www.serd.ait.ac.th/reric

Exploitation of Thermal Properties of Fluids Embedded with Nanostructured Materials

V. Vasu*[1], K. Rama Krishna[+], and A. C. S. Kumar[#]

Abstract – *Nanofluids are new class of heat transfer fluids developed by suspending nano-sized solid particles in liquids. Larger thermal conductivity of solid particles compared to the base fluid such as water, ethylene Glycol, engine oil etc. significantly enhances its thermal properties. Many of phenomenological models have been proposed to explain the heat transfer enhancement in nanofluids. This paper presents systematic literature survey to exploit several characteristic behaviors of nanofluids viz; increase in thermal conductivity, and heat transfer coefficient. An empirical correlation for Al_2O_3 + water and Cu + water nanofluids to evaluate Thermal conductivity and Nusselt number in forced convective internal flow are developed considering the effect of temperature, volume fraction and size of nanoparticle. The improvement in thermophysical characteristics combined with better heat transfer properties makes fluids embedded with nanomaterials as excellent for future applications.*

Keywords – CFD model, empirical relation, nanofluids, Nusselt number, thermal conductivity models.

1. INTRODUCTION

Nanotechnology has been widely used in traditional industry because materials with grain size of nanometers posses unique optical, electrical and chemical properties. An innovative utility of this emerging technology is that nanoparticles can be dispersed in conventional heat transfer fluids such as water, glycol or oil to produce a new class of high efficiency heat exchange media [1]. The key idea is to exploit the very high thermal conductivities of solid particles which can be hundreds or even thousands of times greater than those of fluids. The superior properties of nanoparticle fluid mixtures relative to those of fluids without particle or with large size particle include high thermal conductivities, stability and prevention of clogging in micro channels. It is well known that metals in solid form have thermal conductivities that are higher than those of fluids by orders of magnitude. For example the thermal conductivity of copper at room temperature is about 700 times greater than that of water and about 3000 times greater than that of engine oil. Therefore, fluids containing suspended solid metallic particles are expected to display significantly enhanced thermal conductivities relative to those of conventional heat transfer fluids.

The study of heat transport in solid dispersion is relatively recent. Ahuja [2] showed that sub-micron polystyrene suspensions in aqueous glycerin increased the heat transfer by a factor of two under laminar flow conditions. At the same time, negligible difference was seen for the pressure drop even with a high particle volume fraction of 9%.

With the advent of nanotechnology in the early nineties, it becomes possible to manufacture nano-sized particles. These particles, due to their extreme small sizes can form very stable colloidal systems which are currently known as nanofluids. The first heat transfer enhancement with nano-sized particle was reported by [3] in Japan. They demonstrated that the thermal conductivity of ultra fine suspensions of alumina, silica and other oxides in water increased by a substantial amount for a particle volume fraction of 4.3%.

Choi [1] at the Argonne National Lab proposed to construct a new class of engineered fluids with superior heat transfer capabilities in 1995. Incidentally the term nanofluid was first coined by Choi for denoting this new class of engineered fluids.

Wang *et al.* [4] reported enhanced thermal conductivity for alumina and cupric oxide with a variety of base fluid including water and ethylene glycol. With alumina particles, they observed a maximum of 12% increase in the conductivity with a volume fraction of 3%. The viscosity on the other hand showed an increase of 20-30% for the same volume fraction.

Eastman *et al.* [5] showed that 10nm copper particles in ethylene glycol could enhance the conductivity by 40% with very small particle loading fraction. With cupric oxide (35 nm) the enhancement was 20% for a volume fraction of 4%. These results clearly show the effect of particle size on the conductivity enhancement.

Das *et al.* [6] measured the conductivities of alumina and Cupric oxide for different temperatures ranging from 20°C to 50°C and for linear increase in the conductivity ratio with temperature. However, for the same loading fraction the ratio of increase was higher for cupric oxide than alumina.

Choi *et al.* [7] has reported significant enhancement for multiwalled carbon nanotubes (MWNT) in oil suspension. The MWNT had a mean dia. of 25 nm and length of 50 μm. The results show that unlike with nanopowders the thermal conductivity with nanotubes

* Lecturer in Dept. of Mechanical Engineering, VR Siddhartha Engineering College, Vijayawada, AP, India 520007.

+ Dean and Professor, K L College of Engineering, Vaddeswaram, Guntur District, AP, India 522502.
Email: klce_drkrk@rediffmail.com

Professor, JNT University, Hyderabad, AP, India 500072.
Email: acskumar@yahoo.com

1 Corresponding author; E-mail: vvvasu@rediffmail.com

portrays a quadratic variation with volume fraction. At 1% volume fraction, carbon nanotubes show a remarkable 250% increase in thermal conductivity.

2. MAKING OF NANOFLUIDS

The range of potentially useful combinations of nanoparticle and base fluids is enormous: Nanoparticles of oxides, nitrides, metals, metal carbides, and nonmetals with or without surfactant molecules can be dispersed into base fluids such as water, ethylene glycol, or oils. To be able to produce the most appropriate nanoparticle-fluid combination for a particular application, researchers have developed several methods for nanoparticle production and dispersion. Initial experimental studies [3] employed a two-step process in which nanoparticles are first produced as a dry powder, typically by inert gas–condensation [8], which involves the vaporization of a source material in a vacuum chamber and subsequent condensation of the vapor into nanoparticles via collisions with a controlled pressure of an inert gas such as helium. The resulting nanoparticles are then dispersed into a fluid in a second processing step. Despite the large degree of nanoparticle agglomeration that typically occurs with the gas-condensation process, it works well in some cases, for example, with oxide nanoparticles dispersed in deionized water [3]. Less success has been achieved when producing nanofluids containing heavier metallic nanoparticles by this technique. An advantage of this technique in terms of eventual commercialization of nanofluids is that the inert-gas condensation technique has already been scaled up to economically produce tonnage quantities of nanopowders. A second processing approach, referred to as the direct-evaporation technique, has been used with success to produce nanofluids containing dispersed metal nanoparticles [9], [10], developed by [11], and later improved by [12], synthesizes nanoparticles and disperses them into a fluid in a single step. As with the inert gas condensation technique, the technique involves vaporization of a source material under vacuum conditions. In this case, however, condensation of the vapor to form nanoparticles occurs via contact between the vapor and a liquid. Nanoparticle agglomeration is minimized by flowing the liquid continuously. A significant limitation to the application of this technique is that the liquid must have low vapor pressure, typically less than 1 torr. Higher vapor pressures lead to gas condensation and the associated problems of increased nanoparticle agglomeration. At present, the quantities of nanofluids that can be produced via this direct-evaporation technique are much more limited than with the inert gas–condensation technique, although, if desired, it is likely that the technique could also be scaled to economically produce large quantities of nanofluids. While most studies of the thermal properties of nanofluids to date have used one of the above described processing techniques, other techniques may eventually prove superior, depending on the particular combination of nanoparticle material and fluid. For example, the chemical vapor condensation technique in which nanoparticles are formed by thermal decomposition of a metal-organic precursor entrained in a carrier gas passing through a furnace, has recently been modified to synthesize and disperse non-agglomerated nanoparticles into fluids in a single step. Compared with the direct-evaporation technique, chemical vapor condensation appears to offer advantages in terms of control of particle size, ease of scalability, and the possibility of producing novel core-shell nanostructures. The maximum thermal conductivity for different method are in Table 1.

3. THERMAL PROPERTIES OF NANOFLUIDS

The thermal conductivity measurement of nanofluids was the main focus in the early stages of nanofluid research. Recently, however, studies have been carried out on the heat transfer coefficient of nanofluids in natural [13]-[15] and forced flow. Most studies carried out to date are limited to the thermal characterization of nanofluids without phase change (boiling, evaporation, or condensation). However, nanoparticles in nanofluids can play a vital role in two-phase heat transfer systems, and there is a great need to characterize nanofluids in boiling and condensation heat transfer. Sk Das *et al.* [16] initiated experiments on the boiling characteristics of nanofluids.

In convection heat transfer of nanofluids, the heat transfer coefficient depends not only on the thermal conductivity but also on other properties such as the specific heat, density, and dynamic viscosity of a nanofluid.

The density of a nanofluid can be calculated by:

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_p \quad (1)$$

For typical nanofluids with nanoparticles at a value of volume fraction less than 1%, a change of less than 5% in the fluid density is expected.

Table 1. Methods of nanofluids preparation and maximum thermal conductivity value

Nanomaterials	Base fluid	Dispersion Method	Max K_{nf}/K_f	%Volume Fraction
Cu	Ethylene glycol	One step method	1.4	3
Multi-walledCarbon Nanotubes	Synthetic base oil	Two step method	2.57	1
Al_2O_3	Water	Two step method	1.09	4
Al_2O_3	Ethylene glycol	Two step method	1.19	5
CuO	Water	Two step method	1.12	3.4
CuO	Ethylene glycol	Two step Method	1.22	4
CuO	Ethylene glycol	Two step method with mechanical /ultra sonic mixing	1.54	5
SiC	Di Water	Two step method	1.23	4
Cu	Transformer Oil	Two step method	1.43	7.5
Cu	Water	Two step method Laurate salt	1.76	7.5

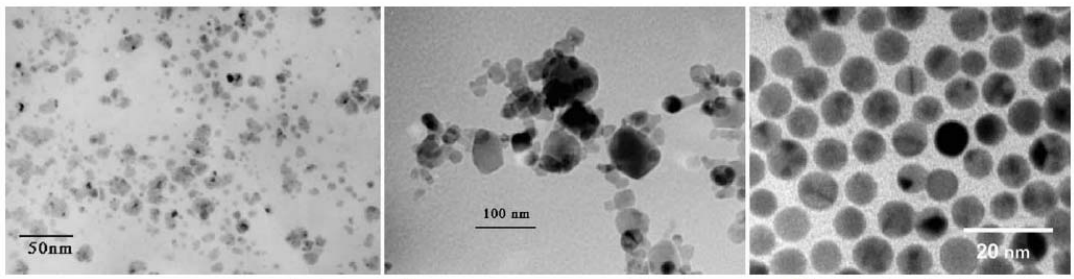

Fig. 1. **Transmission electron micrographs showing (left) Cu nanofluids *(Source: American Institute of Physics)*; (middle) CuO nanoparticles *(MRS)*; and (right) alkanethiolterminated AuPd colloidal particles *(Source:American Physical Society)***

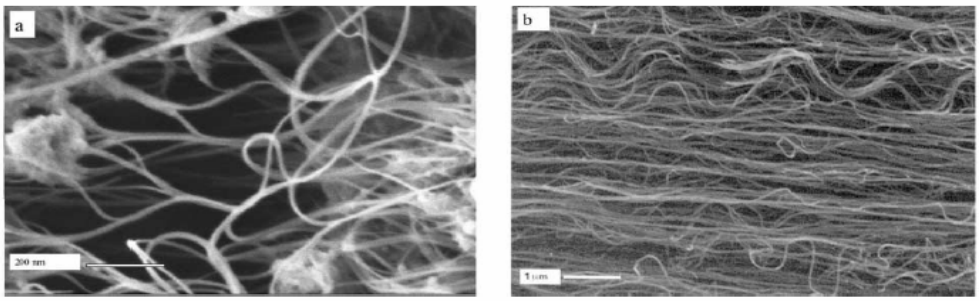

Fig. 2. Scanning electron micrographs of carbon nanotube samples typical of those used in suspensions and composites: (a) single-walled carbon nanotubes obtained by arc discharge, and (b) multiwalled carbon nanotubes obtained by chemical vapor deposition growth. *(Source: P. M. Ajayan, Rensselaer Polytechnic Institute)*

The specific heat C_{nf} of a nanofluid can be calculated by:

$$\rho_{nf} C_{nf} = (1-\phi)\rho_f C_f + \phi \rho_p C_p \quad (2)$$

Using these equations, one can predict that small decreases in specific heat will typically result when solid particles are dispersed in liquids. For example, adding 3 vol. % Al_2O_3 to water would be predicted to decrease the specific heat by approximately 8% compared with that of water alone. The simple equations above may need to be modified if nanoparticles are found to exhibit a size-dependent specific heat.

Wang *et al.* [4] measured the viscosity of water-based nanofluids containing Al_2O_3 nanoparticles dispersed by different dispersion techniques and showed that nanofluids have lower viscosities when the particles are better dispersed. They also showed an increase of ~30% in viscosity at 3 vol.% Al_2O_3, compared with that of water alone. However, the viscosity of the Al_2O_3/water nanofluids prepared by [17] was three times higher than that of water. For metallic nanofluids containing a low volume fraction of nanoparticles (usually <0.01), an Einstein model would predict that the change in the viscosity of a suspension of non-interacting spherical particles is small and linear with the volume fraction.

$$\nu_{nf} = (1+2.5\phi)\nu_f \quad (3)$$

The Einstein equation is valid only for φ<0.05. Sk Das *et al.* [16] measured the viscosity of Al_2O_3 + water and CuO + water nanofluids as a function of shear rate and showed Newtonian behavior of the nanofluids for a range of volume fractions between 1% and 4%.

Brinkman [18] has modified Equation (3) into a more generalized form as

$$\nu_{nf} = \frac{\nu_f}{(1-\phi)^{2.5}} \quad (4)$$

Wang *et al.* [4] gave a correlation for relative visocity as follows and its comparison with various models is shown in Figure 3.

$$\nu_{nf} = 123\phi^2 + 7.3\phi + 1$$

for water – Al_2O_3

$$\nu_{nf} = 306\phi^2 - 0.19\phi + 1$$

for Ethlene Glycol - Al_2O_3.

4. THERMAL CONDUCTIVITY MODELS

Many theoretical and empirical models have been proposed to predict the effective thermal conductivity of Nanofluids. In general, the overall thermal conductivity of a nanofluid depends in a complex fashion on the geometry of the medium. If we assume that conduction in the solid and fluid phases occurs in parallel, then the overall conductivity k_{nf} is the weighted arithmetic mean of the conductivities k_p and k_f.

$$k_{nf} = \phi k_p + (1-\phi)k_f \quad (5)$$

On the other hand, if the structure and orientation of the particles in the fluid are such that the heat conduction takes place in series with all of the heat flux passing through both particles and fluid, then the overall conductivity k_{nf} is the weighted harmonic mean of k_p and k_f :

$$\frac{1}{k_{nf}} = \frac{\phi}{k_p} + \frac{1-\phi}{k_f} \quad (6)$$

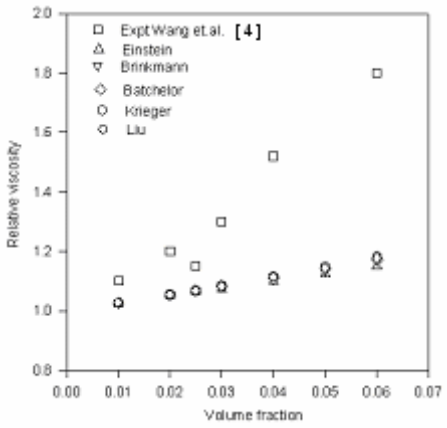

Fig. 3. Effect of nanomaterial fraction on viscosity

In general, these two models will provide upper and lower bounds respectively. On the actual overall conductivity k_{nf}, we always have Harmonic mean value of k <= Arithmetic mean value of k. The equality is valid if and only if $k_p = k_f$. For all practical purposes a rough and ready estimate for k_{nf} is provided by the weighted geometric mean of k_p and k_f defined as:

$$k_{nf} = k_p^{\phi}.k_f^{(1-\phi)} \tag{7}$$

This provides a good estimate so long as k_p and k_f are not too different from each other. More complicated correlation formulas for the conductivity of nanofluids have been proposed. These formulas gave reasonably good results provided when k_f was not significally greater than k_p.

Using potential theory, [19] obtained a simple relationship for the conductivity of randomly distributed and non-interacting homogeneous spheres in a homogeneous medium (Maxwell correlation):

$$\frac{k_{nf}}{k_f} = 1 + \frac{3(\alpha-1)\phi}{(\alpha+2)-(\alpha-1)\phi} \tag{8}$$

where $\alpha = k_p/k_f$.

Hamilton and Crosser [20] correlation is the modified Maxwell correlation below:

$$\frac{k_{nf}}{k_f} = \frac{\left[k_p + (n-1)k_f - (n-1)\phi(k_f - k_p)\right]}{\left[k_p + (n-1)k_f + \phi(k_f - k_p)\right]} \tag{9}$$

where the parameter 'n' is the "shape factor" define as:

$$n = \frac{3}{\psi}$$

where ψ, called the 'sphericity', is defined as the ratio of the surface area of the sphere over that of the particle given the same volume. By assuming spherical particles ψ = 1, and for the cylinders ψ = 0.5. Equation (9) for spherical particle can be reduced to:

$$\frac{k_{nf}}{k_f} = \frac{k_p + 2k_f + 2\phi(k_p - k_f)}{k_p + 2k_f - \phi(k_p - k_f)} \tag{10}$$

Bruggerman [21] model for binary mixture of homogeneous spherical inclusions is:

$$k_{nf} = (3\phi-1)k_p + [3(1-\phi)-1]k_f + \sqrt{\Delta} \tag{11}$$

$$\Delta = (3\phi-1)^2 k_p^2 + [3(1-\phi)-1]^2 k_f + 2[2+9\phi(1-\phi)]k_p k_f$$

Jeffrey [22], interaction of randomly dispersed sphere model for the calculation of the thermal conductivity as:

$$\frac{k_{nf}}{k_f} = 1 + 3\beta\phi + (3\beta^2 + \frac{3\beta^2}{4} + \frac{9\beta^2(\alpha+2)}{16(2\alpha+3)} + ...)\phi^2 \tag{12}$$

Similarly, [23] has developed another model as:

$$\frac{k_{nf}}{k_f} = 1 + \frac{3(\alpha-1)}{(\alpha+2)-(\alpha-1)\phi}\left[\phi + f(\alpha)\phi^2\right] \tag{13}$$

where $f(\alpha) = 2.5$ for $\alpha = 10$, and $f(\alpha) = 0.5$ for $\alpha = \infty$.

Lu–Lin [24] has developed a model for both spherical and non-spherical particles:

$$\frac{k_{nf}}{k_f} = 1 + \alpha\phi + \beta\phi^2 \tag{14}$$

where $\alpha = K_p/K_f$ and $\beta = \frac{\alpha-1}{\alpha+2}$.

The Landau–Lifshitz/Looyenga model describes an isotropic mixture which constitute phases that are not spatially correlated; i.e the mixture is completely random (there is no interfacial surface). The thermal conductivity is given as:

$$\frac{k_{nf}}{k_f} = \left[\left(k_p^{1/3} - k_f^{1/3}\right)\phi + k_f^{1/3}\right]^3 \tag{15}$$

5. PRESENT MODEL FOR THERMAL CONDUCTIVITY

The above thermal conductivity models have shown underestimated values to that of experimental data [6] shown in Figure 4. To over come this [25] found that since the Brownian motion of nanoparticles at the molecular and nanoscale level is a key mechanism governing the thermal behavior of Nanofluids, they theoretical derived a model which considers the concentration, temperature, and size. They considered four modes of energy transport.

1. The first mode is collision between base fluid molecules.
2. Thermal diffusion in nanoparticles. Suspended in fluids.
3. Collision between nanoparticle.
4. Thermal interactions of dynamic or dancing nanoparticles with base fluid molecules.

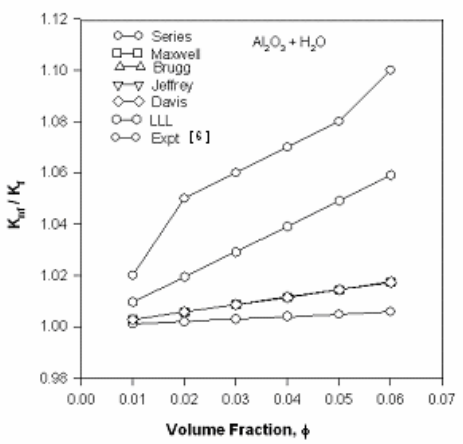

Fig. 4. Comparison of the thermal conductivity of nanofluid model with the experimental data [6]

They derived the thermal conductivity of nanofluid K_{eff} by neglecting third mode, and it is:

$$k_{nf} = k_b(1-\phi) + k_p\phi + 3C_1\frac{d_p}{d_f}k_b \,\mathrm{Re}^2\,\mathrm{Pr}\,\phi \quad (16)$$

where C_1 is constant.

Ravi *et al.* [26] proposed a thermal conductivity equation by considering the Brownian motion as:

$$\frac{k_{nf}}{k_f} = (1 + A\mathrm{Re}^m\,\mathrm{Pr}^{0.333}\,\phi)\left[\frac{(1+2\alpha)+2\phi(1-\alpha)}{(1+2\alpha)-\phi(1-\alpha)}\right] \quad (17)$$

Thus, from the above analysis we can conclude that the thermal conductivity of a nanofluid, k_{nf} is function of:

$$k_{nf} = f\left[v_f, d_p, \rho_p, T, k_p, k_f, \phi\right] \quad (18)$$

These variables can be grouped and can be expressed in non-dimensional terms as:

$$k_{nf} = f\left[\mathrm{Re}_k, \phi, \frac{k_p}{k_f}\right] \quad (19)$$

where, $\mathrm{Re}_k = \frac{1}{v_f}\sqrt{\frac{18k_bT}{\Pi\rho_p d_p}}$

Therefore,

$$\frac{k_{nf}}{k_f} = c.\mathrm{Re}_k^m\,\phi^n\left(\frac{k_p}{k_f}\right)^p \quad (20)$$

The data available in the literatures [5]-[8] and [14]-[17] for $Al_2O_3 + H_2O$ and $Cu + H_2O$ mixtures for different nanoparticle sizes at different volume fraction and at different temperature were used to predict the constants. Using nonlinear regression analysis empirical correlation to predict the k_{nf} of $Al_2O_3 + H_2O$ mixture is obtained as [27:

$$\frac{k_{nf}}{k_f} = \mathrm{Re}_k^{0.175}\,\phi^{0.05}\left(\frac{k_p}{k_f}\right)^{0.2324} \quad (21)$$

for $Al_2O_3 + H_2O$ mixture.

With an average deviation of 0.8% and standard deviation of 1%. The above equation takes care of the diameter of the nanoparticle, concentration and temperature effects. This correlation is applicable to only $Al_2O_3 + H_2O$ mixture with volume fraction greater than zero (limitation of the correlation).

Similarly, for the $Cu + H_2O$ mixture, the correlation is obtained as:

$$\frac{k_{nf}}{k_f} = 0.74\ \mathrm{Re}_k^{0.175}\,\phi^{0.05}\left(\frac{k_p}{k_f}\right)^{0.2324} \quad (22)$$

for $Cu + H_2O$ mixture.

With an average deviation of 0.8% and standard deviation of 1%. The above equation takes care of diameter of the nanoparticle, concentration and temperature effects as shown in Figure 5.

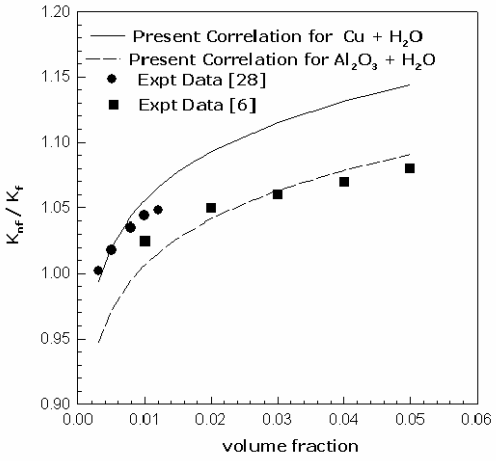

Fig. 5. Comparison of the present K_{nf} correlation with the experimental data

Using the empirical correlation obtained, Figures 6, 7 and 8 are drawn. Figure 6 shows the effect of particle diameter on the thermal conductivity of nanofluid at various volume fractions. It indicates that with increasing particle diameter the thermal conductivity enhancement decreases. Further, it shows that with increasing volume fraction the effective thermal conductivity of a nanofluid increases. The rate of increase of the k value was found to be less at higher volume fractions compared to lower fractions. Moreover, beyond 12% of volume fraction it appears that the improvement in k value may be insignificant.

Figure 7 shows the effect of temperature on the thermal conductivity of a nanofluid. It indicates that with increasing temperature, the k value of the nanofluid increases. Further, we can conclude from the graph that the effect is more dominant in the small-sized particles rather than with large-sized ones. Figure 8 shows the effect of particle diameter on the k_{nf} value of nanofluid. It indicates that with an increase in particle size the thermal conductivity effect decreases. Further, it is evident that the effect of volume fraction as low sized particle is almost nonexistent, and hence it is advisable to go for small-sized low concentration of particles to have a better k value.

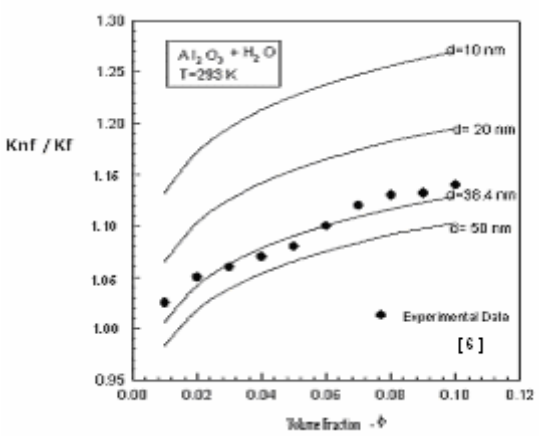

Fig. 6. Effect of volume fraction on thermal conductivity of nanofluid.

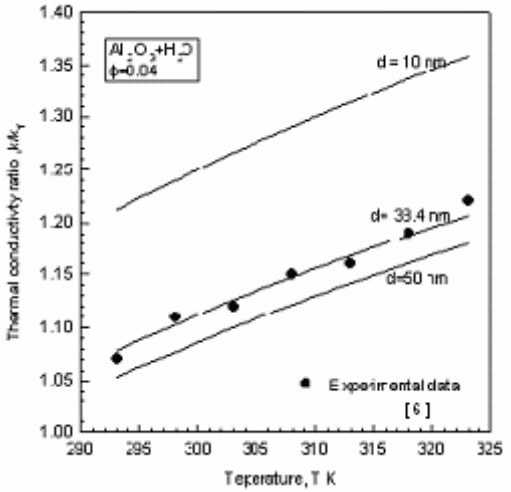

Fig. 7. Effect of temperature on thermal conductivity of nanofluid

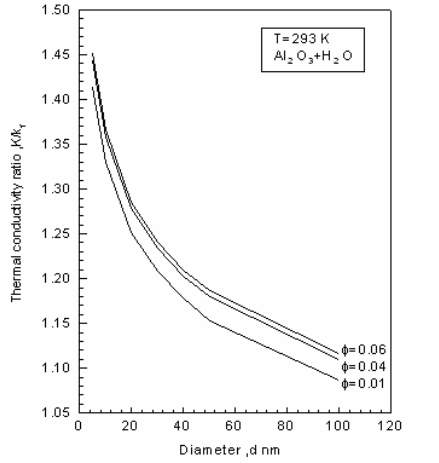

Fig. 8. Effect of diameter of nanoparticle on thermal conductivity of nanofluid.

6. HEAT TRANSFER CHARACTERISTICS OF CONVECTION OF NANOFLUIDS

Analysis was made for steady, forced turbulent convection flow and heat transfer of nanofluid flowing inside a straight tube of circular cross-section. The fluid possesed uniform temperature and axial velocity profile at the outlet section. Also, the flow and the thermal field were assumed symmetrical with respect to the vertical plane passing through the main axis. There exists no formulated theory known to date that could reasonably predict the flow and heat transfer behaviors of a nanofluid by considering it as multi-component material. It is interesting to note that most nanofluids used in practical application are usually composed of oxide particle, finer than 40nm. Therefore, it has been suggested that the particles may be easily fluidized and consequently, can be considered as conventional single-phase fluid, which posses effective physical properties being function of the properties of both constituents and their respective concentrations [17], [28]. As a result, a direct extension from a conventional fluid to nanofluid appears feasible, and one may then expect that the classical theory developed for a convectional single-phase fluid can be applied to nanofluid as well. Thus, all the equations of conservation (mass, momentum and energy) as well known for single-phase fluid can be directly applied to nanofluids.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{23}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{dp}{dx}+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\partial}{\partial y}(\overline{u'v'}) \tag{24}$$

$$\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\alpha\frac{\partial^2 T}{\partial x^2}-\frac{\partial}{\partial y}(\overline{v'T'}) \tag{25}$$

However, [29] proposed Brownian diffusion and thermophoresis effects for the enhancement of heat transfer coefficient of nanofluids as:

$$D_B=\frac{k_B T}{3\pi\mu d_p} \tag{26}$$

$$D_T=\beta\frac{\mu}{\rho}\phi \tag{27}$$

For Turbulent flow conditions, the above conservation equations using CFD K-ε model [30] with zonal method can be solved. Considering the additional factors proposed by [29], Equations 23, 24 and 25 modified as:

For Inner zone which is identified as $y < y_o$ and

$$y_o^+=\frac{y_o u_t}{\nu}$$

the continuity and momentum equations will remain same as that of Equations 23 and 24, but the energy equations changes to:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left((\alpha+D_B)\frac{\partial T}{\partial y}\right) \tag{28}$$

For the outer zone $y > y_o$ continuity and momentum equations will be same. In addition, two more equations are added for turbulence kinetic energy: rate of dissipation equation, and energy as:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left((\varepsilon_H+\frac{D_T}{T})\frac{\partial T}{\partial y}\right) \tag{29}$$

kinetic energy,

$$u\frac{\partial k}{\partial x}+v\frac{\partial k}{\partial y}=\frac{\partial}{\partial y}\left(\frac{\varepsilon_m}{\sigma_\varepsilon}\frac{\partial k}{\partial y}\right)+\varepsilon_m\left(\frac{\partial u}{\partial y}\right)^2-\varepsilon \quad (30)$$

rate of dissipation,

$$u\frac{\partial \varepsilon}{\partial x}+v\frac{\partial \varepsilon}{\partial y}=\frac{\partial}{\partial y}\left(\frac{\varepsilon_m}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial y}\right)+C_{\varepsilon 1}\frac{\varepsilon}{k}\varepsilon_m\left(\frac{\partial u}{\partial y}\right)^2-C_{\varepsilon 2}\frac{\varepsilon^2}{k} \quad (31)$$

The above single-phase fluid assumption, which appears somewhat too simplistic, has its own merit and of course, limits. Due to a striking lack of data permitting to establish a clear picture of the physical mechanism governing the suspended nanoparticles within a continuous liquid phase, it is difficult to assess the exact limit of such an important assumption. It is believed that, under the conditions of negligible slip and thermal equilibrium between the phases, and as long as the particle suspension and their spatial distribution are considered as uniform throughout the domain, the single-phase fluid assumption may reasonably be applied [30].

S B Maiga *et al.* [30] considered the turbulent convection and obtained the Nusselt number by using CFD-based k-ε model applying genetic algorithm as:

$$Nu = 0.085\,\mathrm{Re}^{0.71}\,\mathrm{Pr}_{nf}^{\;0.35} \quad (32)$$

While developing the above expression [30] has considered the properties of nanofluids developed by [17] instead of basefluid. However, he has not considered the Brownian diffusion and thermophoresis effects responsible for enhancement of heat transfer characteristics.

Pak and Cho [17] and Li and Xuan [28] have developed correlations of a form similar to that of the well known Dittus-Boelter formula to characterize nanofluids heat transfer. They proposed correlation for the calculation of Nusselt number as given in Equations 33 and 34. In these correlations the Reynolds number and Prandtl number were calculated by considering the base fluid properties which will give underestimated results compared with experimental values:

$$Nu = 0.021(\mathrm{Re}_b)^{0.8}(\mathrm{Pr}_b)^{0.5} \quad (33)$$

$$Nu = 0.0059\left[1+7.6286\,\phi_b^{0.6886}\left(\mathrm{Re}_b\,\mathrm{Pr}_b\,\frac{d_p}{D}\right)^{0.001}\right]\mathrm{Re}_b^{0.9238}\,\mathrm{Pr}_b^{0.4} \quad (34)$$

Hence, in this paper, the above "single phase fluid" approach was adopted in order to study the thermal behaviors of nanofluids in which the thermophysical properties of the nanofluid itself was considered. The convective heat transfer coefficient and Nusselt number are given as:

$$\mathrm{Nu}_{\mathrm{nf}} = \frac{\mathrm{h}_{\mathrm{nf}}\mathrm{D}}{\mathrm{k}_{\mathrm{nf}}} \quad (35)$$

The heat transfer coefficient of turbulent flow through circular tube can be calculated from Dittus-Boelter equation in the following form:

$$\mathrm{Nu}_{\mathrm{nf}} = \mathrm{c}*(\mathrm{Re}_{\mathrm{nf}})^{0.8}(\mathrm{Pr}_{\mathrm{nf}})^{0.4} \quad (36)$$

Re_{nf} and Pr_{nf} as defined as follows:

$$\mathrm{Re}_{nf} = \frac{\rho_{nf}\,u\,D}{k_{nf}} \quad (37)$$

$$\mathrm{Pr}_{\mathrm{nf}} = \frac{\mathrm{Cp}_{\mathrm{nf}}\mu_{\mathrm{nf}}}{\mathrm{k}_{\mathrm{nf}}} \quad (38)$$

The Nusselt number of the Al_2O_3 + H_2O (Equation 36) mixture which was obtained from the non-linear regression analysis of the data obtained from literature ([1]-[10], [15]-[17] and [28]) is used and the constant 'C' is obtained as 0.0256 for Al_2O_3 + H_2O and 0.027 for Cu + H_2O nanofluids. Thus, the correlations for calculation of Nusselt number were developed as follows:

$$Nu = 0.0256(\mathrm{Re}_{nf})^{0.8}(\mathrm{Pr}_{nf})^{0.4} \quad (39)$$

for Al_2O_3+ H_2O

$$Nu = 0.027*(\mathrm{Re}_{nf})^{0.8}(\mathrm{Pr}_{nf})^{0.4} \quad (40)$$

for Cu + H_2O

With an average deviation of 5% and standard deviation of 6.4 %, the above equation takes care of diameter of the nanoparticle, concentration and temperature effects. The correlations give good agreement with the experimental results as shown in the Figures 9 and 10. The values of Re and Pr were calculated using the properties of nanofluids as developed and presented in Equations 1, 2, 21 and 22.

7. VALIDATION OF PROPOSED CORRELATION

To validate our correlation, two most comprehensive sets of experimental data obtained from single phase turbulent heat transfer in nanofluids were compared and found to be in good agreement. The CFD k-ε model proposed by [30] overestimates the Nusselt number with increase of volume fraction as shown in Figures 9a and 9b. The reason could either be the use of inappropriate thermal conductivity assessment or due to neglecting the Brownian diffusion and thermophoresis effects in the single phase heat transfer analysis.

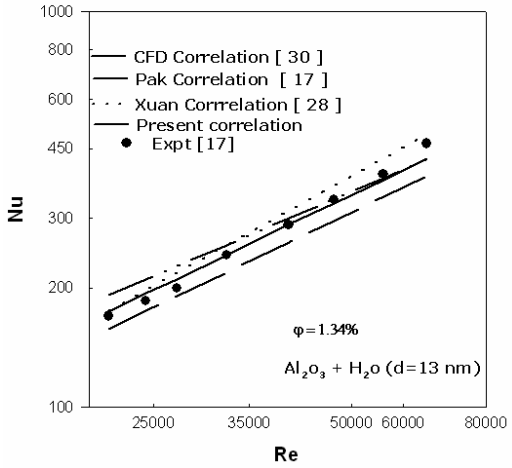

Fig. 9a. Comparison of the heat transfer of Al_2O_3 + water nanofluid correlation with the experimental data [17] at Φ=1.34%

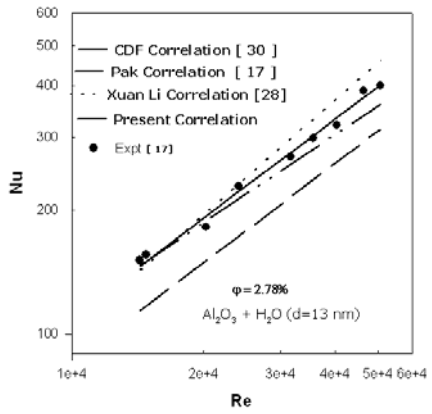

Fig. 9b. Comparison of the heat transfer of Al_2O_3 + water nanofluid correlation with the experimental data [17] at Φ=2.78%

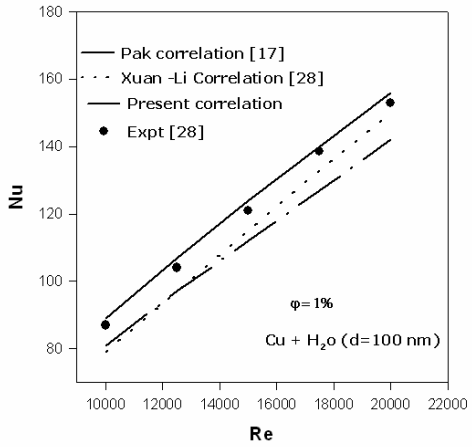

Fig. 10a. Comparison of the heat transfer of Cu + water nanofluid correlation with the experimental data [28] at Φ=1%

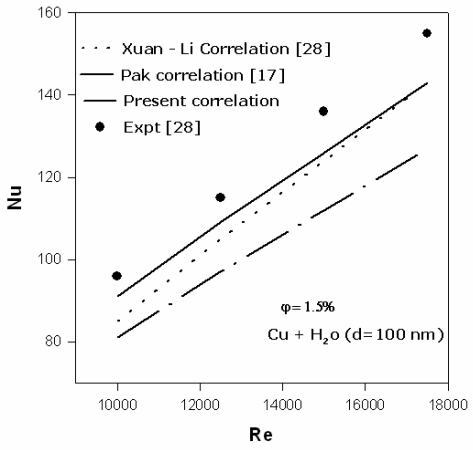

Fig. 10b. Comparison of the heat transfer of Cu + water nanofluid correlation with the experimental data [28] at Φ=1.5%

8. CONCLUSION

- Correlation to calculate nanofluids density, specific heat and viscosity were presented.
- For the calculation of viscosity, Wang *et al.* [4] correlation was found closer to the experimental data.
- Various models to predict thermal conductivity of nanofluid were presented in a chronological order. The comparison of models with experimental data was shown in graphical form.
- A simple empirical correlation to predict *k* of Al_2O_3 + H_2O and Cu + H_2O nanofluid mixture taking into the effect of temperature variation volume fraction and particle size was presented.
- A simple empirical correlation to predict Nu of Al_2O_3 + H_2O and Cu + H_2O nanofluids mixture was presented.
- Single phase fluid treatment with modified properties can be used to predict heat transfer characteristics of a nanofluid with Brownian and thermophoresis.
- Energy equations of turbulent k-ε model were modified by considering Brownian and thermophoresis.

Hence, it is concluded that adding nanostructured materials to base fluids enhances thermal properties makes them more suitable to heat exchanger applications.

NOMENCLATURE

C	specific heat kJ/kg K
D	diameter of tube, m
D_B	brownian diffusion coefficient, m^2/s
D_T	thermal diffusion coefficient, m^2/s
d	diameter, nm.
k	thermal conductivity, W/m K.
k_b	Boltzman constant ($1.3807*10^{-23}$) J/K
Nu	Nusselt number
n	shape factor
Pr	Prandtl number
Re	Reynolds number
t	thickness,nm
T	temperature, K
u	velocity component in x direction, m/s
u_τ	shear velocity, m/s
u`v`ρ	turbulent shear stress in a two dimensional boundary layer, N/m^2
vt`	apparent turbulent heat flux, K. m/s
v	velocity component in y direction, m/s
y_o^+	non-dimensional distance from wall in wall coordinates
y_o	a spatial coordinate in a Cartesian system distance normal to the surface in a boundary layer, m

Greek

ρ	density, kg/m^3
μ	viscosity, Pa.s
β	thermophoretic coefficient
ϕ	volume fraction
ψ	sphericity
ν	viscosity, m^2/s
α	molecular thermal diffusivity, m^2/s
ε	turbulence dissipation rate, $N/s.m^2$
ε_H	eddy diffusivity for heat, m^2/s.
ε_M	eddy diffusivity for momentum, m^2/s
σ	normal stress on an element of fluid, N/m^2

Subscripts

nf	nanofluids
p	nanoparticle
f	basefluid

REFERENCES

[1] Choi, S.U.S. 1995. *Development and applications of non-Newtonian flows.* Vol. FED 231; 99-105, ASME, NewYork.

[2] Ahuja, A. S. 1975. Augmentation of heat transport in laminar flow of polystyrene suspensions. *Journal of Applied Physics* 46(8): 855-903.

[3] Masuda, H.; Ebata, A.; Teramae, K.; Hishinuma, N. 1993. Alteration of thermal conductivity and viscocity of liquid by dispersing ultra-fine particles (dispersions of γ Al_2O_3, SiO_2, and TiO_2 ultra-fine particles). *Netsu Bussei (Japan)* 4: 227-233.

[4] Wang, X.; Xu, X.; and Choi, S.U.S. 1999. Thermal conductivity of nanoparticles fluid mixture. *Journal of ThermoPhysics & Heat Transfer* 13(4): 474-480.

[5] Eastman, J.A.; Choi, S.U.S.; Li, S.; Yu, W.; and Thompson, L.J. 2001. Anomalously increased effective thermal conductivities of ethylene glycol based nanofluids containing copper nanoparticles. *Applied Physic Letters* 78: 718-720.

[6] Das, S. K.; Putra, N.; Thiesen, P.; and Roetzel, W. 2003. Temperature dependence of thermal conductivity enhancement for nanofluids. *ASME Journal of Heat Transfer* 125: 567-574.

[7] Choi, S.U.S.; Zhang, Z.G.; Yu, W.; Lockwood, F. E.; and Grulke, E.A. 2003. Anomalous thermal conductivity enhancement in nanotube suspensions. *Applied Physics Letters* 79: 2252-2254.

[8] Lee, S.; Choi, S.U.S.; Li, S.; and Eastman, J.A. 1999. Measuring thermal conductivity of fluids containing oxide nanoparticles. *Journal of Heat Transfer* 121: 280-289.

[9] Granqvist, C.G. and Buhrman, R.A. 1976. Ultra fine metal particles. *Journal of Applied Physic* 47: 2200-2220.

[10] Eastman, J.A.; Choi, S.U.S.; Li, S.; Yu, W.; Thompson, L.J. 2001. Anomalously Increase Effective Thermal Conductivities Of Ethylene Glycol Based Nanofluids Containing Copper Nanoparticle. *Applied Physic Letter* 78: 718-720.

[11] Yatsuya, S.; Tsukasaki, Y.; Mihama, K.; and Uyeda, R. 1978. Preparation of extremely fine particles by vacuum evaporation onto a running oil substrate. *Journal of Crystalline Growth* 45: 490-497.

[12] Wangener, M. and Gunther, B. 1999. Sputtering in liquids: a versatile process for the production of magnetic suspensions. *Journal of Magnetic Materials* 201: 41-47.

[13] Zhang, X. and Fujii, M. 2000. Simultaneous measurements of the thermal conductivity and thermal diffusivity of molten salts with a transient short hot wire method. *International Journal of Thermophysics* 21: 71-84.

[14] Putra, N.; Roetzel, W.; and Das, S.K. 2003. Natural convection of nanofluids. *Journal Heat and Mass Transfer* 39: 775-784.

[15] Khanafer, K.; Vafai, K.; and Ligthstone, M. 2003. Buoyancy driven heat enhancement in a two dimensional enclosure utilizing nanofluids. *International Journal Heat and Mass Transfer* 46: 3639-3645.

[16] Das,S.K.; Putra, N.; and Roetzel, W. 2003. Pooling boiling characteristics of nanofluids. *International Journal of Heat and Mass Transfer* 46: 851– 862.

[17] Pak, B.C. and Cho, Y.I. 1998. Hydrodynamic and heat transfer study of dispersed fluids with submicron metallic oxide particle. *Experimental Heat Transfer* 11: 151-170.

[18] Brinkman, H.C. 1952. The viscosity of concentrated suspensions and solution. *Journal of Chemistry Physic* 20: 571-581.

[19] Maxwell, J.C. 1881. *A Treatise on Electricity and Magnetism*, 2nd Edition, Vol.1; U.K: Clarendon Press, Oxford.

[20] Hamilton, R. L.; and Crosser, O. K. 1962. Thermal conductivity of heterogeneous two component systems. *Industrial and Engineering Chemistry Fundamentals* 1(3): 187-191.

[21] Bruggeman, D.A.G. 1935. Berechnung Verschiedener Physikalischer Konstanten von Heterogenen Substabzen, I. Dielektrizitatskonstanten und Leitfahigkeiten der Mischkorper aus Isotropen Substanzen. *Annalen der Physik. Leipzig* 430: 285-313.

[22] Jeffrey, D.J. 1973. Conduction through a random suspension of spheres. In *Proceedings of the Royal Society of London*. Series A, 335(1602): 355-367.

[23] Davis, R.H. 1986. Effective thermal conductivity of a composites material with spherical inclusions. *International Journal of Thermophysics* 7(3): 609-620.

[24] Lu, S. and Lin, H. 1996. Effective conductivity of composites containing aligned spheroidal inclusions of finite conductivity. *Journal of Applied Physics* 79: 6761-6769.

[25] Jang, S.P. and Choi, S.U.S. 2004. Role of brownian motion in the enhanced thermal conductivity of nanofluid. *Applied Physic Letters* 84(24): 4316-18.

[26] Ravi, P.; Bhattacharya, P.; and Phelan, P. E. 2005. Thermal conductivity of nanoscale colloidal solutions (nanofluids). *Physical Review Letters* 94: 25901-1-4.

[27] Vasu,V.; Rama, K.K.; and Kumar, A.C.S. 2006. Theoretical evaluation of thermophysical properties of nanofluids. In *Proceedings of ISHMT-ASME Heat and Mass Transfer Conference.* IIT Guwahati, India.

[28] Xuan, Y. and Li, Q. 2003. Investigation of convective heat transfer and flow features of nanofluids. *Journal of Heat Transfer* 125: 151 - 153.

[29] Buongiorno, J. 2006. Convective Transport in nanofluids. *Journal of Heat and Mass Transfer* 128: 240-250.

[30] Maiga, S.B.; Nguyen, C.T.; Galanis, N.; Roy, G.; Mare, T.; and Coqueux, M. 2006. Heat transfer enhancement in turbulent tube flow using Al_2O_3 nanoparticle suspension. *International Journal of Numerical Methods for Heat and Fluid Flow* 16(3): 275-292.