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Exploring Scanning Probe Microscopy with MATHEMATICA

Second, Completely Revised and Enlarged Edition



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