Article

Explicit Expressions for Solar Panel Equivalent Circuit Parameters Based on Analytical Formulation and the Lambert W-Function [†]

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Abstract: Due to the high dependence of photovoltaic energy efficiency on environmental conditions (temperature, irradiation...), it is quite important to perform some analysis focusing on the characteristics of photovoltaic devices in order to optimize energy production, even for small-scale users. The use of equivalent circuits is the preferred option to analyze solar cells/panels performance. However, the aforementioned small-scale users rarely have the equipment or expertise to perform large testing/calculation campaigns, the only information available for them being the manufacturer datasheet. The solution to this problem is the development of new and simple methods to define equivalent circuits able to reproduce the behavior of the panel for any working condition, from a very small amount of information. In the present work a direct and completely explicit method to extract solar cell parameters from the manufacturer datasheet is presented and tested. This method is based on analytical formulation which includes the use of the Lambert W-function to turn the series resistor equation explicit. The presented method is used to analyze commercial solar panel performance (*i.e.*, the current-voltage–*I-V*–curve) at different levels

of irradiation and temperature. The analysis performed is based only on the information included in the manufacturer's datasheet.

Keywords: photovoltaic energy; equivalent circuit; parameters; Lambert W-function; analytical; maximum power point tracking (MPPT)

1. Introduction

Today, the increasing use of renewable energy sources is not only a fact, but it also represents a great concern of modern society, as the effects of global warming are more and more evident and have spread to almost every corner of planet Earth. Some of these renewable energy sources such as wind energy [1] or both solar photovoltaic [2,3] and solar thermal [4], had an enormous development in the recent decades.

Focusing on solar photovoltaic energy current users, there is not a clear and defined user profile, those users ranging from the industrial sector, represented by big power plants, to the private sector represented by citizens who invest in solar panels systems to save costs in terms of electricity consumption. Also, the space sector demands photovoltaic technology, and although it is not as massive as the aforementioned ones, it should be said that it leads the technological advances related to solar cells.

Despite the wide diversity between users, there is a common need among them: to have a better understanding of their photovoltaic systems in order to get the best efficiency from them in terms of costs and revenue. On the other hand, photovoltaic energy is highly dependent on the environmental conditions (temperature, irradiation...) and that complicates its optimization, making the modeling of the panel a necessary tool. Modeling of solar panels is traditionally achieved through numerical fitting to extensive experimental results. However, this is not affordable for small users and it is not practical when a decision among different commercial solar panels/systems has to be made and only the information included in the manufacturers' datasheets is available. Analytical methods represent a solution to this problem, as they are simpler and only require a small amount of data (frequently included in the datasheet) to model the solar panel behavior.

In the present work an analytical methodology to model the behavior (output current, I, and output voltage, V) of a photovoltaic device (cell or solar panel) is presented. It is based on the use of an equivalent circuit, which is an extended and accurate way to model solar panels. The present work is part of a larger research related to the UPMSat-2 (Universidad Politécnica de Madrid Satélite 2) satellite mission. In previous studies some analytical methods for parameter calculation were successfully developed, the aim now being to simplify the equations (with more sophisticated mathematical tools such as the Lambert function [5,6]), and show how these algorithms can be used in a practical application: to modeling a commercial solar panel at different levels of irradiation and temperature. This is a necessity for a correct power optimization, or to include polar panel modeling in bigger electric power simulations (e.g., maximum power point tracking (MPPT) models), but with current methods it is not possible to do it with a small amount of information, and in a way accurate

and simple at the same time. This method aims to solve this problem and enhance analysis capabilities to any user of solar energy.

2. Methods

As it is well known, ideal solar cells behave like a current source connected in parallel with a diode [7–9]. This ideal model is completed with resistors to represent the losses and sometimes with additional diodes that takes into account other phenomena [10,11]. The most popular circuit equivalent to a solar cell/panel is shown in Figure 1, it includes a current source, one diode and two resistors: one in series and one in parallel [12–19]. Each element included in the equivalent circuit implies one parameter that has to be determined (two in the case of the diode whose behavior is represented by the Shockley equation [20]). Therefore, five parameters need to be calculated when using this method [21–33]. The current-voltage curve of a solar cell or panel, hereinafter the *I-V* curve (see Figure 2), is quite well reproduced by this simple equivalent circuit. Three points of the *I-V* curve are also indicated in Figure 2: short circuit, maximum power, and open circuit points. These representative points are, together with their variation as a function of the temperature, the normal information included in manufacturers' datasheets.

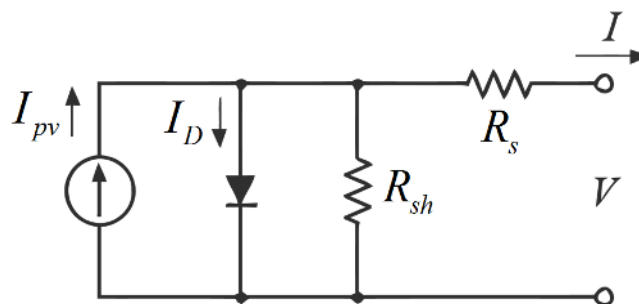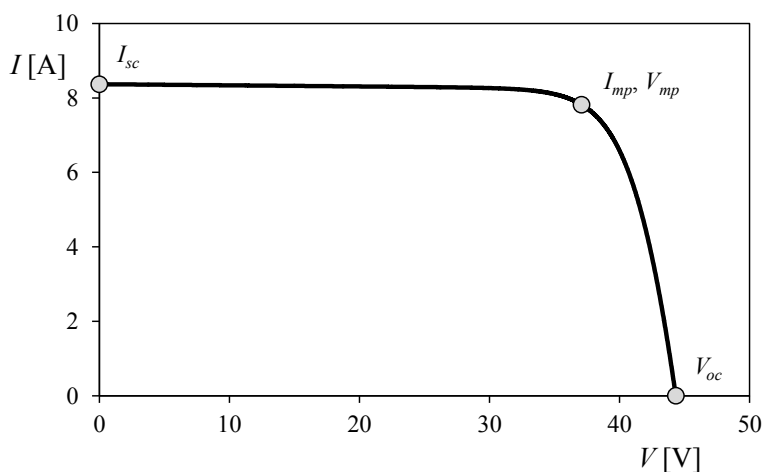**Figure 1.** Equivalent circuit of a solar panel.

Figure 2. *I-V* curve of a solar panel. The three characteristic points (short circuit, maximum power, and open circuit points) are indicated on the curve.

The *I-V* behavior of the circuit model formed by one diode and two resistors (Figure 1) is defined by the following equation [16]:

$$I = I_{pv} - I_0 \left[\exp\left(\frac{V + IR_s}{aV_T} \right) - 1 \right] - \frac{V + IR_s}{R_{sh}} \tag{1}$$

where I_{pv} is the photocurrent delivered by the constant current source; I_0 is the reverse saturation current corresponding to the diode; R_s is the series resistor that takes into account losses in cell solder bonds, interconnection, junction box, *etc.*; R_{sh} is the shunt resistor that takes into account the current leakage through the high conductivity shunts across the p-n junction; and a is the ideality factor that takes into account the deviation of the diodes from the Shockley diffusion theory. V_T is not an unknown parameter; it is the thermal voltage of the diode and depends on the charge of the electron, q, the Boltzmann constant, k; the number of cells in series, n; and the temperature, T:

$$V_T = n\frac{kT}{q} \tag{2}$$

As said, the parameters of the equivalent circuit have to be adjusted to fit the behavior of the equivalent circuit to the solar cell/panel testing results. This adjustment can be done numerically [15,16,34–39] or analytically [9,39–42]; based on the data resulting from *I-V* curves experimentally measured, or based only on the data available from the manufacturers' datasheets.

2.1. Parameter Calculation

In this paper only manufacturer's data is considered to be available for parameter calculation, this is the common situation that faces a regular user from the private sector. In general, commercial datasheets only include information about the three characteristic points of the *I-V* curve for different temperatures, from this three points it is possible to extract four boundary conditions [43]. As these conditions are not enough to determine the five parameters of the equivalent circuit, in absence of more information, some authors propose to estimate the parameter *a*, in order to reduce the number of parameters to four [44]. This parameter is the preferred to be estimated because its value is bound to a small range, it usually is in the bracket [1, 1.5] for a single junction solar cell [19,44,45]. Applying the four boundary conditions to the equivalent circuit Equation (1) it is possible to obtain the following equations [21,46]:

- Short circuit equation:

$$I_{sc} = I_{pv} - I_0 \left[\exp\left(\frac{I_{sc}R_s}{aV_T} \right) - 1 \right] - \frac{I_{sc}R_s}{R_{sh}} \tag{3}$$

- Open circuit equation:

$$0 = I_{pv} - I_0 \left[\exp\left(\frac{V_{oc}}{aV_T} \right) - 1 \right] - \frac{V_{oc}}{R_{sh}} \tag{4}$$

- Maximum power point circuit equation:

$$I_{mp} = I_{pv} - I_0 \left[\exp\left(\frac{V_{mp} + I_{mp}R_s}{aV_T} \right) - 1 \right] - \frac{V_{mp} + I_{mp}R_s}{R_{sh}} \tag{5}$$

- Zero derivative for the power at maximum power point circuit equation:

$$-\frac{I_{mp}}{V_{mp}} = -\frac{I_0}{aV_T}\left(1-\frac{I_{mp}}{V_{mp}}R_s\right)\left[\exp\left(\frac{V_{mp}+I_{mp}R_s}{aV_T}\right)\right]-\frac{1}{R_{sh}}\left(1-\frac{I_{mp}}{V_{mp}}R_s\right)$$ (6)

With some minor and justified simplifications (that is, taking into account order of magnitude of the different terms in the equations), the above equations can be simplified to the following [21]:

- Equation for R_s:

$$\frac{aV_TV_{mp}\left(2I_{mp}-I_{sc}\right)}{\left(V_{mp}I_{sc}+V_{oc}\left(I_{mp}-I_{sc}\right)\right)\left(V_{mp}-I_{mp}R_s\right)-aV_T\left(V_{mp}I_{sc}-V_{oc}I_{mp}\right)}=\exp\left(\frac{V_{mp}+I_{mp}R_s-V_{oc}}{aV_T}\right)$$ (7)

- Equation for R_{sh}:

$$R_{sh}=\frac{\left(V_{mp}-I_{mp}R_s\right)\left(V_{mp}-R_s\left(I_{sc}-I_{mp}\right)-aV_T\right)}{\left(V_{mp}-I_{mp}R_s\right)\left(I_{sc}-I_{mp}\right)-aV_TI_{mp}}$$ (8)

- Equation for I_0:

$$I_0=\frac{\left(R_{sh}+R_s\right)I_{sc}-V_{oc}}{R_{sh}\exp\left(\dfrac{V_{oc}}{aV_T}\right)}$$ (9)

- Equation for I_{pv}:

$$I_{pv}=\frac{R_{sh}+R_s}{R_{sh}}I_{sc}$$ (10)

By estimating parameter a, and solving the previous system, it is possible to determine the values of the equivalent circuit parameters simply, straight forward, and quite accurately. However, although the mathematical system is decoupled, Equation (7) is still implicit for R_s. This equation is not difficult to solve with numerical calculation programs (e.g., Matlab or Maple). Nevertheless, it is possible to transform it into an explicit equation using the Lambert W-function [47]. The Lambert W-function, $W(z)$, is defined as:

$$z=W(z)e^{W(z)}$$ (11)

where z is any complex number. This function is not injective; in the real variable x the relation W is defined only for $x\geq-1/e$ and is double valued in the bracket $[-1/e, 0]$, the two branches of the function being expressed as $W_0(x)$ for $W(x)\geq-1$ and $W_{-1}(x)$ for $W(x)\leq-1$ in the aforementioned bracket. The general strategy to apply the Lambert W-function in solving exponential equations is to use the following equivalence:

$$X=Ye^Y\Leftrightarrow Y=W(X)$$ (12)

Going back to the equivalent circuit equations, Equation (7) can be rewritten as follows:

$$-\frac{V_{mp}\left(2I_{mp}-I_{sc}\right)}{\left(V_{mp}I_{sc}+V_{oc}\left(I_{mp}-I_{sc}\right)\right)}\exp\left(-\frac{2V_{mp}-V_{oc}}{aV_T}+\frac{\left(V_{mp}I_{sc}-V_{oc}I_{mp}\right)}{\left(V_{mp}I_{sc}+V_{oc}\left(I_{mp}-I_{sc}\right)\right)}\right)=$$
$$\left(\frac{I_{mp}R_s-V_{mp}}{aV_T}+\frac{\left(V_{mp}I_{sc}-V_{oc}I_{mp}\right)}{\left(V_{mp}I_{sc}+V_{oc}\left(I_{mp}-I_{sc}\right)\right)}\right)\exp\left(\frac{I_{mp}R_s-V_{mp}}{aV_T}+\frac{\left(V_{mp}I_{sc}-V_{oc}I_{mp}\right)}{\left(V_{mp}I_{sc}+V_{oc}\left(I_{mp}-I_{sc}\right)\right)}\right) \tag{13}$$

Using the mathematical equivalence from Equation (12), Equation (13) turns into:

$$\frac{I_{mp}R_s-V_{mp}}{aV_T}+\frac{\left(V_{mp}I_{sc}-V_{oc}I_{mp}\right)}{\left(V_{mp}I_{sc}+V_{oc}\left(I_{mp}-I_{sc}\right)\right)}=$$
$$W_{-1}\left(-\frac{V_{mp}\left(2I_{mp}-I_{sc}\right)}{\left(V_{mp}I_{sc}+V_{oc}\left(I_{mp}-I_{sc}\right)\right)}\exp\left(-\frac{2V_{mp}-V_{oc}}{aV_T}+\frac{\left(V_{mp}I_{sc}-V_{oc}I_{mp}\right)}{\left(V_{mp}I_{sc}+V_{oc}\left(I_{mp}-I_{sc}\right)\right)}\right)\right) \tag{14}$$

where W_{-1} is the negative branch of the Lambert W-function (as the left part of Equation (14) is lower than −1 for typical cells and solar panels). And then, an explicit expression for R_s is obtained:

$$R_s=A\left(W_{-1}\left(B\exp(C)\right)-(D+C)\right) \tag{15}$$

where:

$$A=\frac{aV_T}{I_{mp}}$$
$$B=-\frac{V_{mp}\left(2I_{mp}-I_{sc}\right)}{\left(V_{mp}I_{sc}+V_{oc}\left(I_{mp}-I_{sc}\right)\right)}$$
$$C=-\frac{2V_{mp}-V_{oc}}{aV_T}+\frac{\left(V_{mp}I_{sc}-V_{oc}I_{mp}\right)}{\left(V_{mp}I_{sc}+V_{oc}\left(I_{mp}-I_{sc}\right)\right)} \tag{16}$$
$$D=\frac{V_{mp}-V_{oc}}{aV_T}$$

In recent years, the Lambert W-function has become a common mathematical tool in solar cell/panel equivalent circuit extraction parameters, as it is quite useful for solving exponential equations like the expression of the *I-V* curve. Commonly, the mathematical relation (12) is directly applied to Equation (1) in order to obtain explicit equations for the solar cell/panel output current or voltage. This method is helpful for the determination of equivalent circuit parameters through numerical methods, that is, when testing data is available and a curve fitting is performed [23,48]. However, it is not convenient to use the Lambert W-function to transform Equation (1) if the aim is to solve the problem analytically, because it is not possible to obtain a decoupled system of equations [49]. In this particular case, the approach used is different, it consists in using the Lambert W-function as the last step of the procedure; clearing R_s from the remaining implicit equation after obtaining explicit equations for the other parameters. Then, a fully decoupled set of explicit equations for the equivalent circuit parameters can be obtained. Data from two solar cells, included in Table 1, have been used to

validate the results obtained following the procedure described in the text. Also, the accuracy of the method can be checked. In Table 2, the values of the equivalent circuit parameters obtained are compared to the ones from [17], obtained through a curve fitting technique and used as reference for the parameter *a*, and to the results from another method based on the Lambert W-function, with the difference that this other one is iterative (and not explicit) [50].

Table 1. Characteristic of blue solar cells and grey solar cells [17].

Blue Solar Cell		Grey Solar Cell	
a	1.51 ± 0.07	*a*	1.72 ± 0.08
I_{mp} (A)	0.0934	I_{mp} (A)	0.485
V_{mp} (V)	0.433	V_{mp} (V)	0.387
I_{sc} (A)	0.1023	I_{sc} (A)	0.561
V_{oc} (V)	0.536	V_{oc} (V)	0.524
T_r (K)	300	T_r (K)	307

Table 2. Characteristic of blue solar cells and grey solar cells [17].

	Blue Solar Cell				Grey Solar Cell		
Equivalent Circuit Parameters	Benchmark [17]	Iterative Method [50]	Present Method	Equivalent Circuit Parameters	Benchmark [17]	Iterative Method [50]	Present Method
R_s (Ω)	0.07 ± 0.009	0.0671	0.0652	R_s (Ω)	0.08 ± 0.01	0.0784	0.0781
R_{sh} (Ω)	1000 ± 50	977	1093	R_{sh} (Ω)	26 ± 1	26.09	26.25
I_{pv} (A)	0.1023 ± 0.0005	0.1023	0.1023	I_{pv} (A)	0.5625 ± 0.0005	0.561	0.5627
I_0 (A)	$110 \pm 50 \times 10^{-9}$	111×10^{-9}	111×10^{-9}	I_0 (A)	$6 \pm 3 \times 10^{-6}$	5.6×10^{-6}	5.4×10^{-6}

2.2. Parameter Dependence on Environmental Conditions

Environmental conditions, such as temperature or irradiation variations, modify the behavior of photovoltaic devices, that is, the *I-V* curve. Therefore, those variations affect the boundary conditions of the equations and, accordingly, the value of the equivalent circuit parameters.

When temperature raises, the current increases and the voltage decreases, and as a result, the maximum power point of the *I-V* curve also decreases. These variations are approximately linear with temperature, and most solar panel manufacturers include the rate of variation of the characteristic *I-V* curve points in the datasheets, either as absolute increase or as percent variation. In the second case the variations with temperature *T* of current and voltage levels at open circuit, maximum power, and short circuit points can be expressed as:

$$V_{oc,T} = V_{oc,T_r}\left(1 + \frac{\beta V_{oc}(T - T_r)}{100}\right)$$

$$V_{mp,T} = V_{mp,T_r}\left(1 + \frac{\beta V_{mp}(T - T_r)}{100}\right)$$

$$I_{sc,T} = I_{sc,T_r}\left(1 + \frac{\alpha I_{sc}(T - T_r)}{100}\right) \qquad (17)$$

$$I_{mp,T} = I_{mp,T_r}\left(1 + \frac{\alpha I_{mp}(T - T_r)}{100}\right)$$

where T_r is the reference temperature; βV_{oc} and βV_{mp} are respectively the percentage variation of the open circuit and maximum power point voltages when the temperature increases one degree; finally, αI_{sc} and αI_{mp} are the percentage variation of the short circuit and maximum power point currents when the temperature increases one degree. Some manufacturers include the percentage variation of the maximum power with temperature, γ, instead of specific variations for current and voltage at that point. If αI_{mp} or βV_{mp} (one of them) is missing, then the variation of the boundary condition can be calculated using the parameter γ and the remaining condition, for example: I_{mp} value in terms of γ and βV_{mp} is:

$$P_{mp,T} = I_{mp,T} V_{mp,T} = P_{mp,T_r} \left(1 + \frac{\gamma(T - T_r)}{100} \right)$$

$$I_{mp,T} = I_{mp,T_r} \frac{100 + \gamma(T - T_r)}{100 + \beta V_{mp}(T - T_r)}$$

(18)

On the other hand, if both αI_{mp} and βV_{mp} are not included in the manufacturer's datasheet and only the parameter γ is provided, it can be assumed that $\beta V_{mp} \approx \beta V_{oc}$ before using Equation (18). With these relations it is possible to directly relate the equivalent circuit parameters and the rate of variations with T [22,44]. Nevertheless, a different approximation is used in the present work: the equivalent circuit parameters are calculated for different temperatures with Equation (17), using the values of the characteristic points at those temperatures. Then, the variation of the parameters as a function of temperature T is empirically determined based on these results. This procedure is very accurate, as the calculated equivalent circuits directly meet the manufacturers' data.

Any information regarding the dependence of the solar cell/panel on irradiation levels is commonly not included in manufacturers' datasheets, as this dependence is well known and easy to be determined. The shape of the *I-V* curve is essentially invariant with intensity in ranges around one solar constant, temperature affects I_{sc} linearly and V_{oc} exponentially, whereas R_s remains not affected for temperature variations [51]. Those conditions lead to the following equation [44]:

$$I_{pv,G} = I_{pv,G_r} \frac{G}{G_r}$$

(19)

where G is the irradiance on the cell/solar panel, $I_{pv,G}$ is the photocurrent delivered by the current source of the equivalent circuit, and G_r and I_{pv,G_r} are the reference values. When data about irradiation dependence is available, it is generally shown as experimental *I-V* curves, in this case it would be possible to extract information about the characteristic points graphically, and therefore define the dependence on irradiation with a similar procedure to the one followed in case of temperature variations.

3. Results and Discussion

In this Section, the equivalent circuits of two commercial solar panels (MSP290AS-36.EU and MSMD290AS-36 by München Solarenergie GmbH, Neu-Ulm, Germany), are calculated as an example of application of the above method. For that calculation only data provided by the manufacturer is used [52,53], see Table 3. The objective is to obtain an equivalent circuit that reproduces the solar panel behavior for any irradiation and temperature levels. The first step is to estimate the ideality factor, *a*. Some authors propose a generic value in the bracket [1, 1.5] for every kind of cell [45],

although different technologies are associated with different values of the aforementioned ideality factor at standard conditions. In [54] $a = 1.2$ is suggested for monocrystalline silicon cells, whereas $a = 1.3$ is suggested for polycrystalline ones. This parameter is normally considered independent of temperature. However, some studies indicate a decreasing linearity with temperature [55]. In any case, the lack of accuracy produced when considering the ideality factor as constant is generally reduced, given that variations of this parameter only affects the curvature of the *I-V* curve. Nevertheless, it should also be pointed out that a not physically possible solution for the equivalent circuit parameters (negative values of the resistors), can be obtained if a value of the ideality factor too detached from the real one is chosen, as the aforementioned curvature is incompatible with the boundary conditions (*i.e.*, the characteristic points). The possibility of those not physically possible solutions increases in large temperature ranges, if the same value of the ideality factor is taken for the lower and the higher temperatures. In those cases a decreasing linear variation law with temperature for parameter *a* could be included [55], or alternatively just values of parameter *a* that result in a valid solution for most of the temperature interval under consideration could be selected. In the present study an ideality factor of $a = 1.1$, has been chosen for the silicon cell panels. Given that the ideality factor *a* decreases as temperature rises, this slightly lower value of *a* with respect to the figures assumed by other authors [54], increases the highest temperature for which physically possible solutions exist. The procedure to extract the equivalent circuit parameters is therefore as follows:

- Estimate the value of the parameter *a*. In this method this parameter is considered as a constant, independent of the temperature and the irradiation. In this case the chosen value is $a = 1.1$.
- Calculate V_t for the panel at that temperature level with Equation (2).
- Determine the boundary conditions at that temperature and irradiation level using manufacturer's data and Equations (17) and/or (18).
- Calculate R_s with Equation (15).
- Calculate R_{sh} with Equation (8).
- Calculate I_0 with Equation (9).
- Calculate I_{pv} with Equation (10).

Table 3. Characteristic points of MSP290AS-36.EU and MSMD290AS-36 solar panels (München Solarenergie GmbH), included in the manufacturer datasheets [52,53] at STC (Standard Test Conditions) (1000 W/m^2 irradiance, 25 °C cell temperature, AM1.5g spectrum according to EN 60904-3).

MSP290AS-36.EU (Multicrystalline)				MSMD290AS-36.EU (Monocrystalline)			
n	72	T_r (°C)	25	n	72	T_r (°C)	25
P_{mp} (W)	290	γ (%/°C)	−0.45	P_{mp} (W)	290	γ (%/°C)	−0.44
I_{mp} (A)	7.82	αI_{mp} (%/°C)	-	I_{mp} (A)	7.70	αI_{mp} (%/°C)	-
V_{mp} (V)	37.08	βV_{mp} (%/°C)	−0.35	V_{mp} (V)	37.66	βV_{mp} (%/°C)	−0.35
I_{sc} (A)	8.37	αI_{sc} (%/°C)	+0.04	I_{sc} (A)	8.24	αI_{sc} (%/°C)	+0.04
V_{oc} (V)	44.32	βV_{oc} (mV/°C)	−0.33	V_{oc} (V)	44.68	βV_{oc} (mV/°C)	−0.31

The values of circuit parameters obtained at AM1.5g (1000 W/m^2) and $T_r = 25$ °C, are included in Table 4.

Table 4. Parameters of MSP290AS-36.EU and MSMD290AS-36.EU (München Solarenergie GmbH) solar panels equivalent circuits at STC (1000 W/m² irradiance, 25 °C cell temperature, AM1.5g spectrum).

MSP290AS-36.EU (Multicrystalline)		MSMD290AS-36.EU (Monocrystalline)	
a	1.10	a	1.10
I_{pv} (A)	8.37	I_{pv} (A)	8.24
I_0 (A)	2.86×10^{-9}	I_0 (A)	2.36×10^{-9}
R_s (Ω)	0.162	R_s (Ω)	0.130
R_{sh} (Ω)	331	R_{sh} (Ω)	316

In order to spread the use of the equivalent circuit to any temperatures inside the range 15 to 85 °C (this bracket includes most of the operational temperatures; it should be taken also into account that the temperature of the panel is higher than the ambient temperature during operation), the process is repeated for temperatures in that range. Parameter values, showing their variation with temperature, are shown in Figure 3. In the figure it can be observed how parameter R_s turns negative for high temperatures (the value of parameter R_{sh} could also turn negative under some conditions). As previously suggested, those mathematically correct but not physically valid solutions at high temperatures are derived in cases where the boundary conditions (slopes of the *I-V* curve at short circuit and open circuit points), are not compatible with the curvature determined by the chosen ideality factor.

Numerical expressions for the circuit parameters as a function of the temperature, *T*, are then derived from this results using polynomial approximation with the least squares method:

- For MSP290AS-36.EU:

$$I_{pv}(T) = 8.37 + 3.62 \cdot 10^{-3} \Delta T - 3.38 \cdot 10^{-6} \Delta T^2 - 7.58 \cdot 10^{-8} \Delta T^3$$

$$R_s(T) = 1.62 \cdot 10^{-1} - 3.21 \cdot 10^{-3} \Delta T + 7.05 \cdot 10^{-7} \Delta T^2 - 3.01 \cdot 10^{-8} \Delta T^3$$

$$R_{sh}(T) = 1/(3.03 \cdot 10^{-3} + 2.65 \cdot 10^{-4} \Delta T + 1.50 \cdot 10^{-6} \Delta T^2 + 1.56 \cdot 10^{-8} \Delta T^3)$$

$$I_0(T) = \exp(-1.97 \cdot 10^1 + 1.44 \cdot 10^{-1} \Delta T - 4.80 \cdot 10^{-4} \Delta T^2 + 1.15 \cdot 10^{-6} \Delta T^3)$$

$$(20)$$

- For MSMD290AS-36.EU:

$$I_{pv}(T) = 8.24 + 3.49 \cdot 10^{-3} \Delta T - 1.68 \cdot 10^{-6} \Delta T^2 - 2.41 \cdot 10^{-8} \Delta T^3$$

$$R_s(T) = 1.30 \cdot 10^{-1} - 1.97 \cdot 10^{-3} \Delta T + 2.53 \cdot 10^{-6} \Delta T^2 - 1.07 \cdot 10^{-8} \Delta T^3$$

$$R_{sh}(T) = 1/(3.18 \cdot 10^{-3} + 2.33 \cdot 10^{-4} \Delta T + 1.27 \cdot 10^{-6} \Delta T^2 + 1.33 \cdot 10^{-8} \Delta T^3)$$

$$I_0(T) = \exp(-1.98 \cdot 10^1 + 1.41 \cdot 10^{-1} \Delta T - 4.69 \cdot 10^{-4} \Delta T^2 + 1.13 \cdot 10^{-6} \Delta T^3)$$

$$(21)$$

The irradiation variation only affects the parameter I_{pv} and can be described with Equation (19), including this variation the following equations can be then derived for the five parameters of the equivalent circuit:

- For MSP290AS-36.EU:

$$I_{pv}(T,G) = \left(8.37 + 3.62 \cdot 10^{-3}\Delta T - 3.38 \cdot 10^{-6}\Delta T^2 - 7.58 \cdot 10^{-8}\Delta T^3\right)\frac{G}{G_r}$$

$$R_s(T) = 1.62 \cdot 10^{-1} - 3.21 \cdot 10^{-3}\Delta T + 7.05 \cdot 10^{-7}\Delta T^2 - 3.01 \cdot 10^{-8}\Delta T^3$$

$$R_{sh}(T) = 1/(3.03 \cdot 10^{-3} + 2.65 \cdot 10^{-4}\Delta T + 1.50 \cdot 10^{-6}\Delta T^2 + 1.56 \cdot 10^{-8}\Delta T^3)$$

$$I_0(T) = \exp(-1.97 \cdot 10^1 + 1.44 \cdot 10^{-1}\Delta T - 4.80 \cdot 10^{-4}\Delta T^2 + 1.15 \cdot 10^{-6}\Delta T^3)$$

(22)

- For MSMD290AS-36.EU:

$$I_{pv}(T,G) = \left(8.24 + 3.49 \cdot 10^{-3}\Delta T - 1.68 \cdot 10^{-6}\Delta T^2 - 2.41 \cdot 10^{-8}\Delta T^3\right)\frac{G}{G_r}$$

$$R_s(T) = 1.30 \cdot 10^{-1} - 1.97 \cdot 10^{-3}\Delta T + 2.53 \cdot 10^{-6}\Delta T^2 - 1.07 \cdot 10^{-8}\Delta T^3$$

$$R_{sh}(T) = 1/(3.18 \cdot 10^{-3} + 2.33 \cdot 10^{-4}\Delta T + 1.27 \cdot 10^{-6}\Delta T^2 + 1.33 \cdot 10^{-8}\Delta T^3)$$

$$I_0(T) = \exp(-1.98 \cdot 10^1 + 1.41 \cdot 10^{-1}\Delta T - 4.69 \cdot 10^{-4}\Delta T^2 + 1.13 \cdot 10^{-6}\Delta T^3)$$

(23)

Figure 3. Calculated values (using the explained methodology) of the equivalent circuit parameters R_s, R_{sh}, I_{pv}, and I_0, for multicrystalline (MSP290AS-36.EU) and monocrystalline (MSMD290AS-36.EU) solar cell panels as a function of the temperature, *T*. Calculated points are indicated with symbols whereas the polynomial approximations fitted to those data (Equations (20) and (21)) have been included in each case as solid lines.
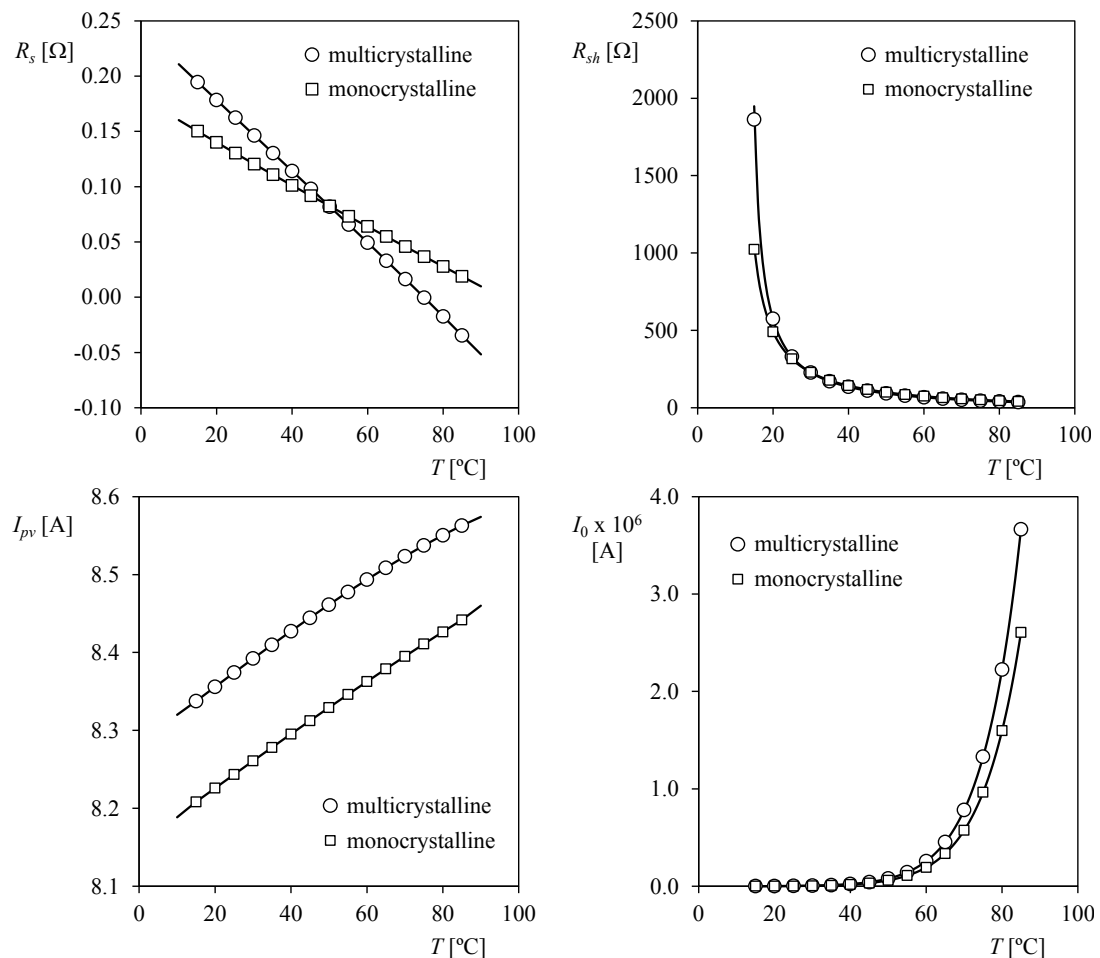
4. Experimental Section

In this Section the behavior of the calculated equivalents circuits is analyzed. The aim is to check if the calculated circuit models reproduce the experimental data of the solar panels included in the manufacturer's datasheet. In order to produce results, the equivalent circuits of the solar panels have been modeled using *LTSpice* (Linear Technology Corporation, Milpitas, CA, USA), a *Simulation Program with Integrated Circuit Emphasis* (*SPICE*) software: Obviously, the mentioned circuits are characterized by the parameters already calculated in Section 3, see Figures 4 and 5. Several simulations have been performed at different temperature and irradiation levels, calculating the *I-V* curve in each case. The results thereof are included in Figures 6–9.

Figure 4. Equivalent circuit of MSP290AS-36.EU (multicrystalline).
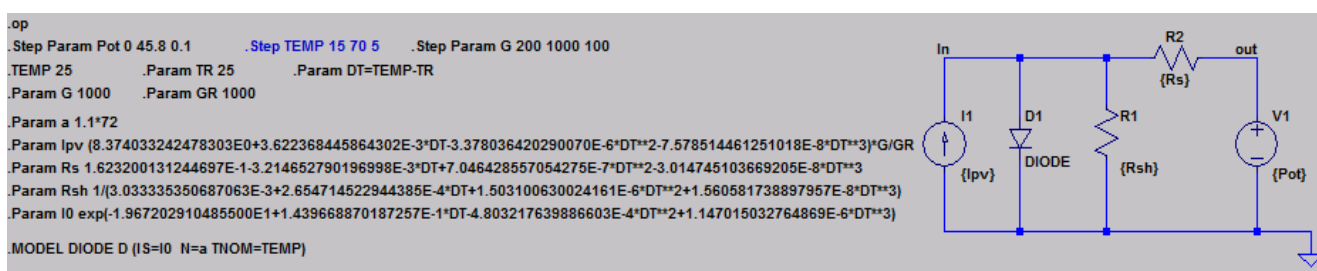
Figure 5. Equivalent circuit of MSMD290AS-36.EU (monocrystalline).
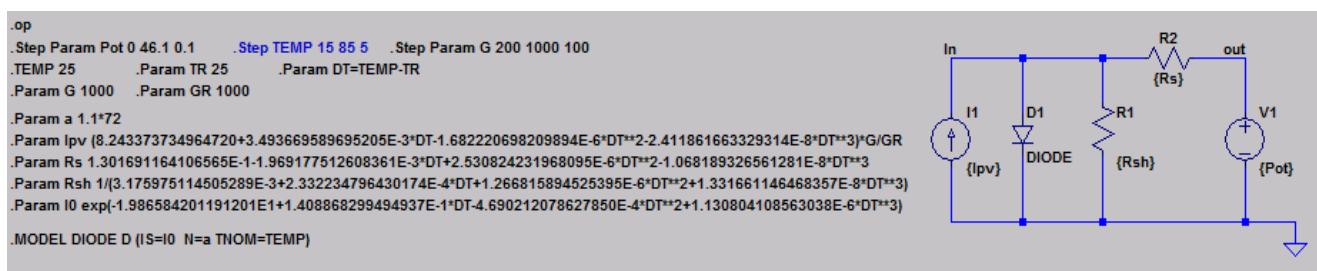
Figure 6. *I-V* curves of panel MSP290AS-36.EU simulated with *LTSpice* calculation of the equivalent circuits for different irradiation levels, from 200 W/m^2 to 1000 W/m^2, at 25 °C.
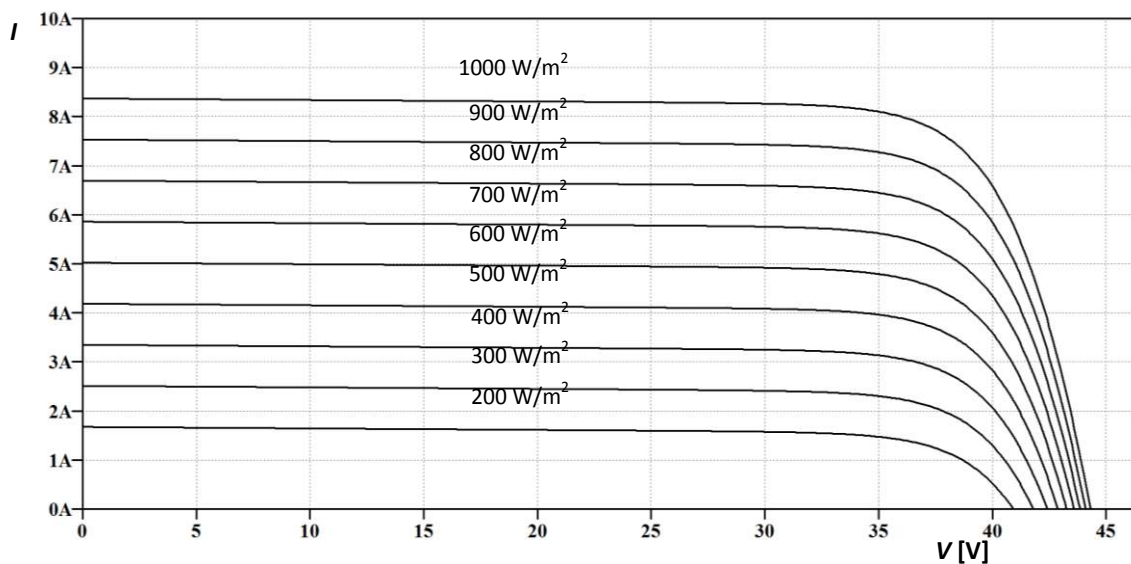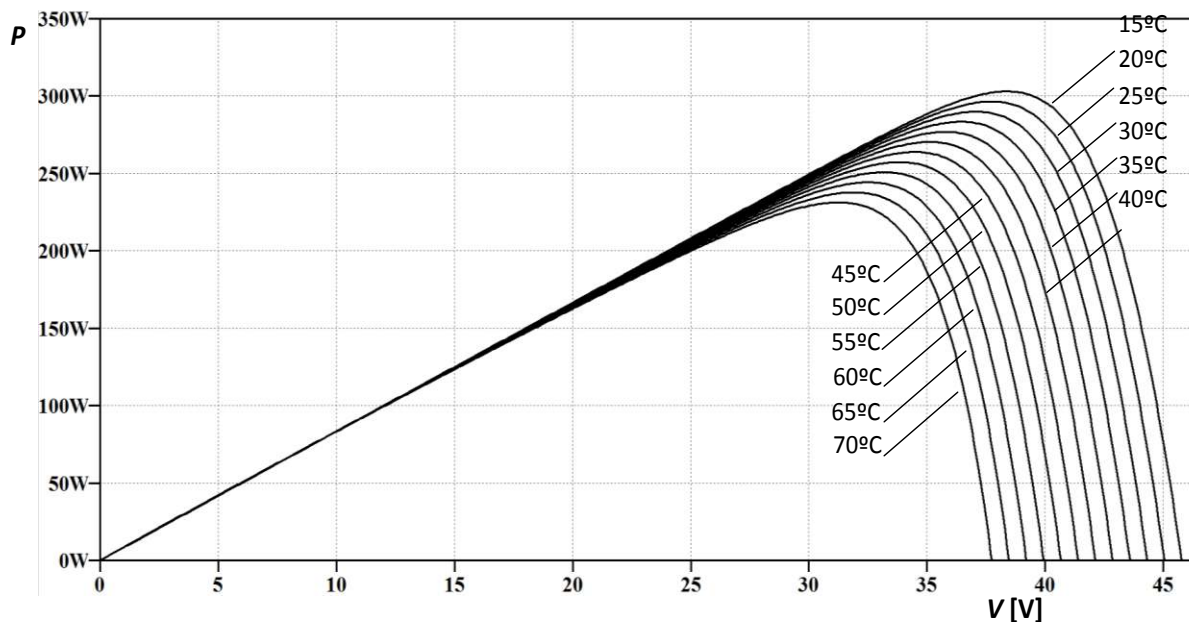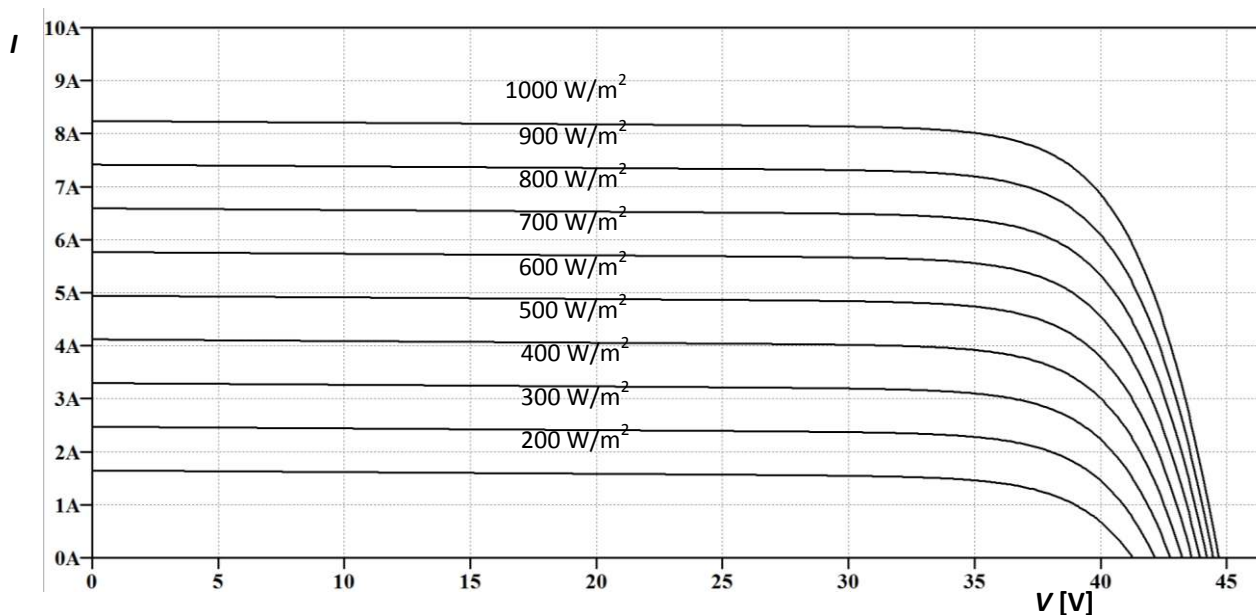
Figure 7. Power curves of panel MSP290AS-36.EU simulated with *LTSpice* calculation of the equivalent circuits for different temperature levels, from 15 °C to 70 °C, with 1000 W/m^2 irradiation level.

Figure 8. *I-V* curves of panel MSMD290AS-36.EU simulated with *LTSpice* calculation of the equivalent circuits for different irradiation levels, from 200 W/m^2 to 1000 W/m^2, at 25 °C.

Finally, the results of the simulations performed reproduce with high accuracy the data from the solar panel datasheets regarding the characteristic points' dependence on the temperature as can be seen in Figure 10, where the manufacturer experimental information about the boundary conditions, Equations (17) and (18), are compared with the characteristic points of the equivalent circuit calculated.

Figure 9. Power curves of panel MSMD290AS-36.EU simulated with *LTSpice* calculation of the equivalent circuits for different temperature levels, from 15 °C to 85 °C, with 1000 W/m^2 irradiation level.
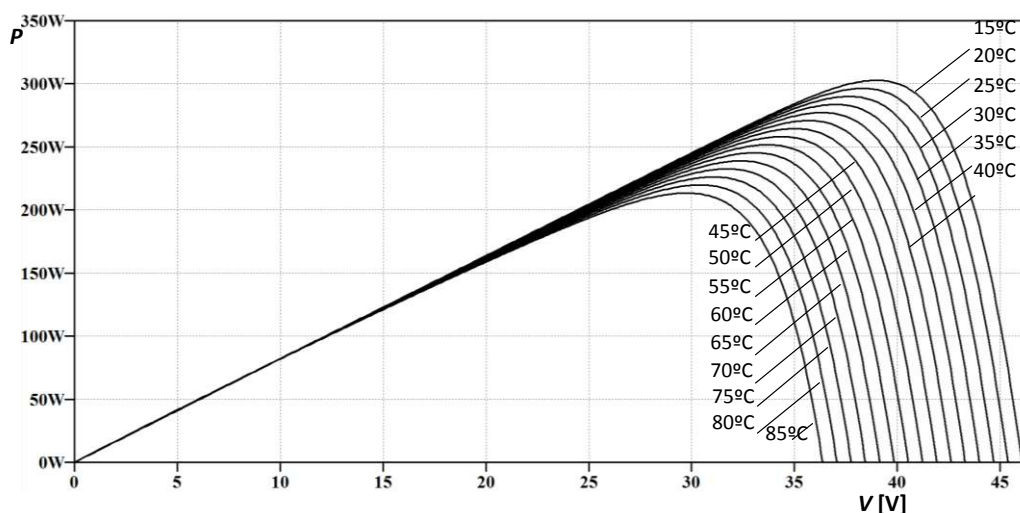

Figure 10. Variation of multicrystalline (MSP290AS-36.EU) and monocrystalline (MSMD290AS-36.EU) solar panel characteristic data, I_{sc}, V_{oc}, P_{mp}, and V_{mp} as a function of the temperature, T. Symbols represent the characteristic points of calculated equivalent circuits (parameters defined in Equations (22) and (23)), whereas solid lines represent the experimental temperature dependence of these points included in the manufacturer datasheet (see Table 3 and Equations (17) and (18)).
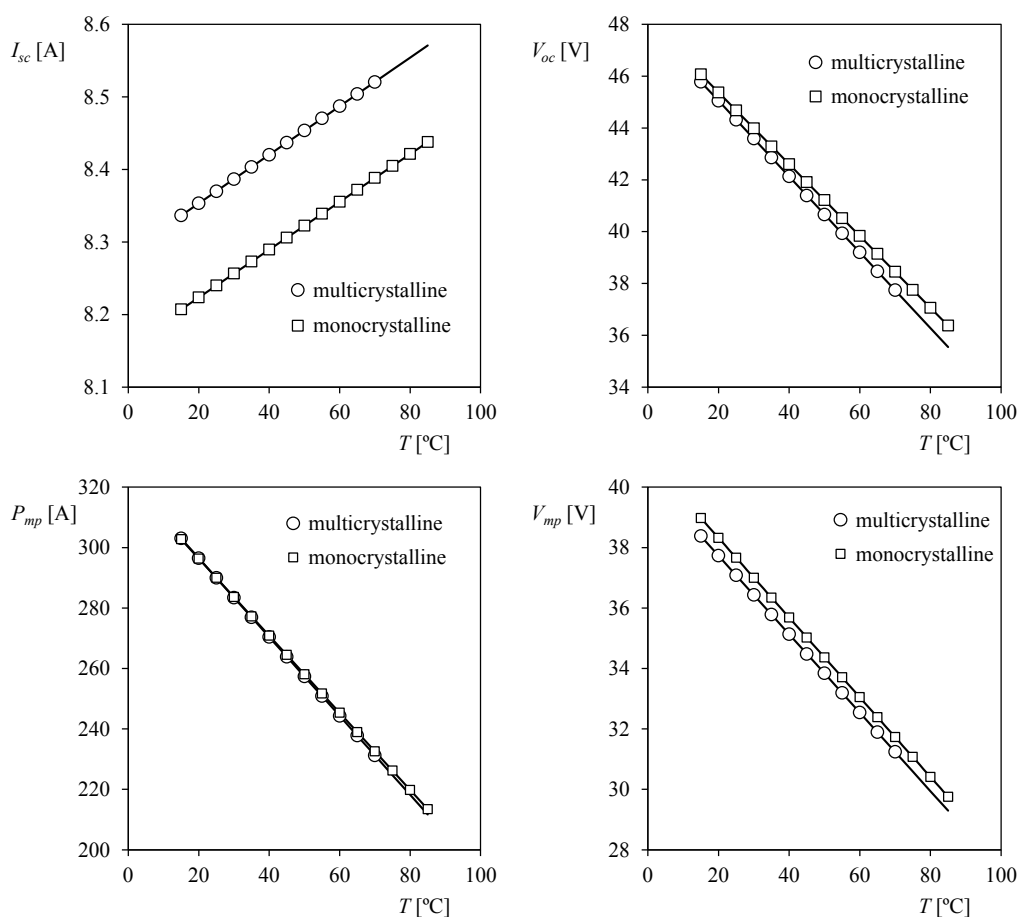
5. Conclusions

In the present work a simple but accurate method to simulate the performances of a photovoltaic device for different working conditions is presented. The method is based in the analytical determination of the parameters of an equivalent circuit. Using the presented methodology it is possible to construct a realistic model of a solar panel that reproduces the experimental data provided by the manufacturer in the datasheet, including variations at different temperatures and irradiations. The method is explicit, non-iterative and straight forward; no iterations or initial values for the parameters are needed. Only data typically included in manufacturers' datasheets are required. There are several possible applications for this method. From the authors' point of view, its simplicity and accuracy would make it interesting for end users with little calculation and testing resources. Also, the procedure described in the present work seems to be very appropriate for analysis that imply profuse calculations of equivalent circuits or for determination of initial values for numerical methods. Finally, an interesting application of this method is to construct realistic models of solar panels that can be used in simulations of MPPT algorithms.

Author Contributions

Javier Cubas and Santiago Pindado developed the idea behind the present research, carrying also out the literature review and manuscript preparation at early stages. The methodology and analytic procedures were carried out by Javier Cubas. Carlos de Manuel programmed the *LTspice* models. Final review, including final manuscript corrections, was done by Santiago Pindado. All authors were in charge of data generation/analysis and post-processing of the results.

Conflicts of Interest

The authors declare no conflict of interest.

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