Exponential Pareto Distribution

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Abstract

In this paper we introduce a new distribution that is dependent on the Exponential and Pareto distribution and present some properties such that the moment generated function, mean, mode, median , variance , the r-th moment about the mean, the r-th moment about the origin, reliability , hazard functions , coefficients of variation ,of sekeness and of kurtosis. Finally, we estimate the parameter .

Keyword: Exponential distribution, Pearson distribution, moment estimation

Introduction

In the last years many researches based on the beta distribution and described as a new distribution like the distributions: Beta-Pareto which is introduced by Akinsete, Famoye and Lee (2008), Barreto-Souza, , Santos, and Cordeiro (2009) constructed Beta generalized exponential, Beta-half-Cauchy is presented by Cordeiro, and Lemonte (2011), Gastellares, Montenegero, and Gauss derived Beta Log–normal **,** while Morais ,Cordeiro ,and Audrey (2011) introduced Beta Generalized Logistic , Beta Burr III model for life time data is introduced by Gomes, da-Silva, Cordeiro, and Ortega, the Beta Burr III Model for Lifetime Data , Beta –hyperbolic Secant(BHS) by Mattheas ,David(2007), Beta Fre ́chet by Nadarajah, and Gupta(2004) by **[8]**, Beta normal distribution and its application Eugene, N., Lee, C. and Famoye, F. (2002) and Beta exponential by Nadarajah, S. and Kotz, S., 2004**.** Exponential Pearson coming from the question that is, why the above distribution used only Beta distribution? we propose another kind of this distribution as the form

$F(x) = \int_a^b f(x)dx$,where $b = \infty$ such that $\infty = \frac{1}{1-F^{\#}(x)}$

, where $F^{\#}(x)$ is any distribution like Normal , Exponential , or Gamma ……. atc. While the previous definition of such distribution is as $G(x) = \int_0^{F(x)} f(x)dx$, where $f(x)$ is the p.d.f. of Beta distribution , and $F(x)$ is the c.d.f. of any distribution.

1.The Probability Density Function of the EPD

The c.d.f. of the EPD is given by the form as

$$F_{e.p}(x) = \int_0^{\frac{1}{1-F^{\#}(x)}} f^{\Im}(x)\, dx$$

where $F^{\#}$is the pareto distribution, $F(X) = 1 - \left(\frac{p}{x}\right)^{\theta}$, and $f^{\Im}$ is the exponential distribution,

$F(X) = e^{-\lambda x}$.

so that

$$F_{e.p}(X;p,\lambda,,\theta) = \int_0^{\frac{1}{1-\left(1-\left(\frac{p}{x}\right)^{\theta}\right)}} \lambda e^{-\lambda x}\, dx$$, where p is a constant of the pareto distribution

$$F(X;p,\lambda,\theta) = \int_0^{\left(\frac{x}{p}\right)^{\theta}} \lambda e^{-\lambda x}\, dx = 1 - e^{-\lambda\left(\frac{x}{p}\right)^{\theta}}$$

So the c.d.f. of the exponential pareto distribution (**E.P.D**) is given by

$$\therefore F_{e.p}(X;p,\lambda,\theta) = 1 - e^{-\lambda\left(\frac{x}{p}\right)^{\theta}} \tag{1}$$

Also the p.d.f. of this distribution is given by :

$$f_{e.p}(X;p,\lambda,\theta) = \frac{dF_{e.p}(X;p,\lambda,\theta)}{dx} = \frac{\lambda\theta}{p}\left(\frac{x}{p}\right)^{\theta-1} e^{-\lambda\left(\frac{x}{p}\right)^{\theta}} I_{(0,\infty)}(x) \tag{2}$$

Which is similar to Weibull distribution that is given as

$$f(x,\theta,\beta) = s\beta x^{\beta-1} e^{-sx^{\beta}} I_{(0,\infty)}(x) \tag{3}$$

That is equal where $\theta = \beta, p = 1, s = \lambda$ .The plot of the p.d.f. and c.d.f. for the **EPD** is given in Figure1,2,…, 6.

2. Limit of the Probability Density and Distribution Functions

The limit of this distribution is given by the form as

$$\lim_{x\to 0} f_{e.p}(X;p,\lambda,\theta) = 0 \tag{4}$$

Because

$$\lim_{x\to 0} \frac{\lambda\theta}{p}\left(\frac{x}{p}\right)^{\theta-1} e^{-\lambda\left(\frac{x}{p}\right)^{\theta}} = \frac{\lambda\theta}{p}\lim_{x\to 0}\left(\frac{x}{p}\right)^{\theta-1} e^{-\lambda\left(\frac{x}{p}\right)^{\theta}} = \underbrace{\frac{\lambda\theta}{p}\left(\frac{0}{p}\right)^{\theta-1}}_{0}\ \underbrace{e^{-\lambda\left(\frac{0}{p}\right)^{\theta}}}_{1} = 0$$

Also

$$\lim_{x\to\infty} f(x;p,\lambda,\theta) = \lim_{x\to\infty} \frac{\lambda\theta}{p}\left(\frac{x}{p}\right)^{\theta-1} e^{-\lambda\left(\frac{x}{p}\right)^{\theta}} = 0\times\infty = 0 \tag{5}$$

Also since the c.d.f. of this distribution is:

$$F_{e.p}(X;p,\lambda,\theta) = 1 - e^{-\lambda\left(\frac{x}{p}\right)^{\theta}}$$

So

$$\lim_{x\to 0} F_{e.p}(X;p,\lambda,\theta) = 0 \tag{6}$$

Because $\lim_{x\to 0}\left[1 - e^{-\lambda\left(\frac{x}{p}\right)^{\theta}}\right] = 1 - 1 = 0$

Where $\lim_{x\to 0} e^{-\lambda\left(\frac{x}{p}\right)^{\theta}} = e^{-\lambda\left(\frac{0}{p}\right)^{\theta}} = 1$

Also

$$\lim_{x\to\infty} F_{e.p}(X;p,\lambda,\theta) = 1 \tag{7}$$

Since

$$\lim_{x\to\infty} F_{e.p}(X;p,\lambda,\theta) = \lim_{x\to\infty}\left[1 - e^{-\lambda\left(\frac{x}{p}\right)^{\theta}}\right] = 1 - e^{-\lambda\left(\frac{\infty}{p}\right)^{\theta}} = 1 - 0 = 1$$

Proposition 1:

The rth moment about the mean of this **EPD** is as follows :

$$\therefore E(X-\mu)^r = \sum_{j=0}^{r} C_j^r \left(\frac{p}{\sqrt[\theta]{\lambda}}\right)^j (-\mu)^{r-j}\Gamma\left(\frac{j}{\theta}+1\right),\ r = 1,2,3,\dots \tag{8}$$

And the r-moment about the origin is

$EX^r = \left(\frac{p}{\sqrt[\theta]{\lambda}}\right)^r \Gamma\left(\frac{r}{\theta}+1\right)$ with $r = 1,2,3,\ldots$ (9)

Proof :-

The r-th moment about the mean is given by

$$E(X-\mu)^r = \int_0^\infty (x-\mu)^r f_{e.p}(X;p,\lambda,\theta)dx$$

So

$$E(X-\mu)^r = \int_0^\infty (x-\mu)^r \frac{\lambda\theta}{p}\left(\frac{x}{p}\right)^{\theta-1} e^{-\lambda\left(\frac{x}{p}\right)^\theta} dx$$

Let

$u = \lambda\left(\frac{x}{p}\right)^\theta \Rightarrow du = \frac{\lambda\theta}{p}\left(\frac{x}{p}\right)^{\theta-1} dx$, $u(0) = 0$ and $u(\infty) = \infty$

Also $x = p\left(\sqrt[\theta]{\frac{u}{\lambda}}\right)$

So we put the above formulas in the integration to have

$$E(X-\mu)^r = \int_0^\infty \left[p\left(\sqrt[\theta]{\frac{u}{\lambda}}\right)-\mu\right]^r e^{-u}du$$

$$= \int_0^\infty \left[\frac{p}{\sqrt[\theta]{\lambda}}u^{\frac{1}{\theta}}-\mu\right]^r e^{-u}du$$

$$= \int_0^\infty \sum_{j=0}^r C_j^r \left(\frac{p}{\sqrt[\theta]{\lambda}}u^{\frac{1}{\theta}}\right)^j (-\mu)^{r-j} e^{-u}du$$

$$= \sum_{j=0}^r C_j^r \left(\frac{p}{\sqrt[\theta]{\lambda}}\right)^j (-\mu)^{r-j} \int_0^\infty u^{\frac{j}{\theta}} e^{-u}du$$

$$= \sum_{j=0}^r C_j^r \left(\frac{p}{\sqrt[\theta]{\lambda}}\right)^j (-\mu)^{r-j}\Gamma\left(\frac{j}{\theta}+1\right)$$

Thus the r-th moment about the meat is

$\therefore E(X-\mu)^r = \sum_{j=0}^r C_j^r \left(\frac{p}{\sqrt[\theta]{\lambda}}\right)^j (-\mu)^{r-j}\Gamma\left(\frac{j}{\theta}+1\right)$, r $= 1,2,3,\ldots$

Where $\mu = EX$, then if $\mu = 0$, thus the r-th moment about the origin is

$EX^r = \left(\frac{p}{\sqrt[\theta]{\lambda}}\right)^r \Gamma\left(\frac{r}{\theta}+1\right)$ $r = 1,2,3,\ldots$ ■

Proposition 2:

The coefficients of variance, of skeweness and of kurtosis are given,respectively, as

$CV. = \frac{\sqrt{\Gamma\left(\frac{2}{\theta}+1\right)-\left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^2}}{\frac{p}{\sqrt[\theta]{\lambda}}\Gamma\left(\frac{1}{\theta}+1\right)}$ (10)

$$CS. = \frac{\left\{\Gamma\left(\frac{3}{\theta}+1\right)-3\Gamma\left(\frac{1}{\theta}+1\right)\Gamma\left(\frac{2}{\theta}+1\right)+2\left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^3\right\}}{\left\{\Gamma\left(\frac{2}{\theta}+1\right)-\left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^2\right\}^{3/2}} \quad (11)$$

$$CK. = \frac{\left\{-3\left(\Gamma\left(\frac{1}{\theta}+1\right)\right)^4+6\left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^2\Gamma\left(\frac{2}{\theta}+1\right)-4\Gamma\left(\frac{1}{\theta}+1\right)\Gamma\left(\frac{3}{\theta}+1\right)+\Gamma\left(\frac{4}{\theta}+1\right)\right\}}{\left\{\Gamma\left(\frac{2}{\theta}+1\right)-\left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^2\right\}^2} - 3 \quad (12)$$

Proof

For $r = 2$ in equation (8) ,we have

$$E(X-\mu)^2 = \sigma^2 = Var(X) = E(X^2) - (EX)^2$$

$$= \left(\frac{p}{\theta\sqrt{\lambda}}\right)^2 \Gamma\left(\frac{2}{\theta}+1\right) - \left[\frac{p}{\theta\sqrt{\lambda}}\Gamma\left(\frac{1}{\theta}+1\right)\right]^2 = \left(\frac{p}{\theta\sqrt{\lambda}}\right)^2 \left\{\Gamma\left(\frac{2}{\theta}+1\right) - \left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^2\right\}$$

$$CV. = \frac{\sigma}{\mu} = \frac{\sqrt{E(X-\mu)^2}}{E(X)} = \frac{\sqrt{\left(\frac{p}{\theta\sqrt{\lambda}}\right)^2\left\{\Gamma\left(\frac{2}{\theta}+1\right)-\left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^2\right\}}}{\frac{p}{\theta\sqrt{\lambda}}\Gamma\left(\frac{1}{\theta}+1\right)} = \frac{\sqrt{\left\{\Gamma\left(\frac{2}{\theta}+1\right)-\left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^2\right\}}}{\Gamma\left(\frac{1}{\theta}+1\right)}$$

Now for $r = 3$,

$$E(X-\mu)^3 = E[(x^2 - 2x\mu + \mu^2)(x-\mu)]$$

$$= \left(\frac{p}{\theta\sqrt{\lambda}}\right)^3 \left\{\Gamma\left(\frac{3}{\theta}+1\right) - 3\Gamma\left(\frac{1}{\theta}+1\right)\Gamma\left(\frac{2}{\theta}+1\right) + 2\left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^3\right\}$$

$$\therefore E(X-\mu)^3 = \left(\frac{p}{\theta\sqrt{\lambda}}\right)^3 \left\{\Gamma\left(\frac{3}{\theta}+1\right) - 3\Gamma\left(\frac{1}{\theta}+1\right)\Gamma\left(\frac{2}{\theta}+1\right) + 2\left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^3\right\}$$

$$CS. = \frac{E(X-\mu)^3}{\sigma^3}\frac{\left\{\Gamma\left(\frac{3}{\theta}+1\right)-3\Gamma\left(\frac{1}{\theta}+1\right)\Gamma\left(\frac{2}{\theta}+1\right)+2\left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^3\right\}}{\left\{\Gamma\left(\frac{2}{\theta}+1\right)-\left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^2\right\}^{3/2}}$$

And if $r = 4$

$$E(X-\mu)^4 = E\left[\sum_{j=0}^{4} C_j^4\, x^j(-\mu)^{4-j}\right] = \sum_{j=0}^{4} C_j^4\, (-\mu)^{4-j}\ E\,(x^j)$$

$$= (-\mu)^4 + 4(-\mu)^3 \frac{p}{\theta\sqrt{\lambda}}\Gamma\left(\frac{1}{\theta}+1\right) + 6(-\mu)^2\left(\frac{p}{\theta\sqrt{\lambda}}\right)^2 \Gamma\left(\frac{2}{\theta}+1\right) - 4\mu\left(\frac{p}{\theta\sqrt{\lambda}}\right)^3 \Gamma\left(\frac{3}{\theta}+1\right) + \left(\frac{p}{\theta\sqrt{\lambda}}\right)^4 \Gamma\left(\frac{4}{\theta}+1\right)$$

$$= \left(\frac{p}{\theta\sqrt{\lambda}}\Gamma\left(\frac{1}{\theta}+1\right)\right)^4 - 4\left(\frac{p}{\theta\sqrt{\lambda}}\Gamma\left(\frac{1}{\theta}+1\right)\right)^3 \frac{p}{\theta\sqrt{\lambda}}\Gamma\left(\frac{1}{\theta}+1\right) + 6\left[\frac{p}{\theta\sqrt{\lambda}}\Gamma\left(\frac{1}{\theta}+1\right)\right]^2\left(\frac{p}{\theta\sqrt{\lambda}}\right)^2 \Gamma\left(\frac{2}{\theta}+1\right) - 4\frac{p}{\theta\sqrt{\lambda}}\Gamma\left(\frac{1}{\theta}+1\right)\left(\frac{p}{\theta\sqrt{\lambda}}\right)^3 \Gamma\left(\frac{3}{\theta}+1\right) + \left(\frac{p}{\theta\sqrt{\lambda}}\right)^4 \Gamma\left(\frac{4}{\theta}+1\right)$$

$$\therefore E(X-\mu)^4 = \left[\frac{p}{\theta\sqrt{\lambda}}\right]^4 \left\{-3\left(\Gamma\left(\frac{1}{\theta}+1\right)\right)^4 + 6\left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^2 \Gamma\left(\frac{2}{\theta}+1\right) - 4\Gamma\left(\frac{1}{\theta}+1\right)\Gamma\left(\frac{3}{\theta}+1\right) + \Gamma\left(\frac{4}{\theta}+1\right)\right\}$$

$$CK. = \frac{E(X-\mu)^4}{\sigma^4} - 3$$

Where σ is the standard deviation

$$= \frac{\left\{-3\left(\Gamma\left(\frac{1}{\theta}+1\right)\right)^4+6\left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^2\Gamma\left(\frac{2}{\theta}+1\right)-4\Gamma\left(\frac{1}{\theta}+1\right)\Gamma\left(\frac{3}{\theta}+1\right)+\Gamma\left(\frac{4}{\theta}+1\right)\right\}}{\left\{\Gamma\left(\frac{2}{\theta}+1\right)-\left[\Gamma\left(\frac{1}{\theta}+1\right)\right]^2\right\}^2} - 3 \qquad \blacksquare$$

The plot of the CS. ,CS. And CK. are given as follows in figure 7,8 and 9 respectively.

Proposition 3

The mode and median of the **EPD** is given as :-

$$mode = p\left(\frac{\theta-1}{\lambda\theta}\right)^{\frac{1}{\theta}} \quad (13)$$

$$median = p\left(\frac{\ln 2}{\lambda}\right)^{\frac{1}{\theta}} \quad (14)$$

Proof

The mode of the **EPD** is given as :

Mode = arg max (f(x))

So that

$$\ln f_{e.p}(X;p,\lambda,\theta) = \ln\left\{\frac{\lambda\theta}{p}\left(\frac{x}{p}\right)^{\theta-1} e^{-\lambda\left(\frac{x}{p}\right)^{\theta}}\right\}$$

$$= \ln\left(\frac{\lambda\theta}{p}\right) + (\theta-1)\ln\left(\frac{x}{p}\right) - \lambda\left(\frac{x}{p}\right)^{\theta}$$

$$\frac{d f_{e.p}(X;p,\lambda,\theta)}{dx} = (\theta-1)\frac{\left(\frac{1}{p}\right)}{\left(\frac{x}{p}\right)} - \frac{\lambda\theta}{p}\left(\frac{x}{p}\right)^{\theta-1}$$

$$\frac{d f_{e.p}(X;p,\lambda,\theta)}{dx} = 0 \Rightarrow (\theta-1)\frac{\left(\frac{1}{p}\right)}{\left(\frac{x}{p}\right)} - \frac{\lambda\theta}{p}\left(\frac{x}{p}\right)^{\theta-1} = 0$$

$$\frac{\theta-1}{x} = \frac{\lambda\theta x^{\theta-1}}{p^{\theta}} \Rightarrow \frac{p^{\theta}(\theta-1)}{\lambda\theta} = x^{\theta}$$

$$\therefore x = p\left(\frac{\theta-1}{\lambda\theta}\right)^{\frac{1}{\theta}}$$

The median of this **EPD** is given by

$$0.5 = \int_0^m f_{e.p}(x;p,\lambda,\theta)\,dx$$

$$0.5 = 1 - e^{-\lambda\left(\frac{m}{p}\right)^{\theta}} \Rightarrow 0.5 = e^{-\lambda\left(\frac{m}{p}\right)^{\theta}} \Rightarrow \ln 2 = \lambda\left(\frac{m}{p}\right)^{\theta}$$

$$\therefore m = p\left(\frac{\ln 2}{\lambda}\right)^{\frac{1}{\theta}}$$

■

Reliability and Hazard Functions

The reliability function and hazard function is given as:

$$R(x) = e^{-\lambda\left(\frac{x}{p}\right)^{\theta}} \quad (15)$$

$$h(x) = \frac{\lambda\theta}{p}\left(\frac{x}{p}\right)^{\theta-1} \quad (16)$$

Proof :-

The reliability function of the **EPD** is given by the form as :

$$R(x) = 1 - F(x) = 1 - \left[1 - e^{-\lambda\left(\frac{x}{p}\right)^{\theta}}\right] = e^{-\lambda\left(\frac{x}{p}\right)^{\theta}}$$

The hazard function is given by

$$h(x) = \frac{f(x)}{F(x)} = \frac{\frac{\lambda\theta}{p}\left(\frac{x}{p}\right)^{\theta-1} e^{-\lambda\left(\frac{x}{p}\right)^{\theta}}}{e^{-\lambda\left(\frac{x}{p}\right)^{\theta}}} = \frac{\lambda\theta}{p}\left(\frac{x}{p}\right)^{\theta-1}$$

The plot of the hazard function is given in **Figures 12,13 and 14**.

3. Estimate of the parameter

Using the MLE, we estimate the parameter of this distribution .

Since

$$f_{e.p}(x; p, \lambda, \theta) = \frac{\lambda\theta}{p}\left(\frac{x}{p}\right)^{\theta-1} e^{-\lambda\left(\frac{x}{p}\right)^{\theta}}$$

The likelihood function is given by :

$$L(x; p, \lambda, \theta) = \frac{\lambda^n \theta^n}{p^n}\left(\frac{\prod_{i=1}^{n} x_i}{p^n}\right)^{\theta-1} e^{-\lambda\frac{\sum_{i=1}^{n} x_i^{\theta}}{p^{\theta}}}$$

Then the natural logarithm of likelihood is

$$\ln L(x; p, \lambda, \theta) = n\ln\lambda + n\ln\theta - n\ln p + (\theta - 1)\ln\prod_{i=1}^{n} x_i - n(\theta - 1)\ln p - \lambda\frac{\sum_{i=1}^{n} x_i^{\theta}}{p^{\theta}}$$

$$\frac{\partial \ln L(x;p,\lambda,\theta)}{\partial\theta} = \frac{n}{\theta} + \ln\prod_{i=1}^{n} x_i - n\ln p - \lambda\sum_{i=1}^{n}\left(\frac{x_i}{p}\right)^{\theta}\ln\left(\frac{x_i}{p}\right)$$

$$\frac{\partial \ln L(x;p,\lambda,\theta)}{\partial\lambda} = \frac{n}{\lambda} - \frac{\sum_{i=1}^{n} x_i^{\theta}}{p^{\theta}}$$

$$\frac{\partial \ln L(x;p,\lambda,\theta)}{\partial p} = -\frac{n}{p} - \frac{n(\theta-1)}{p} + \frac{\lambda\theta\sum_{i=1}^{n} x_i^{\theta}}{p^{\theta+1}}$$

For $\frac{\partial \ln L(x;p,\lambda,\theta)}{\partial\theta} = \frac{\partial \ln L(x;p,\lambda,\theta)}{\partial\lambda} = \frac{\partial \ln L(x;p,\lambda,\theta)}{\partial p} = 0$

$$\frac{n}{\theta} + \ln\prod_{i=1}^{n} x_i - n\ln p - \lambda\sum_{i=1}^{n}\left(\frac{x_i}{p}\right)^{\theta}\ln\left(\frac{x_i}{p}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 1$$

To find the parameter theta using numerical method for solve above equation .

$$\frac{n}{\lambda} - \frac{\sum_{i=1}^{n} x_i^{\theta}}{p^{\theta}} = 0$$

Estimate the parameter lambda when

$$\hat{\lambda} = \frac{np^{\theta}}{\sum_{i=1}^{n} x_i^{\theta}} = \frac{n}{\sum_{i=1}^{n}\left(\frac{x_i}{p}\right)^{\theta}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \quad 2$$

$$-\frac{n}{p} - \frac{n(\theta-1)}{p} + \frac{\lambda\theta\sum_{i=1}^{n} x_i^{\theta}}{p^{\theta+1}} = 0 \Rightarrow -n - n\theta + n + \frac{\lambda\theta\sum_{i=1}^{n} x_i^{\theta}}{p^{\theta}} = 0 \Rightarrow n\theta = \frac{\lambda\theta\sum_{i=1}^{n} x_i^{\theta}}{p^{\theta}}$$

Estimate of parameter $\hat{p}$ when theta and lambda is know:

$$\hat{p} = \sqrt[\theta]{\frac{\lambda \sum_{i=1}^{n} x_i^{\theta}}{n}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \ 3$$

4. Conclusion

We can construct new distribution based on other the beta distribution like Exponentional Pareto with discussion some of its properties.

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3) the Bachelor's degree in Statistics: 1980/1981

4) the Bachelor's degree

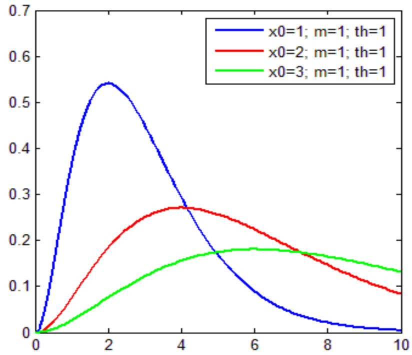

Figure 1: plot the p.d.f. of the **EPD** with the parameters $x0 = 1,2,3$; $m = 2; th = \theta = 1$

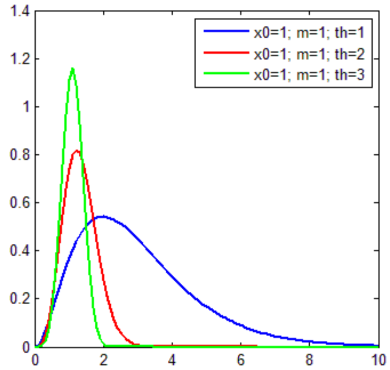

Figure 2: plot of the p.d.f. of the exponential pareto distribution with the parameter $x0 = 1; m = 1, th = \theta = 1,2,3$

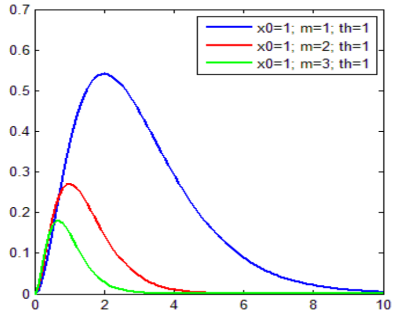

Figure 3: plot the p.d.f. of the **EPD** with the parameter $x0 = 1, m = 1,2,3; th = \theta = 1.$

Figure 1,2,3 show us that the shape of the p.d.f. of **EPD** based on fixed two parameters with change the other parameter, which is similar to Weibull distribution.

Also the c.d.f. of the **E.P.D** is given by :

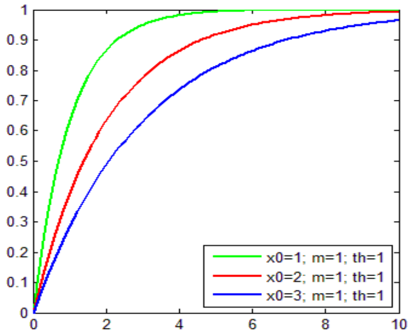

Figure 4:plot of c.d.f of the **EPD** with the parameter $x0 = p = 1,2,3$ and $m = \lambda = 1, th = \theta = 1$

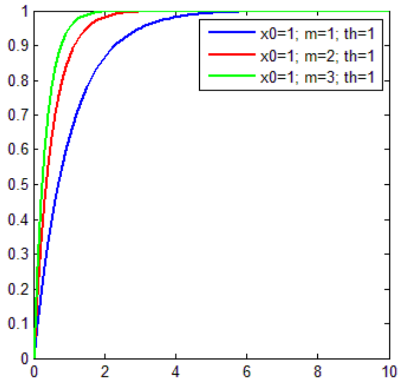

Figure 5: plot of c.d.f. of the **E.P.D** with the parameter $x0 = p = 1, m = \lambda = 1$ and $th = \theta = 1$

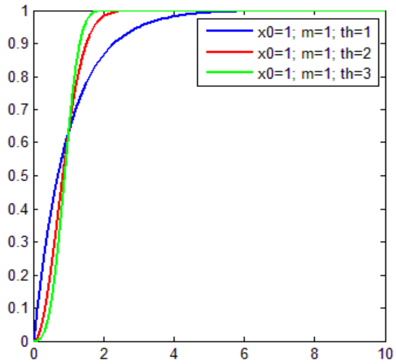

Figure 6: plot of c.d.f of the **E.P.D** with the parameter $x0 = p = 1, m = \lambda = 1, th = \theta = 1,2,3.$

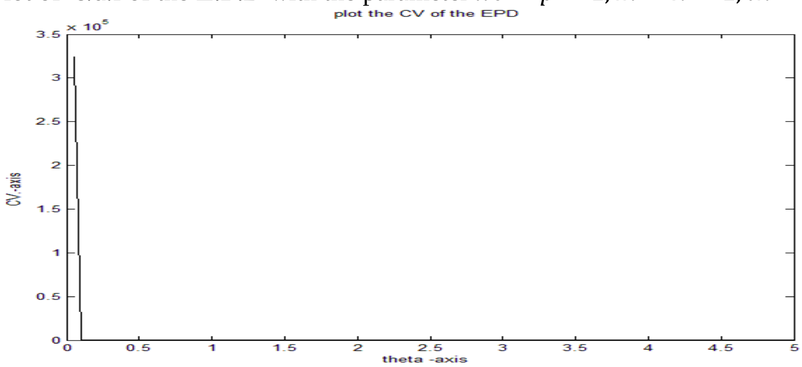

Figure 7: plot of **CV.** Of the EPD .

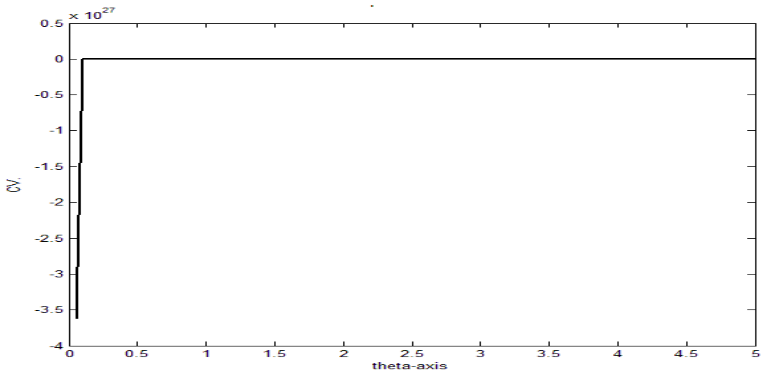

Figure 8: plot of CK with the parameter $\theta = (0,5)$

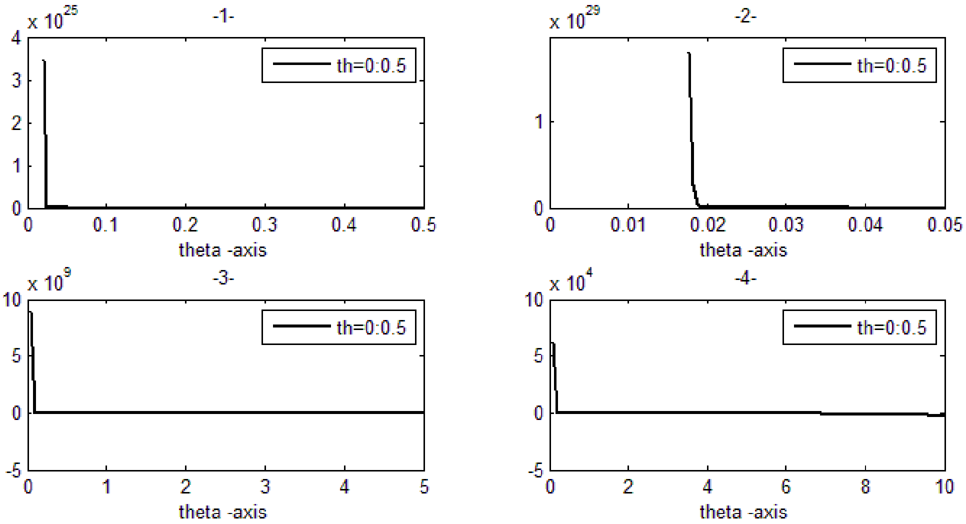

Figure 3.9: plot of CS of the E.P.D with the parameter just theta becose the CS donete dependent on another distribution in figure 1,….,4 the CS is dicresing but with different biginiging in figure -1- bigining with $3.4581 \times 10^{\wedge}25$ in subfigure 2 bigining with infinity ,in subfigure 3 bigine $8.8743 \times 10^{\wedge}9$ and figure -4- bigining with$6.2048 \times 10^{\wedge}4$.

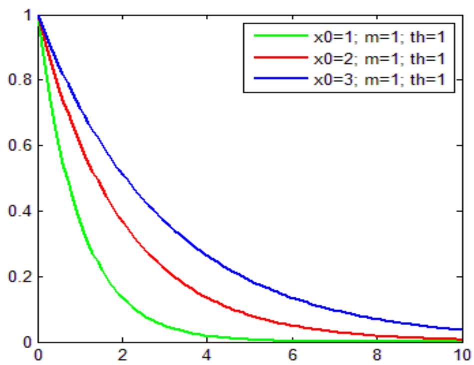

Figure 3.12: plot of R(x) with the change parameter $x0 = p$.

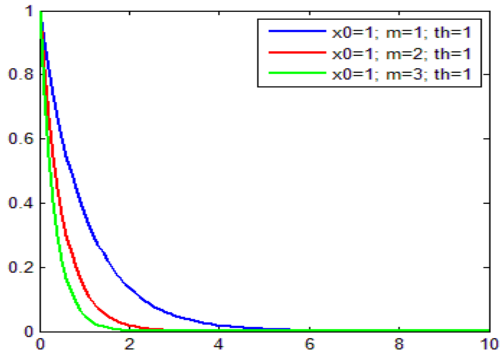

Figure 3.13: plot of R(x) with change parameter $m = \lambda$.

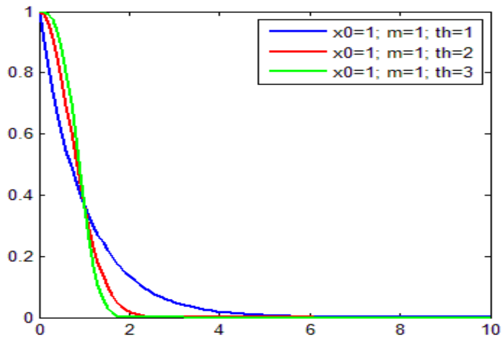

Figure 3.14: plot of R(x) with change parameter th $= \theta$

The Reliability funciton is dicreising with increasing of X that is take the same curve in the change of parameter lembda and p but in the other parameter that is different of the chape .

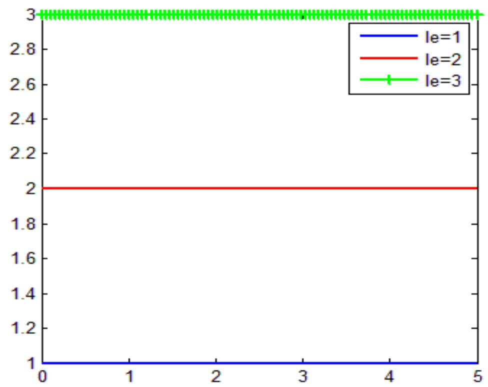

Figure 12: plot of hazard fucntion with fixed point $\theta = p = 1$ and change the parameter $le = \lambda = 1,2,3$

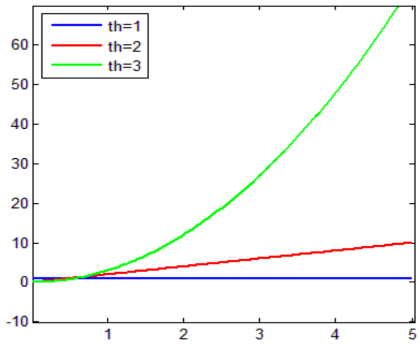

Figure 13 : plot of hazard function of the **EPD** with fixed lambda and p that is equal to one and changing the parameter theta.

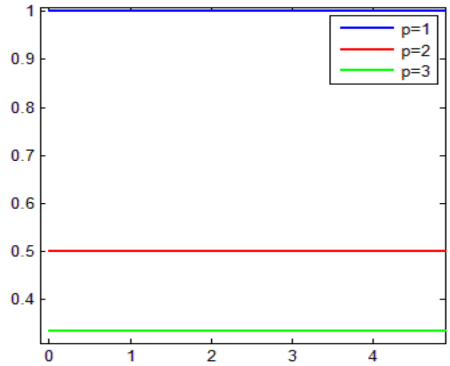

Figure 14: plot of hazard function with change parameter p and $\theta = \lambda = 1$.

As we show that the shape of the hazard function is not similar to the plot of the p.d.f with some of represent by $\frac{1}{R(x)}$ as weighted function, that it is taken the straight line in the **Figures 12 and 14** with the parameter $\theta = 1$ but when the parameter $\theta > 1$the shape of the h(x) is increasing in each figure but when the parameter $0 < \theta < 1$ the shape of the hazard function is decreeing with increasing of x .