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## Explicit Solution of Nonlinear ZK-BBM Wave Equation Using Exp-Function Method

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**Abstract:** This study is devoted to studying the (2+1)-dimensional ZK-BBM (Zakharov-Kuznetsov-Benjamin-Bona-Mahony) wave equation in an analytical solution. The analysis is based on the implementation a new method, called Exp-function method. The obtained results from the proposed approximate solution have been verified with those obtained by the extended tanh method. It shows that the obtained results of these methods are the same; while Exp-function method, with the help of symbolic computation, provides a powerful mathematical tool for solving nonlinear partial differential equations of engineering problems in the terms of accuracy and efficiency.

**Key words:** Benjamin-Bona-Mahony-Burgers (BBMB) equations, Exp-function method

### INTRODUCTION

In the recent decade, the study of nonlinear partial differential equations (NLEEs) modeling physical phenomena, has become an important toll. Seeking exact solutions for (NLEEs) has long been one of the central themes of perpetual interest in Mathematics and Physics. These solutions may well describe various phenomena in physics and other fields and thus may give more insight into the physical aspects of the problems. In this aspect, nonlinear wave equations in mathematical physics play a major role in various fields, such as plasma physics, fluid mechanics, optical fibers, solid state physics, optical fibers, chemical kinetics and geochemistry (Benjamin *et al.*, 1972; Ablowitz and Clarkson, 1991; Rosenau and Hyman, 1993; Hereman *et al.*, 1985; Kadomtsev and Petviashvili, 1970; Zakharov and Kuznestov, 1974; Li *et al.*, 2003; Malfliet, 1992; Malfliet and Hereman, 1996; Monro and Parkes, 1999). But it is well known that except a limited number most of them do not have any precise analytical solutions. So, to solve these nonlinear equations other methods are needed, however, in recent decades numerical methods have well used to analysis the nonlinear partial equations such as (2+1)-dimensional ZK-BBM (Benjamin-Bona-Mahony) equation. Long with the numerical methods, the semi-exact analytical methods have been improved; for instance inverse scattering method (Ablowitz and Clarkson, 1991), Hirota's bilinear method (Hirota, 1971),

homogenous balance method (Wang, 1996), homotopy perturbation method (He, 2005; Ganji and Rafei, 2006; Tolou *et al.*, 2007), variational iteration method (He, 1999, 2000, 2004; Ganji and Rafei, 2006), asymptotic methods (He, 2006), non-perturbative methods (He, 2006), tanh-function method (Zayed *et al.*, 2004; Wazwaz, 2007; Zhang and Xia, 2006a; Zhang, 2007b), algebraic method (Hu, 2005; Zhang and Xia, 2006b), Jacobi elliptic function expansion method (Liu *et al.*, 2001; Zhao *et al.*, 2006), F-expansion method (Zhang, 2006, 2007a) and so on. Recently, He and Wu (2006) proposed a straightforward and concise method, called Exp-function method, to obtain generalized solitary solutions and periodic solutions of NLEEs. The solution procedure of this method, by the help of Matlab or Mathematica, is of utter simplicity and this method can be easily extended to all kinds of NLEEs.

In our previous study we implemented the HPM and VIM to BBMs equations (Fakhari *et al.*, 2007; Tari and Ganji, 2007). Those show the applicability, accuracy and finally efficiency of both VIM and HPM. The Benjamin-Bona-Mahony equation (Benjamin *et al.*, 1972) described by the following:

$$u_t + u_x + uu_x - u_{xxx} = 0 \quad (1)$$

This equation has been proposed as a model for propagation of long waves where nonlinear dispersion is incorporated.

Also we implemented HPM to the fifth-order Korteweg de Vries (KdV) and generalized Hirota-Satsuma coupled KdV equations (Rafei and Ganji, 2006) which again approve the accuracy and efficiency of proposed method to this type of equation. The spatially one-dimensional KdV equation;

$$u_t + auu_x - u_{xxx} = 0 \tag{2}$$

is a model that governs the one-dimensional propagation of small amplitude, weakly dispersive waves and plays a major role in the solitons concepts. The term soliton coined by Zabusky and Kruskal (He, 2006) who found particle like waves which retained their shapes and velocities after collisions. The balance between the nonlinear convection term  $uux$  and the dispersion effect term  $u_{xxx}$  in the KdV Eq. 2 gives rise to solitons. The K (n, n) equation (Rosenau and Hyman, 1993):

$$u_t + a(u^n)_x - u^n_{xxx} = 0 \tag{3}$$

Where, in addition to the nonlinear convection term  $(u^n)_x$ , the dispersion effect term  $u^n_{xxx}$  is genuinely nonlinear as well. The delicate interaction between the convection with the genuine nonlinear dispersion generates solitary waves with exact compact support that are termed compactons. Compactons are defined as solitons with finite wavelengths. Compactons are compact solutions that are usually expressed by powers of trigonometric functions sine and cosine. Unlike soliton that narrows as the amplitude increases, the compacton's width is independent of the amplitude.

In modern physics, a suffix-on is used to indicate the particle property (Wang, 1996), for example phonon, photon, peakon, soliton and compacton. One of the well-known two-dimensional generalizations of the KdV equations is developed, namely the Zakharov-Kuznetsov (ZK) (1974) given by:

$$u_t + auu_x + (\nabla^2 u)_x = 0 \tag{4}$$

Where,  $\nabla^2 = \partial^2 x + \partial^2 y + \partial^2 z$  is the isotropic Laplacian (Zakharov and Kuznetsov, 1974; Li *et al.*, 2003).

The present letter is motivated by the desire to extend these works via implementation the Exp-function method to a modified form of BBMs equations formulated in the ZK sense by examination (2+1) dimensional ZK-BBM problem.

Consider the model describe by the generalized of the ZK-BBM equation (Wazwaz, 2007).

$$u_t + u_x + a(u^n)_x + b(u_{xt} + u_{yy})_x = 0 \tag{5}$$

To clarify the validity of proposed method, the obtained solutions are compared with their corresponding tanh methods.

### MATERIALS AND METHODS

**Implementation the Exp-function method:** In this study, we apply the Exp-function method (He and Wu, 2006) for the solution of the ZK-BBM equations (Eq. 5) (Wazwaz, 2007).

Using the transformation

$$u = u(\eta), \quad \eta = kx + qy + \omega t, \tag{6}$$

Where,  $k$  and  $\omega$  are constants. Eq. 5 becomes:

$$\omega u' + ku' + 3aku^2 u' + b(\omega k^2 u^m + kq^2 u^m) = 0 \tag{7}$$

Where, prime denotes the differential with respect to  $\eta$ . The Exp-function method is based on the assumption that traveling wave solution can be expressed in the following form (He and Wu, 2006):

$$u(\eta) = \frac{\sum_{n=-c}^d a_n \exp(n\eta)}{\sum_{m=-p}^q b_m \exp(m\eta)} \tag{8}$$

Where,  $c, d, p$  and  $q$  are positive integers which are unknown to be further determined,  $a_n$  and  $b_m$  are unknown constants. Equation 7 can be re-written in an alternative form (He and Wu, 2006) as follows:

$$u(\eta) = \frac{a_c \exp(c\eta) + \dots + a_{-d} \exp(-d\eta)}{a_q \exp(p\eta) + \dots + a_{-p} \exp(-p\eta)} \tag{9}$$

In order to determine the values of  $c$  and  $p$ , we balance the linear term of highest order in Eq. 7 with the highest order nonlinear term (He and Wu, 2006). By calculation, obtained:

$$u'' = \frac{c_1 \exp[(7p+c)\eta] + \dots}{c_2 \exp[8p\eta] + \dots} \tag{10}$$

and

$$u^2 u' = \frac{c_3 \exp[(p+3c)\eta] + \dots}{c_4 \exp[4p\eta] + \dots} = \frac{c_3 \exp[(5p+3c)\eta] + \dots}{c_4 \exp[8p\eta] + \dots} \tag{11}$$

Where,  $c_i$  are determined coefficients only for simplicity. Balancing highest order of Exp-Function in Eq. 10 and 11 we have:

$$7p+c = 2(3p+c) \tag{12}$$

This leads to the following result:

$$p = c \tag{13}$$

Similarly, to determine the values of d and q, we balance the linear term of lowest order in Eq. 7 with the lowest order nonlinear term.

$$u''' = \frac{\dots + d_1 \exp[-(7q+d)\eta]}{\dots + d_2 \exp[-8q\eta]} \tag{14}$$

and

$$u^2 u' = \frac{\dots + d_3 \exp[-(q+3d)\eta]}{\dots + d_4 \exp[-(-4q)\eta]} = \frac{\dots + d_3 \exp[-(5q+d)\eta]}{\dots + d_4 \exp[-(-8q)\eta]} \tag{15}$$

Where,  $d_i$  are determined coefficients only for simplicity. Balancing lowest order of Exp-Function in Eq. 14 and 15:

$$-(7q+d) = -2(3q+d) \tag{16}$$

which leads

$$q = d \tag{17}$$

**Case 1:**  $p = c = 1, d = 1$ : We can freely choose the values of c and d, but we illustrate that the final solution does not strongly depend upon the choice of values c and d (He and Wu, 2006). By setting  $p = c = 1$  and  $q = d = 1$  the trial function Eq. 9 becomes:

$$u(\eta) = \frac{a_1 \exp(\eta) + a_0 + a_{-1} \exp(-\eta)}{\exp(\eta) + b_0 + b_{-1} \exp(-\eta)} \tag{18}$$

Substituting Eq. 18 into 7 we have:

$$\frac{1}{A} [C_{-3} e^{-3\eta} + C_{-2} e^{-2\eta} + C_{-1} e^{-\eta} + C_0 + C_1 e^{\eta} + C_2 e^{2\eta} + C_3 e^{3\eta}] = 0, \tag{19}$$

Where:

$A = (e^{\eta} + b_0 + b_{-1} e^{-\eta})^4$  and  $C_i$  are as below:

$$\begin{aligned} &2ka_{-1}b_{-1} + 2ka_1b_0^2 + 8b\omega k^2a_1b_{-1} - 4b\omega k^2a_1b_0^2 - 2\omega a_{-1} - 2ka_0b_0 \\ &- 2\omega a_0b_0 + 6aka_1^2b_{-1} - 4bkq^2a_1b_0^2 + 4bkq^2a_0b_0 - 2ka_{-1} \\ &+ 4b\omega k^2a_0b_0 - 6aka_1a_0^2 + 6aka_1^2a_0b_0 + 8bkq^2a_1b_{-1} - 8b\omega k^2a_{-1} \\ &- 8bkq^2a_{-1} - 6aka_1^2a_{-1} + 2\omega a_1b_0^2 + 2\omega a_1b_{-1} = 0 \\ &- b\omega k^2a_0 + 3aka_1^2b_0 + ka_1b_0 + \omega a_1b_0 - \omega a_0 - bkq^2a_0 \\ &+ b\omega k^2a_1b_0 + bkq^2a_1b_0 - 3aka_1^2a_0 - ka_0 = 0 \end{aligned}$$

$$\begin{aligned} &-ka_{-1}b_0^3 + k^2a_0b_{-1}b_0^2 + \omega a_0b_{-1}b_0^2 + 3aka_0^2b_{-1} - 15aka_{-1}^2a_0 \\ &- 6ka_{-1}b_0b_{-1} + 5ka_1b_{-1}^2b_0 + 5\omega a_1b_0b_{-1}^2 - 6\omega a_{-1}b_0b_{-1} + \omega a_0b_{-1}^2 \\ &+ ka_1b_{-1}^2 - \omega a_{-1}b_0^3 - b\omega k^2a_{-1}b_0^3 + b\omega k^2a_0b_{-1}b_0^2 + bkq^2a_0b_{-1}b_0^2 \\ &- bkq^2a_{-1}b_0^3 - 23b\omega k^2a_0b_{-1}^2 - 23bkq^2a_0b_{-1}^2 + 18b\omega k^2a_{-1}b_{-1}b_0 \\ &+ 5b\omega k^2a_1b_0b_{-1}^2 + 5bkq^2a_1b_0b_{-1}^2 + 18bkq^2a_{-1}a_1b_0 - 3aka_{-1}^2a_1b_0 \\ &+ 18aka_1a_0a_{-1}b_{-1} - 3aka_0^2a_{-1}b_0 = 0 \\ &- 4bkq^2a_0b_{-1}^2b_0 + 8bkq^2a_1b_{-1}^3 + 2\omega a_0b_{-1}^2b_0 + 2ka_0b_{-1}^2b_0 \\ &- 4b\omega k^2a_0b_{-1}^2b_0 + 2ka_1b_{-1}^3 - 8bkq^2a_{-1}b_{-1}^2 - 2\omega a_{-1}b_0^2b_{-1} \\ &+ 4bkq^2a_{-1}b_{-1}b_0^2 - 2ka_{-1}b_0^2b_{-1} - 2ka_{-1}b_{-1}^2 + 6aka_1^2a_1b_{-1} \\ &- 6aka_1^2a_0b_0 + 8b\omega k^2a_1b_{-1}^3 - 6aka_{-1}^2 - 2\omega a_{-1}b_{-1}^2 \\ &- 8b\omega k^2a_{-1}b_{-1}^2 + 6aka_1a_0^2b_{-1} + 2\omega a_1b_{-1}^3 + 4b\omega k^2a_{-1}b_{-1}b_0^2 = 0 \\ &- bkq^2a_{-1}b_{-1}^2b_0 + b\omega k^2a_0b_{-1}^3 - \omega a_{-1}b_0b_{-1}^2 - b\omega k^2a_{-1}b_{-1}^2b_0 \\ &+ 3aka_{-1}^2b_0b_{-1} - 3aka_{-1}^3b_0 + bkq^2a_0b_{-1}^3 - ka_{-1}b_0b_{-1}^2 + \omega a_0b_{-1}^3 \\ &+ ka_0b_{-1}^3 = 0 \\ &6\omega a_1b_0b_{-1} + 6ka_1b_{-1}b_0 + ka_1b_0^3 - \omega a_0b_{-1} + \omega a_1b_0^3 - ka_0b_0^2 \\ &- \omega a_0b_0^2b\omega k^2a_1b_0^3 + bkq^2a_1b_0^3 - bkq^2a_0b_0^2 - 5\omega a_{-1}b_0 \\ &- b\omega k^2a_0b_0^2 - 3aka_0^3 - 5ka_{-1}b_0 + 23bkq^2a_0b_{-1} + 23b\omega k^2a_0b_{-1} \\ &+ 3aka_0^2a_1b_0 - 5bkq^2a_{-1}b_0 - 5b\omega k^2a_{-1}b_0 - 18b\omega k^2a_1b_0b_{-1} \\ &+ 15aka_1^2a_0b_{-1} - 18aka_1a_0a_{-1} - 18bkq^2a_1b_0b_{-1} + 3aka_1^2a_{-1}b_0 = 0 \\ &- 4\omega a_{-1}b_{-1} - 4ka_{-1}b_{-1} - 32b\omega k^2a_1b_{-1}^2 + 32bkq^2a_{-1}b_{-1} \\ &+ 32b\omega k^2a_{-1}b_{-1} - 12aka_{-1}a_0^2 - 4ka_{-1}b_0^2 - 4\omega a_{-1}b_0^2 \\ &+ 4\omega a_1b_0^2b_{-1} - 32bkq^2a_1b_{-1}^2 + 4b\omega k^2a_1b_0^2b_{-1} + 4bkq^2a_1b_{-1}b_0^2 \\ &- 4b\omega k^2a_{-1}b_0^2 - 4bkq^2a_{-1}b_0^2 + 4ka_1b_{-1}^2 + 4\omega a_1b_{-1}^2 + 12aka_1^2a_{-1}b_{-1} \\ &+ 12aka_1a_0^2b_{-1} + 4ka_1b_0^2b_{-1} - 12aka_{-1}^2a_1 = 0 \end{aligned}$$

Solving the system of algebraic equations, we obtain the following results:

$$\begin{aligned} a_{-1} &= -\frac{1}{8}a_1b_0^2 & a_0 &= 0 & b_0 &= b_0 & b_{-1} &= \frac{1}{8}b_0^2 \\ a &= -\frac{1}{2}\frac{k + \omega}{ka_1^2} & b &= \frac{1}{2}\frac{k + \omega}{k(\omega k + q^2)} \end{aligned} \tag{20}$$

Where,  $a_1, b_0$  and  $k$  are free parameters.

Substituting these results into Eq. 18, we obtain the following exact solution.

$$\begin{aligned} u &= \frac{a_1 e^{(kx+qy+\omega t)} + \frac{1}{8}a_1b_0^2 e^{-(kx+qy+\omega t)}}{e^{(kx+qy+\omega t)} + b_0 + \frac{1}{8}b_0^2 e^{-(kx+qy+\omega t)}} \\ &= a_1 - \frac{8a_1b_0}{8e^{(kx+qy+\omega t)} + 8b_0 + b_0^2 e^{-(kx+qy+\omega t)}} \end{aligned} \tag{21}$$

When  $k$  is an imaginary number, the obtained solitary solution can be converted in to periodic solution (He and Wu, 2006). We write  $k = ik$ ,  $q = iq$  and  $\omega = i\omega$  using the transformation

$$e^{i(kx+qy+\omega t)} = \cos[kx + qy + \omega t] + i \sin[kx + qy + \omega t] \quad (22)$$

and

$$e^{-i(kx+qy+\omega t)} = \cos[kx + qy + \omega t] - i \sin[kx + qy + \omega t] \quad (23)$$

Then Eq. 21 becomes

$$u = a_1 - \frac{8a_1 b_0}{(8+b_0^2)\cos(kx + qy + \omega t) + 8b_0 + (8-b_0^2)\sin(kx + qy + \omega t)} \quad (24)$$

If we search for a periodic solution or compact solution, the imaginary part in Eq. (24) must be zero, that requires:

$$8 - b_0^2 = 0 \quad (25)$$

From Eq. 25 we obtain

$$b_0 = \pm 2\sqrt{2} \quad (26)$$

Substituting Eq. 26 in Eq. 24 yields two periodic solutions:

$$u = a_1 - \frac{16\sqrt{2}a_1}{16\cos(kx + qy + \omega t) + 16\sqrt{2}} \quad (27)$$

and

$$u = a_1 + \frac{16\sqrt{2}a_1}{16\cos(kx + qy + \omega t) - 16\sqrt{2}} \quad (28)$$

It can be seen good approximation with the extended tanh method (Wazwaz, 2007).

**Case 2:**  $p = c = 2$ ,  $q = d = 2$ : As mentioned above the values of  $c$  and  $d$  can be freely chosen, we set  $p = c = 2$  and  $q = d = 2$  then the trial function, Eq. 9 becomes:

$$u(\eta) = \frac{a_2 \exp(2\eta) + a_1 \exp(\eta) + a_0 + a_{-1} \exp(-\eta) + a_{-2} \exp(-2\eta)}{b_2 \exp(-2\eta) + b_1 \exp(\eta) + b_0 + b_{-1} \exp(-\eta) + b_{-2} \exp(-2\eta)} \quad (29)$$

There are some free parameters in Eq. 29, we set  $b_2 = 1$ ,  $b_1 = 0$  and  $b_{-1} = 0$  for simplicity, then the trial function, Eq. (29) is simplified as follows:

$$u(\eta) = \frac{a_2 \exp(2\eta) + a_1 \exp(\eta) + a_0 + a_{-1} \exp(-\eta) + a_{-2} \exp(-2\eta)}{\exp(-2\eta) + b_0 + b_{-2} \exp(-2\eta)} \quad (30)$$

By the manipulation as illustrated above, we obtain

$$\begin{aligned} a_{-2} &= -\frac{1}{4} \frac{a_2^2 b_0^2 - a_0^2}{a_2} & a_0 &= a_0 & b_0 &= b_0 & b_{-2} &= \frac{1}{4} \frac{a_2^2 b_0^2 - a_0^2}{a_2^2} \\ a &= -\frac{k + \omega}{ka_2^2} & b &= \frac{1}{2} \frac{k + \omega}{k(\omega k + q^2)} & a_1 &= 0 & a_{-1} &= 0 \end{aligned} \quad (31)$$

Substituting Eq. 31 in Eq. 30 yields to the following solution

$$u = \frac{a_2 e^{2(kx+qy+\omega t)} + a_0 - \frac{1}{4} \frac{(a_2^2 b_0^2 - a_0^2)}{a_2} e^{-2(kx+qy+\omega t)}}{e^{2(kx+qy+\omega t)} + b_0 + \frac{1}{4} \frac{(a_2^2 b_0^2 - a_0^2)}{a_2^2} e^{-2(kx+qy+\omega t)}} \quad (32)$$

It can be proved that the obtained solution (Eq. 29) is equivalent to the solution obtained in case 1.

**Case 3:**  $p = c = 2$ ,  $q = d = 1$ : We consider the case  $p = c = 2$  and  $q = d = 1$ , Eq. 9 can be expressed as:

$$u(\eta) = \frac{a_2 \exp(2\eta) + a_1 \exp(\eta) + a_0 + a_{-1} \exp(-\eta)}{b_2 \exp(-2\eta) + b_1 \exp(\eta) + b_0 + b_{-1} \exp(-\eta)} \quad (33)$$

There are some free parameters in Eq. 33 we set  $b_2 = 1$  for simplicity, by the same manipulation as illustrated above we obtained:

$$\begin{aligned} a_0 &= a_0 & a_{-1} &= -\frac{1}{8} \frac{\gamma_1}{a_2^2} & a_1 &= a_1 \\ b_0 &= \frac{1}{2} \frac{\gamma_2}{a_2^2} & b_{-1} &= \frac{1}{8} \frac{\gamma_1}{a_2^2} & b_1 &= b_1 \\ a &= -\frac{\omega + k}{ka_2^2} & b &= \frac{2(\omega + k)}{k(\omega k + q^2)} & \gamma_i &= \text{const } (i=1,2) \end{aligned} \quad (34)$$

By Substituting Eq. 34 in Eq. 33 we obtained a solution that easily proved that this equation is same with that obtained in case 1.

### CONCLUSION

In this survey, our objective has been to show that exact solution of the (2+1)-dimensional ZK-BBM

(Benjamin-Bona-Mahony) equation can be obtained by Exp-Function method. The method is used to finding the traveling wave solutions of ZK-BBM nonlinear partial differential equation. We also found new exact solution that is not obtained by other existed method. Furthermore, the method leads to both the generalized solitary solutions and periodic solutions. The results obtained from proposed method have been compared and verified with those obtained by the extended tanh method. The results revealed that Exp-function method is powerful mathematical tool for solutions of nonlinear partial differential equations in the terms of accuracy and efficiency while systems of nonlinear partial differential equations having wide applications in engineering.

### REFERENCES

- Ablowitz, M.J. and P.A. Clarkson, 1991. Solitons, Nonlinear Evolution Equations and Inverse Scattering. Cambridge University Press, Cambridge.
- Benjamin, R.T., J.L. Bona and J.J. Mahony, 1972. Model equations for long waves in nonlinear dispersive systems. *Philos. Trans. R. Soc. London*, 272: 47-78.
- Fakhari, A., D.D. Ganji and Ebrahimpour, 2007. Approximate explicit solutions of nonlinear BBMB equations by homotopy, analysis method and comparison with the exact solution. *Phys. Lett. A*, 368 (1-2): 64-68.
- Ganji, D.D. and M. Rafei, 2006. Solitary wave solutions for a generalized Hirota-Satsuma coupled KdV equation by homotopy perturbation method. *Phys. Lett. A*, 356 (2): 131-137.
- He, J.H., 1999. Variational iteration method: A kind of nonlinear analytical technique: Some examples. *Int. J. Nonl. Mech.*, 344: 699.
- He, J.H., 2000. Variational iteration method for autonomous ordinary differential systems. *Applied Math. Comput.*, 114 (1-2): 115.
- He, J.H., 2004. Variational principles for some nonlinear partial differential equations with variable coefficients. *Chaos, Solitons and Fractals*, 19 (4): 847.
- He, J.H., 2005. Application of homotopy perturbation method to nonlinear wave equations. *Chaos, Solitons Fractals*, 26 (3): 695.
- He, J.H., 2006. Non-Perturbative Methods for Strongly Nonlinear Problems. Dissertation, de-Verlag im Internet GmbH, Berlin.
- He, J.H. and X.H. Wu, 2006. Exp-function method for nonlinear wave equations. *Chaos, Solitons Fractals*, 30 (3): 700.
- Hereman, W., A. Korpel and P.P. Banerjee, 1985. A general physical approach to solitary wave construction from linear solutions. *Wave Motion*, 7 (3): 283-289.
- Hirota, R., 1971. Exact solution of the Kdv equation for multiple collisions of solitons. *Phys. Rev. Lett.*, 27: 1192-1194.
- Hu, J.Q., 2005. An algebraic method exactly solving two high-dimensional nonlinear evolution equations. *Chaos, Solitons Fractals*, 23 (2): 391.
- Kadomtsev, B.B. and V.I. Petviashvili, 1970. On the stability of solitary waves in weakly dispersive media. *Sov. Phys. Dokl.*, 15: 539-541.
- Li, B., Y. Chen and H. Zhang, 2003. Exact travelling wave solutions for Kuznetsov equation. *Applied Math. Comput.*, 146 (1-2): 653-666.
- Liu, S.K., Z.T. Fu, S.D. Liu and Q. Zhao, 2001. Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations. *Phys. Lett. A*, 289 (1-2): 69-74.
- Malfliet, W., 1992. Solitary wave solutions of nonlinear wave equations. *Am. J. Phys.*, 60 (7): 650-654.
- Malfliet, W. and W. Hereman, 1996. The Tanh Method: II. Perturbation technique for conservative systems. *Phys. Ser.*, 54: 569-575.
- Monro, S. and E.J. Parkes, 1999. The derivation of a modified Zakharov-Kuznetsov equation and the stability of its solutions. *J. Plasma Phys.*, 62 (3): 305-317.
- Rafei, M. and D.D. Ganji, 2006. Explicit solutions of Helmholtz equation and fifth-order KdV equation using homotopy perturbation method. *Int. J. Nonl. Sci. Numer. Simul.*, 7 (3): 321-328.
- Rosenau, P. and J.M. Hyman, 1993. Compactons: Solitons with finite wavelengths. *Phys. Rev. Lett.*, 70 (5): 564-567.
- Tari, H. and D.D. Ganji, 2007. Approximate explicit solutions of nonlinear BBMB equations by He's methods and comparison with the exact solution. *Phys. Lett. A*, 367 (1-2): 95-101.
- Tolou, N., D.D. Ganji, M.J. Hosseini and Z.Z. Ganji, 2007. Application of Homotopy Perturbation Method in Nonlinear Heat Diffusion-Convection-Reaction Equations. *Open Mech. J.*, 1: 20-25.
- Wang, L., 1996. Exact solutions for a compound KdV-Burgers equation. *Phys. Lett. A*, 213 (5-6): 279-287.
- Wazwaz, A.M., 2007. The extended tanh method for new compact and noncompact solutions for the KP-BBM and the ZK-BBM equations. *Chaos, Solitons Fractals*.

- Zakharov, V.E. and E.A. Kuznetsov, 1974. On three-dimensional solitons. *Soviet Phys.*, 39 (1): 285-288.
- Zayed, E.M.E., H.A. Zedan and K.A. Gepreel, 2004. Group analysis and modified extended tanh-function to find the invariant solutions and soliton solutions for nonlinear Euler equations. *Int. J. Nonl. Sci. Numer. Simul.*, 5 (1): 221.
- Zhang, S. and T.C. Xia, 2006a. Symbolic computation and new families of exact non-travelling wave solutions to (3+1)-dimensional Kadomstev-Petviashvili equation. *Applied Math. Comput.*, 181 (1): 319-331.
- Zhang, S. and T.C. Xia, 2006b. Further improved extended Fan sub-equation method and new exact solutions of the (2+1)-dimensional Broer-Kaup-Kupershmidt equations. *Applied Math. Comput.*, 182 (2): 1651-1660.
- Zhang, S., 2006. New exact solutions of the KdV-Burgers-Kuramoto equation. *Phys. Lett. A*, 358 (5-6): 414-420.
- Zhang, S., 2007a. Symbolic computation and new families of exact non-travelling wave solutions of (2+1)-dimensional Konopelchenko-Dubrovsky equations. *Chaos, Solitons and Fractals*, 31 (4): 951-959.
- Zhang, S., 2007b. Further improved F-expansion method and new exact solutions of Kadomstev-Petviashvili Equation. *Chaos, Solitons Fractals*, 32 (4): 1375-1383.
- Zhao, X.Q., H.Y. Zhi and H.Q. Zhang, 2006. Improved Jacobi-function method with symbolic computation to construct new double-periodic solutions for the generalized Ito system. *Chaos Solitons Fractals*, 28 (1): 112-126.