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# Exploring the Limits of Preclassical Mechanics

A Study of Conceptual Development in  
Early Modern Science: Free Fall  
and Compounded Motion in the Work  
of Descartes, Galileo, and Beeckman

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With 159 Illustrations



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