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Exploring exchange rate returns at different time horizons

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Abstract

This paper explores and compares the empirical distribution of the US dollar–deutsche mark exchange rate returns with well-known continuous-times processes at different frequencies. We use a variety of parametric models to simulate the unconditional density of the exchange rate returns at different frequencies, and show that the studied models do not fit the empirical distribution of exchange rate returns at both the high and low frequencies.

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1. Introduction

Many models in continuous-time finance rely on the assumption of a specific stochastic process, while little attention is paid to the empirical fit of an assumed process to the actual data across different time scales. Consequently, the application of such statistical models to financial data would result in specification errors when the underlying data-generating process scales differently across time. This assumption can also lead to mispricing of financial assets, and can have serious implications on portfolio selection and risk management. There is now evidence that investors do have heterogeneous expectations differentiated according to their time dimensions (see Ref. [4]). The co-existence of short-term as well as long-term traders indicates that there are different time scales for different traders in the market. Therefore, different time scales can lead

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to different price formation processes, which have other effects such as volatility clustering and foreign exchange adjustment. However, investigating the scaling properties of foreign exchange returns and modelling its dynamics is far away from being trivial, and one recent promising attempt is the wavelet multi-scaling approach (see Ref. [7]).

The aim of this study is to investigate the performance of the well-known stochastic processes in fitting the empirical distribution of the exchange rate returns at different time scales. We start with estimating the parameters of the candidate processes at different time scales and proceed with simulating the empirical distributions of exchange rate returns from selected candidate processes. The theoretical distributions are then compared with the empirical distribution via a Kolmogorov–Smirnov goodness-of-fit test.

2. Candidate processes for the exchange rate returns

2.1. Random walk GARCH(1,1)

Consider the following representation of the continuous-time logarithmic price process P_t, where P_t is a dollar price of the foreign currency at time t:

$$\mathrm{d}P_t/P_t = \alpha\,\mathrm{d}t + \sigma\,\mathrm{d}W_t\,, \tag{1}$$

where $t \geqslant 0$ and W_t denotes a standard Brownian motion. The mean, α, and the variance, σ, are both defined per unit time. Moreover, consider the following continuous-time representation of the returns, defined as $X_t = \ln P_t - \ln P_{t_0}$:

$$\mathrm{d}X_t = \mu\,\mathrm{d}t + \sigma\,\mathrm{d}W_t\,. \tag{2}$$

The drift part in Eq. (2), $\mu = \alpha - \frac{1}{2}\sigma^2$, can be omitted since the expected returns are equal to zero for all return horizons. Our estimation shows further that the drift is significantly equal to zero. In fact, Andersen et al. [1] argue that it is straightforward to allow for a drift or more general forms of conditional mean predictability but the assumption of no conditional mean provides a very good approximation for the high-frequency returns over 20 min. In order to capture volatility clustering, we use the GARCH process for conditional volatility. Therefore, the random walk model with GARCH(1,1) specification for the exchange rate returns is presented as follows:

$$\mathrm{d}X_t = \sigma_t\,\mathrm{d}W_t = \sigma_t\varepsilon_t\,,$$
$$\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\sigma_{t-1}^2\,, \tag{3}$$

where ε_t is assumed to follow a probability distribution with zero mean and unit variance, such as the standard normal distribution or the Student-t distribution.

2.2. Random walk with stochastic volatility

The next model considered is the random walk with stochastic volatility. The volatility of the returns is assumed to follow an Ornstein–Uhlenbeck process, hence the returns