Extended depth of field through wave-front coding

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We designed an optical–digital system that delivers near-diffraction-limited imaging performance with a large depth of field. This system is the standard incoherent optical system modified by a phase mask with digital processing of the resulting intermediate image. The phase mask alters or codes the received incoherent wave front in such a way that the point-spread function and the optical transfer function do not change appreciably as a function of misfocus. Focus-independent digital filtering of the intermediate image is used to produce a combined optical–digital system that has a nearly diffraction limited point-spread function. This high-resolution extended depth of field is obtained through the expense of an increased dynamic range of the incoherent system. We use both the ambiguity function and the stationary-phase method to design these phase masks.

Key words: Extended depth of field, extended depth of focus, wave-front coding.

1. Introduction

Extending the depth of field of incoherent optical systems has been an active research topic for many years. The majority of the literature on this topic has concerned methods of employing an optical power-absorbing apodizer, with possible $\pm\pi$ phase variations, on a standard incoherent optical system as a means to increase the depth of field.[1–5] These methods have all suffered from two significant deficiencies: a decrease of optical power at the image plane and a decrease of image resolution. A unique method of achieving an extended depth of field without an apodizer[6] was described in 1972. The major shortcoming of this method is that the focus must be varied during exposure.

We describe a novel method for extending the depth of field of incoherent optical systems that does not suffer from the significant deficiencies of earlier methods. Our method employs a phase mask to modify the incoherent optical system in such a way that the point-spread function (PSF) is insensitive to misfocus, while the optical transfer function (OTF) has no regions of zero values within its passband. The PSF of the modified optical system is not directly comparable to that produced from a diffraction-limited PSF. However, because the OTF has no regions of zeros, digital processing can be used to restore the sampled intermediate image. Further, because the OTF is insensitive to misfocus, the same digital processing restores the image for all values of misfocus. This combined optical–digital system produces a PSF that is comparable to that of the diffraction-limited PSF but over a far larger region of focus. We term the general process of modifying the incoherent optical system and the received incoherent wave front, by means of a phase mask, wave-front coding. By modifying only the phase of the received wave front, general wave-front coding techniques maximize optical power at the image plane.

When designing extended-depth-of-field systems, we make two main assumptions. The first is that the incoherent optical system is being modified by a rectangularly separable phase mask. This leads to a rectangularly separable PSF and OTF. Second, we assume that any resulting image will be an intermediate image. This intermediate image will require digital processing. This second assumption follows from our belief that the best performance is obtained by optimum preprocessing through optics, followed by optimum digital postprocessing.[7] These preprocessing and postprocessing stages are optimum in the sense that each is matched to the other in order to solve an interesting problem.

Our solution to extended-depth-of-field systems relies on the theory of the ambiguity function[8–10] and the stationary-phase[11–13] method. The ambiguity function can be used as a polar display of the OTF's of a rectangularly separable incoherent optical system as a function of misfocus.[10] Extended-depth-of-field systems can be noticed almost by inspection of their

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corresponding ambiguity functions. The stationary-phase method permits the design of phase masks whose corresponding ambiguity functions have desired extended-depth-of-field qualities.

In Section 2 we outline this method of designing phase masks for extended-depth-of-field systems. This design leads to the cubic-phase-modulation (cubic-pm) mask, which is introduced in Section 3. Through simulated measurement of the width of the combined optical–digital PSF, as well as through simulations of imaging a spoke target, we show that this method can produce an incoherent optical system with a large depth of field with near-diffraction-limited imaging performance. A complete mathematical derivation of the cubic-pm mask function, based on the stationary-phase method, can be found in Appendix A.

2. Design of Extended-Depth-of-Field Systems

Through the use of the ambiguity function and the stationary-phase method, phase masks for an extended-depth-of-field incoherent optical system are readily found. The ambiguity function is an analytical tool that permits us to observe and to design OTF's for all values of misfocus at the same time. The stationary-phase method provides the analytical flexibility needed to consider only phase masks in this design process.

Consider a one-dimensional unit-power phase mask or phase function, in normalized coordinates, such as

$$P(x) = \begin{cases} \dfrac{1}{\sqrt{2}} \exp[\, j\theta(x)] & \text{for } |x| \leq 1 \\ 0 & \text{otherwise} \end{cases}, \tag{1}$$

where $j = \sqrt{-1}$ and $\theta(x)$ is some unspecified nonlinear function. Knowledge of this phase function determines the PSF and the OTF of the incoherent optical system for all values of misfocus.[14,15] We assume that a two-dimensional rectangularly separable phase mask will be used in practice. The one-dimensional OTF, as a function of misfocus, is given by

$$H(u, \psi) = \int \{P(x + u/2)\exp[\, j(x + u/2)^2\psi]\} \times \{P^*(x - u/2)\exp[-j(x - u/2)^2\psi]\}\mathrm{d}x \tag{2}$$

with spatial frequency u and misfocus parameter ψ. The symbol * denotes the complex conjugate. The misfocus parameter, ψ, is dependent on the physical lens size as well as the focus state:

$$\psi = \frac{\pi L^2}{4\lambda}\left(\frac{1}{f} - \frac{1}{d_o} - \frac{1}{d_i}\right) = \frac{2\pi}{\lambda} W_{20} = kW_{20}, \tag{3}$$

where L is the one-dimensional length of the lens aperture and λ is the wavelength of the light. The distance d_o is measured between the object and the first principal plane of the lens, and d_i is the distance between the second principal plane and the image plane. The quantity f is the focal length of the lens. The wave number is given by k, and the traditional misfocus aberration constant is given by W_{20}. The traditional or Hopkins criterion for misfocus[16] is equivalent to $\psi \approx 1$.

The ambiguity function related to this general mask can be used as a polar display of the OTF for all values of misfocus.[10] The ambiguity function of the mask $P(x)$ is given by [8,9]

$$A(u, v) = \int P(x + u/2)P^*(x - u/2)\exp(\, j2\pi vx)\mathrm{d}x. \tag{4}$$

From Eqs. (2) and (4) the ambiguity function can be shown to be related to the OTF of the system generated by $P(x)$, as[10]

$$H(u, \psi) = A(u, u\psi/\pi). \tag{5}$$

Or, the projection of the point $(u, u\psi/\pi)$ of the ambiguity function onto the horizontal u axis yields the OTF for spatial frequency u and misfocus ψ. In this way the two-dimensional ambiguity function can be used to determine the one-dimensional OTF for all values of misfocus.

As an example of the utility of the ambiguity function approach to visualizing misfocus OTF's, consider the standard rectangularly separable incoherent optical system. Such a system is formed with a rectangular-pupil or mask function. Calculation of the magnitude of the ambiguity function of this one-dimensional rectangular function leads to the image shown in Fig. 1. In this image, regions of large power are given by dark shades. The majority of power in the ambiguity function of the rectangular aperture is concentrated along the $v = 0$ axis, which corresponds to the in-focus OTF. The radial line in this figure has a slope of $\pi/2$.

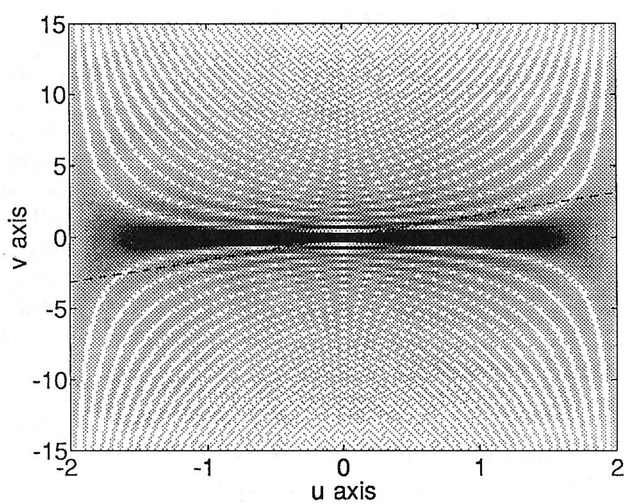

Fig. 1. Ambiguity function of a rectangular aperture. The radial line has a slope of $\pi/2$, corresponding to an OTF with misfocus $\psi = \pi^2/2$.

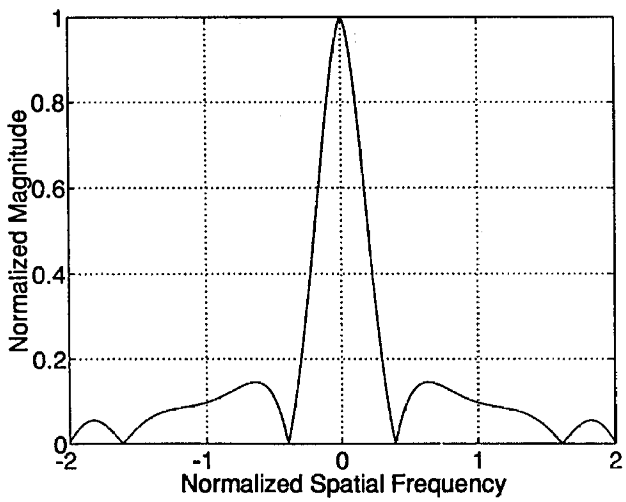

Fig. 2. Misfocus OTF of the standard optical system with misfocus parameter $\psi = \pi^2/2$.

Figure 2 shows a misfocused OTF related to the rectangular aperture, or standard optical system. The misfocus parameter for this OTF is $\psi = \pi^2/2$. From Eq. (5) the ambiguity function of Fig. 1 along the radial line with a slope of $\pi/2$ describes this OTF. By inspection of these two figures we can confirm this relationship between the OTF and the ambiguity function.

Extended-depth-of-field systems, or systems that are insensitive to changes of focus, have ambiguity functions that are not a function of the second parameter, here given as v. From Eq. (5), ambiguity functions that are independent of the second parameter, v, lead to OTF's that are invariant to misfocus ψ. In practice, extended-depth-of-field systems are those with ambiguity functions approximately independent of v over a relatively wide angular region about the u axis. From the ambiguity function of Fig. 1 we can immediately notice that a rectangular-pupil function does not describe an extended-depth-of-field system.

By careful selection of the nonlinear function $\theta(x)$ of the general mask given in Eq. (1) a phase function that produces an ambiguity function with the desired extended-depth-of-field characteristics can be found. We term this mask the cubic-phase-modulation (cubic-pm) mask. Section 3 describes this mask. See Appendix A for a derivation.

3. Cubic-Phase-Modulation Phase Mask

Modification of a standard incoherent optical system by a cubic-pm phase mask produces intermediate images that are insensitive to misfocus. Conceptually simple filtering techniques applied to these intermediate images form a complete system that images with high resolution and a large depth of field. The cubic-pm mask, in normalized coordinates, is given by

$$P(x) = \begin{cases} \dfrac{1}{\sqrt{2}} \exp(j\alpha x^3) & \text{for } |x| \le 1 \\ 0 & \text{otherwise} \end{cases} \qquad |\alpha| \gg 20, \tag{6}$$

where the constant α controls the phase deviation. The OTF of the incoherent system related to this function can be approximated as

$$H(u, \psi) \approx \begin{cases} \left(\dfrac{\pi}{12|\alpha u|}\right)^{1/2} \exp\left(j\dfrac{\alpha u^3}{4}\right) & |\alpha| \gg 20 \quad u \ne 0 \\ 1 & u = 0 \end{cases}. \tag{7}$$

See Appendix A for a derivation of this result. The approximation of the OTF is independent of misfocus. This can be inferred from the ambiguity function related to the cubic-pm mask, with $\alpha = 90$, shown in Fig. 3. The cubic-pm ambiguity function has uniform nonzero values distributed about the u axis. Radial lines through the origin of this ambiguity function have nearly the same values as a function of angle for a broad range of angles. Hence the cubic-pm mask should form an extended-depth-of-field incoherent optical system. Figure 4 shows a comparison between the stationary-phase approximation and the actual calculated OTF obtained by Eq. (2). The smooth curve in this figure is the approximation of the magnitude of the OTF; the other is the calculated magnitude of the OTF. For this figure the constant α of Eq. (6) was also selected as 90, and the misfocus parameter, ψ, was set to 15. The approximation holds for other values of misfocus as well as for the phase of the OTF. See Fig. 5. This figure is a plot of the magnitude of three misfocused OTF's related to the cubic-pm mask, with $\alpha = 90$. The misfocus values of the three OTF's are ψ equal to 0, 15, and 30. These OTF's are nearly constant with misfocus and have no zeros. This is what makes it possible to use one focus-independent digital filter to restore the intermediate image. Figure 6 shows the dramatic variation of the OTF's of the standard optical system with the same misfocus values. Also, the vertical scale in Fig. 6 is different from that of Fig. 5.

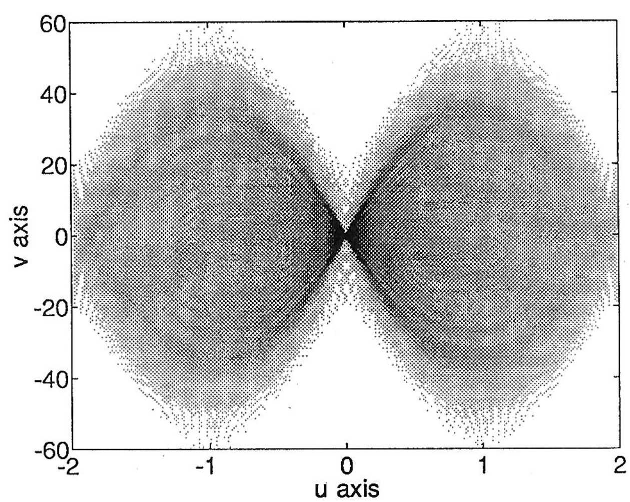

Fig. 3. Magnitude of the ambiguity function of the cubic-pm function with $\alpha = 90$. Radial lines through this function are insensitive to angle for a broad range of angles.

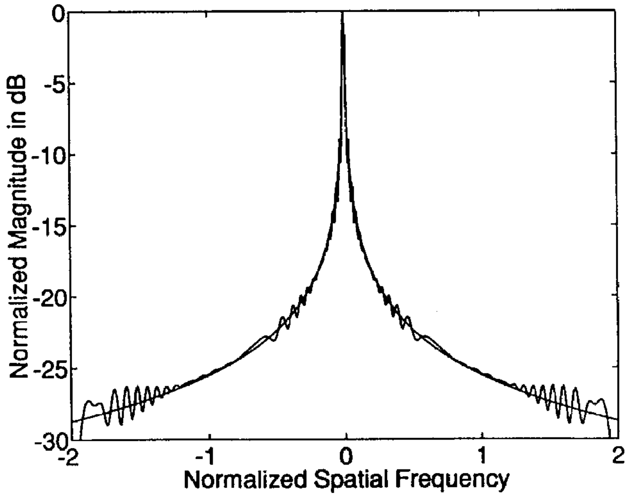

Fig. 4. Magnitude of the OTF of the cubic-pm system with $\alpha = 90$ and misfocus $\psi = 15$. The smooth curve is the stationary-phase approximation of the OTF. The other curve is the calculated OTF.

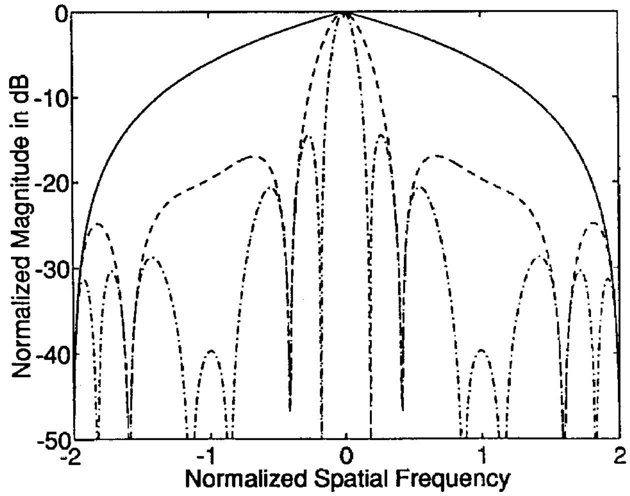

Fig. 6. Magnitude of OTF's from the standard optical system. The solid curve denotes the OTF with misfocus ψ of 0, the dashed curve is for ψ of 15, and the dashed–dotted curve is for ψ of 30. The vertical scale is different than that of Fig. 5.

In order to illustrate the performance of the optical–digital cubic-pm system for extended-depth-of-field imaging, we present two methods of comparison. These are the simulated measurement of the full width at half-maximum amplitude (FWHM) of the PSF as a function of misfocus and simulated imaging of a spoke target at different misfocus values. Comparison is made to the standard optical system in both cases.

Figure 7 illustrates the FWHM criterion applied to the standard optical system and the cubic-pm optical–digital system. The width of the standard system with no misfocus is normalized to unity. The width of the PSF from the cubic-pm system after focus-independent digital filtering is essentially constant out to the normalized misfocus value of $\psi = 30$. From Eq. (3) we can show that the normalized misfocus of $\psi = 30$ is nearly 30 times that of the Hopkins criterion for misfocus,[16] where $W_{20} = \lambda/6$. As expected, the width of the PSF of the standard system greatly increases with misfocus. The width of the unfiltered or intermediate PSF of the cubic-pm system would be much wider than that of the in-focus PSF fom the standard system.

Figure 8 illustrates simulated imaging of a spoke target with the cubic-pm optical–digital system, along with a comparison of images from the standard optical system. The cubic-pm optical–digital system includes both the formation of the incoherent intermediate image and the focus-independent digital filtering of this image. Without digital filtering the intermediate images would be unrecognizable. The digital filter used for this example was a simple inverse filter that, when combined with the intermediate OTF of approximation (7), resulted in a triangular system OTF, in a least-squares sense. The left column of

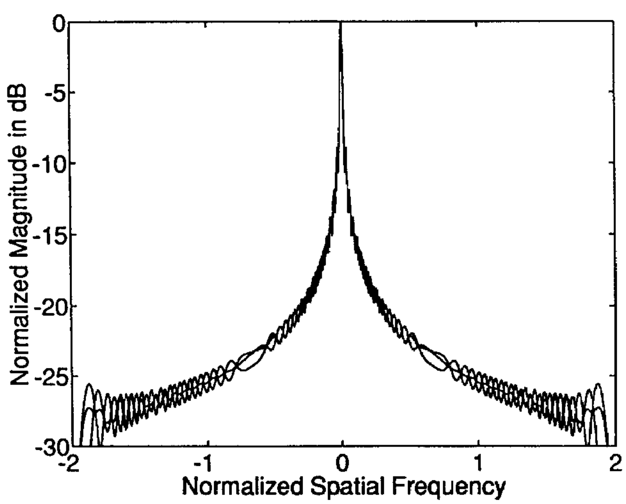

Fig. 5. Magnitude of the OTF's from the cubic-pm system with $\alpha = 90$ and misfocus ψ of 0, 15, and 30.

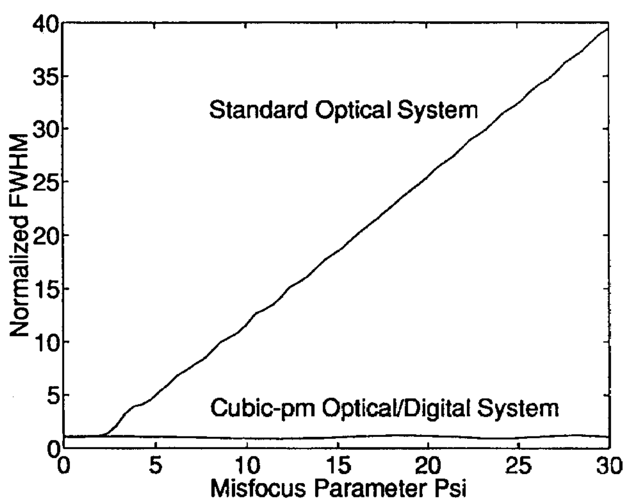

Fig. 7. Normalized full width at half-maximum amplitude (FWHM) of the PSF of the cubic-pm optical–digital system with comparison to that of the standard optical system.

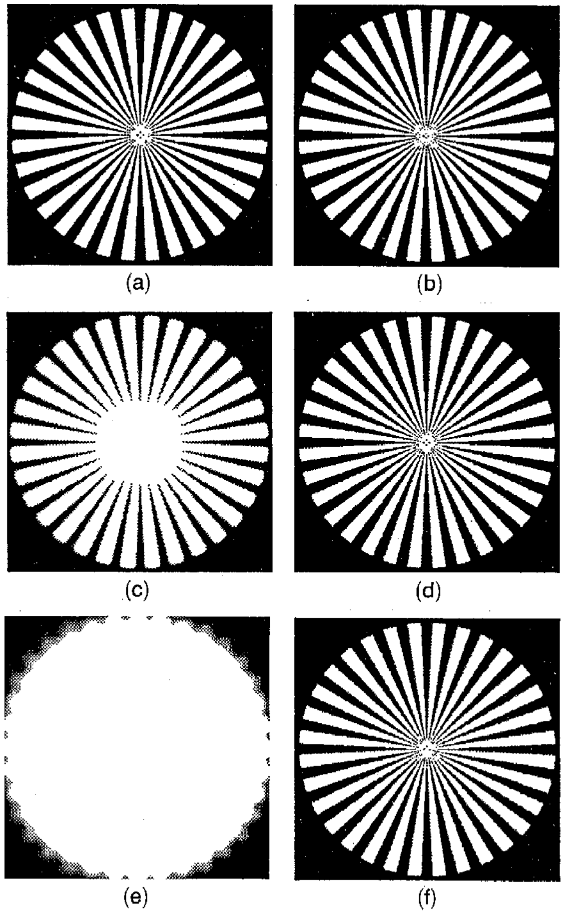

Fig. 8. Simulated images of a spoke target from a standard optical system (first column) and a cubic-pm optical–digital system (second column). (a), (b) (Geometrically in focus; (c), (d) mild misfocus; (e), (f) extreme misfocus.

this figure simulates imaging of a spoke target with a standard optical system under varying degrees of misfocus. The right column shows a simulation of the same imaging conditions with the cubic-pm optical–digital system. The term mild misfocus corresponds to $\psi = 5$ or ~ 5 times the Hopkins criterion for misfocus. The term extreme misfocus corresponds to $\psi = 30$ or ~ 30 times the Hopkins limit. The image of the spoke target from the standard system is severely degraded for even mild misfocus. The images from the cubic-pm system are essentially constant with misfocus, and the image quality is nearly the same as that from the standard system with no misfocus. Only a single digital filter is used for all values of misfocus with the cubic-pm system. No single filter can be applied to the misfocused images from the standard system to correct for the effects of misfocus.

These simulations assumed a noise-free optical–digital system. In practice, restoration of the intermediate image through digital filtering will alter the noise properties of the final image. As in other restorative schemes, a signal-to-noise-ratio or dynamic-range premium is required at the image. Different filtering schemes require different signal-to-noise-ratio premiums. The simple inverse filtering used here requires the largest premium. Other more complex filtering schemes would require less. The least-squares inverse filter used for the simulations of Figs. 7 and 8 has a transfer function, which is given in Fig. 9. From approximation (7) the phase of this filter is approximately cubic. The zero spatial frequency component of this filter is normalized to unity. With this filter the maximum magnification of any spatial frequency component is approximately 20 dB. An exaggerated estimate of the required signal-to-noise-ratio premium for this simple filter is then approximately 20 dB; required extra dynamic range would be approximately 3.5 bits.

An algorithm-independent measure of the increase in performance of the cubic-pm optical–digital system over the standard system can be found from the Fisher information of misfocus. Fisher information is a measure used to describe the information content of a given signal pertaining to a certain parameter.[17,18] For an ideal focus-invariant system the Fisher information of misfocus would be zero. In other words the ideal focus-invariant system would produce an image that contains no information pertaining to the focus state. Such an image would not be a function of misfocus. A system whose OTF has a large variation with misfocus cannot employ a single focus-independent digital filter to correct for misfocus. A focus-dependent digital filter can be used if the focus state is known *a priori*.

Assume that a general incoherent system is imaging a point object, or one with a flat spatial frequency spectrum. We can show that the Fisher information of misfocus from this assumed application is

$$J(\psi) = \int \left| \frac{\partial}{\partial \psi} H(u, \psi) \right|^2 \mathrm{d}u, \tag{8}$$

where $J(\psi)$ is the traditional notation for the Fisher information of the misfocus parameter ψ and $H(u, \psi)$ is the OTF.

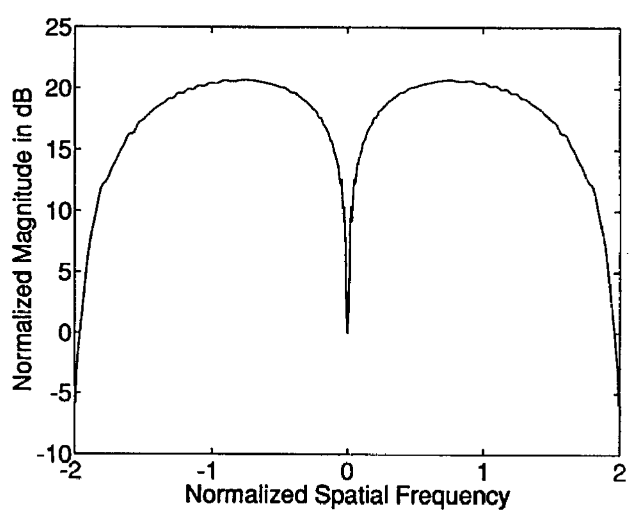

Fig. 9. Magnitude of the digital filter transfer function used in simulations of the cubic-pm optical–digital system.

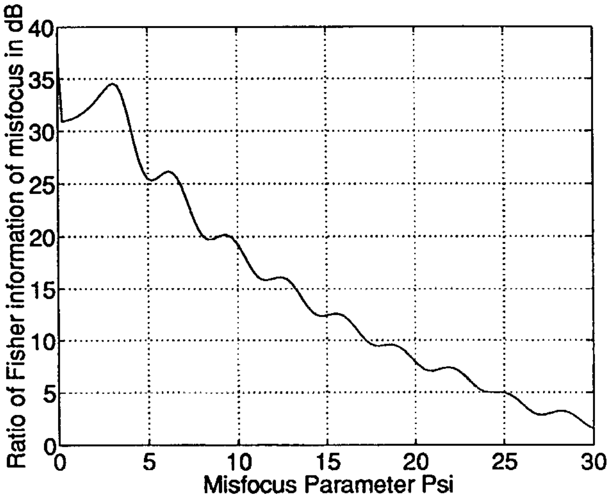

Fig. 10. Ratio of the Fisher information of misfocus, assuming a point object, of the standard optical system over the Fisher information of misfocus for the cubic-pm optical–digital system.

A ratio of the Fisher information related to the standard system over the Fisher information related to the cubic-pm system can be used as a measure of performance of the cubic-pm system. When this ratio is greater than unity, or 0 dB, the theoretical variation with misfocus of the standard system exceeds that of the cubic-pm system. Again, the cubic-pm system was chosen with the constant α from Eq. (6) equal to 90. This ratio of the Fisher information is given in Fig. 10. For example, the variation of the OTF of the standard system at misfocus of $\psi = 10$ is 20 dB larger than the variation of the OTF for the cubic-pm optical–digital system. Increasing the constant α of the cubic-pm system increases this difference in the variation of the OTF; decreasing α decreases the difference. The misfocus value at which the Fisher information of misfocus is equal for the standard and the cubic-pm system is monotonically related to the parameter α. Other methods of characterizing the performance of the cubic-pm optical–digital system are currently under investigation.

4. Conclusion

We have presented a method of modifying the phase of an incoherent wave front to produce an incoherent optical system with an extended depth of field. The general method of modifying the phase of an incoherent wave front is termed wave-front coding. When combined with digital filtering of intermediate images, this wave-front coded system delivers near-diffraction-limited performance with a large depth of field and with maximum optical power at the image plane.

Appendix A: Stationary-Phase Derivation of the Cubic-Phase-Modulation Optical Transfer Function

Through the stationary-phase method applied to the ambiguity function we can find an ambiguity function (and its associated phase function) that is independent of the second parameter, here called v. Such ambiguity functions define incoherent optical systems insensitive to misfocus.

The ambiguity function of the general phase mask or function, given in Eq. (1), is

$$A(u, v) = \frac{1}{2}\int_{-(1-|u|/2)}^{(1-|u|/2)} \exp[j\theta(x + u/2)] \times \exp[-j\theta(x - u/2)]\exp(j2\pi v x)\mathrm{d}x, \quad |u| \le 2. \tag{A1}$$

Let us assume that the nonlinear function $\theta(x)$ is some monomial

$$\theta(x) = \alpha x^{\gamma}, \qquad \gamma \neq \{0, 1\}, \qquad \alpha \neq 0. \tag{A2}$$

This form of $\theta(x)$ will result in a mathematically tractable solution. We can then rewrite Eq. (A1) as

$$\begin{aligned} A(u, v) &= \frac{1}{2}\int_{-(1-|u|/2)}^{(1-|u|/2)} \exp[j\alpha(x + u/2)^{\gamma}] \times \exp[-j\alpha(x - u/2)^{\gamma}]\exp(j2\pi v x)\mathrm{d}x, \quad |u| \le 2 \\ &= \frac{1}{2}\int_{-(1(|u|/2)}^{(1-|u|/2)} \exp[j\vartheta(x)]\exp(j2\pi v x)\mathrm{d}x, \quad |u| \le 2; \end{aligned} \tag{A3}$$

where

$$\vartheta(x) = \alpha[(x + u/2)^{\gamma} - (x - u/2)^{\gamma}]. \tag{A4}$$

If the phase term $\vartheta(x)$ varies fast enough, the above integral can be approximated through the stationary point of $[\vartheta(x) + 2\pi v x]$. This is the general idea behind the stationary-phase method, first described by Lord Kelvin. Contemporary researchers have applied the stationary-phase method to the ambiguity function. The stationary-phase approximation for $A(u, v)$ is given by[11–13]

$$\begin{aligned} A(u, v) &\approx \frac{1}{2}\left[\frac{2\pi}{|\vartheta''(x_i)|}\right]^{1/2} \exp[j\phi(v)] \\ &= \frac{1}{2}\left(\left|\frac{\partial x_i}{\partial v}\right|\right)^{1/2} \exp[j\phi(v)], \end{aligned} \tag{A5}$$

where x_i is the stationary point and

$$\phi(v) = 2\pi v x_i + \vartheta(x_i). \tag{A6}$$

From approximation (A5) the magnitude of the ambiguity function will be independent of its second parameter v when the second derivative of $\vartheta(x_i)$ with respect to x_i is independent of v, or, equivalently, when stationary point x_i is linear in v. In order to find stationary point x_i, we can begin by taking the derivative of Eq. (A6) and setting the result equal to zero. We obtain

$$(\partial/\partial x_i)[2\pi\nu x_i + \vartheta(x_i)] = 0,$$

$$2\pi\nu + \gamma\alpha(x_i + u/2)^{\gamma-1} - \gamma\alpha(x_i - u/2)^{\gamma-1} = 0. \quad \text{(A7)}$$

We can show that the solution for x_i above, as a function of γ, will be linear in ν if and only if $\gamma = 3$. The needed mask will then have a cubic phase profile. We term this cubic phase modulation, or a cubic-pm mask. This cubic-pm function has a stationary point of

$$x_i = \frac{-\pi\nu}{3\alpha u}, \qquad u \neq 0. \quad \text{(A8)}$$

The stationary-phase approximation to the magnitude of the ambiguity function of the cubic-pm system is then

$$|A(u, \nu)| \approx \frac{1}{2}\left(\left|\frac{\partial x_i}{\partial \nu}\right|\right)^{1/2} = \left(\frac{\pi}{12|\alpha u|}\right)^{1/2}, \quad u \neq 0. \quad \text{(A9)}$$

Using Eqs. (A4), (A6), and (A8), we can find that the phase term of this ambiguity function, $\phi(\nu)$, from Eq. (A6), is given by

$$\phi(\nu) \approx \frac{\alpha u^3}{4} - \frac{\pi^2\nu^2}{3\alpha u}, \quad u \neq 0. \quad \text{(A10)}$$

Combining both the magnitude and the phase approximations, we have

$$A(u, \nu) \approx \left(\frac{\pi}{12|\alpha u|}\right)^{1/2} \exp\left(j\,\frac{\alpha u^3}{4}\right) \exp\left(-j\,\frac{\pi^2\nu^2}{3\alpha u}\right), \quad u \neq 0. \quad \text{(A11)}$$

From Eq. (5) the resulting approximation to the OTF of the cubic-pm system is then given by

$$H(u, \psi) \approx \left(\frac{\pi}{12|\alpha u|}\right)^{1/2} \exp\left(j\,\frac{\alpha u^3}{4}\right) \exp\left(-j\,\frac{\psi^2 u}{3\alpha}\right), \quad u \neq 0. \quad \text{(A12)}$$

The magnitude of the approximate OTF above is independent of the misfocus parameter, ψ. The phase approximation contains two terms, however. One term is independent of misfocus, the other is not. Specifically, the second of the phase terms, $\exp(-j\psi^2 u/3\alpha)$, is a function of misfocus ψ and is a linear phase term in u. Such a term has the effect of merely shifting the location of the resulting point-spread function (PSF) with large misfocus. Fortunately, this term can be controlled through the constant α. Large values of α, from Eq. (A2), minimize the sensitivity of the cubic-pm system to movement of the PSF with misfocus. In practice this misfocus-dependent term can be effectively controlled so as to be negligible. The final approximation for the OTF is then

$$H(u, \psi) \approx \left(\frac{\pi}{12|\alpha u|}\right)^{1/2} \exp\left(j\,\frac{\alpha u^3}{4}\right) \text{ for large } |\alpha|, \quad u \neq 0. \quad \text{(A13)}$$

It is easy to show from Eq. (A1) that $H(0, \psi) = 1$. The stationary-phase approximations are valid for large space–bandwidth-product (SBP) functions.[8,9] The definition of a large SBP is usually accepted to be greater than 100. With the general mask of Eq. (1) the spatial extent is 2. The bandwidth of this general mask is given by its maximum instantaneous frequency. Because instantaneous frequency is the derivative of phase, the bandwidth of the general mask is

$$\mathrm{BW} = \max_x \frac{\partial}{\partial x}\,\theta(x) = \max_x \frac{\partial}{\partial x}\,\alpha x^3 = 3\alpha. \quad \text{(A14)}$$

The SBP of the cubic-pm mask must then satisfy

$$\mathrm{SBP} = 2(3\alpha) = 6\alpha \gg 100$$

or approximately

$$\alpha \gg 20.$$

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References

1. M. Mino and Y. Okano, "Improvement in the optical transfer function of a defocused optical system through the use of shaded apertures," Appl. Opt. **10,** 2219–2225 (1971).
2. J. Ojeda-Castañeda, P. Andres, and A. Diaz, "Annular apodizers for low sensitivity to defocus and to spherical aberation," Opt. Lett. **11,** 487–489 (1986).
3. J. Ojeda-Castañeda, R. Ramos, and A. Noyola-Isgleas, "High focal depth by apodization and digital restoration," Appl. Opt. **27,** 2583–2586 (1988).
4. J. Ojeda-Castañeda, E. Tepichin, and A. Diaz, "Arbitrary high focal depth with a quasioptimum real and positive transmittance apodizer," Appl. Opt. **28,** 2666–2670 (1989).
5. J. Ojeda-Castañeda and L. R. Berriel-Valdos, "Zone plate for arbitrarily high focal depth," Appl. Opt. **29,** 994–997 (1990).
6. G. Häusler, "A method to increase the depth of focus by two step image processing," Opt. Commun. **6,** 38–42 (1972).
7. W. T. Cathey, B. R. Frieden, W. T. Rhodes, and C. K. Rushforth, "Image gathering and processing for enhanced resolution," Opt. Soc. Am. A **1,** 241–250 (1984).
8. C. E. Cook and M. Bernfeld, *Radar Signals* (Academic, New York, 1967), Chap. 4, pp. 59–108.
9. A. W. Rihaczek, *Principles of High Resolution Radar* (McGraw-Hill, New York, 1969), Chaps. 4 and 5, pp. 87–157.
10. K. Brenner, A. Lohmann, and J. O. Casteñeda, "The ambiguity function as a polar display of the OFT," Opt. Commun. **44,** 323–326 (1983).
11. E. L. Key, E. N. Fowle, and R. D. Haggarty, "A method of designing signals of large time–bandwidth product," IRE Int. Conv. Rec. **4,** 146–155 (1961).

12. E. N. Fowle, "The design of FM pulse compression signals," IEEE Trans. Inf. Theory **IT-10,** 61–67 (1964).
13. M. Born and E. Wolf, *Principles of Optics* (Pergamon, New York, 1989), App. III, pp. 747–754.
14. J. W. Goodman, *Introduction to Fourier Optics* (McGraw-Hill, New York, 1968), Chap. 6, pp. 101–140.
15. W. T. Cathey, *Optical Information Processing and Holography* (Wiley, New York, 1974), Chap. 8, pp. 219–255.
16. H. Bartelt, J. O. Casteñeda, and E. S. Enrique, "Misfocus tolerance seen by simple inspection of the ambiguity function," Appl. Opt. **23,** 2693–2696 (1984).
17. T. M. Cover and J. A. Thomas, *Elements of Information Theory* (Wiley, New York, 1991), Chap. 12, 279–335.
18. L. L. Scharf, *Statistical Signal Processing* (Addison-Wesley, Reading, Mass. 1991), Chap, 6, pp. 209–276.