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Research paper

Exploration on traveling wave solutions to the 3rd-order klein–fock-gordon equation (KFGE) in mathematical physics

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Abstract

In this paper, the -expansion method has been applied to find the new exact traveling wave solutions of the nonlinear evaluation equations (NLEEs) by utilizing 3rd-order Klein–Gordon Equation (KFGE). With the collaboration of symbolic commercial software maple, the competence of this method for inventing these exact solutions has been more exhibited. As an upshot, some new exact solutions are obtained and signified by hyperbolic function solutions, different combinations of trigonometric function solutions, and exponential function solutions. Moreover, the -expansion method is a more efficient method for exploring essential nonlinear waves that enrich a variety of dynamic models that arises in nonlinear fields. All sketching is given out to show the properties of the innovative explicit analytic solutions. Our proposed method is directed, succinct, and reasonably good for the various nonlinear evaluation equations (NLEEs) related treatment and mathematical physics also.

Keywords: *The Expansion Method; Nonlinear Evolution Equation; Exact Solution; 3rd-Order Klein–Gordon Equation; Mathematical Physics.*

1. Introduction

Physical phenomena and processes that occurred in nature generally have tangled nonlinear features. Nonlinear problems (NLPs) are of interest to engineers, biologists, physicists, mathematicians, and many other scientists because most systems are inherently nonlinear in nature. From there nonlinear evaluation equations have been the topic of concern in different branches of nonlinear sectors such as physics, optical fibers, plasma physics, neural physics, solid-state physics, propagation of shallow water wave, mathematical fluid dynamics, electromagnetism, signal processing, chaos, viscoelasticity, heat flow and wave propagation phenomena, applied mathematics, protein chemistry, geochemistry, chemical kinematics, chemically reactive materials and meteorology, etc. That is why the exploration of traveling wave solutions is becoming a matter of concerning issue day by day.
Our proposed Klein-Fock-Gordon equation (KFGE) is an important class of NLEEs that arise in the theory of relativity, relativistic quantum mechanics and quantum field theory, which is also of great significance for the high energy particle physics and is applied to model various types of matter, including the spread of deviation in crystals and the properties of elementary particles. Sometimes it is defined as the equation of relativistic wave related to Schrodinger equation.
Herewith the Klein-Fock-Gordon equation (KFGE) [32] is defined as the form

$$\frac{\partial^2 u}{\partial t^2}+\alpha\frac{\partial^2 u}{\partial x^2}+\beta u+\partial u^m=f(x,t) \tag{1}$$

In this work, we mainly study a special case of nonlinear Klein–Fock-Gordon equation by using $m=3$ in Eq. (1).This m^{th} Order Klein–Fock-Gordon equations Eq. (1) has an m^{th} order source term, a linear source term, and a 2nd order derivative in the initial value variable and spatial variables. When $m=2$ the equation is called the 2nd order or quadratic Klein–Fock-Gordon equation and when $m=3$ the equation is tern into the cubic or 3rd order Klein–Fock-Gordon equation.
The main goal of this article is to apply the $\exp(-\varphi(\xi))$ -expansion method to find the exact solutions of nonlinear 3rd order Klein–Fock-Gordon equation.
Nowadays, the exact traveling wave solution for nonlinear evaluation equations (NEEs) has been explored by many authors and they have been used many powerful methods. Many powerful method have been presented such as Hirota's bilinear transformation method [1], [2], The $\exp(-\varphi(\xi))$ -expansion method[3-5], The exp-function expansion method [6], The Extend $\exp(-\varphi(\xi))$ -expansion method [7], [8], The extended tanh-function method [9], [10], Lie symmetry method [11-14], The modified simple equation method [15], [16], The complex hyperbolic function method [17], The Bernoulli's Sub-ODE method [18], The extended sinh-cosh and sin-cos methods [19], The (G'/G)-

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expansion method [20-22], The enhance (G'/G)-expansion method [23], The Jacobi elliptic function method [24], Homogeneous balance method [25, 26], He's polynomial [27], Asymptotic methods and Nano mechanics [28], The extended multiple Riccati equations expansion method [29], The variational iteration method [30] and so on.

M.G. Hafez [31] explored the coupled Klein–Gordon–Zakharov equation using $\exp(-\varphi(\xi))$ -expansion method but he didn't discuss the 3rd-Order or cubic Klein– Fock-Gordon equations. For this, we firmly intended our self to solve the 3rd-Order or cubic Klein–Fock-Gordon equations for the better solution of mathematical and physical treatment.

The rest of this article is prepared as follows: In section 2, the $\exp(-\varphi(\xi))$ -expansion method has been discussed. In section 3, we applied this method to the proposed nonlinear evolution equations pointed out above. In section 4, we provide some graphical representations among the obtained solutions. Finally, conclusions are given in section 5.

2. Formation of the $\exp(-\varphi(\xi))$-expansion method:

In this section, we will describe $\exp(-\varphi(\xi))$ -expansion method by term. Let us consider a nonlinear partial differential equation in the following form,

$$\Re(U, U_{xx}, U_{xz}, U_{xx}, U_{xy}, U_{xtt}, \ldots\ldots) = 0. \tag{2}$$

Where $U = U(x, y, z, t)$ is an unknown function, $\Re$ is a polynomial of U and its different type partial derivatives, in which the nonlinear terms and the highest order derivatives are involved.

Step-1. Now we consider a transformation variable to convert all independent variable into one variable, such as $U(x,t) = u(\xi)$,

$$\xi = kx + ly + mz \pm Vt. \tag{3}$$

By implementing this variable Eq. (3) permits us reducing Eq. (2) in an ODE for $u(x,t) = u(\xi)$

$$P(u, u', u'', \cdots \cdots\cdots) = 0 \tag{4}$$

Step-2. Suppose that the solution of ODE Eq. (4) can be expressed by a polynomial in $\exp(-\varphi(\xi))$ as follows

$$u = \sum_{i=0}^{m} a_i \exp(-\varphi(\xi))^i , \tag{5}$$

Where the derivative of $\varphi(\xi)$ satisfies the ODE in the following form

$$\varphi'(\xi) = \exp(-\varphi(\xi)) + \mu \exp(\varphi(\xi)) + \lambda. \tag{6}$$

Then the solutions of ODE Eq. (7) are

Case I:

Hyperbolic function solution (when $\lambda^2 - 4\mu > 0, \mu \neq 0$):

$$\varphi(\xi) = \ln\left(\frac{-\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + C)\right) - \lambda}{2\mu} \right).$$

And

$$\varphi(\xi) = \ln\left(\frac{-\sqrt{\lambda^2 - 4\mu}\coth\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + C)\right) - \lambda}{2\mu} \right).$$

Case II:

Trigonometric function solution (when $\lambda^2 - 4\mu < 0, \mu \neq 0$):

$$\varphi(\xi) = \ln\left(\frac{\sqrt{4\mu - \lambda^2}\tan\left(\frac{\sqrt{4\mu - \lambda^2}}{2}(\xi + C)\right) - \lambda}{2\mu} \right).$$

And

$$\varphi(\xi)=\ln\left(\frac{\sqrt{4\mu-\lambda^2}\cot(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+C))-\lambda}{2\mu}\right).$$

Case III:
Exponential function solution (when $\lambda^2-4\mu>0,\mu=0$):

$$\varphi(\xi)=-\ln\left(\frac{\lambda}{\exp(\lambda(\xi+C))-1}\right).$$

Case IV:
Rational function solution (when $\lambda^2-4\mu=0,\mu\neq 0,\lambda\neq 0$):

$$\varphi(\xi)=\ln\left(-\frac{2(\lambda(\xi+C)+2)}{\lambda^2(\xi+C)}\right).$$

Case V:
Other solution (when $\lambda^2-4\mu=0,\mu=\lambda=0$):
$\varphi(\xi)=\ln(\xi+C)$.
Where $a_i,V,\lambda;\ i=0,\cdots\cdots,\mathrm{m}$ and μ are constants to be determined later. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in ODE (3).
Step-3. By substituting Eq. (5) into Eq. (4) and using the ODE (6), collecting all same order of $\exp(-\varphi(\xi))$ together, then we execute an polynomial form of $\exp(-\varphi(\xi))$. Equating each coefficients of this polynomial to zero, yields a set of algebraic system for $a_i,\cdots V,\lambda;\ i=0,\cdots\cdots,\mathrm{m}$ and μ .
Step-4. Assuming that the constants $a_i,\cdots V,\lambda;\ i=0,\cdots\cdots,\mathrm{m}$ and μ can be obtained by solving the algebraic system, since the general solutions of the auxiliary ODE (6) have been well known for us, then substituting $a_i,\cdots V;\ i=0,\cdots\cdots,\mathrm{m}$, and the general solutions of Eq.(5) into Eq.(6). Thus we attain exact and explicit traveling wave solutions of nonlinear partial differential equation (2).

3. Application of the suggested method

In this section we apply our proposed $\exp(-\varphi(\xi))$ -expansion method to find the exact solution of cubic Klein–Fock-Gordon equation. Here for our convenient we consider $m=3$ to the m^{th} order Klein–Fock-Gordon equation [31].

$$u_{tt}+\alpha u_{xx}+\beta u+\delta u^3=f(x,t), \tag{7}$$

where α,β,δ are nonzero constants.
The travelling wave equation is of the form

$$u=u(x,t),\quad \xi=mx-bt,\quad u=u(\xi),\quad u(x,t)=u(\xi). \tag{8}$$

Using travelling wave equation Eq. (8) and integrating Eq. (7) with respect to ξ , we obtain the following ordinary differential equation.

$$\left(b^2+\alpha m^2\right)u''+\beta u+\delta u^3=0. \tag{9}$$

Now considering the homogeneous balance between the highest order derivative u'' and nonlinear term u^3, then we get $n=1$. Therefore, our suggested method allows us to use the auxiliary solution in the following form:

$$u(\xi)=A_0+A_1e^{-\varphi(\xi)}, \tag{10}$$

Where A_0 and A_1 are arbitrary constant to be determined such that $A_1\neq 0$, while λ,μ are arbitrary constants.
Now differentiating Eq. (10) and using Eq. (6) we get

$$u''(\xi)=-A_1\left(-\left(\exp(-\varphi(\xi))+\mu\exp(\varphi(\xi))+\lambda\right)\exp(-\varphi(\xi))+\mu\left(\exp(-\varphi(\xi))+\mu\exp(\varphi(\xi))+\lambda\right)\exp(\varphi(\xi))\right)\exp(-\varphi(\xi))$$

$$+A_1\left(\exp(-\varphi(\xi))+\mu\exp(\varphi(\xi))+\lambda\right)^2\exp(-\varphi(\xi)).$$

And also calculate $u^3(\xi)=A_0^3+\frac{3A_0^2A_1}{\exp(\varphi(\xi))}+\frac{3A_0A_1^2}{(\exp(\varphi(\xi)))^2}+\frac{A_1^3}{(\exp(\varphi(\xi)))^3}$.

Now putting the value of u, u'', u^3 in Eq. (9) and coefficient of $e^{i\varphi(\xi)}, i = 0, \pm 1, \pm 2, \cdots\cdots$, to zero, we get

$$A_1 \mu\lambda\alpha\, m^2 + A_1 \mu\lambda b^2 + \delta A_0^3 + \beta A_0 = 0. \tag{11}$$

$$\alpha\lambda^2 m^2 A_1 + 2\alpha\, m^2 \mu A_1 + b^2 \lambda^2 A_1 + 2b^2 \mu A_1 + 3\delta A_0^2 A_1 + \beta A_1 = 0. \tag{12}$$

$$3\alpha\,\lambda m^2 A_1 + 3b^2 \lambda A_1 + 3\delta A_0 A_1^2 = 0. \tag{13}$$

$$2\alpha\, m^2 A_1 + \delta A_1^3 + 2b^2 A_1 = 0. \tag{14}$$

Solving the polynomial (11)-(14) by using maple we get the following sets:
Set-1:

$$b = \pm\sqrt{\frac{\alpha m^2 \lambda^2 + 4\alpha m^2 \mu - 2\beta}{\lambda^2 - 4\mu}}, \quad m = m, \; A_0 = \pm\frac{\beta\lambda}{\left(\lambda^2 - 4\mu\right)\delta\sqrt{-\frac{\beta}{\delta\lambda^2 - 4\delta\mu}}}, \; A_1 = \pm 2\sqrt{-\frac{\beta}{\delta\lambda^2 - 4\delta\mu}},$$

Set-2:

$$b = \pm\sqrt{\frac{\alpha m^2 \lambda^2 + 4\alpha m^2 \mu - 2\beta}{\lambda^2 - 4\mu}}, \quad m = m, \; A_0 = \frac{\beta\lambda}{\left(\lambda^2 - 4\mu\right)\delta\sqrt{-\frac{\beta}{\delta\lambda^2 - 4\delta\mu}}}, \; A_1 = -2\sqrt{-\frac{\beta}{\delta\lambda^2 - 4\delta\mu}},$$

Where μ and λ are arbitrary constants.
Now substituting the values of b, m, A_0, A_1 into Eq. (10) we get

$$u(\xi) = \pm\frac{\beta\lambda}{\left(\lambda^2 - 4\mu\right)\delta\sqrt{-\frac{\beta}{\delta\lambda^2 - 4\delta\mu}}} \pm 2\sqrt{-\frac{\beta}{\delta\lambda^2 - 4\delta\mu}}\exp\left(-\varphi(\xi)\right),$$

Where, $\xi = -\sqrt{\frac{\alpha m^2 \lambda^2 + 4\alpha m^2 \mu - 2\beta}{\lambda^2 - 4\mu}}\, t + mx.$

Case-I: (when $\lambda^2 - 4\mu > 0, \mu \neq 0$) we get following hyperbolic solution
Family-1

$$u_{1,2}(\xi) = \pm\frac{\beta\lambda}{\left(\lambda^2 - 4\mu\right)\delta\sqrt{-\frac{\beta}{\delta\lambda^2 - 4\delta\mu}}} + \frac{2 \pm 2\sqrt{-\frac{\beta}{\delta\lambda^2 - 4\delta\mu}}\,\mu}{-\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{1}{2}\sqrt{\left(\lambda^2 - 4\mu\right)}(\xi + C)\right) - \lambda},$$

$$u_{3,4}(\xi) = \pm\frac{\beta\lambda}{\left(\lambda^2 - 4\mu\right)\delta\sqrt{-\frac{\beta}{\delta\lambda^2 - 4\delta\mu}}} + \frac{2 \pm 2\sqrt{-\frac{\beta}{\delta\lambda^2 - 4\delta\mu}}\,\mu}{\sqrt{-\lambda^2 + 4\mu}\tan\left(\frac{1}{2}\sqrt{\left(-\lambda^2 + 4\mu\right)}(\xi + C)\right) - \lambda},$$

Where, $\xi = -\sqrt{-\frac{\alpha m^2 \lambda^2 - 4\alpha m^2 \mu - 2\beta}{\lambda^2 - 4\mu}}\, t + mx,$ and C is an arbitrary constant.

Case-II: (when $\lambda^2 - 4\mu > 0, \quad \mu \neq 0$) get following trigonometric solution
Family-2

$$u_{5,6}(\xi) = \pm\frac{\beta\lambda}{\left(\lambda^2 - 4\mu\right)\delta\sqrt{-\frac{\beta}{\delta\lambda^2 - 4\delta\mu}}} + \frac{2 \pm 2\sqrt{-\frac{\beta}{\delta\lambda^2 - 4\delta\mu}}\,\mu}{-\sqrt{\lambda^2 - 4\mu}\coth\left(\frac{1}{2}\sqrt{\left(\lambda^2 - 4\mu\right)}(\xi + C)\right) - \lambda},$$

$$u_{7,8}(\xi) = \pm\frac{\beta\lambda}{\left(\lambda^2 - 4\mu\right)\delta\sqrt{-\frac{\beta}{\delta\lambda^2 - 4\delta\mu}}} + \frac{2 \pm 2\sqrt{-\frac{\beta}{\delta\lambda^2 - 4\delta\mu}}\,\mu}{\sqrt{-\lambda^2 + 4\mu}\cot\left(\frac{1}{2}\sqrt{\left(-\lambda^2 + 4\mu\right)}(\xi + C)\right) - \lambda},$$

Where, $\xi=-\sqrt{-\frac{\alpha m^2\lambda^2-4\alpha m^2\mu-2\beta}{\lambda^2-4\mu}}t+mx,$ and C is an arbitrary constant.

Case-III: (when $\lambda^2-4\mu>0,\quad \mu=0,\quad \lambda\neq 0$) we get following exponential solution
Family-3

$$u_{9,10}(\xi)=\pm\frac{\beta}{\lambda\delta\sqrt{-\frac{\beta}{\delta\lambda^2}}}-\frac{1}{2}\frac{\pm 2\sqrt{-\frac{\beta}{\delta\lambda^2}}\lambda^2(\xi+C)}{\lambda(\xi+C)+2},$$

Where, $\xi=-\sqrt{-\frac{\alpha m^2\lambda^2-2\beta}{\lambda^2}}t+mx,$ and C is an arbitrary constant.

Case IV & Case V:

When $\lambda^2-4\mu=0,$ the executing value of A_0 is undefined. So the solution cannot be obtained. For this purpose Case IV is rejected. Similarly when $\lambda^2-4\mu=0,\mu=0,\lambda=0$ the executing value of A_0,A_1 are undefined. So the solution cannot be obtained. So Case V is also rejected.

4. Results and discussions

4.1. Physical explanation

In this part, we discus about the physical depiction of obtained and solitary wave solutions for the Klein–Fock-Gordon equation with respect to the $\exp(-\varphi(\xi))$ -expansion method. Obviously the 3rd order Klein–Fock-Gordon equation has solitary wave solutions that have exponentially decaying wings. There is difference type of traveling wave solutions that one of special interest in solitary wave theory. For some different physical parameters, solitary wave solutions are developed from the acquired exact solutions. Figure 1 represents Bright kink shape solution of u_1 for the parameters $\alpha=-2,\ \beta=-2,\ \delta=2,\ C=-1,\ m=2,\ \lambda=3,\ \mu=2$ within the interval $-10\le x,t\le 10$. Figure 2 represents the Dark kink shape solution of u_2 for the parameters $\alpha=-2,\beta=-2,\delta=2,C=-1,m=2,\quad \lambda=3,\mu=2$ within the interval $-10\le x,t\le 10.$ Figure 3 represents the Singular kink shape solution of u_3 for the parameters $\alpha=-2,\ \beta=-2,\ \delta=1,\ C=-1,\ m=2,\lambda=3,\mu=2$ within the interval $-10\le x,t\le 10$. Figure 4 represents the Periodic shape solution of u_6 for the parameters $\alpha=-4,\ \beta=-3,\ \delta=1,\ C=-1,\ m=2,\lambda=3,\mu=6$ within the interval $-10\le x,t\le 10$ and Figure 5 represents the Soliton shape solution of u_{10} for $\alpha=2,\ \beta=-2,\ \delta=-1,\ C=-1,\ m=2,\lambda=2,\mu=0$ within the same interval $-10\le x,t\le 10.$

4.2. Graphical explanation

This sub-section represents the graphical representation of 3rd order Klein-Fock-Gordon equation. By using mathematical software Maple 18, Contour, 3D and 2D plots of some achieved solutions have been shown in Fig.-1-Fig.-5 to envisage the essential instrument of the original equations.

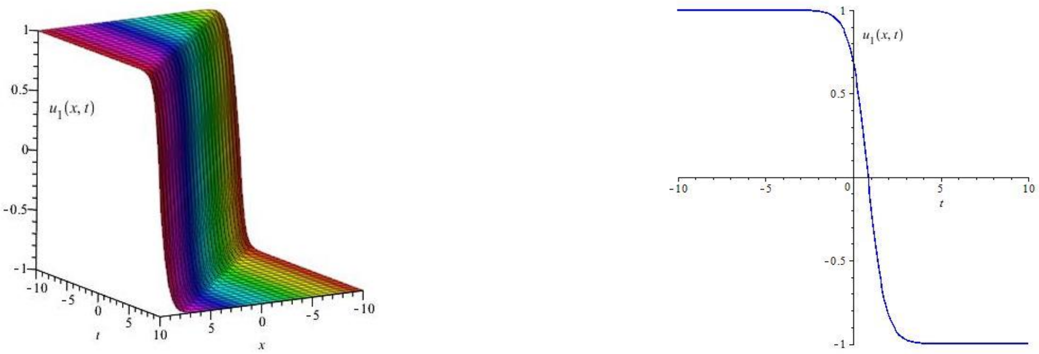

Fig. 1: Bright Kink Shape Solution of u_1 For $\alpha=-2,\beta=-2,\delta=2,C=-1,m=2,\lambda=3,\mu=2$ within $-10\le x\le 10$ and $-10\le t\le 10.$ the Left Figure Shows the 3D Plot and the Right Figure Shows the 2D Plot for $x=1$.

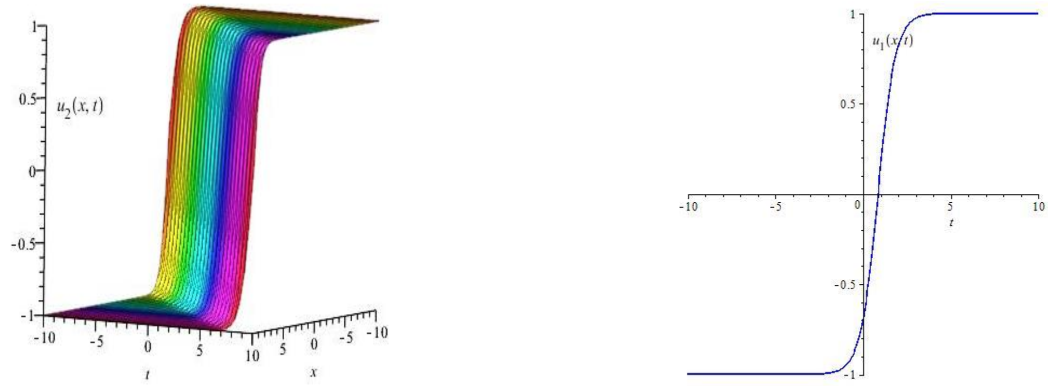

Fig. 2: Dark Kink Shape Solution of u_2 For $\alpha = -2, \beta = -2, \delta = 2, C = -1, m = 2, \lambda = 3, \mu = 2$. the Left Figure Shows the 3D Plot and the Right Figure Shows the 2D Plot for $x = 1$.

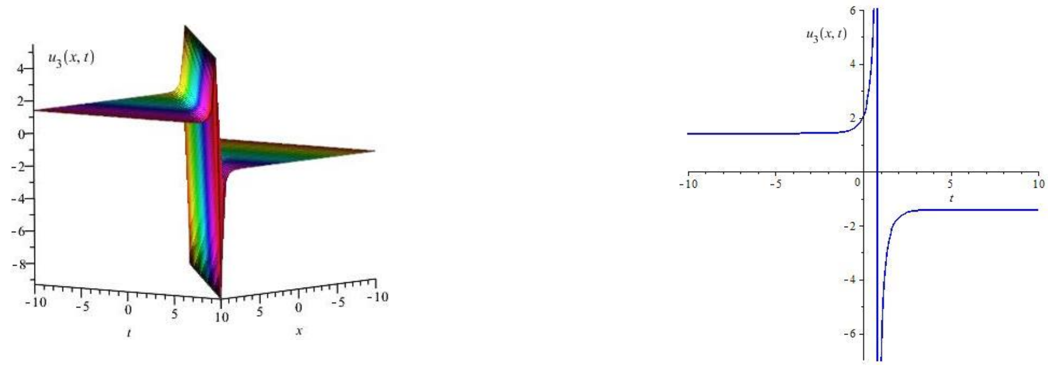

Fig. 3: Singular Kink Shape Solution of u_3 For $\alpha = -2, \beta = -2, \delta = 1, C = -1, m = 2, \lambda = 3, \mu = 2$. the Left Figure Shows the 3D Plot and the Right Figure Shows the 2D Plot for $x = 1$.

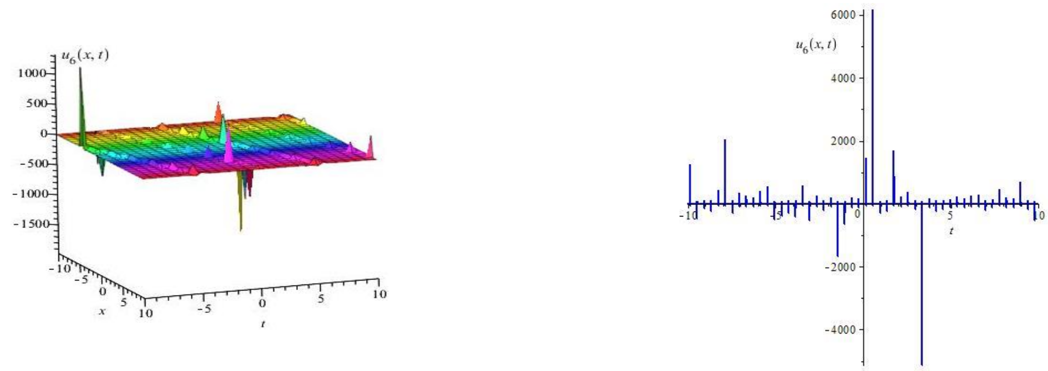

Fig. 4: Periodic Shape Solution of u_6 for $\alpha = -4, \beta = -3, \delta = 1, C = -1, m = 2, \lambda = 3, \mu = 6$. the Left Figure Shows the 3D Plot and the Right Figure Shows the 2D Plot for $x = 1$.

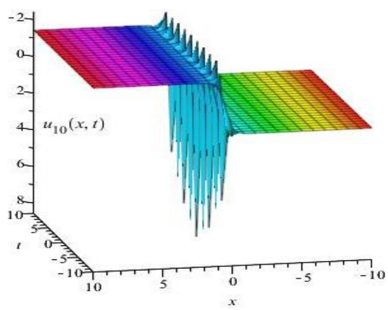

Fig. 5: Soliton Shape Solution of u_{10} for $\alpha = 2, \beta = -2, \delta = -1, C = -1, m = 2, \lambda = 2, \mu = 0$.

5. Conclusion

In this study, the $\exp(-\varphi(\xi))$ -expansion has been successfully applied to find new traveling wave solutions for nonlinear wave equation of 3rd-Order Klein-Fock–Gordon equation (KFGE). We get some new traveling wave solutions including hyperbolic function solutions, trigonometric function solutions and exponential solutions. Therefore, we successfully got the Bright kink shape solution, Dark kink shape solution, Singular kink shape solution, Periodic shape solution and other Soliton shape solution which has given the proper geometrical explanation. The results is clear to us that our proposed method is reliable, effective and reasonable good for nonlinear evolution equations.

Likewise, the solutions of the proposed nonlinear evolution equation in this paper have numerous potential applications in engineering field.

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Competing interests

The authors declare that they have no competing interests.

Author's contributions

All authors contributed equally to this work. All authors read and approved the final manuscript.

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