests have been performed for uncoupled and coupled vibrations of tube bundles in liquids. The experimental data and analytical results for natural frequencies, mode shapes, and forced responses are in good agreement. Therefore, the analytical method developed earlier [1,3] is basically sound and can be used with confidence for tube bundles vibrating in liquid.

Although extensive studies of vibration of tube bundles have been reported, this study is the first known systematic experimental investigation to obtain the detailed information of coupled tube/fluid vibration. In the past, predictions of tube responses in liquid were frequently based on uncoupled mode of a single tube. From the analytical and experimental results presented in this paper, it is clear that in predicting tube bundle responses coupled modes should be employed.

ACKNOWLEDGMENTS

This work was performed under the sponsorship of the Division of Reactor Development and Demonstration, U. S. Energy Research and Development Administration.

Credit is due to Mr. H. N. Cornelius for his help in performing the test work.

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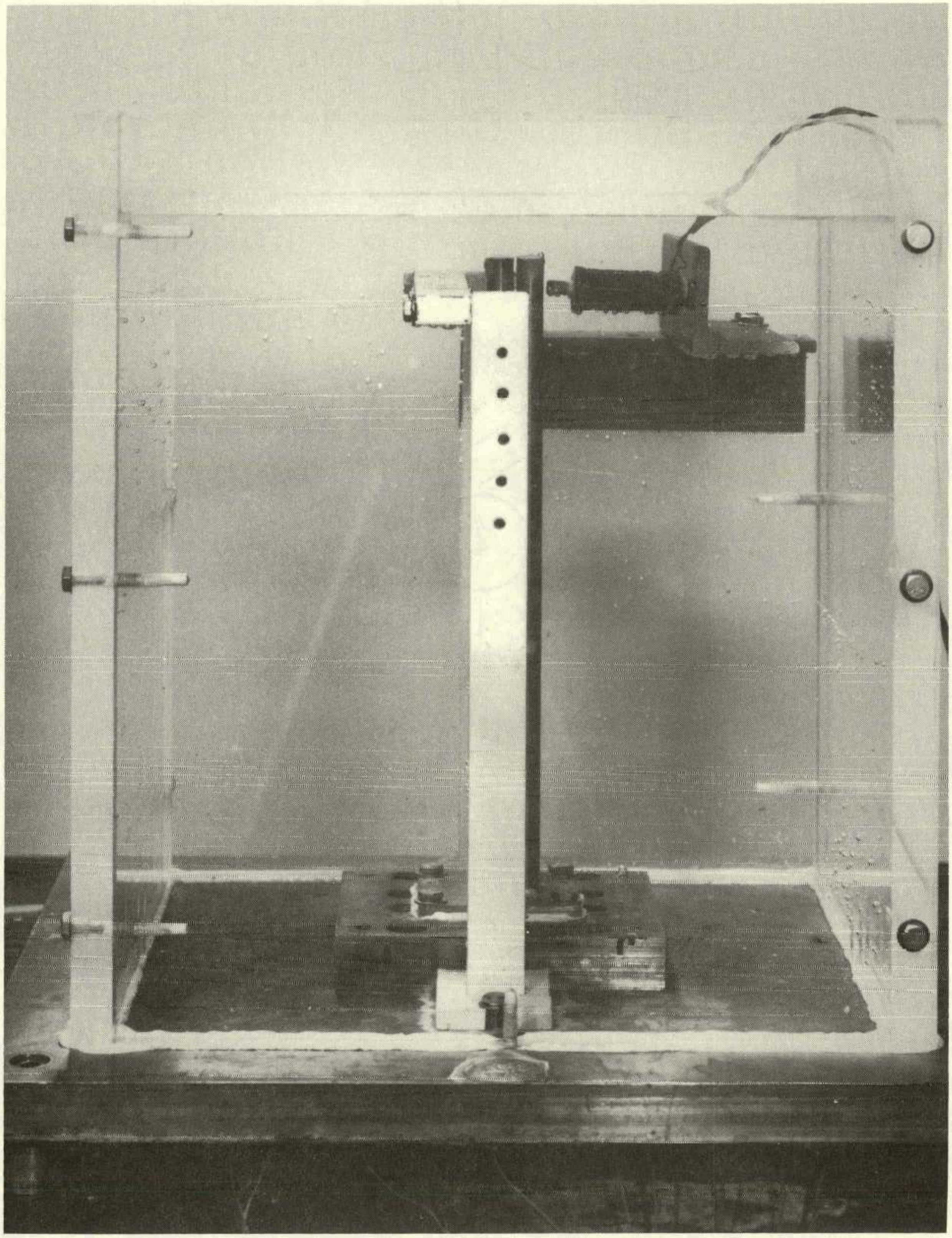

Fig. 1. Test assembly

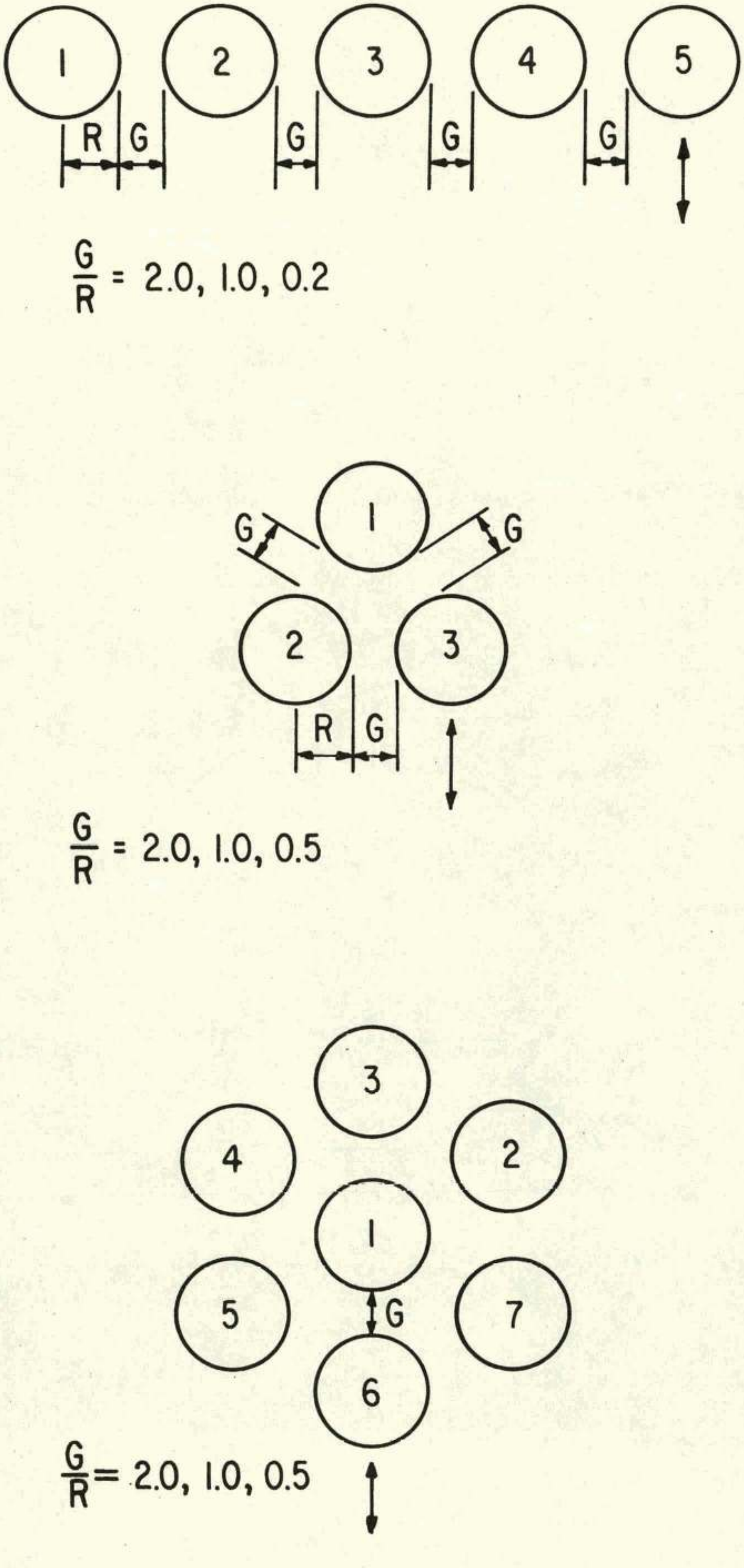

Fig. 2a. Tube arrangement

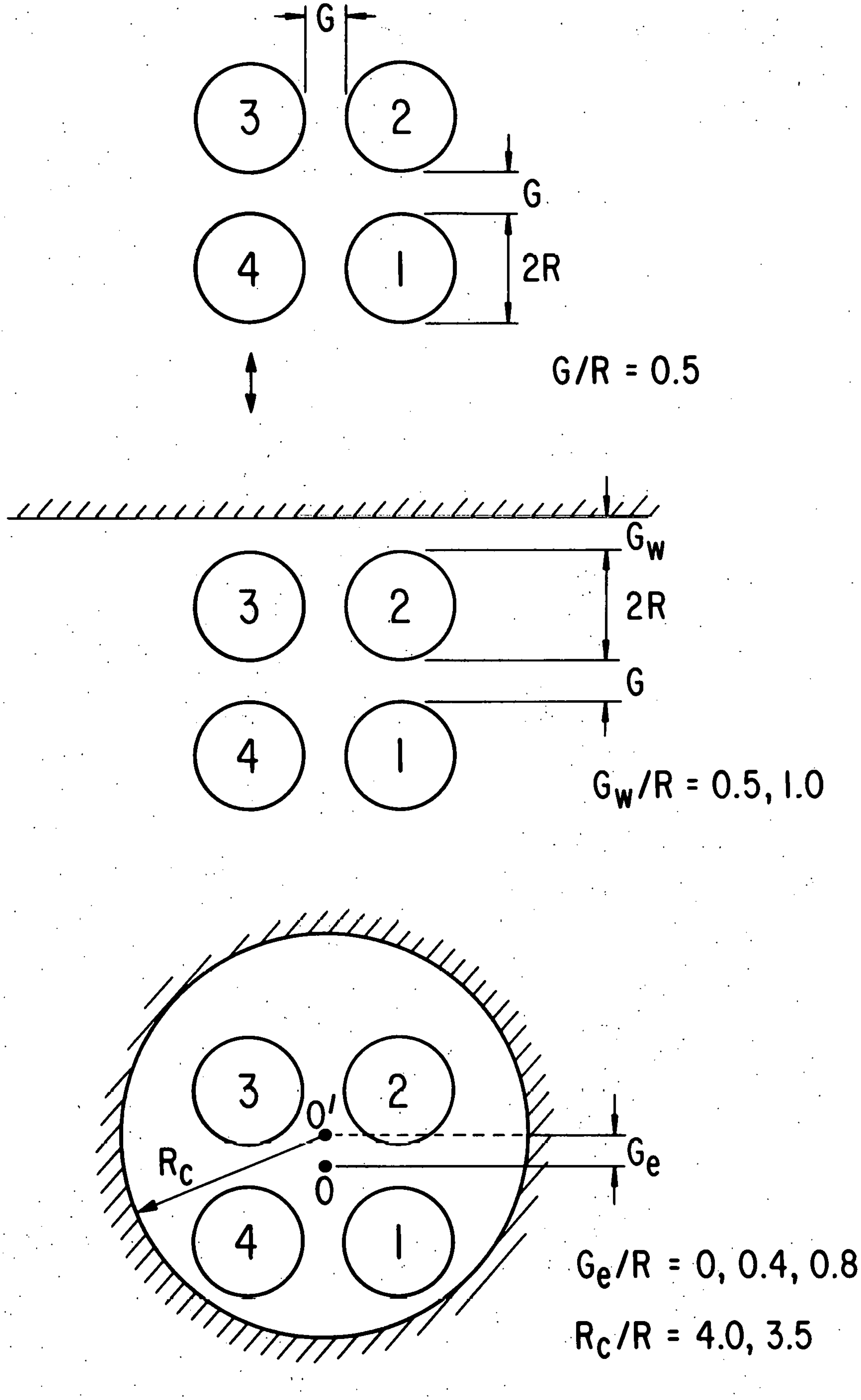

Fig. 2b. Tube arrangement

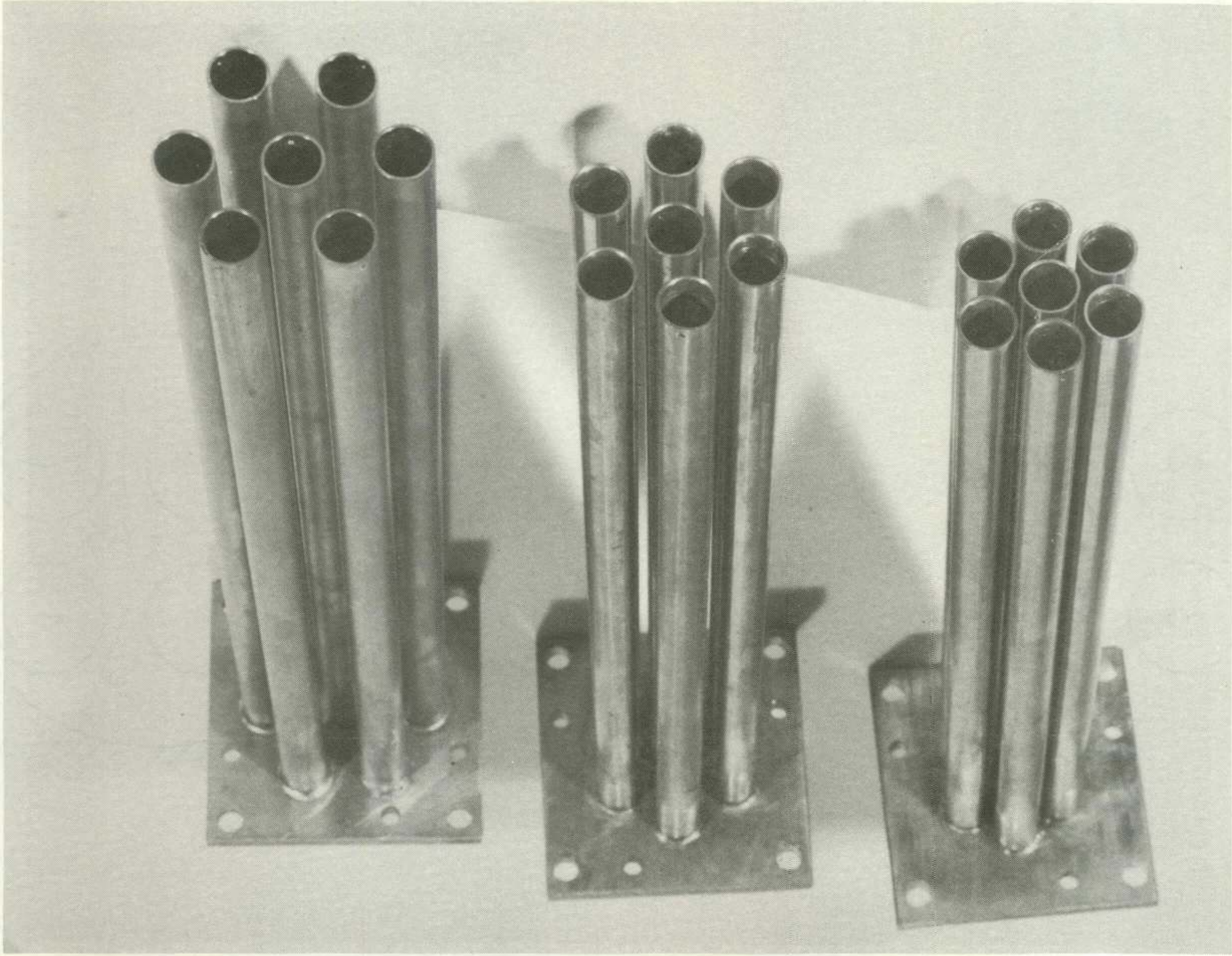

Fig. 3. Test elements

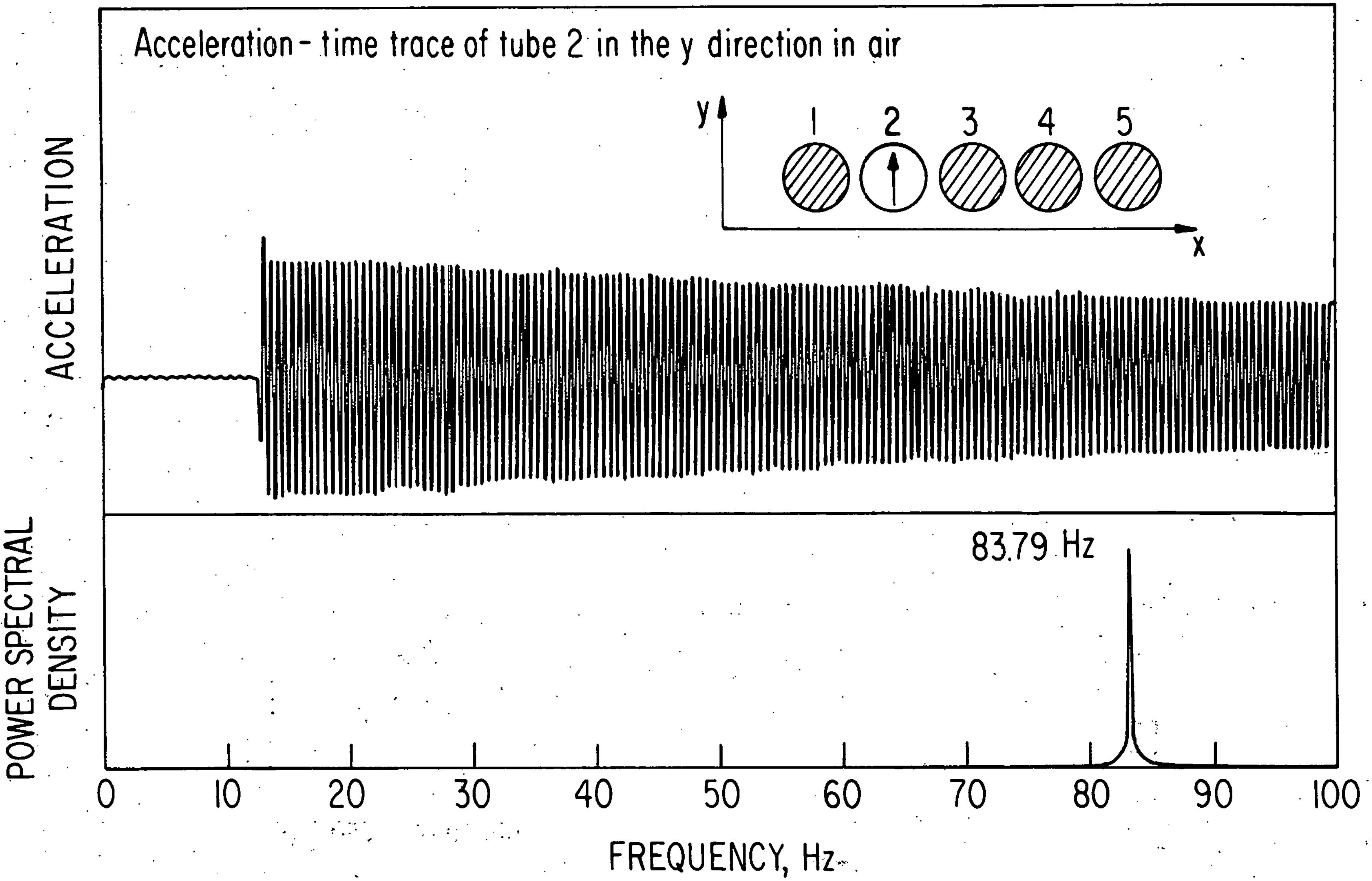

Fig. 4. Acceleration-time trace and power spectral density of tube acceleration for tube 2 in air (G/R = 0.25)

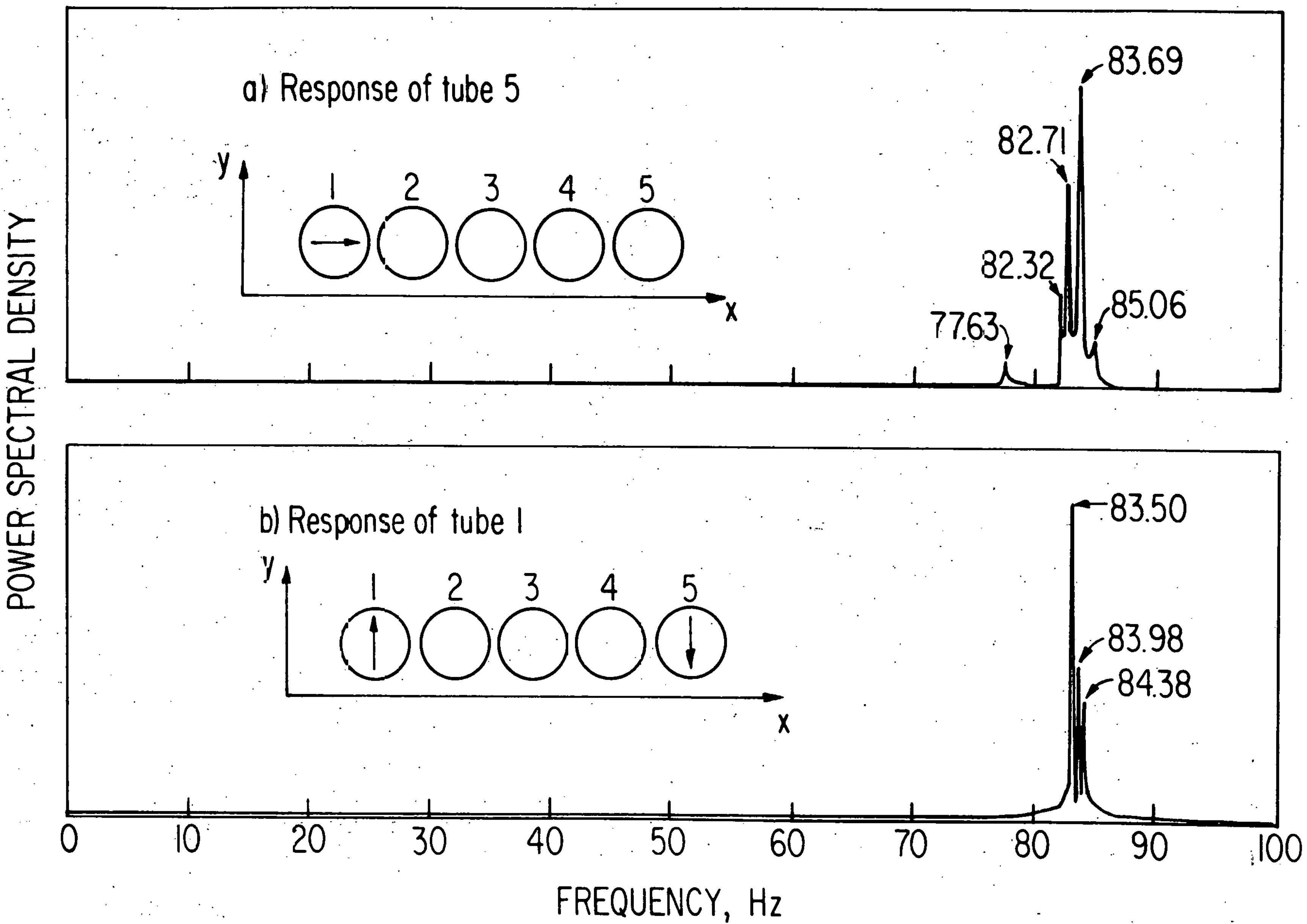

Fig. 5. Power spectral density of tube acceleration for a row of five tubes in air (G/R = 0.25)

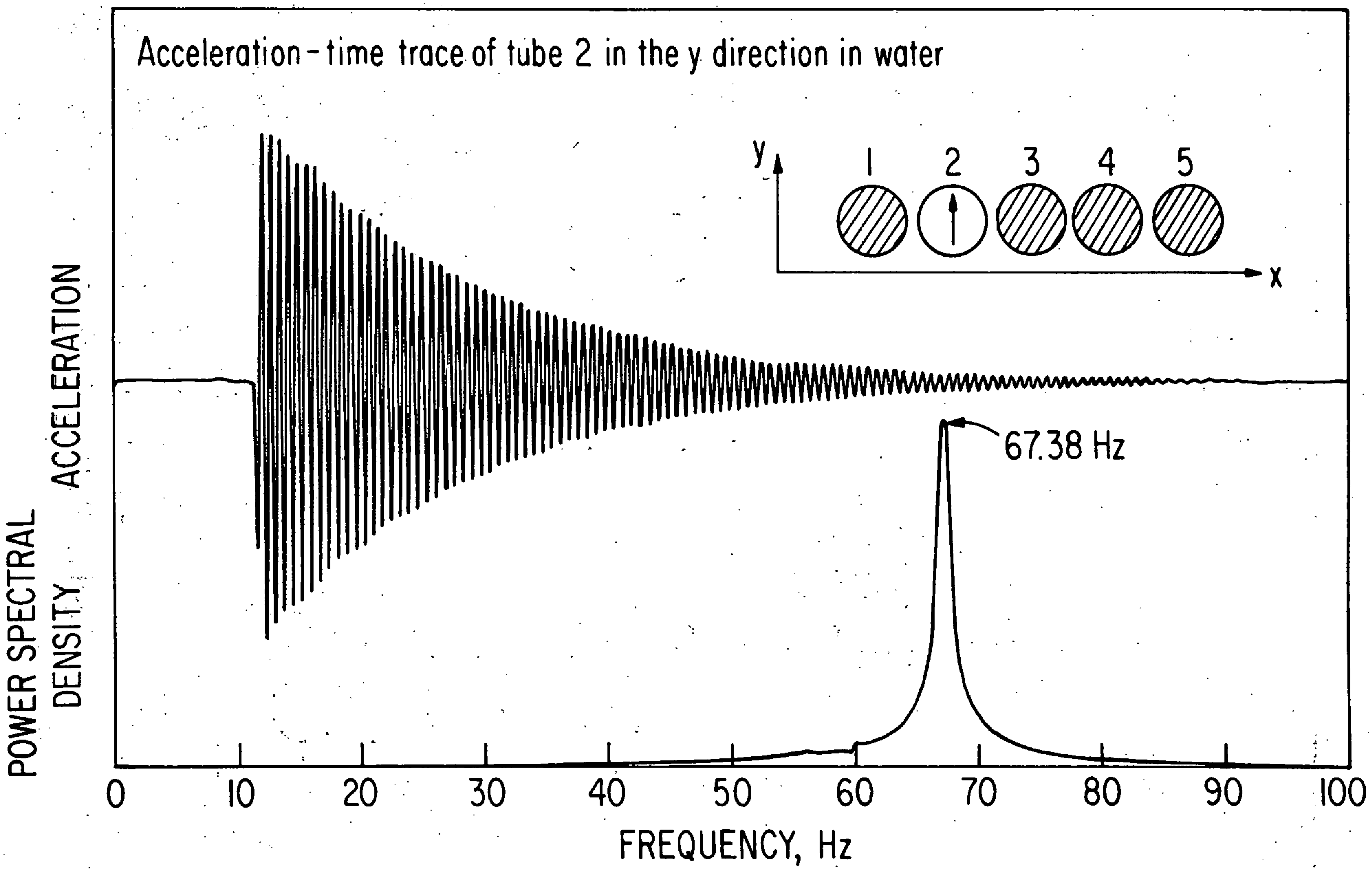

Fig. 6. Acceleration-time trace and power spectral density of acceleration for tube 2 in water (G/R = 0.25)

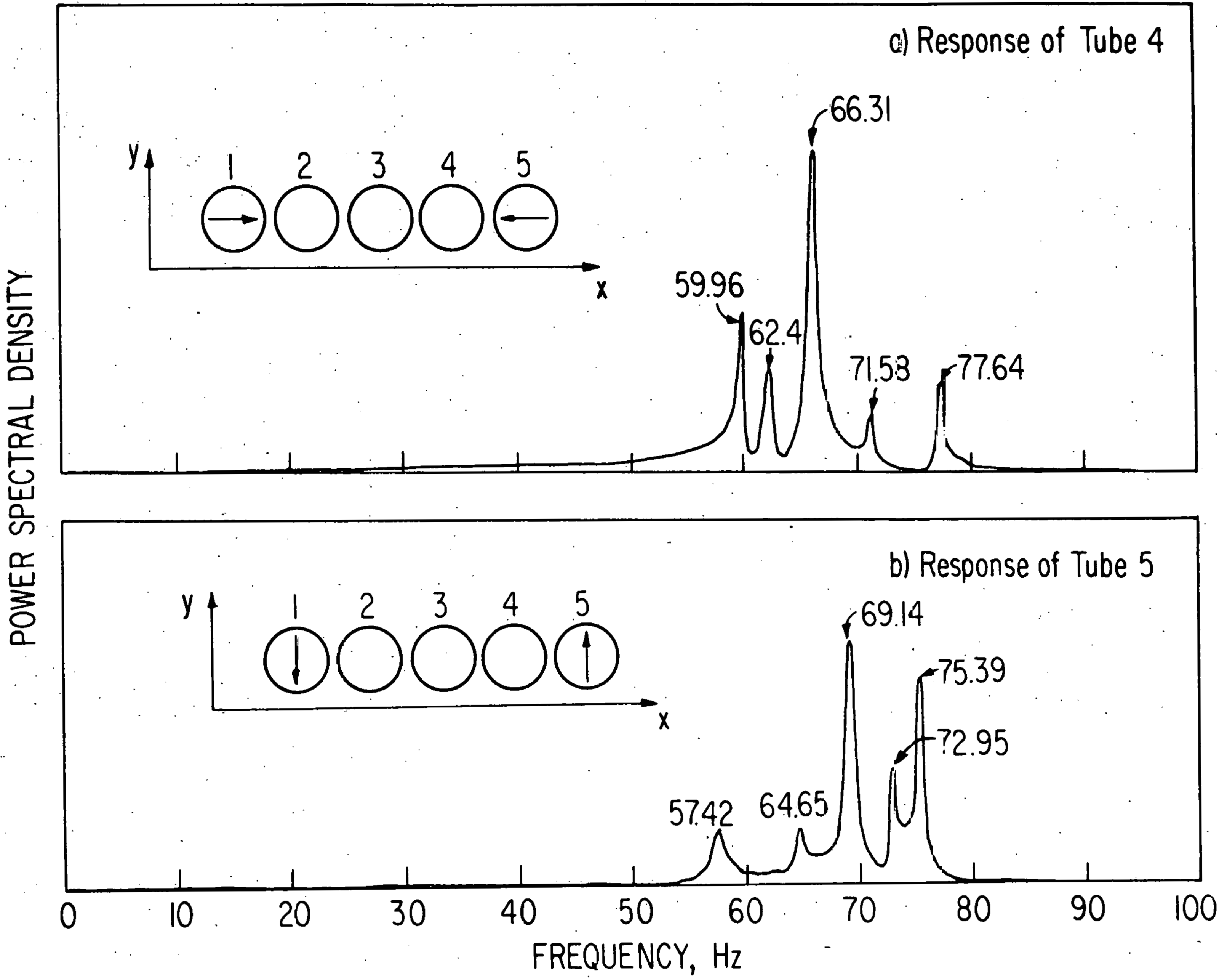

Fig. 7. Power spectral density of tube acceleration in water (G/R = 0.25)

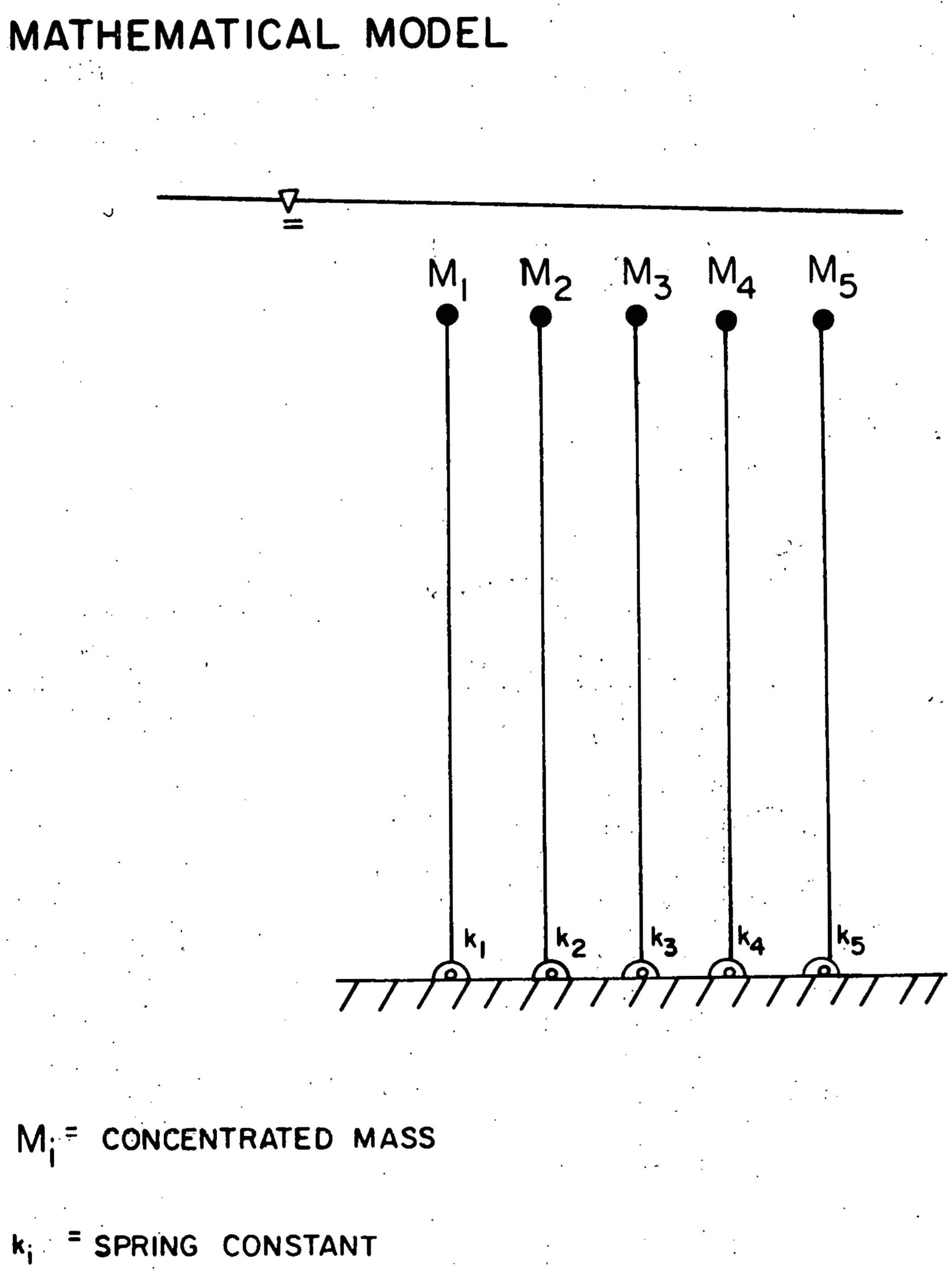

Fig. 8. Mathematical model used in analysis

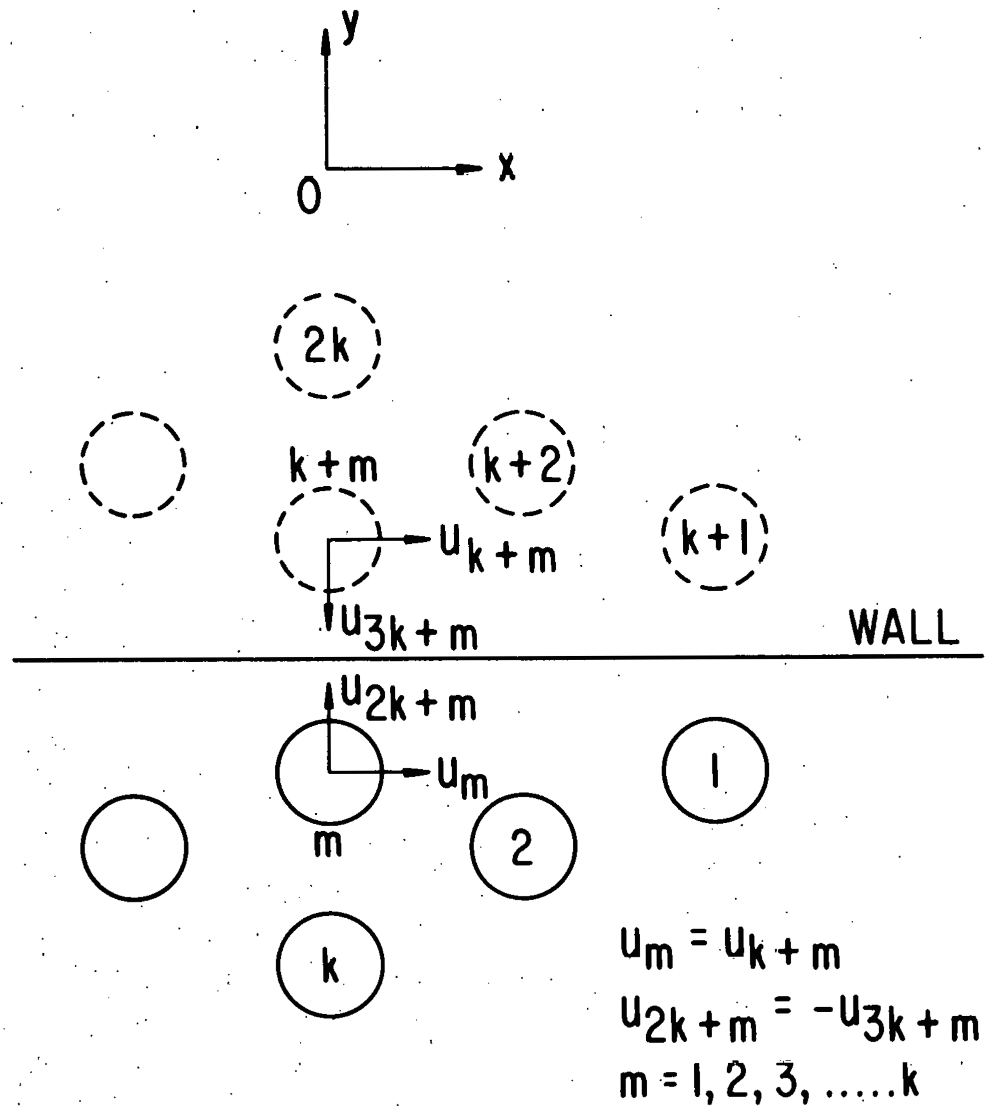

Fig. 9. Tube array near a wall

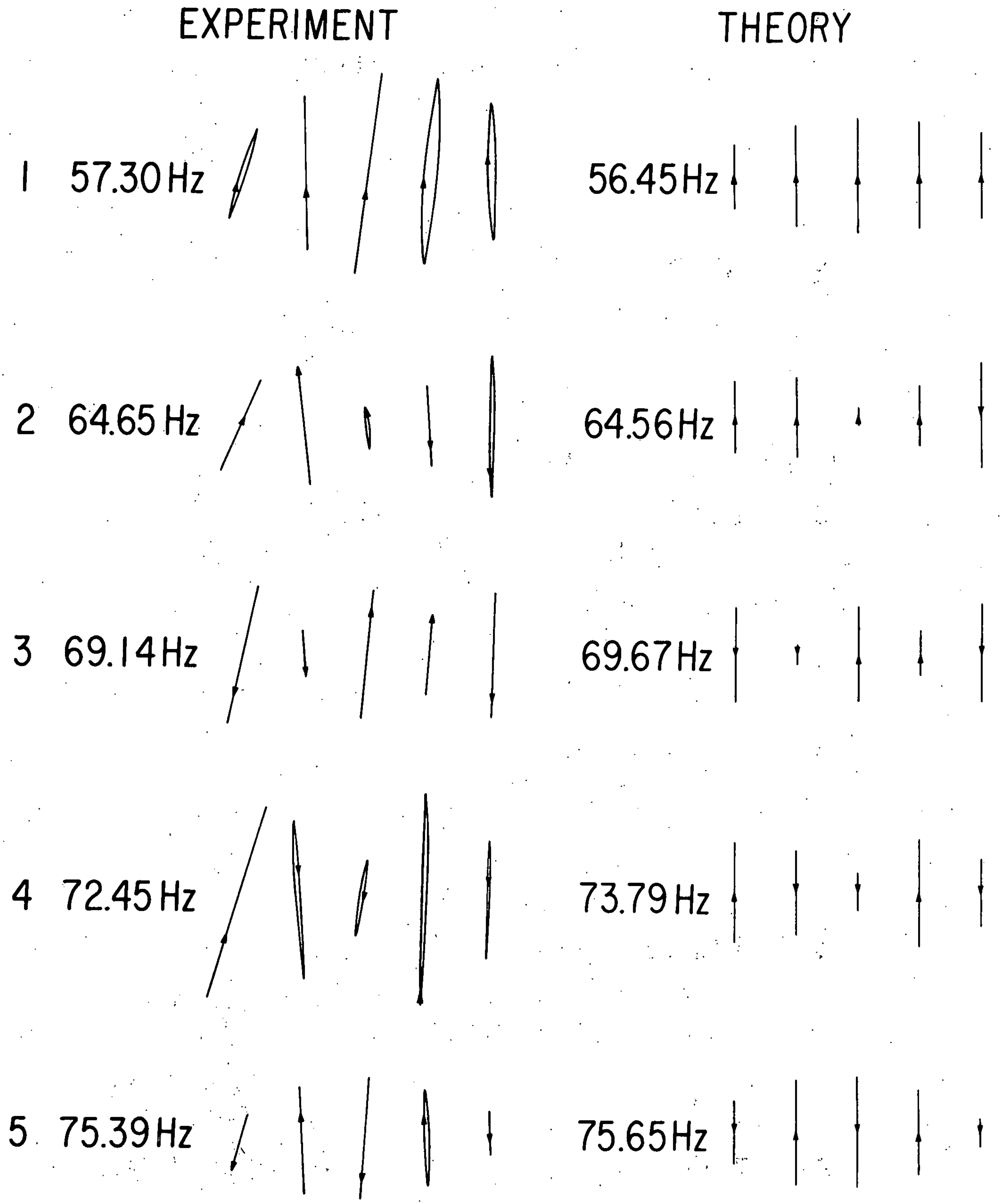

Fig. 10. Mode shapes of a row of 5 tubes with G/R = 0.25

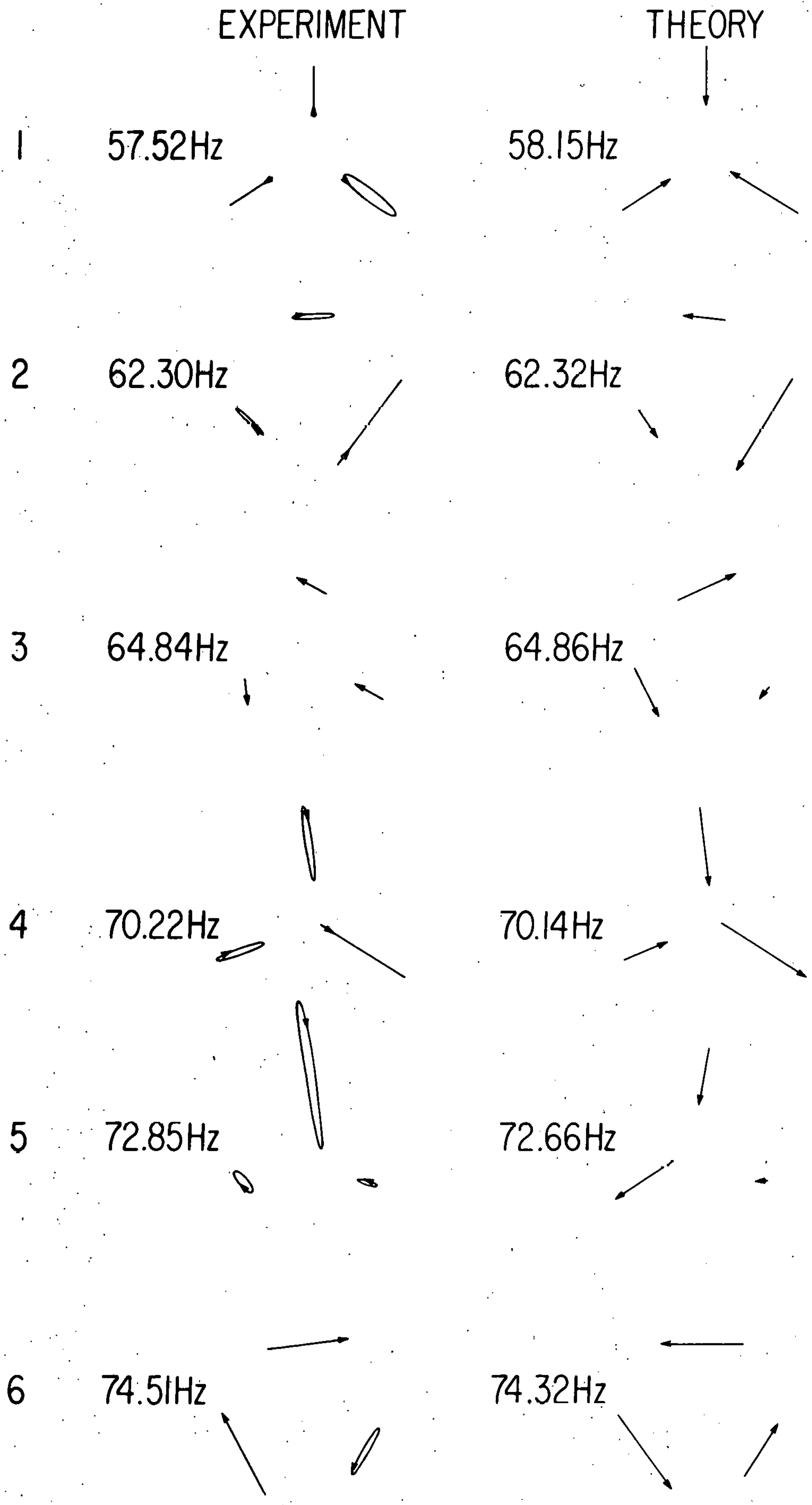

Fig. 11. Mode shapes of a group of 3 tubes with G/R = 0.5

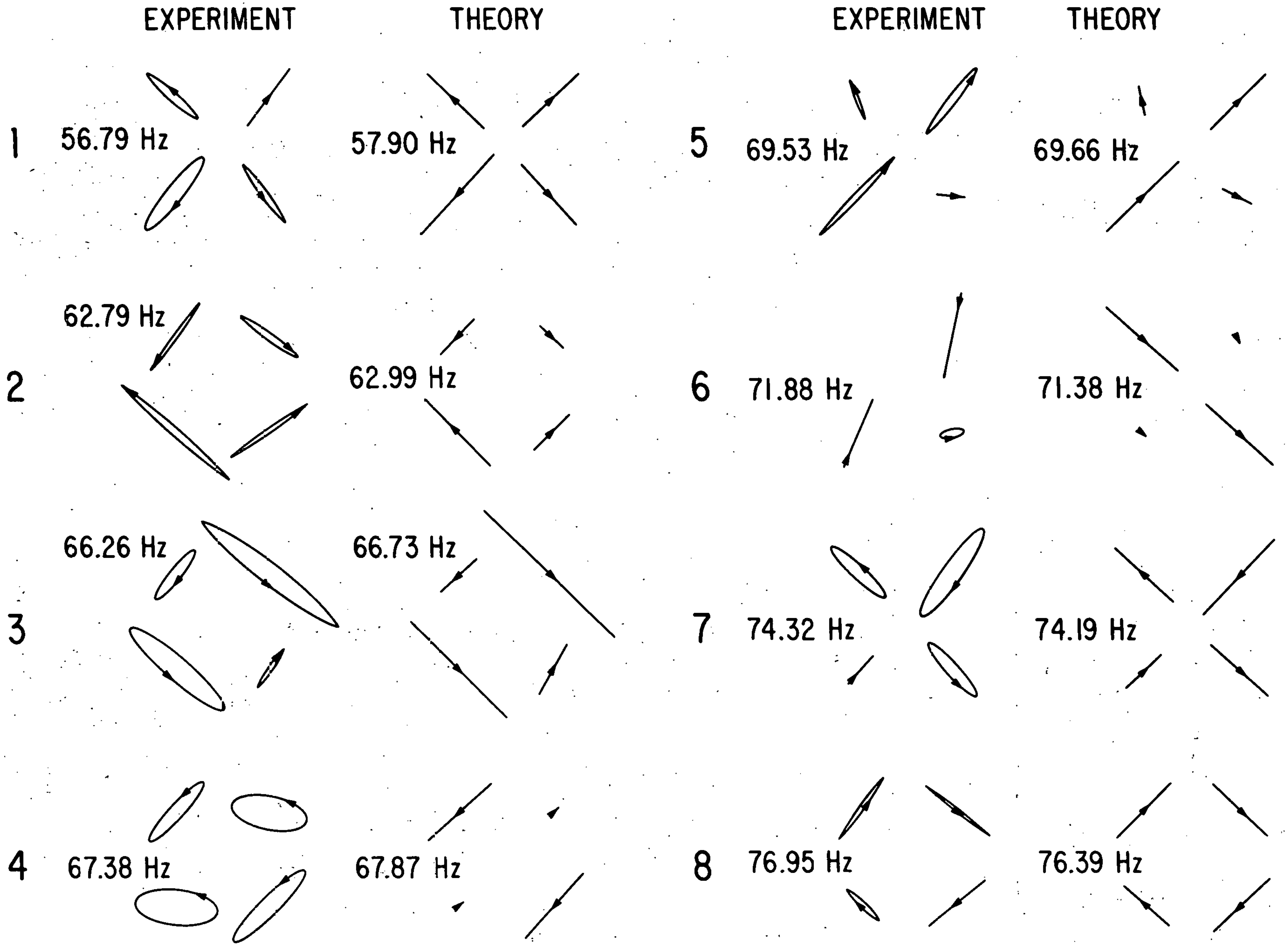

Fig. 12. Mode shapes of a group of four tubes in unconfined water

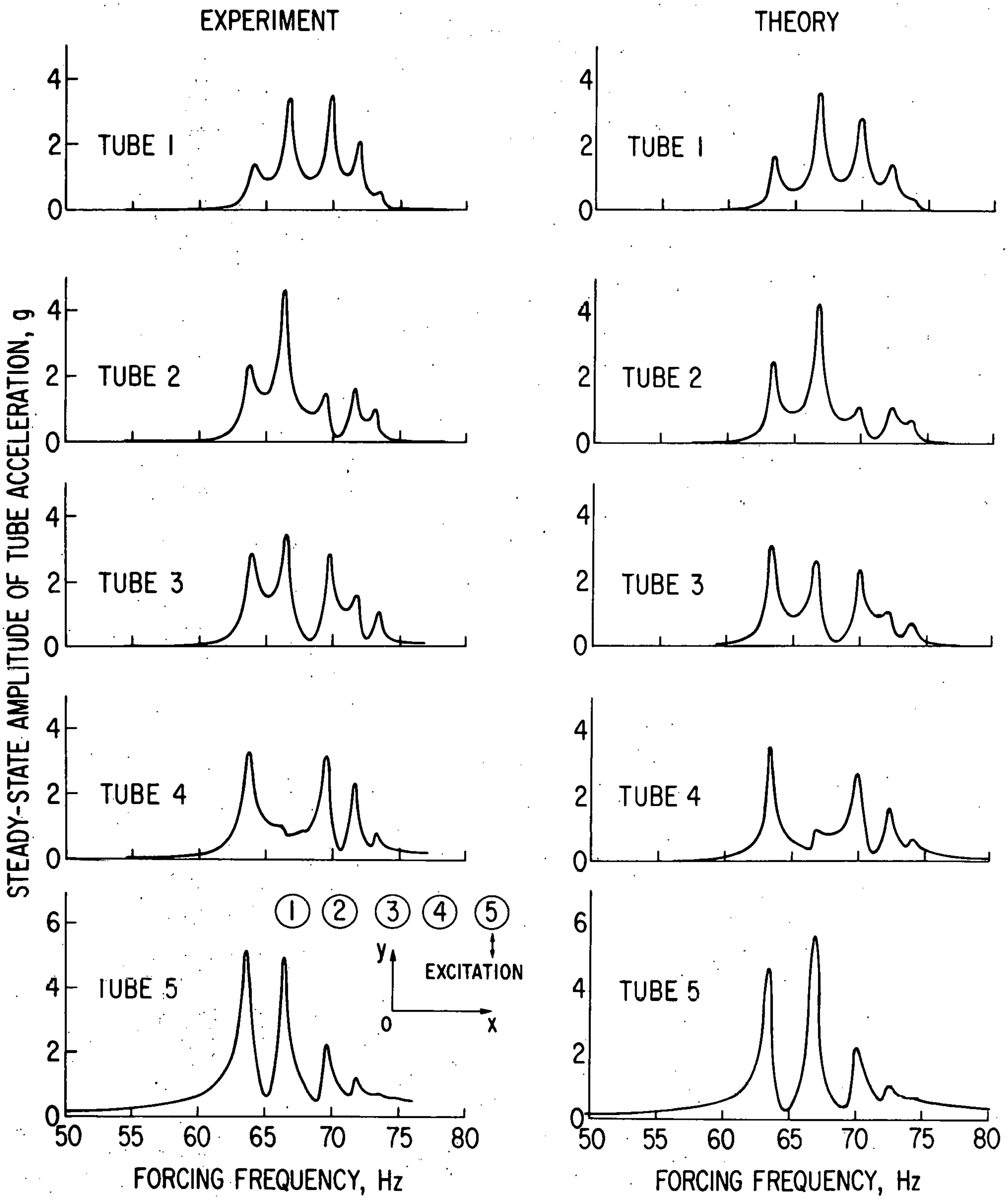

Fig. 13. Steady-state responses of a row of 5 tubes to an excitation on tube 5 with G/R = 1.0

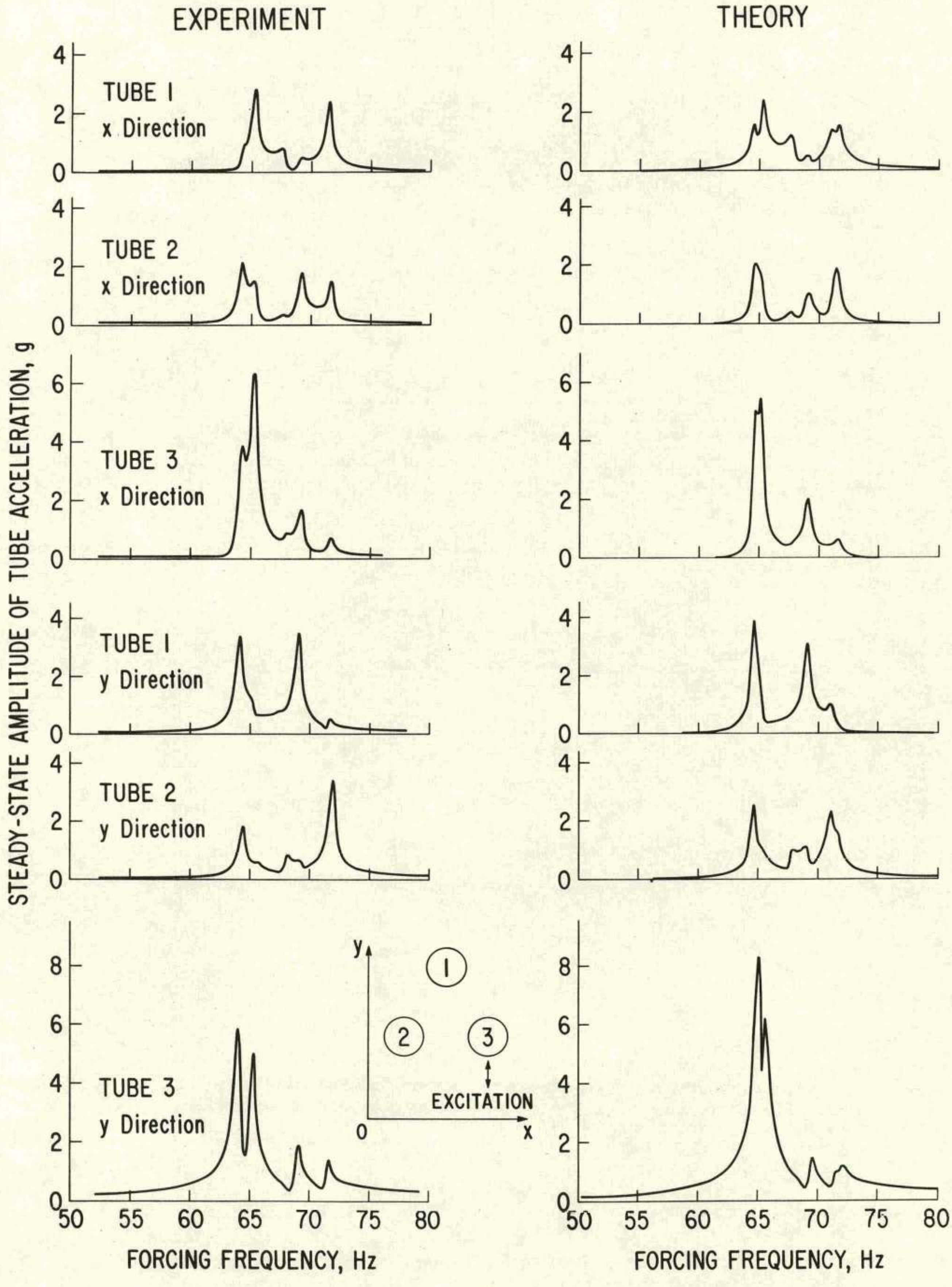

Fig. 14. Steady-state responses of a group of 3 tubes to an excitation on tube 3 with G/R = 2.0

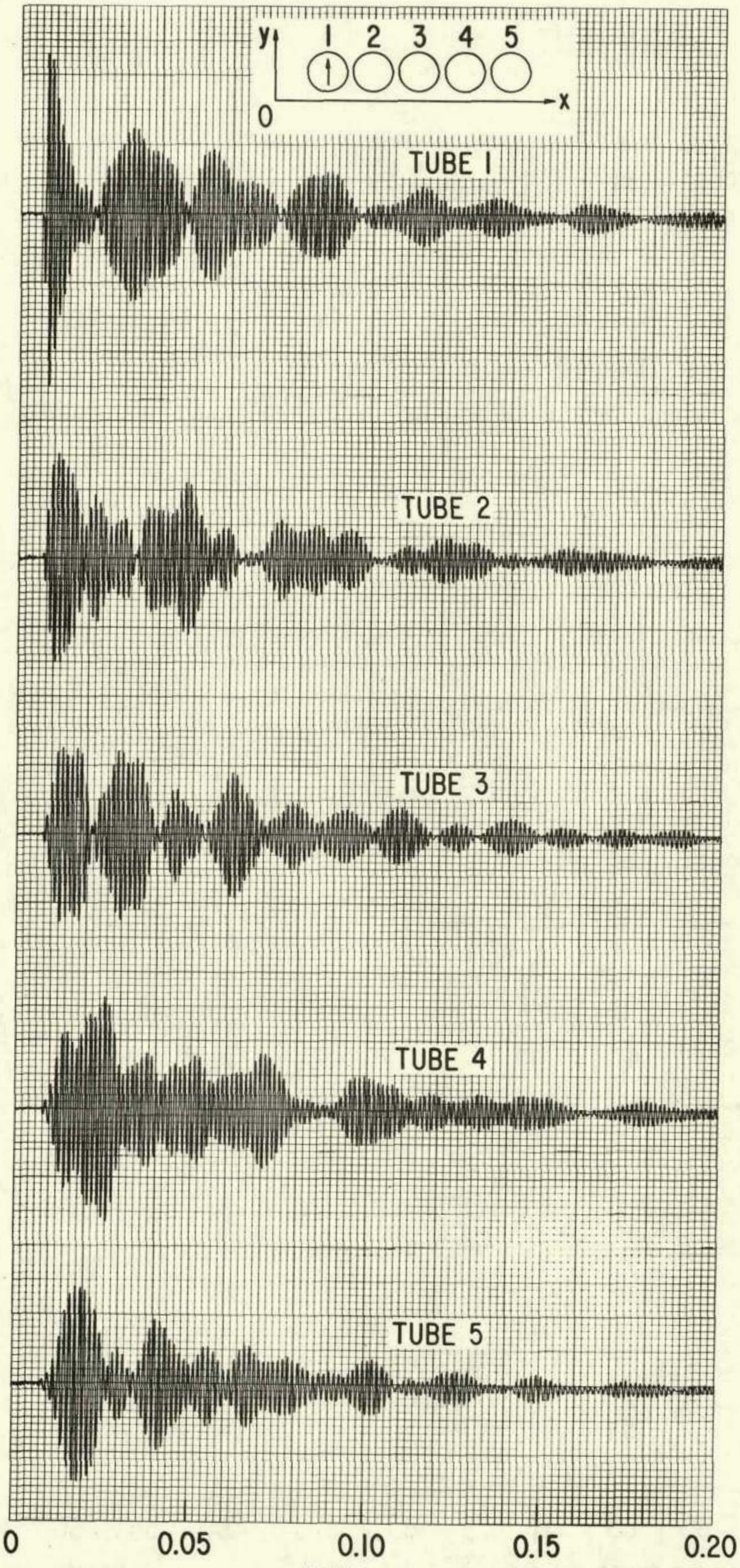

Fig. 15. Acceleration-time traces of a row of 5 tubes to an initial disturbance on tube 1 with G/R = 0.2

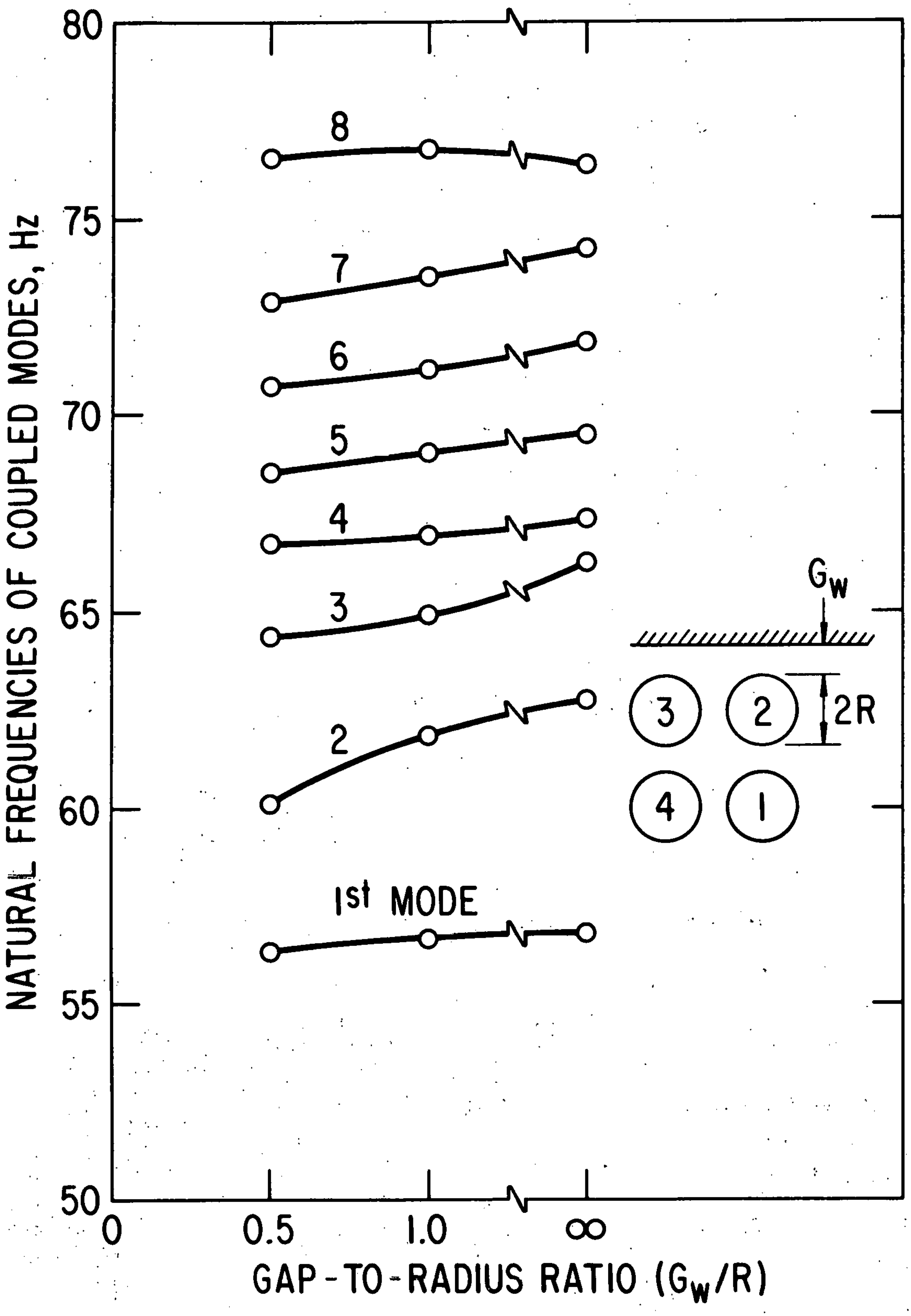

Fig. 16. Natural frequencies as a function of the gap to radius ratio, G_w/R.

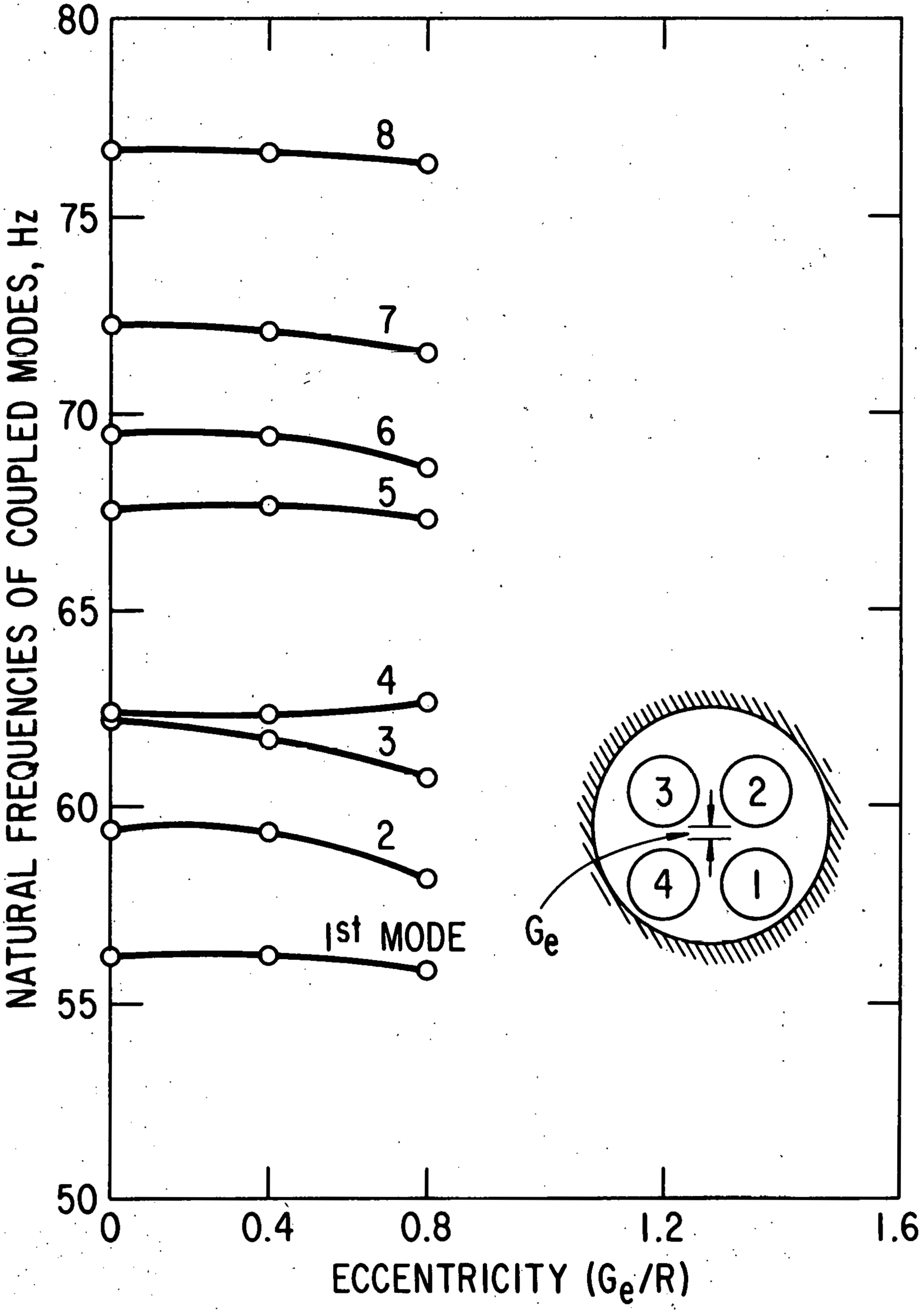

Fig. 17. Natural frequencies as a function of the eccentricity, G_e/R

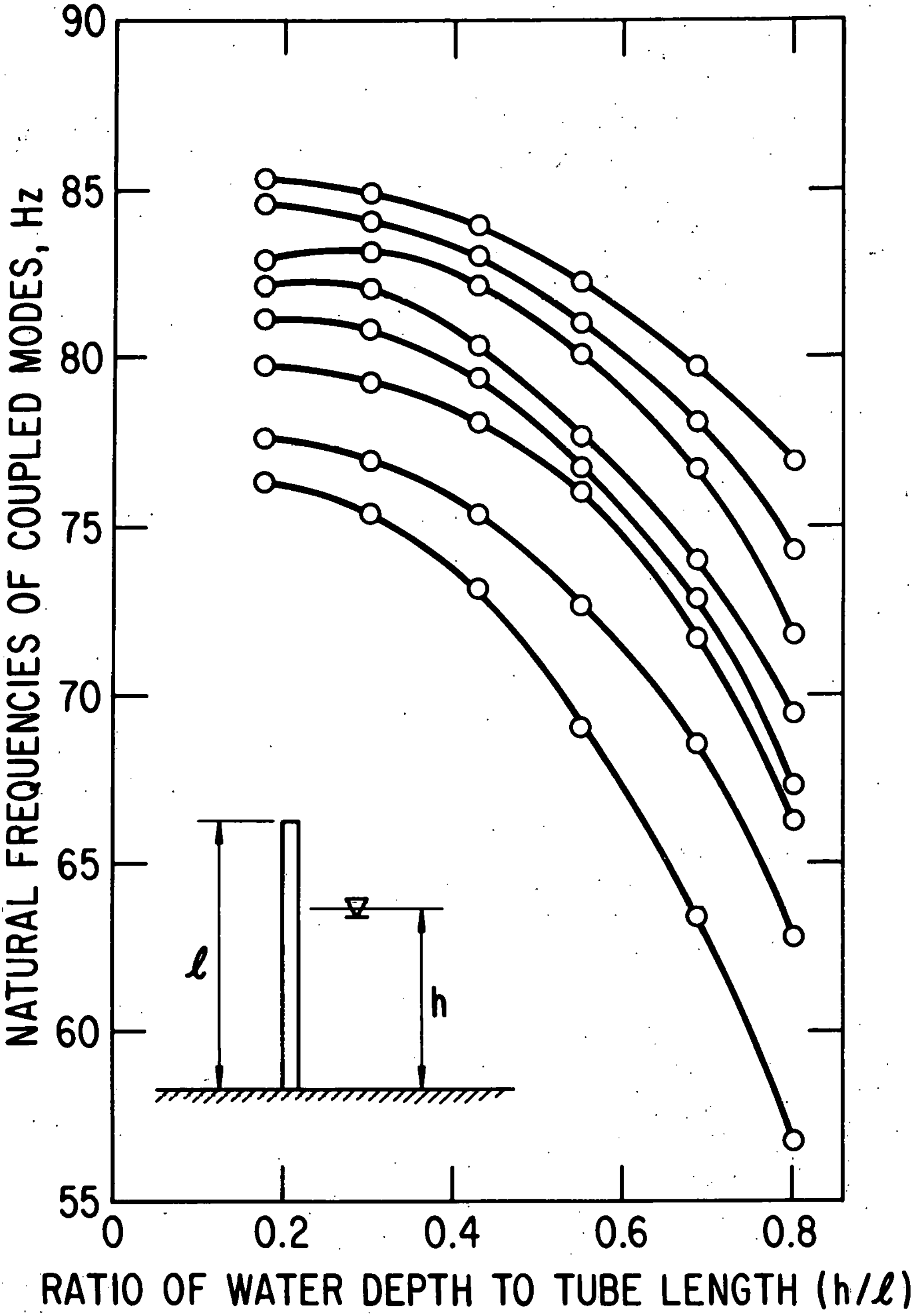

Fig. 18. Natural frequencies as a function of the ratio of water depth to tube length (h/ℓ)

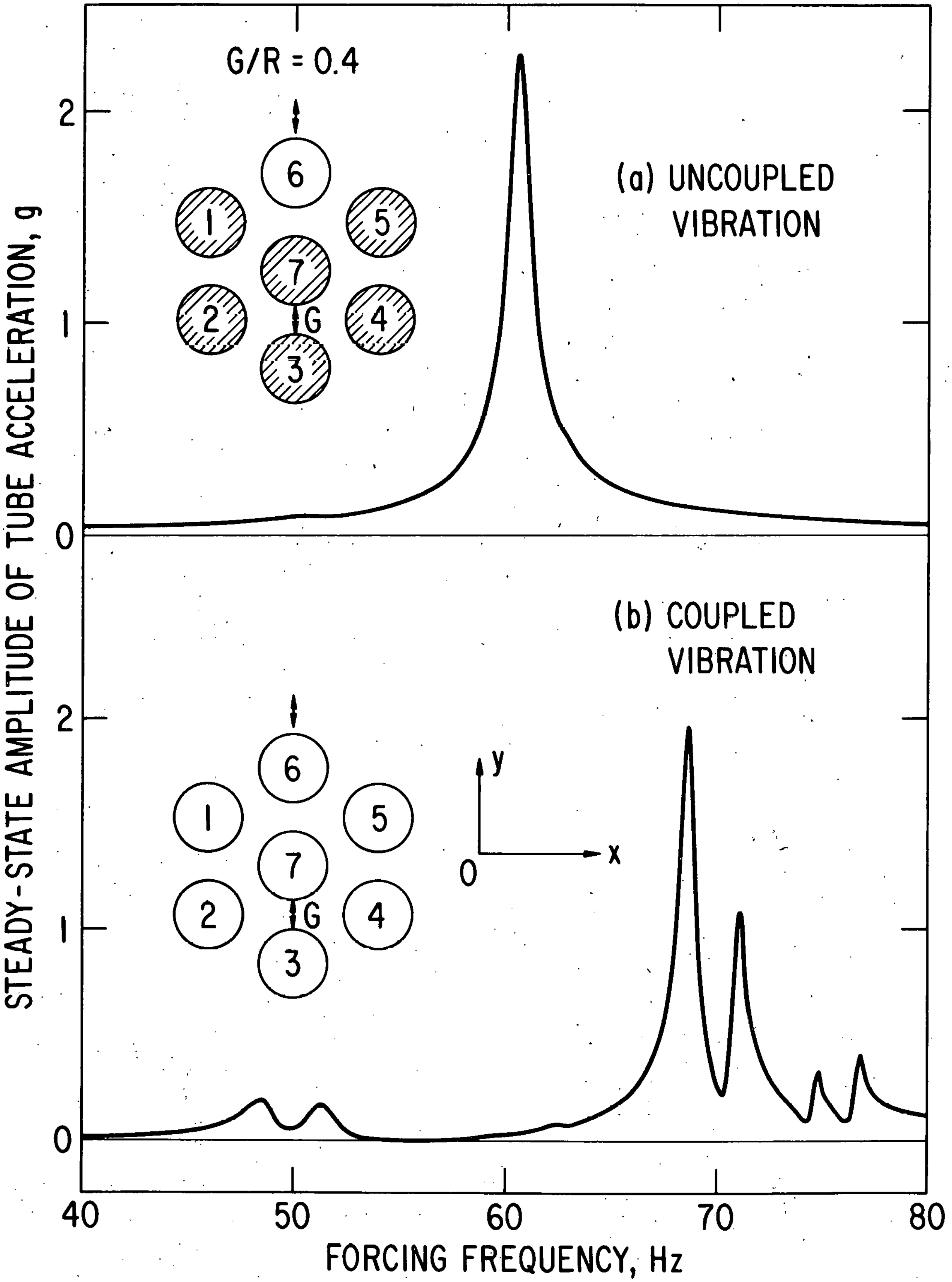

Fig. 19. Response of tube 6 in the y direction based on uncoupled and coupled vibrations for a 7-tube array (G/R = 0.4) in water

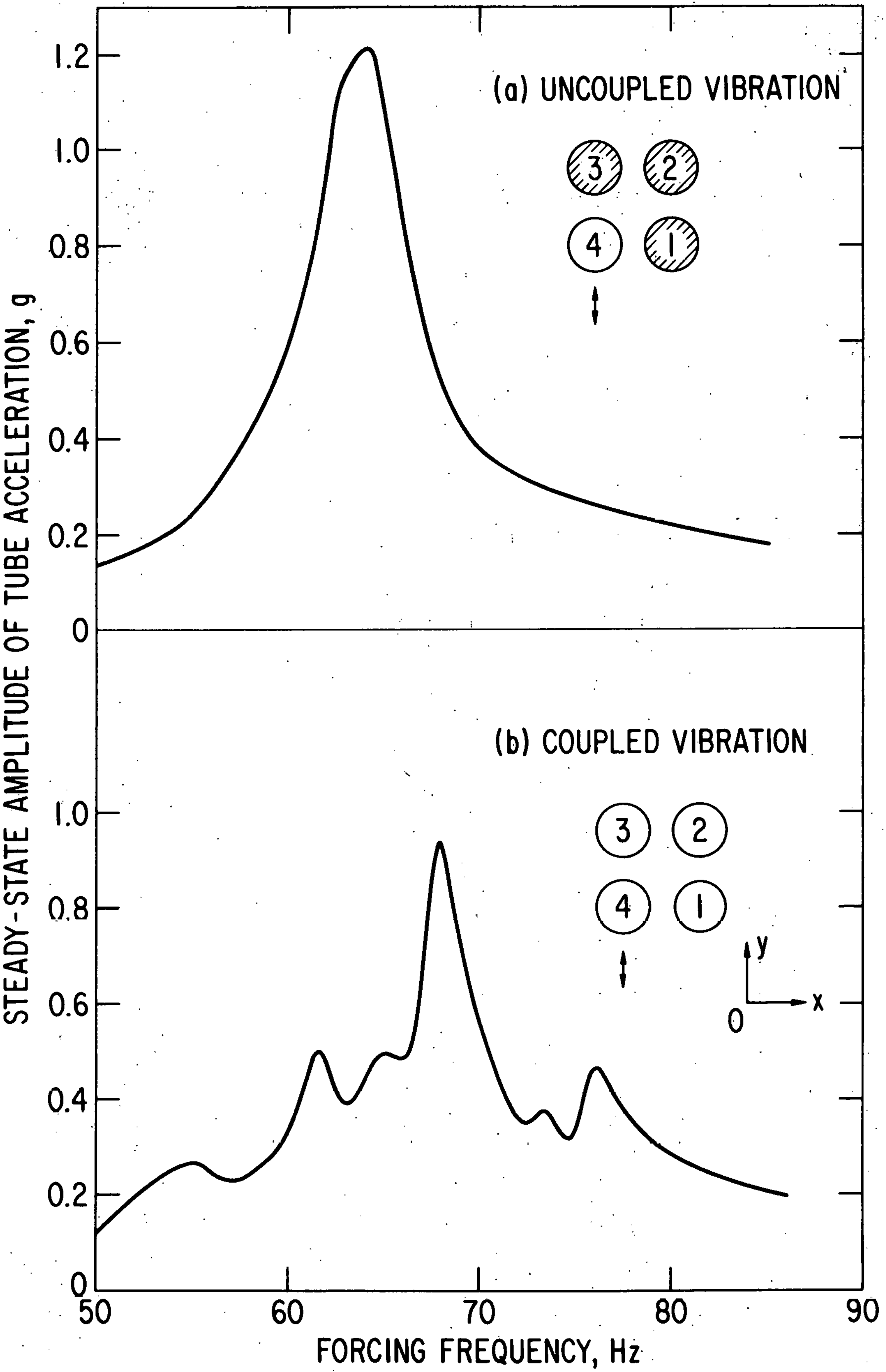

Fig. 20. Response of tube 4 in the y direction based on uncoupled and coupled vibrations for the 4-tube array in unconfined mineral oil

Table 1. Experimental and analytical results for uncoupled vibration of a row of five tubes

Gap-to-Radius Ratio (G/R)	Direction of Motion	Tube Number	Dimensionless Spring Constant (p_i)	Measured Uncoupled Natural Frequency, Hz		Measured Damping Ratio		Calculated Uncoupled Natural Frequency in Water, Hz
				In Air	In Water	In Air	In Water	
2.0 (1.988)	x	1	104.3	83.79	69.63	0.00118	0.0044	70.07
		2	99.0	84.67	70.41	0.00095	0.0049	70.58
		3	128.5	84.27	69.92	0.00031	0.0053	70.39
		4	155.2	84.38	70.21	0.00028	0.0045	70.57
		5	354.0	77.93	65.50	0.00062	0.0043	66.87
	y	1	129.0	84.08	70.02	0.00113	0.0042	70.31
		2	129.3	85.05	70.90	0.00032	0.0036	70.89
		3	242.6	84.86	70.51	0.00103	0.0044	70.86
		4	288.5	84.86	70.80	0.00078	0.0034	70.96
		5	354.0	77.93	66.70	0.00148	0.0042	66.87
1.0 (0.981)	x	1	105.1	83.40	69.24	0.00044	0.0063	69.54
		2	84.2	83.79	68.75	0.00052	0.0078	69.39
		3	98.3	83.69	68.85	0.00051	0.0088	69.43
		4	86.5	83.01	68.26	0.00063	0.0099	68.96
		5	186.6	77.05	64.94	0.00103	0.0047	65.93
	y	1	155.8	83.89	69.92	0.00032	0.0037	69.91
		2	114.1	84.28	69.53	0.00054	0.0043	69.72
		3	154.9	84.28	69.63	0.00073	0.0044	69.82
		4	127.4	83.59	69.04	0.00152	0.0051	69.37
		5	253.4	77.25	65.53	0.00092	0.0039	66.07
0.25 (0.248)	x	1	99.2	83.40	68.55	0.00093	0.0073	68.45
		2	85.2	83.50	66.99	0.00045	0.0124	67.08
		3	92.0	83.50	67.29	0.00146	0.0137	67.23
		4	97.2	83.40	67.38	0.00131	0.0190	67.28
		5	325.2	77.73	64.55	0.00108	0.0081	65.63
	y	1	131.5	83.79	69.24	0.00046	0.0062	68.52
		2	101.1	83.79	67.38	0.00055	0.0076	66.70
		3	97.2	83.59	66.80	0.00088	0.0070	66.53
		4	103.6	83.50	67.19	0.00069	0.0076	66.75
		5	930.5	78.02	65.42	0.00226	0.0046	65.66

Table 2. Experimental and analytical results for uncoupled vibration of a group of three tubes

Gap-to-Radius Ratio (G/R)	Direction of Motion	Tube Number	Dimensionless Spring Constant (p_i)	Measured Uncoupled Natural Frequency, Hz		Measured Damping Ratio		Calculated Uncoupled Natural Frequency in Water, Hz
				In Air	In Water	In Air	In Water	
2.0 (1.933)	x	1	53.2	82.03	68.35	0.00152	0.0038	68.53
		2	141.4	84.38	70.21	0.00090	0.0041	70.35
		3	386.0	77.24	65.82	0.00341	0.0041	66.22
	y	1	55.2	82.13	68.55	0.00103	0.0037	68.59
		2	79.1	83.50	69.53	0.00095	0.0045	69.63
		3	123.2	76.46	65.33	0.00217	0.0034	65.55
1.0 (0.983)	x	1	62.6	82.23	69.26	0.00072	0.0047	68.28
		2	79.5	83.50	69.14	0.00109	0.0087	69.08
		3	115.8	76.76	65.04	0.00073	0.0071	65.36
	y	1	71.1	82.52	68.36	0.00095	0.0042	68.37
		2	63.4	83.01	68.55	0.00174	0.0075	68.75
		3	77.4	76.17	64.55	0.00215	0.0063	64.91
0.5 (0.475)	x	1	58.5	82.42	67.58	0.00076	0.0044	67.62
		2	94.0	83.98	67.68	0.00183	0.0051	68.26
		3	82.9	76.07	63.09	0.00226	0.0052	63.76
	y	1	52.6	82.13	66.99	0.00125	0.0048	66.75
		2	59.2	83.01	67.77	0.00233	0.0053	67.79
		3	88.3	76.17	63.87	0.00119	0.0055	64.12

Table 3. Experimental and analytical results for uncoupled vibration of a group of seven tubes

Gap-to-Radius Ratio (G/R)	Direction of Motion	Tube Number	Dimensionless Spring Constant (p_i)	Measured Uncoupled Natural Frequency, Hz		Measured Damping Ratio		Calculated Uncoupled Natural Frequency in Water, Hz
				In Air	In Water	In Air	In Water	
1.5 (1.384)	x	1	52.2	82.42	67.91	0.00079	0.0049	68.29
		2	60.1	83.11	68.96	0.00044	0.0103	68.72
		3	73.6	83.06	68.75	0.00086	0.0043	69.94
		4	57.3	81.93	67.48	0.00050	0.0061	67.97
		5	55.9	81.98	68.02	0.00074	0.0054	68.03
		6	223.4	77.15	65.38	0.00052	0.0040	65.77
		7	47.4	81.78	67.19	0.00099	0.0047	67.29
	y	1	64.1	82.96	68.80	0.00107	0.0040	68.81
		2	79.5	83.74	69.17	0.00089	0.0045	69.32
		3	55.2	82.37	67.63	0.00140	0.0055	68.21
		4	72.0	82.47	68.46	0.00110	0.0033	68.49
		5	73.3	82.62	68.75	0.00054	0.0043	68.64
		6	106.4	76.46	64.45	0.00094	0.0039	65.05
		7	49.8	81.93	66.80	0.00109	0.0047	67.41
1.0 (0.867)	x	1	52.1	81.15	65.63	0.00104	0.0049	66.20
		2	54.7	81.84	66.21	0.00063	0.0040	66.61
		3	91.9	82.81	67.77	0.00083	0.0043	67.96
		4	75.7	82.62	67.19	0.00084	0.0040	67.48
		5	52.0	82.23	67.38	0.00085	0.0042	67.20
		6	88.3	75.49	63.39	0.00180	0.0043	63.68
		7	50.6	81.35	64.84	0.00092	0.0051	65.18
	y	1	71.5	81.93	67.19	0.00061	0.0044	67.09
		2	72.5	82.52	67.19	0.00063	0.0055	67.42
		3	61.2	81.98	66.60	0.00062	0.0049	66.75
		4	75.7	82.62	67.97	0.00084	0.0040	67.74
		5	63.2	82.28	68.16	0.00120	0.0041	67.49
		6	74.2	75.20	62.60	0.00134	0.0048	63.00
		7	48.9	81.25	64.84	0.00078	0.0055	65.10
0.4 (0.394)	x	1	66.2	82.62	62.70	0.00253	0.0072	64.11
		2	66.8	83.06	62.82	0.00263	0.0073	64.27
		3	61.1	82.62	65.14	0.00143	0.0056	65.72
		4	95.9	83.01	63.55	0.00121	0.0064	64.52
		5	91.2	82.81	61.91	0.00130	0.0010	64.40
		6	76.5	75.98	61.77	0.00048	0.0060	62.39
		7	48.8	81.64	59.13	0.00168	0.0103	60.13
	y	1	56.6	82.23	65.72	0.00250	0.0052	64.85
		2	53.2	82.62	66.11	0.00200	0.0050	64.99
		3	49.3	82.03	61.87	0.00082	0.0088	63.14
		4	77.1	82.62	69.77	0.00126	0.0050	65.26
		5	91.2	82.81	65.53	0.00118	0.0054	65.45
		6	62.6	75.59	59.82	0.00045	0.0066	60.28
		7	40.8	81.05	58.94	0.00062	0.0112	59.69

Table 4. Experimental and analytical results for uncoupled vibration of the four-tube array in unconfined water

Gap-to-Radius Ratio (G/R)	Direction of Motion	Tube Number	Dimensionless Spring Constant (p_1)	Measured Uncoupled Natural Frequency, Hz		Measured Damping Ratio		Calculated Uncoupled Natural Frequency in Water, Hz
				In Air	In Water	In Air	In Water	
0.5 (0.585)	x	1	98.1	83.79	68.65	0.00099	0.0079	68.57
		2	60.6	83.98	68.85	0.00169	0.0138	68.42
		3	78.1	83.59	68.55	0.00566	0.0125	68.41
		4	1030.0	77.83	65.33	0.00091	0.0080	65.55
	y	1	75.0	83.30	67.77	0.00131	0.0072	68.17
		2	74.9	84.28	68.95	0.00286	0.0147	68.82
		3	86.4	83.78	68.46	0.00290	0.0089	68.57
		4	228.1	77.34	64.16	0.00633	0.0117	65.13

Table 5. Experimental and analytical results for uncoupled vibration of the four-tube array near a flat wall

Gap-to-Radius Ratio (G_w/R)	Direction of Motion	Tube Number	Measured Uncoupled Natural Frequency, Hz	Measured Damping Ratio	Calculated Uncoupled Natural Frequency, Hz
1.0	x	1	68.46	0.0066	68.27
		2	67.24	0.0071	66.87
		3	66.75	0.0059	66.88
		4	65.14	0.0101	65.29
	y	1	67.58	0.0078	67.93
		2	67.04	0.0065	67.38
		3	66.55	0.0087	67.15
		4	64.06	0.0070	64.93
0.5	x	1	68.51	0.0059	68.14
		2	65.53	0.0138	65.31
		3	65.38	0.0143	65.31
		4	65.19	0.0094	65.18
	y	1	67.48	0.0038	67.85
		2	66.16	0.0087	66.15
		3	66.06	0.0096	65.91
		4	64.21	0.0103	64.86

Table 6. Experimental and analytical results for uncoupled vibration of the four-tube array contained in a cylinder

Radius Ratio R_c/R	Eccentricity G_e/R	Direction of Motion	Tube Number	Measured Uncoupled Natural Frequency, Hz	Measured Damping Ratio	Calculated Uncoupled Natural Frequency, Hz
4.0	0.0	x	1	65.33	0.0095	66.07
			2	65.77	0.0082	65.89
			3	65.92	0.0071	65.91
			4	62.26	0.0055	63.43
		y	1	64.70	0.0079	65.68
			2	65.53	0.0073	66.28
			3	65.87	0.0068	66.06
			4	62.26	0.0053	63.02
	0.4	x	1	65.48	0.0060	65.47
			2	65.14	0.0070	66.19
			3	65.38	0.0068	66.21
			4	62.60	0.0054	62.92
		y	1	64.94	0.0069	65.07
			2	65.33	0.0060	66.62
			3	64.84	0.0106	66.40
			4	62.55	0.0054	62.50
	0.8	x	1	65.33	0.0069	64.25
			2	62.84	0.0093	66.30
			3	63.82	0.0082	66.31
			4	62.79	0.0074	61.87
		y	1	65.82	0.0057	63.89
			2	64.21	0.0091	66.79
			3	62.94	0.0111	65.57
			4	62.45	0.0075	61.49
3.5	0.0	x	1	63.67	0.0079	64.43
			2	64.31	0.0092	64.25
			3	64.75	0.0094	64.27
			4	60.84	0.0105	62.02
		y	1	62.21	0.0101	64.05
			2	64.50	0.0091	64.63
			3	63.82	0.0100	64.42
			4	59.91	0.0102	61.62
	0.4	x	1	64.65	0.0069	62.73
			2	63.67	0.0085	64.94
			3	63.92	0.0070	64.96
			4	61.13	0.0128	60.56
		y	1	63.78	0.0085	62.36
			2	62.45	0.0073	65.40
			3	63.92	0.0090	65.19
			4	61.67	0.0067	60.17

Table 7. Experimental and analytical results for coupled vibration of 5 tubes

Gap-to-Radius Ratio (G/R)	Direction of Motion	Mode Number	Measured Coupled Natural Frequencies, Hz	Calculated Coupled Natural Frequencies, Hz	Damping Ratio
2.0 (1.988)	x	1	66.16	66.45	0.0043
		2	68.12	68.50	0.0047
		3	69.34	69.46	0.0047
		4	71.12	71.15	0.0046
		5	73.29	73.28	0.0051
	y	1	66.08	66.18	0.0040
		2	68.64	68.44	0.0039
		3	70.21	70.54	0.1040
		4	71.80	72.08	0.0038
		5	72.68	73.04	0.0041
1.0 (0.981)	x	1	63.77	64.56	0.0061
		2	65.72	66.24	0.0072
		3	68.09	68.16	0.0075
		4	70.98	70.91	0.0080
		5	74.83	74.48	0.0088
	y	1	63.77	63.35	0.0040
		2	66.67	66.74	0.0039
		3	69.80	69.98	0.0043
		4	71.90	72.31	0.0046
		5	73.52	73.88	0.0046
0.25 (0.248)	x	1	59.96	61.02	0.0126
		2	62.50	62.92	0.0105
		3	66.29	66.59	0.0102
		4	71.53	71.55	0.0127
		5	77.64	77.19	0.0152
	y	1	57.33	56.45	0.0058
		2	64.67	64.56	0.0059
		3	69.14	69.67	0.0063
		4	72.95	73.39	0.0074
		5	75.39	75.65	0.0081

Table 8. Experimental and analytical results for coupled vibration of 3 tubes

Gap-to-Radius Ratio (G/R)	Mode Number	Measured Coupled Natural Frequencies, Hz	Calculated Coupled Natural Frequencies, Hz	Damping Ratio
2.0 (1.933)	1	64.02	64.61	0.0036
	2	65.20	65.23	0.0038
	3	67.65	67.76	0.0039
	4	69.04	69.10	0.0039
	5	71.28	71.10	0.0043
	6	71.52	71.61	0.0042
1.0 (0.983)	1	61.21	62.04	0.0061
	2	64.02	63.84	0.0063
	3	66.25	66.42	0.0060
	4	69.52	69.68	0.0065
	5	72.10	71.78	0.0069
	6	72.85	72.72	0.0070
0.5 (0.475)	1	57.42	58.15	0.0046
	2	62.29	62.32	0.0051
	3	64.76	64.86	0.0047
	4	70.24	70.14	0.0055
	5	72.88	72.66	0.0054
	6	74.47	74.32	0.0055

Table 9. Experimental and analytical results for coupled vibration of 7 tubes

Gap-to-Radius Ratio (G/R)	Mode Number	Measured Natural Frequencies, Hz	Calculated Natural Frequencies, Hz	Calculated Damping Ratio
1.5 (1.384)	1	60.25	61.18	0.0043
	2	61.91	62.66	0.0045
	3	62.70	62.76	0.0052
	4	64.65	64.74	0.0040
	5	66.06	66.33	0.0045
	6	66.46	66.94	0.0048
	7	67.53	68.31	0.0043
	8	68.99	68.79	0.0055
	9	69.87	70.22	0.0051
	10	70.75	71.07	0.0060
	11	71.58	72.03	0.0066
	12	72.41	73.13	0.0050
	13	73.44	73.32	0.0056
	14	74.36	74.47	0.0049
1.0 (0.867)	1	55.61	56.87	0.0041
	2	58.20	58.73	0.0044
	3	58.89	58.86	0.0042
	4	62.06	62.12	0.0041
	5	64.45	64.43	0.0043
	6	65.33	64.99	0.0042
	7	68.36	67.68	0.0050
	8	69.29	68.23	0.0044
	9	70.85	70.12	0.0049
	10	71.63	71.26	0.0047
	11	72.66	72.69	0.0048
	12	74.07	74.04	0.0052
	13	74.46	74.27	0.0056
	14	75.54	75.79	0.0050
0.4 (0.394)	1	48.39	49.69	0.0048
	2	50.96	51.35	0.0077
	3	51.41	51.70	0.0066
	4	59.47	59.30	0.0032
	5	60.99	61.35	0.0054
	6	62.40	62.09	0.0050
	7	68.41	67.99	0.0067
	8	69.14	68.30	0.0057
	9	70.80	70.74	0.0074
	10	72.46	72.09	0.0062
	11	74.46	74.22	0.0079
	12	76.07	76.02	0.0094
	13	76.46	76.09	0.0099
	14	78.56	78.28	0.0065

Table 10. Experimental and analytical results for coupled vibration of the four-tube array in unconfined water

Gap-to-Radius Ratio (G/R)	Mode Number	Measured Coupled Natural Frequencies, Hz	Calculated Coupled Natural Frequencies, Hz	Damping Ratio
0.5 (0.585)	1	56.79	57.90	0.0090
	2	62.79	62.99	0.0094
	3	66.26	66.73	0.0123
	4	67.38	67.87	0.0091
	5	69.53	69.66	0.0117
	6	71.88	71.38	0.0095
	7	74.32	74.19	0.0124
	8	76.95	76.39	0.0121

Table 11. Experimental and Analytical Results for Coupled Vibration of the Four-Tube Array Near a Flat Wall

Gap-to-Radius Ratio G_w/R	Mode Number	Measured Coupled Natural Frequency, Hz	Calculated Coupled Natural Frequency, Hz	Damping Ratio
1.0	1	56.69	57.78	0.0066
	2	61.87	62.08	0.0071
	3	64.84	64.75	0.0077
	4	66.94	66.69	0.0071
	5	69.04	68.95	0.0079
	6	71.19	70.32	0.0077
	7	73.54	73.35	0.0080
	8	76.81	76.35	0.0083
0.5	1	56.25	57.17	0.0094
	2	60.11	60.52	0.0093
	3	64.36	64.08	0.0098
	4	66.70	66.11	0.0091
	5	68.51	68.30	0.0097
	6	70.68	69.60	0.0112
	7	72.99	72.82	0.0084
	8	76.51	76.13	0.0115

Table 12. Experimental and Analytical Results for Coupled Vibration of the Four-Tube Array Contained in a Cylinder

Radius Ratio R_c/R	Eccentricity G_e/R	Mode Number	Measured Coupled Natural Frequency, Hz	Calculated Coupled Natural Frequency, Hz	Damping Ratio
4.0	0.0	1	56.15	57.86	0.0061
		2	59.52	59.98	0.0058
		3	62.21	62.11	0.0074
		4	62.25	62.37	0.0070
		5	67.53	68.31	0.0070
		6	69.43	69.27	0.0080
		7	72.27	72.22	0.0083
		8	76.66	76.43	0.0086
	0.4	1	56.25	57.81	0.0058
		2	59.40	59.12	0.0054
		3	61.67	61.96	0.0062
		4	62.26	62.80	0.0075
		5	67.68	68.06	0.0066
		6	69.43	69.15	0.0073
		7	72.11	72.34	0.0079
		8	76.61	76.43	0.0081
	0.8	1	55.86	57.29	0.0064
		2	58.11	57.63	0.0071
		3	60.74	61.25	0.0075
		4	62.65	63.14	0.0092
		5	67.29	67.33	0.0076
		6	68.60	68.79	0.0084
		7	71.53	72.34	0.0100
		8	76.32	76.35	0.0099
3.5	0.0	1	55.57	57.43	0.0091
		2	57.03	57.70	0.0090
		3	59.33	59.56	0.0085
		4	59.77	59.83	0.0087
		5	66.31	67.40	0.0105
		6	68.07	68.25	0.0100
		7	70.90	70.59	0.0102
		8	76.22	76.18	0.0113
	0.4	1	55.76	55.33	0.0076
		2	57.37	57.23	0.0084
		3	59.20	59.03	0.0078
		4	59.72	60.87	0.0076
		5	66.70	66.42	0.0088
		6	68.21	67.91	0.0087
		7	70.31	70.99	0.0089
		8	76.17	76.12	0.0099

Table 13. Experimental and Analytical Results for Coupled Vibrations of the Four-Tube Array Partially Submerged in Water

Ratio of Water Depth to Tube Length (h/ℓ)	Mode Number	Natural Frequencies of Coupled Modes, Hz	
		Experiment	Theory
1.00	1	56.79	57.90
	2	62.79	62.99
	3	66.26	66.73
	4	67.38	67.87
	5	69.53	69.66
	6	71.88	71.38
	7	74.32	74.19
	8	76.95	76.39
0.875	1	63.48	63.84
	2	68.55	68.21
	3	71.68	71.85
	4	72.85	73.02
	5	74.02	73.69
	6	76.66	75.83
	7	78.13	77.89
	8	80.08	79.26
0.75	1	69.04	70.24
	2	72.66	72.62
	3	76.07	76.53
	4	76.76	76.69
	5	77.64	77.57
	6	79.98	79.25
	7	81.05	80.50
	8	82.32	81.14
0.625	1	73.15	74.60
	2	75.39	75.44
	3	78.10	79.43
	4	79.49	80.11
	5	80.47	80.68
	6	82.42	81.54
	7	83.01	82.23
	8	83.98	82.51
0.50	1	75.39	76.47
	2	76.95	76.98
	3	79.30	81.77
	4	81.25	82.25
	5	82.08	82.52
	6	83.04	82.85
	7	84.08	83.17
	8	84.96	83.46
0.375	1	76.37	77.12
	2	77.59	77.61
	3	79.83	82.96
	4	81.12	83.22
	5	82.13	83.40
	6	82.96	83.50
	7	84.57	83.61
	8	85.35	83.99

Table 14. Experimental Results for Uncoupled Vibration of the Four-Tube Arrays in Viscous Fluids

Conditions	Direction of Motion	Tube Number	Measured Uncoupled Natural Frequency, Hz		Measured Damping Ratio	
			Water	Mineral Oil	Water	Mineral Oil
In unconfined fluid	x	1	68.65	68.31	0.0079	0.0262
		2	68.85	69.14	0.0138	0.0318
		3	68.55	68.41	0.0125	0.0285
		4	65.33	65.04	0.0080	0.0309
	y	1	67.77	67.48	0.0072	0.0284
		2	68.95	69.14	0.0147	0.0336
		3	68.46	68.56	0.0089	0.0285
		4	64.16	64.99	0.0117	0.0290
Near a flat wall ($G_w/R = 0.5$)	x	1	68.51	68.56	0.0059	0.0254
		2	65.53	66.31	0.0138	0.0311
		3	65.38	65.58	0.0143	0.0308
		4	65.19	64.70	0.0094	0.0375
	y	1	67.48	67.48	0.0038	0.0350
		2	66.16	65.82	0.0087	0.0268
		3	66.06	66.02	0.0006	0.0323
		4	64.21	64.84	0.0103	0.0243

Table 15. Experimental and Analytical Results for Natural Frequencies of Coupled Modes in Viscous Fluids

Conditions	Mode Number	Measured Coupled Natural Frequency, Hz		Calculated Coupled Natural Frequency, Hz	
		Water	Mineral Oil	Water	Mineral Oil
In unconfined fluid	1	56.79	56.93	57.90	58.82
	2	62.79	62.65	62.99	63.76
	3	66.26	63.28	66.73	67.47
	4	67.38	67.58	67.87	68.62
	5	69.53	69.75	69.66	70.23
	6	71.88	74.22	71.38	72.01
	7	74.32	76.57	74.19	74.70
	8	76.95	77.08	76.39	76.78
Near a flat wall ($G_w/R = 0.5$)	1	56.25	57.09	57.17	58.12
	2	60.11	61.15	60.52	61.44
	3	64.36	63.00	64.08	64.80
	4	66.70	66.31	66.11	66.92
	5	68.51	68.65	68.30	68.99
	6	70.68	70.13	69.60	70.26
	7	72.99	72.79	72.82	73.36
	8	76.51	76.52	76.13	76.53