Explicit Approximate Model Predictive Control of Constrained Nonlinear Systems with Quantized Input

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Abstract : In this paper, a Model Predictive Control problem for constrained nonlinear systems with *quantized* input is formulated and represented as a multi-parametric Nonlinear Integer Programming (mp-NIP) problem. Then, a computational method for *explicit* approximate solution of the resulting mp-NIP problem is suggested, which consists in constructing a feasible piecewise constant approximation to the optimal solution on the state space of interest. The proposed approximate mp-NIP approach is applied to the design of an *explicit* approximate MPC controller for a clutch actuator with on/off valves.

1 Introduction

In several control engineering problems, the system to be controlled is characterized by a finite set of possible control actions. Such systems are referred to as systems with *quantized* control input and the possible values of the input represent the levels of *quantization.* For example, hydraulic systems using on/off valves are systems with *quantized* input. In order to achieve a high quality of the control system performance it would be necessary to take into account the effect of the control input *quantization.* Thus, in [7] receding horizon optimal control ideas were proposed for synthesizing *quantized* control laws for *linear* systems with *quantized* inputs and quadratic optimality criteria. Further in [1], a method for *explicit* solution of optimal control problems with *quantized* control input was developed. It is based on solving multi-parametric Nonlinear Integer Programming (mp-NIP) problems, where the cost function and the constraints depend linearly on the vector of parameters. In this paper, a Model Predictive Control (MPC) problem for constrained *nonlinear* systems with *quantized* input is formulated and represented as an mp-NIP problem. Then, a computational method for *explicit* approximate solution of the resulting mp-NIP problem is suggested. The mp-NIP method proposed here is more general compared to the mp-NIP method in [1], since it allows the cost function and the constraints to depend nonlinearly on the vector of parameters (the state variables).

The following notation is used in the paper. $A \succ 0$ means that the square matrix A is positive definite. For $x \in \mathbb{R}^n$, the Euclidean norm is $\|x\| = \sqrt{x^T x}$ and the weighted norm is defined for some symmetric matrix $A \succ 0$ as $\|x\|_A = \sqrt{x^T A x}$.

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2 Formulation of quantized Nonlinear Model Predictive Control problem

Consider the discrete-time nonlinear system:

$$x(t+1) = f(x(t), u(t)) \tag{1}$$
$$y(t) = Cx(t), \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state variable, $y(t) \in \mathbb{R}^p$ is the output variable, and $u(t) \in \mathbb{R}^m$ is the control input, which is constrained to belong to the finite set of values $U^A = \{\overline{u}_1, \overline{u}_2, \dots, \overline{u}_L\}$, $\overline{u}_i \in \mathbb{R}^m$, $\forall i = 1, 2, \dots, L$, i.e. $u \in U^A$. Here, $\overline{u}_1, \overline{u}_2, \dots, \overline{u}_L$ represent the levels of *quantization* of the control input u. In (1), $f : \mathbb{R}^n \times U^A \longmapsto \mathbb{R}^n$ is a nonlinear function.

Here, we consider a reference tracking problem where the goal is to have the output variable $y(t)$ track the reference signal $r(t) \in \mathbb{R}^p$. Suppose that a full measurement of the state $x(t)$ is available at the current time t. For the current $x(t)$, the reference tracking *quantized* NMPC solves the following optimization problem:

Problem P1:

$$V^*(x(t), r(t)) = \min_U J(U, x(t), r(t)) \tag{3}$$

subject to $x_{t|t} = x(t)$ and:

$$y_{\min} \leq y_{t+k|t} \leq y_{\max},\ k = 1, \dots, N \tag{4}$$
$$u_{t+k} \in U^A = \{\overline{u}_1, \overline{u}_2, \dots, \overline{u}_L\},\ k = 0, 1, \dots, N-1 \tag{5}$$
$$\left\| y_{t+N|t} - r(t) \right\| \leq \delta \tag{6}$$
$$x_{t+k+1|t} = f(x_{t+k|t}, u_{t+k}),\ k \geq 0 \tag{7}$$
$$y_{t+k|t} = Cx_{t+k|t},\ k \geq 0 \tag{8}$$

Here, $U = [u_t, u_{t+1}, \dots, u_{t+N-1}] \in \mathbb{R}^{Nm}$ is the set of free control moves, $U \in U^B$, where $U^B = (U^A)^N = U^A \times \dots \times U^A$ and the cost function given by:

$$J(U, x(t), r(t)) = \sum_{k=0}^{N-1} \left[\left\| y_{t+k|t} - r(t) \right\|_Q^2 + \left\| h(x_{t+k|t}, u_{t+k}) \right\|_R^2 \right] + \left\| y_{t+N|t} - r(t) \right\|_P^2 \tag{9}$$

Here, N is a finite horizon and $h : \mathbb{R}^n \times U^A \longmapsto \mathbb{R}^s$ is a nonlinear function. It is assumed that $P, Q, R \succ 0$. From a stability point of view it is desirable to choose δ in (6) as small as possible. However, the feasibility of (3)–(9) will rely on δ being sufficiently large. A part of the NMPC design will be to address this tradeoff. We introduce an extended state vector:

$$\tilde{x}(t) = [x(t),\ r(t)] \in \mathbb{R}^{\tilde{n}},\ \ \tilde{n} = n + p \tag{10}$$

Let $\tilde{x}$ be the value of the extended state at the current time t. Then, the optimization problem P1 can be formulated in a compact form as follows:

Problem P2:

$$V^*(\tilde{x}) = \min_U J(U, \tilde{x}) \ \text{ subject to } \ G(U, \tilde{x}) \leq 0 \tag{11}$$

The *quantized* NMPC problem defines a multi-parametric Nonlinear Integer Programming problem (mp-NIP), since it is a Nonlinear Integer Programming problem in U parameterized by $\tilde{x}$. An optimal solution to this problem is denoted $U^* = [u_t^*, u_{t+1}^*, \dots, u_{t+N-1}^*]$ and the control input is chosen according to the receding horizon policy $u(t) = u_t^*$. Define the set of N-step feasible initial states as follows:

$$X_f = \{\tilde{x} \in \mathbb{R}^{\tilde{n}} \mid G(U, \tilde{x}) \leq 0 \text{ for some } U \in U^B\} \tag{12}$$

If δ in (6) is chosen such that the problem P1 is feasible, then X_f is a non-empty set.

In parametric programming problems one seeks the solution $U^*(\tilde{x})$ as an *explicit* function of the parameters $\tilde{x}$ in some set $\underline{X} \subseteq X_f \subseteq \mathbb{R}^{\tilde{n}}$ [2]. In this paper we suggest a computational method for constructing an *explicit* piecewise constant (PWC) approximate solution of the reference tracking *quantized* NMPC problem.

3 Approximate mp-NIP approach to explicit quantized NMPC

3.1 Computation of feasible PWC solution

Definition 1 (Feasibility):
Let $\bar{X} \subset \mathbb{R}^{\tilde{n}}$ be a hyper-rectangle and $V_{\bar{X}} = \{v_1, v_2, \dots, v_Q\} \subset \bar{X}$ be a discrete set. A function $U(\tilde{x})$ is feasible on $V_{\bar{X}}$ if $G(U(v_i), v_i) \leq 0$, $i \in \{1, 2, \dots, Q\}$. Further, the feasibility of $U(\tilde{x})$ on $V_{\bar{X}} \subset \bar{X}$ will be referred to as feasibility of $U(\tilde{x})$ on $\bar{X}$.
We restrict our attention to a hyper-rectangle $X \subset \mathbb{R}^{\tilde{n}}$ where we seek to approximate the optimal solution $U^*(\tilde{x})$ to problem P2. We require that the state space partition is orthogonal and can be represented as a $k-d$ tree. The main idea of the approximate mp-NIP approach is to construct a feasible *piecewise constant* (PWC) approximation $\widehat{U}(\tilde{x})$ to $U^*(\tilde{x})$ on X, where the constituent constant functions are defined on hyper-rectangles covering X. The solution of problem P2 is computed at the $2^{\tilde{n}}$ vertices of a considered hyper-rectangle X_0 by solving up to $2^{\tilde{n}}$ NIPs, as well as at some interior points. These additional points represent the vertices and the facets centers of one or more hyper-rectangles contained in the interior of X_0. The following procedure is used to generate a set of points $V_0 = \{v_0, v_1, v_2, \dots, v_{N_1}\}$ associated to a hyper-rectangle X_0:
Procedure 1 (Generation of set of points):
Consider any hyper-rectangle $X_0 \subseteq X$ with vertices $\Lambda^0 = \{\lambda_1^0, \lambda_2^0, \dots, \lambda_{N_\lambda}^0\}$ and center point v_0. Consider also the hyper-rectangles $X_0^j \subset X_0$, $j = 1, 2, \dots, N_0$ with vertices respectively $\Lambda^j = \{\lambda_1^j, \lambda_2^j, \dots, \lambda_{N_\lambda}^j\}$, $j = 1, 2, \dots, N_0$. Suppose $X_0^1 \subset X_0^2 \subset \dots \subset X_0^{N_0}$. For each of the hyper-rectangles X_0 and $X_0^j \subset X_0$, $j = 1, 2, \dots, N_0$, denote the set of its facets centers with $\Phi^j = \{\phi_1^j, \phi_2^j, \dots, \phi_{N_\phi}^j\}$, $j = 0, 1, 2, \dots, N_0$. Define the set of all points $V_0 = \{v_0, v_1, v_2, \dots, v_{N_1}\}$, where $v_i \in \left\{\bigcup_{j=0}^{N_0} \Lambda^j\right\} \bigcup \left\{\bigcup_{j=0}^{N_0} \Phi^j\right\}$, $i = 1, 2, \dots, N_1$.
The global solution $U^*(v_i)$ of problem P2 at a point $v_i \in V_0$ is computed by using the routine 'glcSolve' of the TOMLAB optimization environment in Matlab

[4]. The routine 'glcSolve' implements an extended version of the DIRECT algorithm [5], that handles problems with both nonlinear and integer constraints. The DIRECT algorithm (DIviding RECTangles) [5] is a deterministic sampling algorithm for finding the global minimum of a multivariate function subject to constraints, using no derivative information. It is a modification of the standard Lipschitzian approach that eliminates the need to specify a Lipschitz constant.

Based on the global solutions $U^*(v_i)$ at all points $v_i \in V_0$, a feasible local constant approximation $\widehat{U}_0(\tilde{x}) = K_0$ to the optimal solution $U^*(\tilde{x})$, valid in the whole hyper-rectangle X_0, is determined by applying the following procedure:

Procedure 2 (Computation of explicit approximate solution):
Consider any hyper-rectangle $X_0 \subseteq X$ with a set of points $V_0 = \{v_0, v_1, v_2, \dots, v_{N_1}\}$ determined by applying Procedure 1. Compute K_0 by solving the following NIP:

$$\min_{K_0} \sum_{i=0}^{N_1} (J(K_0, v_i) - V^*(v_i)) \text{ subject to } G(K_0, v_i) \leq 0, \ \forall v_i \in V^0 \tag{13}$$

3.2 Estimation of error bounds

Suppose that a constant function $\widehat{U}_0(\tilde{x}) = K_0$ that is feasible on $V^0 \subseteq X_0$ has been determined by applying Procedure 2. Then, for the cost function approximation error in X_0 we have:

$$\varepsilon(\tilde{x}) = \widehat{V}(\tilde{x}) - V^*(\tilde{x}) \leq \varepsilon_0 \ , \ \ \tilde{x} \in X_0 \tag{14}$$

where $\widehat{V}(\tilde{x}) = J(\widehat{U}_0(\tilde{x}), \tilde{x})$ is the sub-optimal cost and $V^*(\tilde{x})$ denotes the cost corresponding to the global solution $U^*(\tilde{x})$, i.e. $V^*(\tilde{x}) = J(U^*(\tilde{x}), \tilde{x})$. The following procedure can be used to obtain an estimate $\widehat{\varepsilon}_0$ of the maximal approximation error ε_0 in X_0.

Procedure 3 (Computation of error bound approximation):
Consider any hyper-rectangle $X_0 \subseteq X$ with a set of points $V_0 = \{v_0, v_1, v_2, \dots, v_{N_1}\}$ determined by applying Procedure 1. Compute an estimate $\widehat{\varepsilon}_0$ of the error bound ε_0 through the following maximization:

$$\widehat{\varepsilon}_0 = \max_{i \in \{0,1,2,\dots,N_1\}} (\widehat{V}(v_i) - V^*(v_i)) \tag{15}$$

3.3 Approximate mp-NIP algorithm

Assume the tolerance $\bar{\varepsilon} > 0$ of the cost function approximation error is given. The following algorithm is proposed to design *explicit* reference tracking *quantized* NMPC:

Algorithm 1 (explicit reference tracking quantized NMPC)

1. Initialize the partition to the whole hyper-rectangle, i.e. $\Pi = \{X\}$. Mark the hyper-rectangle X as unexplored.

2. Select any unexplored hyper-rectangle $X_0 \in \Pi$. If no such hyper-rectangle exists, terminate.

3. Generate a set of points $V_0 = \{v_0, v_1, v_2, \dots, v_{N_1}\}$ associated to X_0 by applying Procedure 1.

4. Compute a solution to problem P2 for $\tilde{x}$ fixed to each of the points v_i , $i = 0, 1, 2, \dots, N_1$ by using routine 'glcSolve' of TOMLAB optimization environment. If problem P2 has a feasible solution at all these points, go to step

6. Otherwise, go to step 5.

5. Compute the size of X_0 using some metric. If it is smaller than some given tolerance, mark X_0 infeasible and explored and go to step 2. Otherwise, split X_0 into hyper-rectangles X_1, X_2, ... , X_{N_s} by applying the heuristic rule 1 from [3]. Mark X_1, X_2, ... , X_{N_s} unexplored, remove X_0 from Π, add X_1, X_2, ... , X_{N_s} to Π, and go to step 2.

6. Compute a constant function $\widehat{U}_0(\tilde{x})$ using Procedure 2, as an approximation to be used in X_0. If no feasible solution was found, split X_0 into two hyper-rectangles X_1 and X_2 by applying the heuristic rule 3 from [3]. Mark X_1 and X_2 unexplored, remove X_0 from Π, add X_1 and X_2 to Π, and go to step 2.

7. Compute an estimate $\widehat{\varepsilon}_0$ of the error bound ε_0 in X_0 by applying Procedure 3. If $\widehat{\varepsilon}_0 \leq \bar{\varepsilon}$, mark X_0 as explored and feasible and go to step 2. Otherwise, split X_0 into two hyper-rectangles X_1 and X_2 by applying Procedure 4 from [3]. Mark X_1 and X_2 unexplored, remove X_0 from Π, add X_1 and X_2 to Π, and go to step 2.

4 Explicit quantized NMPC of an electropneumatic clutch actuator using on/off valves

Here, a pneumatic actuator of an electropneumatic clutch system is considered. The pneumatic actuator acts on the clutch plates through the clutch spring, and the state of the clutch directly depends on the actuator position. The actuator is controlled by using on/off valves. In comparison to proportional valves, the on/off valves are smaller and cheaper. In [8] the case when only fully open and closed are possible states of the valves is considered. Then, a controller is designed to govern switches between these states based on backstepping and Lyapunov theory. It should be noted however, that the method in [8] can not handle the constraints imposed on the clutch actuator position. On the other hand, Model Predictive Control (MPC) is an optimization based method for control which can handle both state and input constraints. This makes the MPC methodology very suitable to the optimal control of the clutch actuator. The fast dynamics of the clutch actuator, characterized with sampling time of about 0.01 [s] requires the design of an *explicit* MPC controller, where the only computation performed on-line would be a simple function evaluation.

4.1 Description of the electropneumatic clutch actuator

The clutch actuator system is shown in Figure 1. To control both supply to and exhaust from the clutch actuator chamber, at least one pair of on/off valves are needed. As we only allow these to be fully open or closed, with two valves and under the assumption of choked flow, we restrict the flow of the clutch actuator to three possible values, maximum flow into the volume, maximum flow out of the volume, or no flow [8]. The electronic control unit (ECU) calculates and sets voltage signals to control the on/off valves. These signals control whether the valve should open or close, and thus also the flow into the actuator. A position sensor measures position and feeds it back to the ECU. To calculate the control signals, knowledge of other states of the system are also needed, and these can be obtained either by sensors or by estimation.

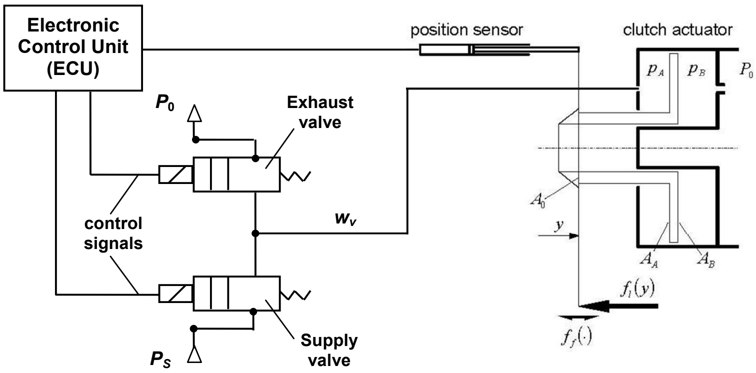

Figure 1: Electropneumatic clutch actuator ([6], [8]).

The full 5-th order model of the clutch actuator dynamics is the following [6]:

$$\dot{y} = v \tag{16}$$

$$\dot{v} = \frac{1}{M}(A_0 P_0 + A_A p_A - A_B p_B - f_f(v,z) - f_l(y)) \tag{17}$$

$$\dot{p}_A = -\frac{A_A}{V_A(y)} v p_A + \frac{RT_0}{V_A(y)} w_v(p_A, u) \tag{18}$$

$$\dot{p}_B = \frac{A_B}{V_B(y)} v p_B + \frac{RT_0}{V_B(y)} w_r(p_B) \tag{19}$$

$$\dot{z} = v - \frac{K_z}{F_C}|v|_q z \tag{20}$$

where y is the position, v is the velocity, p_A is the pressure in chamber A, p_B is the pressure in chamber B, z is the friction state, $w_v(p_A, u)$ is the flow to/from chamber A, $w_r(p_B)$ is the flow to/from chamber B, and u is an integer control variable introduced below. The meaning of the parameters is the following: A_A and A_B are the areas of chambers A and B, $A_0 = A_B - A_A$ is piston area, M is piston mass, P_0 is the ambient pressure, T_0 is the temperature, R is the gas constant of air, K_z is asperity stiffness, F_C is Coulomb friction. In (18), (19), $V_A(y) = V_{A0} + A_A y$ and $V_B(y) = V_{B0} - A_B y$ are the volumes of chambers A and B, respectively, and V_{A0}, V_{B0} are the dead volumes of these chambers. In (20), $|v|_q = \sqrt{v^2 + \sigma^2}$, where $\sigma > 0$ is an arbitrary small design parameter. In (17), $f_l(y)$ and $f_f(v,z)$ are the clutch load and the friction force, described by:

$$f_l(y) = K_l(1 - e^{-L_l y}) - M_l y\,, \quad f_f(v,z) = D_v v + K_z z + D_{\dot{z}} \dot{z}(v,z) \tag{21}$$

An integer control variable $u \in U^A = \{1,2,3\}$ is introduced which is related to the flow $w_v(p_A, u)$ to/from chamber A in the following way:

$$u = 1 \Rightarrow w_v(p_A, 1) = -\rho_0 C_{v,out} \psi(r, B_{v,out}) p_A\,, \quad r = \frac{P_0}{p_A} \tag{22}$$

$$u = 2 \Rightarrow w_v(p_A, 2) = 0 \tag{23}$$

$$u = 3 \Rightarrow w_v(p_A, 3) = \rho_0 C_{v,in} \psi(r, B_{v,in}) P_S\,, \quad r = \frac{p_A}{P_S} \tag{24}$$

In (24), P_S is the supply pressure. Therefore, $u = 1$ corresponds to maximal flow from chamber A, $u = 2$ means no flow, and $u = 3$ corresponds to maximal flow to chamber A. The expressions for the valve flow function $\psi(r, B_{v,in/out})$, as well as for the flow $w_r(p_B)$ to/from chamber B can be found in [6].

4.2 Design of explicit quantized NMPC

In order to reduce the computational burden, the design of the *explicit quantized* NMPC controller is based on a simplified 3-rd order model of the clutch actuator, where the states are the actuator position y^s, the velocity v^s and the pressure p_A^s in chamber A:

$$\dot{y}^s = v^s \tag{25}$$

$$\dot{v}^s = \frac{1}{M}(-A_A P_0 + A_A p_A^s - f_f^*(v^s) - f_l(y^s)) \tag{26}$$

$$\dot{p}_A^s = -\frac{A_A}{V_A(y^s)} v^s p_A^s + \frac{RT_0}{V_A(y^s)} w_{v^s}(p_A^s, u) \tag{27}$$

In (26), $f_f^*(v^s) = D_v v^s + F_C \frac{v^s}{\sqrt{{v^s}^2 + \sigma^2}}$ is a static sliding friction characteristic [6]. We discretize the model (25)–(27) using a sampling time $T_s = 0.01$ [s]. The control objective is to have the actuator position y^s track a reference signal $r(t) > 0$, which is achieved by minimizing the following cost function:

$$J(U, y^s(t), r(t)) = \sum_{k=0}^{N-1} \left[Q \left(\frac{y^s_{t+k|t} - r(t)}{r(t)} \right)^2 + R \left(\frac{w_{v^s}(p^s_{A,t+k|t}, u_{t+k})}{w_{v^s,max} - w_{v^s,min}} \right)^2 \right] + P \left(\frac{y^s_{t+N|t} - r(t)}{r(t)} \right)^2 \tag{28}$$

where $N = 10$ is the horizon, $Q = 1$, $R = 0.1$, $P = 1$ are the weighting coefficients, and $w_{v^s,max}$ and $w_{v^s,min}$ are the maximal and the minimal flows to/from chamber A. The following constraints are imposed:

$$y_{\min} \le y^s_{t+k|t} \le y_{\max},\ k = 1, ..., N\,;\ u_{t+k} \in U^A = \{1, 2, 3\},\ k = 0, 1, ..., N-1 \tag{29}$$

where $y_{\min} = 0$, $y_{\max} = 0.025$ [m]. In (28), $U \in U^B = (U^A)^N$. The *quantized* NMPC minimizes the cost function (28) subject to the system equations (25)–(27) and the constraints (29). The extended state vector is $\tilde{x}(t) = [e(t), v^s(t), p_A^s(t), r(t)] \in \mathbb{R}^4$, where the state $e(t)$ denotes the projected reference tracking error defined as:

$$e(t) = \begin{cases} r(t) - y^s(t)\,,\ \text{if}\ \ -0.005 \le r(t) - y^s(t) \le 0.005 \\ -0.005\,,\ \text{if}\ r(t) - y^s(t) < -0.005 \\ 0.005\,,\ \text{if}\ r(t) - y^s(t) > 0.005 \end{cases} \tag{30}$$

The state space to be partitioned is 4-dimensional and it is defined by $X = [-0.005; 0.005] \times [-0.05; 0.15] \times [P_0; P_S] \times [0.0001; 0.024]$. The cost function approximation tolerance is chosen as $\bar{\varepsilon}(X_0) = \max(\bar{\varepsilon}_a, \bar{\varepsilon}_r \min_{\tilde{x} \in X_0} V^*(\tilde{x}))$, where $\bar{\varepsilon}_a = 0.001$ and $\bar{\varepsilon}_r = 0.02$ are the absolute and the relative tolerances, respectively. The partition has 10871 regions and 17 levels of search. Thus, 17 arithmetic operations are needed in real-time to compute the control input (17 comparisons).

The performance of the *explicit quantized* NMPC controller was simulated for a typical clutch reference signal and the resulting response is depicted in Figure 2. The simulations of the closed-loop system are based on the full 5-th order model (16)–(20) of the clutch actuator dynamics.

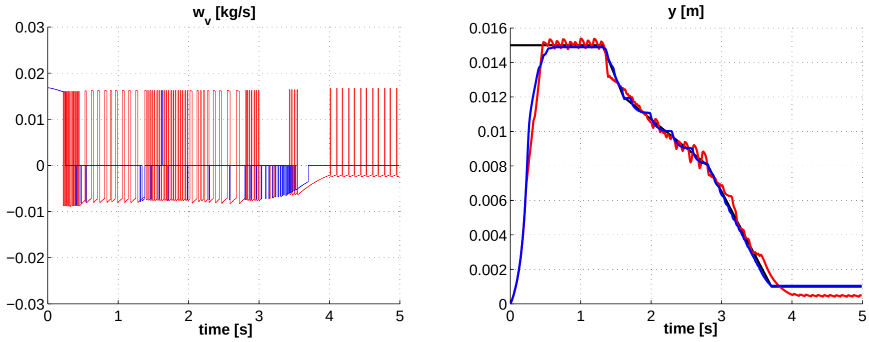

Figure 2: Left: The valve flow $w_v(p_A, u)$. Right: The clutch actuator position y. The red line is with the approximate explicit quantized NMPC, the blue line is with the exact quantized NMPC and the black line is the reference signal.

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