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Exploring the Characteristics of an Optimal Design of Digital Materials for Concept Learning in  
Mathematics: Multimedia Learning and Variation Theory

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### **Abstract**

Design principles emerging from the cognitive theory of multimedia learning can be applied to engage cognitive processing, and teaching methods that have evolved from variation theory can encourage thinking through comparisons in mathematics education. Applying these principles and teaching methods in designing digital material should be a sound proposition. However, there is a disconnection between research in digital educational material and classroom practices. Teachers often have doubts about the effectiveness of the materials. Thus, this paper presents a design-based research of developing a digital material for algebra concept learning. We collaborated with two experienced teachers and a subject expert from a university, and designed some digital learning material that was presented to 68 students through an iterative redesign development cycle; the effectiveness of the final product was tested on another group of 66 students the following year. Characteristics of an optimal design generated from the data collected are presented in this paper. The characteristics may have useful practical implications for instructional designers and teachers and contribute to improvements in the design of digital learning materials.

*Keywords:* design-based research, cognitive processing, multimedia learning, concept learning, variation theory

## 1. Introduction

Digital educational materials for mathematics learning currently used in schools often incorporate mainstream teaching methods that focus primarily on improving procedural knowledge. These materials provide students with great learning opportunities through online exercises and quizzes that instantly reward responses with answers and solutions; further, teachers receive detailed analysis of student performance. While this timely feedback for students and teachers is useful, the materials offer training platforms focusing on assessment rather than learning. Balanced mathematics learning encompasses another type of knowledge – conceptual (CDC & HKEAA, 2007; Rittle-Johnson, Siegler, & Alibali, 2001). Conceptual knowledge comprises ideas retrieved from conceptual understanding (Rabinowitz, 1988). Due to the different nature of procedural and conceptual knowledge, digital educational materials that foster the development of conceptual knowledge are therefore often considered as cognitive tools to improve students' active involvement in the learning process – that is, active learning (Churchill, 2007, 2011, 2013, 2014; Mayer, 2009). These materials are designed to elicit thinking, and focus on understanding rather than memorizing (Churchill, 2011, 2013, 2014). The design of tools influences learning processes and outcomes (Ainsworth, 2006; Churchill, 2007; Mayer, 2009). Therefore, targeting the design to engage learners' cognitive processing is important (Churchill, 2011; Mayer, 2009). The present study focused on this issue. Multimedia learning design principles suggested by Mayer (2009) were primarily used to cater for learners' cognitive processing needs; and the way learning messages in the content were presented evolved from variation theory (Marton et al., 2004; Gu, Huang, & Marton, 2004). The main goal of this study was to explore the optimal design of mute digital material for concept learning in

algebra, resulting in design characteristics. Moreover, many studies often focus on investigating the effects of Mayer's multimedia learning principles in design (Moreno & Mayer, 1999; Moreno, R & Mayer, 2000; Harskamp, Mayer, & Suhre, 2007). Design recommendations in the literature on the application of multimedia learning principles are scarce (Churchill, 2013). This paper also exemplifies the application of a combination of the multimedia learning principles including mute in the design of digital educational materials.

### **1.1. Digital material in school algebra**

Conceptual understanding is considered to be a hard-to-teach mathematical idea (Hoyles, Noss, Vahey, & Roschelle, 2013). Teachers lack of effective learning strategies for developing deeper understanding of mathematics concepts in students (Wong, 2007). The use of digital materials in school mathematics can support students in developing conceptual understanding (Churchill, 2011; Churchill & Hedberg, 2008; Hoyles, Noss, Vahey, & Roschelle, 2013). Computer aided algebra resources whose core comprises symbolic manipulators, were originally designed to complete algebraic procedures accurately and quickly. These resources let students observe the relationship between quantities and graphs, and support different views and representations of the mathematical concept (Heid, 1995; Yerushalmy & Chazan, 2008). However, most of the materials typically neglect mathematical and instructional issues (Yerushalmy 1999; Yerushalmy & Chazan, 2008), and cognitive processing when visual representation is applied (Churchill, 2013). For examples, Churchill and Hedberg (2008) designed the material for concept learning by representing one or more related mathematical ideas in an interactive and visual way. The material allowed students to explore mathematics properties by manipulation; Caglayan (2014) suggested the materials should visualize algebraic

expressions or numbers to construct mathematical formulae meaningfully; and Vahey and colleagues (2013) developed a system called SimCalc and suggested the materials should provide dynamic representation environments which embed mathematical relationships. The SimCalc system linked algebraic expression, tabular expression, narrative and graphical representation through a visualization of motion. These materials focused on how to visualize mathematical ideas, but not on student cognitive processing and a domain specific instructional strategy. These may not result in optimal learning outcomes.

## **1.2. Design for cognitive processing**

Presentation using words and images to promote active learning should be considered when designing digital educational materials to foster concept learning (Churchill, 2007, 2011, 2013, 2014; Mayer, 2009, 2014). Designs promoting active learning effectively facilitate a level of understanding that can be referred to as mental representation (Mayer, 2009). During active learning, learners utilize three types of cognitive processing when engaging with learning messages (Mayer, 2009, 2014): generative, essential and extraneous processing (Mayer, 2009). Mayer and colleagues (2009) developed the cognitive theory of multimedia learning to explain what is involved in processing, and suggest twelve design principles to apply when presenting learning messages using words and images. Generative processing functions to build relationships among learning messages, and is closely related to the learner's motivation level. Presenting words and images together can enhance this processing (Mayer's multimedia principle refers), and better enable learning than through use of words alone (Mayer, 2009; Plass, Chun, Mayer, & Leutner, 1998), while also catering for different learning styles (Cole, et. al,

1998; Plass, Chun, Mayer, & Leutner, 1998). Moreover, essential processing helps learners to select thinking-related learning messages from the presentation (Mayer, 2009). Allowing learners to learn at their own pace (Mayer's segmenting principle refers and naming the key messages can engage essential processing (Mayer's pre-training principle refers). Furthermore, extraneous processing does not contribute to the learning process and wastes learners' cognitive capacity (Mayer, 2009); the heavier the processing required by the learning material, the more likely the learning will fail. They suggest ways to reduce extraneous processing: (1) deleting irrelevant words and graphics (Mayer's coherence principle refers); (2) highlighting important words and graphics (Mayer's multimedia principle refers); and (3) presenting words next to corresponding graphics simultaneously (Mayer's spatial and temporal contiguity principles refer). These principles are intended to maximize the available cognitive capacity of learners and engage cognitive processing when learning with written words and images. Thus, presentation of learning materials should be designed to free cognitive capacity by engaging generative and essential processing, and consuming less extraneous processing.

Moreover, other research-based recommendations on design presentation in digital materials for concept learning draw on ideas similar to Mayer's (2009). For example, recommendations to present learning information visually (Churchill, 2007, 2011, 2014; Seufert, 2003) draw on the notion of generative processing. Recommendations to involve interactive features (Collins, 1996; Churchill, 2011, 2014; Salomon, Perkins, & Globerson, 1991) relate to essential processing. Mindfulness of extraneous processing informs recommendations to use a single screen, the same font style (so as not to distract learners), moderate color and a holistic scenario, to divide the screen area logically (Churchill, 2011, 2014), and to avoid decorative pictures and words (Collins, 1996; Churchill, 2011, 2014).

### **1.3. Multimedia messages in mathematics**

What are the words and images in mathematics? Words comprise signs describing learning messages in content. Consider, for example, the sentence “Concepts are abstract.” The noun and adjective are entities, the verb shows how they connect to each other. Correspondingly, in the equation  $z = x + 2y + 1$ , the variables  $x$ ,  $y$  and  $z$  are entities, while the operators  $=$  and  $+$  connect the variables (Schnotz, 2002; Schnotz & Bannert, 2003; Schnotz & Kürschner, 2008). Moreover, images have no signs to describe the relations among different learning messages in the content. For example, a curve presented in a coordinate plane shows how the value of  $x$  relates to that of  $y$  and nothing in the curve explicitly points out the relationships (Schnotz, 2002; Schnotz & Bannert, 2003; Schnotz & Kürschner, 2008). In the mathematics domain, equations, expressions, numbers and symbols, theorems, notation, symbolic expressions, formulae and figures are classified as words; graphical representation, diagrams, tables and lines are classified as images (Schnotz & Bannert, 2003).

### **1.4. Instructional learning messages in algebra**

While cognitive processing is an important design consideration, in the context of a specific subject domain the learning messages in the content should also be a primary focus. Instructionally providing appropriate and relevant learning messages can be another key component in design (Brophy, 2001; Marton et al., 2004; NCTM, 2000). Effective teaching presentations and methods can also be considered in design - the content should be selected and displayed in a way that is compatible with how competent teachers present in classrooms. In algebra learning and teaching, an effective teaching method has emerged from variation theory



(Marton et al., 2004; Gu, Huang, & Marton, 2004; Ling & Marton, 2011). The theory describes learning as a process that helps students develop abilities to think in different ways by seeing and experiencing (Marton et al., 2004; Gu, Huang, & Marton, 2004; Ling & Marton, 2011).

Mathematics learning and teaching activities should be designed to assist students to see the relations among different forms of the same problem (Gu, Huang & Marton, 2004; Mok, 2009; Mok & Lopez-Real, 2006), allowing them to approach the past from different perspectives (Gu, Huang & Marton, 2004; Ling & Marton, 2011). In algebra, concepts should be presented numerically, graphically, algebraically and descriptively simultaneously (NTCM, 2000); and the description can be presented implicitly. Students are more likely to construct a more complete understanding when they build relationships among the four representations (NTCM, 2000). This is the basis for the idea of multiple representations (Ainsworth, 1999, 2006; Bodemer & Faust, 2006; Moreno & Durán, 2004). Multiple representations lead to a more complete representation than one source of learning information by compensating each other (Bodemer & Faust, 2006).

### **1.5. Problem proposition**

There is a disconnection between research in digital educational materials and application of those materials in practice (Amiel & Reeves, 2008; Hjalmarson & Lesh, 2008; Yerushalmy & Chazan, 2008). Much research in educational technology ignores the complicated interaction between educational bodies and technological interventions, and even the main point of educational research (Amiel & Reeves, 2008). Educational technology researchers generally focus on the value of technology itself, rather than its effect on learning in real teaching environments, often failing to take into account different learning variables (Amiel & Reeves,

2008; Cuban, 2001; Kent & McNergney, 1999; Wang & Hannafin, 2005). In schools, teachers are commonly skeptical about the effectiveness of classroom application of the tools designed or provided by a publisher, learning materials provider and/or the academic community (Hjalmarson & Lesh, 2008; Yerushalmy & Chazan, 2008). Instead of basing designs purely on theory, researchers should work with teachers directly when studying how to design educational technologies

### **1.6. The present study**

The present study adopted a design-based approach and aimed to explore the characteristics of a design of learning material that is desirable for researchers and practitioners in order to foster concept learning in secondary school algebra. The presentation of the material was designed according to multimedia learning design principles; and learning content was displayed using an instructional strategy.

## **2. Method**

### **2.1. Design**

Design-based research is an appropriate methodology to bridge the gap between designers and practitioners in educational technology (Amiel & Reeves, 2008; Anderson & Shattuck, 2012; Wang & Hannafin, 2005). The tools developed using design-based research methods can increase the impact of learning materials in educational practice (Amiel & Reeves, 2008; Anderson & Shattuck, 2012; Wang & Hannafin, 2005). The research should (i) be theory-

driven; (ii) involve both researchers and practitioners; (iii) adopt an iterative redesign cycle; (iv) conduct in real situation; and (v) develop knowledge or principles that can contribute to both theory and practice.

This study comprised two stages: development and examining. In the development stage, we developed the digital material in an iterative redesign development cycle. The cycle included four stages: (1) review of the literature, current digital educational materials and student learning problems; (2) design and development; (3) testing in real situations; and (4) analysis of the participants' responses and evaluations (see Figure 1). In the examining stage, we investigated whether the material would improve conceptual understanding leading to better conceptual and procedural knowledge than a traditional material would, and how students learned with the material with semi-structured individual interviews.

## **2.2. Participants**

Table 1 shows the participants involved in the study. In the development stage, participants comprised 68 Level Four secondary students from two classes in a Hong Kong secondary school, two experienced mathematics teachers and one subject expert from a university. The students were aged 16-18. One class comprised 32 students with comparatively low academic performance in mathematics, and the other, 36 students with comparatively high academic performance in mathematics. One of the teachers had more than 10 years of teaching experience and worked in an examination and assessment authority; the other had more than 25 years of teaching experience and was a mathematics panel head. In the examining stage in the second year of the study, another two groups of total 66 students from secondary Level Four - a

traditional group (32 students) and an intervention group (34 students)- participated in the pre- and post-tests. Five students in the intervention group participated in semi-structured individual interviews.

### **2.3. Procedure**

The study lasted two academic years, during the first of which we spent almost one month developing the material in the iterative redesign development cycle. There were 15 lessons, but only nine involved the material. Initially, we conducted the review with the two teachers and the subject expert. We used Flash to redevelop different versions of the material and tested each version in the classrooms. Then, the teachers taught their lessons as usual with the assistance of different versions of the material developed; a student demonstrated them to his classmates; and students used some of the versions in a computer room. During each trial, students comments were collected and short evaluative talks were conducted with the teachers. The redesign factored in their comments.

In the second year, we invited 72 students were randomly divided into two groups – traditional and intervention. They were taught the essential concepts covered in the designed material by the teacher with 10 years of teaching experience. Two students in the traditional group did not finish the experiment; and two students from each group scored zero in the tests. These six samples were removed from the analysis. The students had 40 minutes to complete paper-based procedural and conceptual knowledge pre-tests before our experiment in their classroom. We conducted the experiment in a 100-minute lesson in a computer room. All the students were assigned to their own computer and learned through manipulating one of the materials. After the experiment, the students completed the paper-based post-test in their

classroom. The tests were graded by the two mathematics teachers. Furthermore, the 5 students from the intervention group completed the 20-minute interviews. Two main questions were: a) How did you learn with the material?; and b) What design features or learning messages in the model helped you learn?. The interviews were conducted in Cantonese, recorded, transcribed and translated into English.

#### **2.4. Materials**

Two learning materials were used in the experiment. The material researched was in the area of senior secondary level mathematics - specifically, quadratic equations. The material was used to consolidate (redevelop) students' conceptual understanding. The students manipulated the material for further concept learning after receiving essential conceptual and other relevant knowledge in the classroom. In the intervention group, the material was the final version developed in the cycle. In the traditional group, the material was a digital material the teachers used to teach with before - interactive and visual representation (Churchill & Hedberg, 2008). The material included a graph and allowed the students to manipulate different values of coefficients of quadratic equation.

The two tests were based on the study of Schneider and Stern (2010). We validated the relatedness and quality of the questions in both tests with the two teachers to ensure their relevance to the learning activities. In the procedural knowledge test, questions comprised solving quadratic equations with different methods, forming a quadratic equation from roots given, and identifying roots from a graph. For examples, (a) Solve the following equations  $(x-1)(x-2)=0$ , (b) Form a quadratic equation in  $x$  with roots 1 and -2, and (c) Solve the given

equations and determine the signs of the value of discriminants (positive, zero or negative) graphically. In the conceptual knowledge test, questions involved understanding the properties of graphs of quadratic functions, understanding relationships between knowledge and concepts, and justifying the validation of a solution to a problem. For examples, (a) Sketch two possible graphs  $y=f(x)$  and  $y=g(x)$  if the roots of the quadratic equations  $f(x)=0$  and  $g(x)=0$  are 2 and 1, (b) Consider the quadratic equation  $-x^2+3x-3 = 0$ , please give comment on the statements provided by the following graph; (c) The solution of the quadratic equation  $-x^2-x = 2$  is -2 or 1 (a related graph was given). Do you agree with this solution? Please explain. The two tests were scored out of a possible 36 points.

### 3. Results

The design of the material improved, and we gained knowledge about the characteristics of an optimal design. The process was documented in more than 180 pages including the teacher and student's comments (evaluation), learning activities and tests, and audio scripts. We first reported on data collected, focusing on the design modifications, in the development cycle and then on students' performance in the tests. We followed this with an analysis of the interview data.

#### 3.1. Development cycle

The first stage in the iterative redesign development cycle, before starting the design, was to review the literature on design issues and educational tools for quadratic equation learning used in schools, and discuss student learning problems. The teachers expressed that even though the students knew how to answer some problems, they often lacked a complete conceptual

understanding. The students remembered how to solve quadratic equation problems using the methods their teachers had taught them or that they had used before. Moreover, the two teachers and the subject expert assisted in reviewing three educational tools commonly used in schools. The first was a “Quadratic equation calculator” (Figure 2 refers), an electronic calculator that provided the solutions of a quadratic equation immediately after the values of coefficients were typed in. This enabled students to verify their solutions and reinforced their understanding of the properties of solutions (Math, 2012). Secondly, the “Graphical drawer” (Figure 3 refers), provided by a publisher, allowed the teachers and students to type in coefficients to obtain the graph of an equation. The tool provided different degrees of equations - i.e. not only quadratic equations - and there was potential for the students to become confused when manipulating it. The third tool reviewed was GeoGebra, which Figure 4 shows to be a powerful, flexible and complex multi-functional educational tool, allowing teachers to design their own learning and teaching materials and offering many graphs of different equations. However, as with the “Graphical drawer”, the different functions offered were confusing for the students and did not serve instructional goals unless the teachers provided a purpose-fit design. Thus, GeoGebra risked being under-exploited and used merely as a “Graphical drawer. The reviews were in line with the literature discussed earlier.

Based on the data collected in the review, we developed Version 1 of the material (Figure 5 refers). By the coherence principle, essential concepts were identified to serve instructional goals. Four essential learning messages were selected and displayed in the four sections: a graph (top left), quadratic equation (top right), solving method/algebraic forms (bottom right) and description (bottom left). The interface was divided equally into the four sections. In the top right section, different forms of a quadratic equation were presented, and in the bottom right section,

different equation solving methods. The relationships between different sections were not explicitly shown. Moreover, in applying temporal and spatial contiguity principles, the four sections were placed next to each other on the same interface. Each adjacent section was related. The graph section was directly related to the equation section, as one of the equation solving methods is graphical presentation, and similarly the method solving section. In accordance with the segmenting principle, the control sliders in the top right section allowed the students to manipulate and learn at their own pace. Finally, the background color of the four sections was blue; and the parameter range of the control slides was -100 to 100.

In the cycle, we took multimedia learning principles into account when redesigning the materials, see Figure 7, 8 and 9. Table 2 shows some of student responses, their corresponding modifications and multimedia learning principles applied if any. The responses showed what in the materials caused difficulties in students learning, which were considered as their requests on the presentations of the materials. The data showed that the material (1) should guide the students to focus their thoughts on building connections among learning messages (see Modification a, b, c and d); (2) should output information facilitating seeing changes (see Modification c); (3) should make the learning messages mathematically meaningful (see Modification f and g); (4) should allow manipulation of the graph (see Modification i); and (5) should exclude the extended concepts (see Modification h). Finally, more than 60 % of the students complained that the background color was too bright before they had started learning, resulting in changing to grey, see Modification e.

Further, the following excerpts showed what in the materials facilitated their learning. These confirmed the modifications we made.

*“The control value is better now.”*



*“Numbers and components in the equation solving methods are clear now because of the colors.”*

*“I finally know the names of equation solving methods now.”*

*“It is discriminant.”*

*“I know it is quadratic formula now.”*

*“I like the background color”*

Final, this paper attaches a short video that presented the final version of the material and explained some of what we changed in the cycle. The material was also award in a electronic educational resources design scheme organized by the Hong Kong Education City. The judge team included teachers, researchers and government officers.

### **3.2. Pre-test and post-test**

After the development, in the second year of the study, pre- and post- tests were conducted with the two groups. Table 3 shows the ANCOVA results of the two post-tests. The analysis of homogeneity of the regression coefficient showed that two groups had no difference in procedural knowledge,  $F(1, 64) = 2.87$ ,  $p = 0.095$ , and conceptual knowledge,  $F(1, 64) = 1.24$ ,  $p = 0.27$ . These confirm the hypothesis of homogeneity. Following that, analyses of covariance (ANCOVAs) were conducted to analyze the scores in the two post-tests by excluding the effect of their pre-test scores.

For the dependent variable procedural knowledge, the adjusted means of the intervention and traditional groups were 28.14 and 24.54 respectively. There was a significant difference in

the post-test scores between the two groups,  $F(1, 64) = 5.28$ ,  $p < 0.05$ ,  $\eta^2 = 0.08$ , showing a medium effect size.

For the dependent variable conceptual knowledge, the adjusted means of the intervention and traditional groups were 28.14 and 24.54 respectively. The post-test scores of the two groups reached a significant level with  $F(1, 64) = 14.45$ ,  $p < 0.001$ ,  $\eta^2 = 0.19$ , showing a large effect size.

We concluded that the students who learned better with the material designed in the cycle. They redeveloped a more complete conceptual understanding during learning. In other words, the design of the material was more effective in the area of concept learning.

### **3.3. Interview**

The interview data showed that all the students were able to acquire a concept when they were able to build the relations among the learning messages, see point a, b and c in Table 4. The materials offered them opportunities to think through the comparison, “alerting” and “mathematical change”. The comparison refers to the various forms, see point d, e and f in Table 4, the “alerting” refers to the color-matching, dots and color changes, see g, h, i and j in Table 4, that informed the students where and what to think; and the “mathematical change” refers to the different messages presenting different important mathematical concept, see k, l and m in Table 4, for example, the sign of the value of discriminant.

## **4. Discussion**

The main goal of this study was to use a design-based approach to explore the characteristics of an optimal design for concept learning in algebra. The results suggest that

researchers can improve design by working with practitioners in classroom. Researchers can apply theories to the design of digital materials, and refine the design based on feedback from practitioners. Moreover, applying multimedia principles and variation theory in designing the material appears to be beneficial, and supports the contention that reconstructing conceptual change can improve procedural and conceptual knowledge performance (Tillema & Knol, 1997; Vamvakoussi & Vosniadou, 2004; Vosniadou & Verschaffel, 2004). The results also confirmed that design should take account of selection of meaningful learning messages and cognitive processing. The results suggest the following main characteristics.

#### **4.1. The characteristics of a meaningful design**

The material should present meaningful learning messages in the subject domain. First, instructional strategies or effective teaching methods should be used to identify the relevant and essential learning messages. Detail information, such as long paragraph, may not be essential learning messages in designing material for developing concepts. For example, steps of solving methods, sentences and paragraphs should not be presented. These could be helpful in redeveloping procedural skills, but irrelevant in concept learning. Second, a simplified or abbreviated form of a learning message is sometimes presented in classroom teaching, but the material should display the exact or complete forms to help the students understand different components. For example,  $b^2-4ac$  should be used in a quadratic formula instead of its symbol  $\Delta$ . Final, learning message should be labelled although the students had already known their names. The data showed labelling was not redundant.

#### **4.2. The characteristics of a building relation design**

The interview data suggested that students are required to understand the relations between different learning messages to acquire a concept. We concluded that the materials should be designed to optimize opportunities to encourage building the relations. First, different or various forms of a piece of learning information facilitating comparisons should be provided. These comparisons engage the student thinking. Second, the material should assist students to successfully experience the meaningful changes can facilitate building relationships among messages. Reasonable control parameter ranges should be selected. For example, when a large range was provided, the students may not see the changes from negative to positive after many times of manipulation. This did not contribute to the redevelopment of concepts, but rather discourages students. Third, the design should enable both graphs and coefficients to be manipulated; therefore, the students could see changes from different perspectives. Fourth, concepts are network-structured. Different components in the material are related. Instead of highlighting, color matching should be adopted to show the obvious links or relations among the components. This can guide the students in where to look and what to think, and highlights what happens in the mathematical relationships when appropriate variables are selected. Final, color changes should be made when the implicit concepts are shown. This alerts students to stop manipulating and focus their thoughts on what has happened to graphs, equations, solving methods, symbols and numbers in the material.

### **4.3. The characteristics of a cognitive capacity design**

The presentation of the material should be designed to maximize cognitive capacity available (refers to Mayer extraneous cognitive processing). First, group the learning content of a mathematical idea (Churchill, 2011). This grouping is more likely reduce learners' extraneous cognitive processing when processing information during learning (Churchill, 2011). The different grouped learning content should be presented in equal measure in different sections, each featuring a particular mathematical idea. Second, present the sections together simultaneously. Third, each section should be in different levels of a color. Fourth, position section optimally - related sections that have instructional implications should be placed next to other on the same screen. For example, graphs should be placed next to equations; a graph should also be placed next to its textual description. Fifth, the amount of content from the extended curriculum was too much for the students. Large numbers of mathematics ideas do not constitute learning material or a reference for students to read, and are thus inappropriate to a learning context. The acquisition of mathematics concepts involves heavy cognitive processing. Too many concepts can demotivate students and disengage cognitive processing. This is a similar idea to the Mayer's coherence principle. Final, teachers review showed that a learning material provided by publisher catered different topics under a domain was ineffective; therefore, the material should be designed for topic-specific rather than domain-specific.

#### **4.4. The characteristics of an appearance-friendly design**

Data showed that the appearance of the material was irritating before the students began to learn. It was very important to gain student attention in the first place. Irritating appearance could demotivate student. The background color should be less bright.

### **5. Conclusions**

The results showed that the design used in the study led to better learning outcomes. Design of digital educational should take account of student cognitive processing. The characteristics we suggested not only can assist instructional designers in developing digital materials for concept learning, but also assist teachers in designing their learning content using authoring software and choosing effective materials from the resources offered to them. Moreover, we also exemplified the application of the combination of the multimedia learning principles in designing digital educational materials.

Researchers may not be able to apply the theories and principles in designing digital materials well, perhaps because they lack current classroom experience. This study has demonstrated that involving users in a real learning environment can contribute to the design process of a digital material. Thus, we suggest the process to design the most desirable learning materials should involve students and teachers, and include testing in classrooms. Through this, teachers would be more likely to accept the digital materials and integrate them in their teaching. This is supported by the award given to this material.

Finally, while this study appears to support the characteristics we proposed, more studies are needed to validate them and confirm the effectiveness of digital materials designed accordingly. We are engaged in further research of other conceptual subjects to refine the characteristics arising from this study and provide outcomes to extend the cognitive theory of multimedia learning from which the design principles applied here emerged.

## References

- Ainsworth, S. (1999). The functions of multiple representations. *Computers & Education*, 33(2), 131–152.
- Ainsworth, S. (2006). DeFT: A conceptual framework for considering learning with multiple representations. *Learning and Instruction*, 16(3), 183-198.
- Amiel, T., & Reeves, T. C. (2008). Design-Based Research and Educational Technology: Rethinking Technology and the Research Agenda. *Educational Technology & Society*, 11(4), 29-40.
- Anderson, T., & Shattuck, J. (2012). Design-Based Research A Decade of Progress in Education Research?. *Educational Researcher*, 41(1), 16-25.
- Bodemer, D., & Faust, U. (2006). External and mental referencing of multiple representations. *Computers in Human Behavior*, 22(1), 27-42.
- Brophy, J. E. (2001). *Subject specific instructional methods and activities*. Bingley, UK: Emerald Group Publishing
- Caglayan, G. (2014). Visualizing number sequences: Secondary preservice mathematics teachers' constructions of figurate numbers using magnetic color cubes. *The Journal of Mathematical Behavior*, 35, 110-128.
- Churchill, D. (2007). Towards a useful classification of learning objects. *Education Technology Research and Development*, 55, 479–497
- Churchill, D. (2011). Conceptual model learning objects and design recommendations for small screens. *Educational Technology & Society*, 14(1), 203–216.
- Churchill, D. (2013). Conceptual model design and learning uses. *Interactive Learning Environments*, 21(1), 54–67.



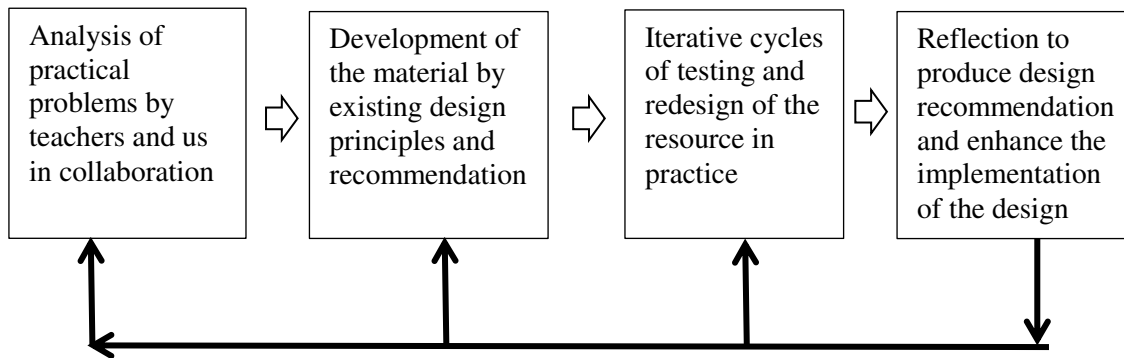
- Churchill, D. (2014). Presentation design for “conceptual model” learning objects. *British Journal of Education Technology*, 45 (1), 136-148
- Churchill, D., & Hedberg, G. (2008). Learning Object Design Considerations for Small-Screen Handheld Devices. *Computers & Education*, 50(3), 881-893.
- Cole, R., Mariani, J., Uszkoreit H, Varile, G., Zaenen, A., & Zampolli, A. (1998). *Survey of the state of the art in human language technology*. Cambridge, Massachusetts: Cambridge University Press.
- Collins, A. (1996). Design issues for learning environments. In S. Vosniadou, E. De Corte, & R. Glasser (Eds.), *International perspectives on the design of technology-supported learning environments*, Mahwah, NJ: Lawrence Erlbaum.
- Cuban, L. (2001). *Oversold and underused: Computers in the classroom*. Cambridge, Massachusetts: Harvard University Press.
- Curriculum Development Council, & Hong Kong Examinations and Assessment Authority. (2007). *The New Senior Secondary Mathematics Curriculum and Assessment Guide* (Secondary 4- 6). Hong Kong, China: The Government Printer.
- Design-Based Research Collective. (2003). Design-based research: An emerging paradigm for educational inquiry. *Educational Researcher*, 32(1), 5–8.
- Globerson, T, Salomon, G., & Perkins, D.N. (1991). Partners in cognition: Extending human intelligence with intelligent technologies. *Educational Researcher*, 20(3), 2-9.
- Gu, L., Huang, R., & Marton, F. (2004). Teaching with variation: A Chinese way of promoting effective mathematics learning. In L. Fan, N. Y. Wong, J. Cai, & S. Li (Eds.), *How Chinese Learn Mathematics: Perspectives from Insiders* (pp. 309–347). Singapore: World Scientific.

- Harskamp, E. G., Mayer, R. E., & Suhre, C. (2007). Does the modality principle for multimedia learning apply to science classrooms?. *Learning and Instruction, 17*(5), 465-477.
- Heid, M. K. (1995). *Algebra in a Technological World. Curriculum and Evaluation Standards for School Mathematics Addenda Series, Grades 9-12*. Reston, VA: National Council of Teachers of Mathematics
- Hoyles, C., Noss, R., Vahey, P., & Roschelle, J. (2013). Cornerstone mathematics: designing digital technology for teacher adaptation and scaling. *ZDM, 45*(7), 1057-1070.
- Hjalmarson, M. A., & Lesh, R. (2008). Design research: Engineering, systems, products, and processes for innovation. In *Handbook of International Research in Mathematics Education* (pp. 520-534). New York, NY: Routledge.
- Kent, T. W., & McNergney, R. F. (1999). *Will technology really change education?: From blackboard to Web*. Thousand Oaks, CA: Corwin Press.
- Ling, L. M., & Marton, F. (2011). Towards a science of the art of teaching: Using variation theory as a guiding principle of pedagogical design. *International Journal for Lesson and Learning Studies, 1*(1), 7-22.
- Marton, F., Tsui, A. B. M., Chik, P., Ko, P. Y., Lo, M. L., & Mok, I. A. C. (2004). *Classroom discourse and the space of learning*. Mahwah, NJ: Lawrence Erlbaum.
- Math. (2012, March 4) Quadratic equations calculator website. *The world of math online*. Retrieved from <http://www.math.com/students/calculators/source/quadratic.htm>
- Mayer, R. E. (2009). *Multimedia learning*. New York, NY: Cambridge Press.
- Mayer, R. E. (2014). Multimedia instruction. In *Handbook of Research on Educational Communications and Technology* (pp. 385-399). New York, NY: Springer.

- Mok, I. A. C. (2009). *Learning of algebra inspiration from students' understanding of the distributive law*. Hong Kong, China: Hong Kong Association for Mathematics Education.
- Mok, I. A. C., & Lopez-Real, F. (2006). A tale of two cities: A comparison of six teachers in Hong Kong and Shanghai. In D. Clarke, C. Keitel, & Y. Shimizu (Eds.), *Mathematics classrooms in 12 countries: The insiders' perspective* (pp. 237–246). Rotterdam, Netherlands: Sense Publishers B.V.
- Moreno, R., & Durán, R. (2004). Do Multiple Representations Need Explanations? The Role of Verbal Guidance and Individual Differences in Multimedia Mathematics Learning. *Journal of Educational Psychology, 96*(3), 492.
- Moreno, R., & Mayer, R. E. (1999). Cognitive principles of multimedia learning: The role of modality and contiguity. *Journal of Educational Psychology, 91*(2), 358.
- Moreno, R., & Mayer, R. E. (2000). A coherence effect in multimedia learning: The case for minimizing irrelevant sounds in the design of multimedia instructional messages. *Journal of Educational Psychology, 92*(1), 117.
- National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- Rabinowitz, M. (1988). On teaching cognitive strategies: The influence of accessibility of conceptual knowledge. *Contemporary Educational Psychology, 13*(3), 229-235.
- Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology, 93*(2), 346.

- Plass, J. L., Chun, D. M., Mayer, R. E., & Leutner, D. (1998). Supporting visual and verbal learning preferences in a second-language multimedia learning environment. *Journal of Educational Psychology, 90*(1), 25.
- Salomon, G., Perkins, D. N., & Globerson, T. (1991). Partners in cognition: Extending human intelligence with intelligent technologies. *Educational Researcher, 20*(3), 2-9.
- Seufert, T. (2003). Supporting coherence formation in learning from multiple representations. *Learning and Instruction, 13*(2), 227-237.
- Schneider, M., & Stern, E. (2010). The developmental relations between conceptual and procedural knowledge: A multimethod approach. *Developmental Psychology, 46*(1), 178.
- Schnotz, W. (2002). Commentary - towards an integrated view of learning from text and visual displays. *Educational Psychology Review, 14*(1), 101-120.
- Schnotz, W., & Bannert, M. (2003). Construction and interference in learning from multiple representation. *Learning and Instruction, 13*(2), 141-156.
- Schnotz, W., & Kürschner, C. (2008). External and internal representations in the acquisition and use of knowledge: visualization effects on mental model construction. *Instructional Science, 36*(3), 175-190.
- Tillema, H. H., & Knol, W. E. (1997). Promoting student teacher learning through conceptual change or direct instruction. *Teaching and Teacher Education, 13*(6), 579-595.
- Vahey, P., Knudsen, J., Rafanan, K., & Lara-Meloy, T. (2013). Curricular activity systems supporting the use of dynamic representations to foster students' deep understanding of mathematics. In C. Mouza & N. Lavigne (Eds.), *Emerging technologies for the classroom: A learning sciences perspective* (pp. 15–30). New York, NY: Springer

- Vamvakoussi, X., & Vosniadou, S. (2004). Understanding the structure of the set of rational numbers: A conceptual change approach. *Learning and Instruction, 14*(5), 453-467.
- Vosniadou, S., & Verschaffel, L. (2004). Extending the conceptual change approach to mathematics learning and teaching. *Learning and Instruction, 14*(5), 445-451.
- Wang, F., & Hannafin, M. J. (2005). Design-based research and technology-enhanced learning environments. *Educational Technology Research and Development, 53*(4), 5-23.
- Wong, N. Y. (2007). Hong Kong teachers' views of effective mathematics teaching and learning. *ZDM, 39*(4), 301-314.
- Yerushalmy, M. (1999). Making exploration visible: On software design and school algebra curriculum. *International Journal for Computers in Mathematical Learning, 4*(2), 169-189.
- Yerushalmy, M., & Chazan, D. (2008). Technology and curriculum design: The ordering of discontinuities in school algebra. *Handbook of International Research in Mathematics Education* (pp. 806-837). New York, NY: Routledge.



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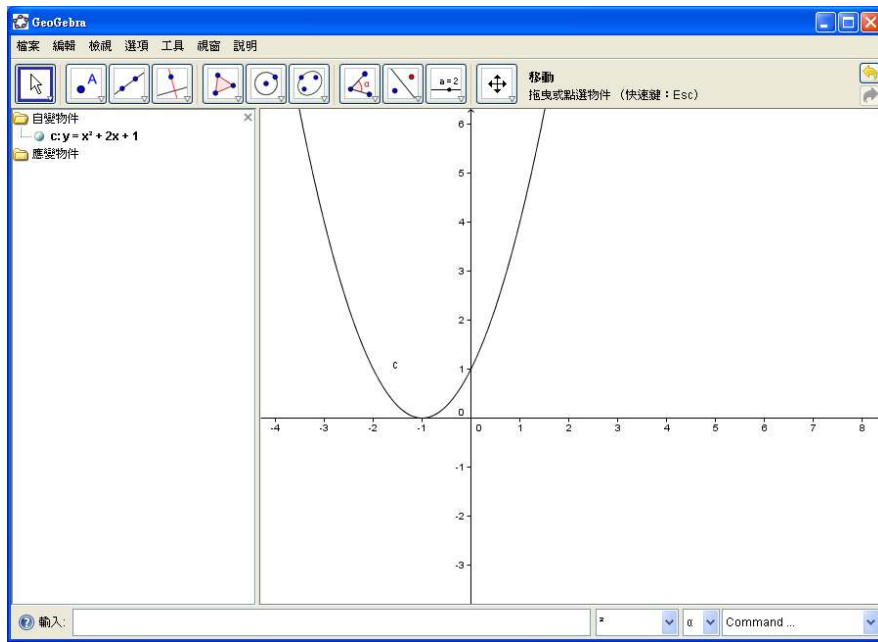
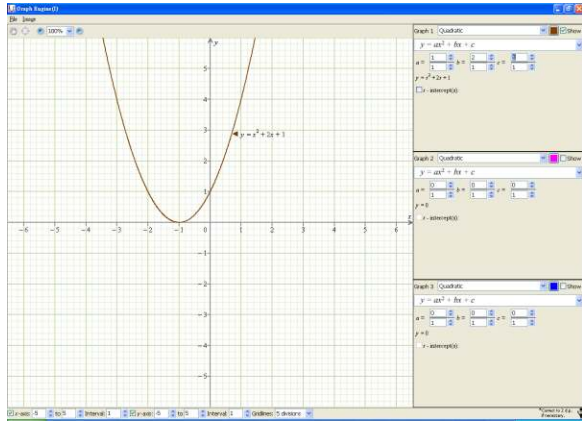
Pre-K Kindergarten First Grade Second Grade Third Grade Fourth Grade Fifth Grade Sixth Grade

**Quadratic Equation**

Enter the coefficients for the  $Ax^2 + Bx + C = 0$  equation and Quadratic Equation will output the solutions (if they are not imaginary).

Quadratic Equation		
$Ax^2 + Bx + C = 0$		
A = <input type="text" value="1"/>	:- Solve ->	<input type="text" value=""/>
B = <input type="text" value="2"/>		X1 = <input type="text" value="-1"/>
C = <input type="text" value="1"/>		X2 = <input type="text" value="-1"/>

If A=0, the equation is not quadratic.



The screenshot shows a software window titled "solve quadratic" with a menu bar (File, View, Control, Debug). The interface is divided into several sections:

- Graph:** A coordinate plane showing a blue parabola opening upwards. The vertex is at (-1, 0). The x-axis ranges from -9 to 9, and the y-axis ranges from -10 to 9.
- Control sliders:** A section titled "Coefficients a, b and c in quadratic equation" with three sliders:
  - a: slider set to 1
  - b: slider set to 2
  - c: slider set to 1
- General Form:**
  - $1X^2 + 2X + 1 = 0$
  - $1X^2 + 2X = -1$
  - $(X)(1X + 2) = -1$
- Descriptions:**
  - $\Delta = 0$
  - $X = -1.000$
  - $X = -1.000$
- Other Algebraic Forms:**
  - $(2(1)X + 2)^2 = (-2)^2 - (4)(1)(1)$
  - $(2X + 2)^2 = 0$
  - $(X + 1)^2 = 0$
  - $(X+1)(X+1) = 0$

Annotations with arrows point to these sections:

- "Images: graph" points to the coordinate plane.
- "Words: description" points to the "Descriptions" section.
- "Control sliders" points to the sliders for coefficients a, b, and c.
- "Words: different form of a quadratic equation" points to the "General Form" section.
- "Words: different solving methods" points to the "Other Algebraic Forms" section.



The screenshot shows a software window titled "draw\_decimal\_signal" with a menu bar (File, View, Control, Debug). The interface is divided into several sections:

- Graph:** A coordinate plane showing a blue parabola opening upwards. A red dot is placed at the vertex of the parabola, which is at the point (-1, 0). An arrow points from the text "The dot indicates the solution." to this red dot.
- Coefficients:** A section titled "Coefficients a, b and c in quadratic equation" with three input fields:
  - a: 1
  - b: 2
  - c: 1
- Standard Form:** The equation  $1X^2 + 2X + 1 = 0$  is displayed. Below it, the equation is shown in factored form:  $(X + 1)(X + 1) = 0$ . The numbers 1, 2, and 1 in the standard form are color-coded (orange, blue, green respectively). The corresponding numbers in the factored form are also color-coded to match: (X + 1)(X + 1). An arrow points from the text "The color matching indicates the link between the roots and the equation." to these color-coded numbers.
- Other Algebraic Form:** This section shows the derivation of the roots:
 
$$(2(1)X + 2)^2 = (-2)^2 - (4)(1)(1)$$

$$(2X + 2)^2 = 0$$

$$(X + 1)^2 = 0$$

$$(X + 1)(X + 1) = 0$$
- Descriptions:** A section titled "Descriptions" containing:
  - $\Delta = 0$  equal to zero (=0)
  - X = -1.00
  - X = -1.00
 An arrow points from the text "Description for discriminant" to the  $\Delta = 0$  line.

The screenshot shows a software window titled "draw\_decimal\_signal5" with a menu bar (File, View, Control, Debug). The interface is divided into several sections:

- Graph:** A coordinate plane showing a blue parabola opening upwards with its vertex at (-1, 0). The x-axis ranges from -9 to 9, and the y-axis from -10 to 9.
- Coefficients a, b and c in quadratic equation:** Three sliders are shown. 'a' is set to 1, 'b' to 2, and 'c' to 1.
- Algebraic Form:**
  - 1)  $1X^2 + 2X + 1 = 0$
  - 2)  $1X^2 + 2X = -1$
  - 3)  $(X + 1)(X + 1) = 0$
- Solving methods:**
  - 1) quadratic formula:  $X = \frac{-+2 \pm \sqrt{(+2)^2 - 4(1)(1)}}{2(1)}$
  - 2) taking square:  $(1X + \frac{+2}{2})^2 = \frac{0}{4}$
  - 3) factorization:  $(X + 1)(X + 1) = 0$
- Descriptions:**
  - $\Delta = 0$  equal to zero (=0)
  - $X = -1.00$
  - $X = -1.00$

The exact form of quadratic formula.

Names of the solving methods

**Coefficients a, b and c in quadratic equation**

a = 1  
b = 2  
c = 1

**Algebraic Form**

1)  $1x^2 + 2x + 1 = 0$   
2)  $1x^2 + 2x = -1$   
3)  $(x+1)(x+1) = 0$

**Descriptions**

$\Delta = 0$  equal to zero (=0)  
X = -1.00  
X = -1.00  
Sum of roots = -2  
Product of roots = 1  
Max / Min = 0  
Direction of opening = Upwards

**Solving methods**

1) quadratic formula  $x = \frac{-+2 \pm \sqrt{(+2)^2 - (4)(1)(1)}}{2(1)}$   
2) taking square  $(1x + \frac{+2}{2})^2 = \frac{0}{4}$   
3) factorization  $(x+1)(x+1) = 0$

The additional information

Slider for the manipulation of the graph

Adobe Flash Player 10

檔案(F) 檢視(V) 控制(C) 說明(H)

9  
8  
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-3  
-4  
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-7  
-8  
-9  
-10

-9 -8 -7 -6 -5 -4 -3 -2 -1 1 2 3 4 5 6 7 8 9 1

Slider for the manipulation of the graph

Coefficients a, b and c in quadratic equation

a

b

c

Algebraic Form

1)  $1X^2 + 2X + 1 = 0$

2)  $1X^2 + 2X = -1$

3)  $(X)(1X + 2) = -1$

Descriptions

$\Delta = 0$  equal to zero (=0)

$X = -1.00$

$X = -1.00$

Solving methods

1) quadratic formula  $X = \frac{-+2 \pm \sqrt{(+2)^2 - (4)(1)(1)}}{2(1)}$

2) taking square  $(1X + \frac{+2}{2})^2 = \frac{0}{4}$

3) factorization  $(X + 1)(X + 1) = 0$

*Table 1: Participants involved in the development and examining stages.*

Stage	Participants
Development (first year )	32 students (Low academic performance class)
	36 students (High academic performance class)
Examining (second year)	32 students (traditional group)
	34 students (intervention group)

Table 2: Student response and modification in the cycle

Version	Student response	Modification	Multimedia principle
2	I do not understand what the numbers in equations and methods mean.	a. Color-matching linked numbers	Signaling
	I don't know where the roots in the graph are	b. Show the dots at the interceptions of the graph	Signaling
	I cannot see the change of the graph when the values were changed. The ranges of the parameters are too large. The parabola disappears (the issue arising from the values of parameters). Where is the graph? (the issue arising from the large values of parameters).	c. Make the range of control parameters smaller	Coherence
	I do not know what the number (the value of discriminant) means.	d. Provide description of discriminant	N/A
	I don't like the color (background color) **	e. Change the background color to grey	N/A
3	We do not know what algebraic forms (solving methods) are.	f. Labeling the solving method	Pre-training method
	We cannot recognize the first algebraic form as the quadratic formula.	g. Display the exact form	N/A
4	It is too packed. The information (learning messages) is too much. I do not know what they (the additional information) are.	h. Reduce the amount of learning messages displayed	N/A
5	I want to control the graph.	i. Add the slider for the manipulation of the graph.	N/A

\*\* more than 60 students mentioned it.

*Table 3:* Descriptive data and ANCOVA results of the two post-tests

Variable	Group	N	Mean	SD	Adjusted mean	SE	F	$\eta^2$
Procedural knowledge	Intervention	34	29.65	6.94	28.14	1.07	5.28*	0.08
	Traditional	32	22.93	8.74	24.54	1.10		
Conceptual knowledge	Intervention	34	24.03	7.38	24.12	0.80	14.45***	0.19
	Traditional	32	19.88	7.49	19.78	0.82		

\*p<0.05, \*\* p<0.01, \*\*\*p<0.001

*Figure 1:* Redesign development cycle in this study

*Figure 2:* Quadratic equation calculator (Math, 2012)

*Figure 3.* Graphical drawer from a publisher

*Figure 4:* GeoGebra

*Figure 5:* Layout of Version 1 of the material

*Figure 6:* Layout of Version 2 of the material

*Figure 7:* Layout of Version 3 of the material

*Figure 8:* Layout of Version 4 of the material

*Figure 9:* Layout of Version 5 of the material