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# Extended EDAS Methods for Multi-Criteria Group Decision-Making Based on IV-CFSWAA and IV-CFSWGA Operators With Interval-Valued Complex Fuzzy Soft Information

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**ABSTRACT** In multi-criteria decision making (MCDM), it is difficult for decision-makers to give accurate evaluation values, and one-dimensional fuzzy set theory cannot capture periodic and seasonal information. The interval-valued complex fuzzy soft set (IV-CFSS) has these advantages in describing the information, which extends the evaluation value to the interval value, and extends the membership degree from the real number to the complex number, and it is not limited by the parameterization. MCDM methods can identify the best alternatives involving multiple criteria, and Evaluation based on Distance from Average Solution (EDAS) method is one of the MCDM methods, which simplifies the traditional decision-making process. In the real world, multi-criteria group decision making (MCGDM) is more realistic than MCDM. The purpose of this manuscript is to propose new EDAS method for MCGDM in interval-valued complex fuzzy soft environment. In the current work, the aggregation operators for IV-CFSS have not been applied to the ranking of alternatives in MCGDM problems. For this proposed work, the interval-valued complex fuzzy soft weighted arithmetic averaging (IV-CFSWAA) operator and the interval-valued complex fuzzy soft weighted geometric averaging (IV-CFSWGA) operator are proposed. Then, the related properties of these operators are studied. Based on these two operators, the interval-valued complex fuzzy soft EDAS methods in MCGDM environment are proposed. Finally, an example of economic problem is provided to test the feasibility and applicability of the proposed methods.

**INDEX TERMS** Interval-valued complex fuzzy soft set, interval-valued complex fuzzy soft weighted arithmetic averaging operator, interval-valued complex fuzzy soft weighted geometric averaging operator, evaluation based on distance from average solution, multi-criteria group decision-making.

## I. INTRODUCTION

The concept of interval-valued complex fuzzy soft set (IV-CFSS) is proposed by Selvachandran [1] in 2017, which is a combination of interval-valued fuzzy set (IVFS) [2], soft set (SS) [3], and complex fuzzy set (CFS) [4]. IVFS extends the membership value from the determined value to the interval value, which avoids the influence of personal preference in the given evaluation, and the described information is more reliable. SS overcomes the limitation of parameterization of fuzzy sets and has outstanding advantages in multi-parameter information description. CFS extends the membership value from the real number to the complex number, increasing the phase term that captures the periodicity and seasonality of the information. This makes it easy to describe two-dimensional data, and it can more rationally describe some special scenes in the real world, such as economic applications, physics analysis, and so on. IV-CFSS has the advantages of the above three, so the application prospect is broad. Subsequently, Selvachandran and Singh [5] proposed the application

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of IV-CFSS. Fan *et al.* [6] defined the distance measures of interval-valued complex fuzzy soft sets (IV-CFSSs) and gave their application.

The problem of aggregation operator is a research hotspot in decision science. In terms of fuzzy set theory, Xu and Yager [7] proposed some geometric aggregation operators based on intuitionistic fuzzy sets. Atanassov [8] studied operators over interval valued intuitionistic fuzzy sets. Xia and Xu [9] proposed hesitant fuzzy information aggregation in decision making. Yu *et al.* [10] provided dual hesitant fuzzy aggregation operators. Liu *et al.* [11] gave generalized Pythagorean fuzzy aggregation operators and applications in decision making. Garg and Nancy [12] proposed some hybrid weighted aggregation operators under neutrosophic set environment and applied them to multi-criteria decision-making (MCDM). In terms of SS theory, Garg and Arora [13] studied Bonferroni mean aggregation operators under intuitionistic fuzzy soft set environment and applied them to decision-making. Arora and Garg [14] defined the prioritized averaging/geometric aggregation operators under the intuitionistic fuzzy soft environment.. Arora and Garg [15] provided robust aggregation operators for MCDM with intuitionistic fuzzy soft environment and applied it to decision-making. Garg and Arora [16] proposed the maclaurin symmetric mean aggregation operators based on t-norm operations for the dual hesitant fuzzy soft set. Garg and Arora [17] introduced dual hesitant fuzzy soft aggregation operators and applied them in decision-making. Jana and Pal [18] proposed a robust single-valued neutrosophic soft aggregation operators in MCDM. In terms of CFS theory, Bi *et al.* [19] defined complex fuzzy arithmetic aggregation operators. Subsequently, Bi *et al.* [20] defined complex fuzzy geometric aggregation operators. Garg and Rani [21] presented some generalized complex intuitionistic fuzzy aggregation operators and applied them to MCDM process. Rani and Garg [22] provided complex intuitionistic fuzzy power aggregation operators and their applications in MCDM. Garg and Rani [23] presented complex interval-valued intuitionistic fuzzy sets and their aggregation operators.

The method of Evaluation based on Distance from Average Solution (EDAS) [24] is a new MCDM method proposed by Keshavarz Ghorabaee et al. in 2015. The core idea of the EDAS method is to use average solution to evaluate alternatives without computing positive and negative ideal solutions. In this method, two measures called PDA (positive distance from average) and NDA (negative distance from average) are used, and the evaluation is based on the higher PDA value and the lower NDA value. Kahraman *et al.* [25] extended the EDAS method to intuitionistic fuzzy set. Peng *et al.* [26] proposed interval-valued fuzzy soft decision making methods based on Multi-Attributive Border Approximation area Comparison (MABAC), similarity measure and EDAS. Peng and Chong [27] presented algorithms for neutrosophic soft decision making based on EDAS and new similarity measure. Liang *et al.* [28] developed an integrated EDAS Elimination and Choice Translating Reality (EDAS-ELECTRE) method with picture fuzzy information for cleaner production evaluation in gold mines. Karasan and Kahraman [29] presented a novel interval-valued neutrosophic EDAS method. Galina *et al.* [30] developed the decision analysis with classic and fuzzy EDAS modifications. Feng *et al.* [31] introduced EDAS method for extended hesitant fuzzy linguistic MCDM.

In terms of group decision making (GDM) method. Li *et al.* [32] introduced a GDM model for integrating heterogeneous information. Song and Li [33] presented a large-scale GDM with incomplete multi-granular probabilistic linguistic term sets and its application in sustainable supplier selection. Song and Li [34] proposed handling GDM model with incomplete hesitant fuzzy preference relations and its application in medical decision. Song and Li [35] proposed consensus constructing in large-scale GDM with multi-granular probabilistic 2-tuple fuzzy linguistic preference relations. Liu et al. [36] proposed the multi-attribute group decision making based on Intuitionistic uncertain linguistic Hamy mean operators with linguistic scale functions and its application to health-care waste treatment technology selection. Lin *et al.* [37] presented GDM model with hesitant multiplicative preference relations based on regression method and feedback mechanism.

The EDAS method differs from the traditional Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [38] and Vlsekriterijumska Optimizacija I Kompromisno Resenje (VIKOR) [39] methods which require the calculation of positive and negative ideal solutions to select the best alternative. The EDAS method only needs to consider the distance from the average solution to obtain the best alternative. It is greatly simplified in the calculation procedure, and the results obtained are consistent with the results calculated by the above method. Multi-criteria group decision-making (MCGDM) is an unavoidable problem in the decision-making field. To extend the EDAS method to MCGDM, it is necessary to use aggregation operator. IV-CFSS has many advantages in describing information, which can overcome the influence of subjective preference of experts and capture the periodic characteristics of information. So this paper extends the EDAS method to MCGDM environment with interval-valued complex fuzzy soft information. In this paper, interval-valued complex fuzzy soft weighted arithmetic averaging (IV-CFSWAA) operator and interval-valued complex fuzzy soft weighted geometric averaging (IV-CFSWGA) operator are proposed. Then, the related properties of these operators are studied. Based on the above aggregation operator for IV-CFSS, the interval-valued complex fuzzy soft EDAS methods in MCGDM environment are proposed. In these methods, we use the generalized entropy for IV-CFSS to determine the weight vector of parameters for the case where the weight vector of experts is known and the weight vector of parameters is unknown.

The rest of the paper is organized as follows. Section II recalls the basic concepts of interval-valued fuzzy set, interval-valued fuzzy soft set, interval-valued complex fuzzy set, and IV-CFSS. Section III defines the IV-CFSWAA operator and the IV-CFSWGA operator, and studies the related properties of these operators. Section IV proposes EDAS methods built on IV-CFSWAA operator and the IV-CFSWGA operator in MCGDM environment. Section V gives an illustrative example to show the validity of the interval-valued complex fuzzy soft EDAS methods. Section VI compares the proposed methods with the existing methods. Finally, in Section VII, conclusions and future work are stated.

## II. PRELIMINARIES

In this section, some basic concepts and operations of IV-CFSS are reviewed.

*Definition 1 [2]:* Let $U$ be an initial universal set. $A$ is an interval-valued fuzzy set (IVFS) over $U$, which is defined as

$$A=\left\{\left(x,\left(\mu_A^-(x),\mu_A^+(x)\right)\right):x\in U\right\}$$

where $\mu_A^-(x),\mu_A^+(x)\in[0,1]$ and $0\leq\mu_A^-(x)+\mu_A^+(x)\leq 1$.

*Definition 2 [40]:* Let $P(U)$ be the power set over initial universal set $U$, $E$ be the set of parameters, and $A\subset E$. $(F,A)$ is called an interval-valued fuzzy soft set (IVFSS) over $U$, where $F$ is a mapping given by $F:A\to P(U)$. Mathematically, IVFSS can be defined as follows:

$$(F,A)=\left\{\left(x,\left(\mu_{F_a}^-(x),\mu_{F_a}^+(x)\right):x\in U,a\in A\right)\right\}$$

where $\mu_{F_a}^-(x),\mu_{F_a}^+(x)\in[0,1]$ and $0\leq\mu_{F_a}^-(x)+\mu_{F_a}^+(x)\leq 1$.

*Definition 3 [41]:* Let $U$ be an initial universal set. $A$ is an interval-valued complex fuzzy set (IVCFS) over $U$, which is defined as

$$\begin{aligned}A&=\left\{\left(x,\left(\mu_A^-(x),\mu_A^+(x)\right)\right):x\in U\right\}\\&=\left\{\left(x,\left[r_A^-(x),r_A^+(x)\right]\cdot e^{i\left[\omega_A^-(x),\omega_A^+(x)\right]}\right):x\in U\right\}\end{aligned}$$

where $\mu_A^-:U\to\{a:a\in C,|a|\leq 1\}$ and $\mu_A^+:U\to\{a:a\in C,|a|\leq 1\}$ are the mappings of lower and upper bounds of complex membership functions, respectively. The amplitude terms $r_A^-(x),r_A^+(x)$ and the phase terms $\omega_A^-(x),\omega_A^+(x)$ satisfy the conditions $r_A^-(x),r_A^+(x)\in[0,1]$ and $\omega_A^-(x),\omega_A^+(x)\in[0,2\pi]$, respectively, with $i=\sqrt{-1}$.

*Definition 4 [1]:* Let $F(U)$ be a power set over initial universe set $U$, $E$ be a set of parameters, and $(F,A)$ is an IV-CFSS over $U$, where $F$ is a mapping given by $F:A\to F(U)$. Mathematically, IV-CFSS can be defined as follows:

$$\begin{aligned}F_{a_i}(x_j)&=\left\{\left(x,\left(\mu_{F(a_j)}^-(x_i),\mu_{F(a_j)}^+(x_i)\right)\right):x\in U\right\}\\&=\left\{\left(x,\left[r_{F(a_j)}^-(x_i),r_{F(a_j)}^+(x_i)\right]\cdot e^{i\left[\omega_{F(a_j)}^-(x_i),\omega_{F(a_j)}^+(x_i)\right]}\right):x\in U\right\},\end{aligned}$$

where $i=\sqrt{-1}$, $r_{F(a_j)}^-(x_i),r_{F(a_j)}^+(x_i)\in[0,1]$, $\omega_{F(a_j)}^-(x_i),\omega_{F(a_j)}^+(x_i)\in[0,2\pi]$.

For convenience, we define $\alpha=\left[r_{F(a_j)}^-(x_i),r_{F(a_j)}^+(x_i)\right]\cdot e^{i[\omega_{F(a_j)}^-(x_i),\omega_{F(a_j)}^+(x_i)]}$ as IV-CFSN, denoted by $\alpha_{ij}=\left[r_{ij}^-,r_{ij}^+\right]\cdot e^{i\left[\omega_{ij}^-,\omega_{ij}^+\right]}$.

*Definition 5:* For IV-CFSN $\alpha_{ij}=\left[r_{ij}^-,r_{ij}^+\right]\cdot e^{i\left[\omega_{ij}^-,\omega_{ij}^+\right]}$, the score function of $\alpha_{ij}$is defined as

$$S\left(\alpha_{ij}\right)=r_{ij}^-+r_{ij}^+-1+\frac{1}{2\pi}\left(\omega_{ij}^-+\omega_{ij}^+-2\pi\right)\tag{1}$$

and the accuracy function of $\alpha_{ij}$ is defined as

$$H\left(\alpha_{ij}\right)=r_{ij}^--r_{ij}^++1+\frac{1}{2\pi}\left(\omega_{ij}^--\omega_{ij}^+-2\pi\right)\tag{2}$$

Based on these two functions, $\alpha_{ij}$ and $\beta_{ij}$ are two IV-CFSNs, then the comparison between $\alpha_{ij}$ and $\beta_{ij}$ is stated as

1) If $S\left(\alpha_{ij}\right)>S\left(\beta_{ij}\right)$, then $\alpha_{ij}>\beta_{ij}$;
2) If $S\left(\alpha_{ij}\right)=S\left(\beta_{ij}\right)$, then
   2a) If $H\left(\alpha_{ij}\right)>H\left(\beta_{ij}\right)$, then $\alpha_{ij}>\beta_{ij}$;
   2b) If $H\left(\alpha_{ij}\right)=H\left(\beta_{ij}\right)$, then $\alpha_{ij}=\beta_{ij}$.

*Definition 6:* Let $U=\{x_1,x_2,\ldots,x_m\}$ be a universal set, $E=\{e_1,e_2,\ldots,e_n\}$ be a set of parameters. $(F,E)=\left\{F\left(e_j\right)|j=1,2,\ldots,n\right\}$ and $(G,E)=\left\{G\left(e_j\right)|j=1,2,\ldots,n\right\}$ are two IV-CFSSs. The generalized entropy for $(F,E)$ is defined in (3), as shown at the bottom of this page.

## III. INTERVAL-VALUED COMPLEX FUZZY SOFT WEIGHTED AVERAGING OPERATOR

In this section, we proposed the IV-CFSWAA operator and IV-CFSWGA operator for IV-CFSS.

### A. INTERVAL-VALUED COMPLEX FUZZY SOFT WEIGHTED ARITHMETIC AVERAGING OPERATOR

In this section, we defined the IV-CFSWAA operator and studied its properties.

$$M(F,E)=\frac{1}{n}\sum_{j=1}^{n}\left(1-\sqrt[\frac{1}{\lambda}]{\frac{1}{2m}\sum_{i=1}^{m}\left(\begin{array}{l}\left|r_{F(e_j)}^-(x_i)+r_{F(e_j)}^+(x_i)-1\right|^{\lambda}\\+\left(\frac{1}{2\pi}\left|\omega_{F(e_j)}^-(x_i)+\omega_{F(e_j)}^+(x_i)-2\pi\right|\right)^{\lambda}\end{array}\right)}\right),\quad\lambda>0\tag{3}$$

*Theorem 1:* For two IV-CFSNs $\alpha_{11} = \left[r_{11}^{-}, r_{11}^{+}\right] \cdot e^{i\left[\omega_{11}^{-}, \omega_{11}^{+}\right]}$, $\alpha_{12} = \left[r_{12}^{-}, r_{12}^{+}\right] \cdot e^{i\left[\omega_{12}^{-}, \omega_{12}^{+}\right]}$, the operations of them are defined as

1)

$$\alpha_{11} \oplus \alpha_{12} = \left[1 - \prod_{j=1}^{2}\left(1 - r_{1j}^{-}\right), 1 - \prod_{j=1}^{2}\left(1 - r_{1j}^{+}\right)\right] \cdot e^{i\left[2\pi\left(1-\prod_{j=1}^{2}\left(1-\frac{\omega_{1j}^{-}}{2\pi}\right)\right), 2\pi\left(1-\prod_{j=1}^{2}\left(1-\frac{\omega_{1j}^{+}}{2\pi}\right)\right)\right]}.$$

2)

$$\alpha_{11} \otimes \alpha_{12} = \left[\prod_{j=1}^{2} r_{1j}^{-}, \prod_{j=1}^{2} r_{1j}^{+}\right] \cdot e^{i\left[2\pi\left(\prod_{j=1}^{2}\frac{\omega_{1j}^{-}}{2\pi}\right), 2\pi\left(\prod_{j=1}^{2}\frac{\omega_{1j}^{+}}{2\pi}\right)\right]}.$$

3)

$$\lambda\alpha_{11} = \left[1 - \left(1 - r_{11}^{-}\right)^{\lambda}, 1 - \left(1 - r_{11}^{+}\right)^{\lambda}\right] \cdot e^{i\left[2\pi\left(1-\left(1-\frac{\omega_{11}^{-}}{2\pi}\right)^{\lambda}\right), 2\pi\left(1-\left(1-\frac{\omega_{11}^{+}}{2\pi}\right)^{\lambda}\right)\right]}.$$

4)

$$\alpha_{11}^{\lambda} = \left[\left(r_{11}^{-}\right)^{\lambda}, \left(r_{11}^{+}\right)^{\lambda}\right] \cdot e^{i\left[2\pi\left(\frac{\omega_{11}^{-}}{2\pi}\right)^{\lambda}, 2\pi\left(\frac{\omega_{11}^{+}}{2\pi}\right)^{\lambda}\right]}.$$

*Definition 7:* Let $\alpha_{ij} = \left[r_{ij}^{-}, r_{ij}^{+}\right] \cdot e^{i\left[\omega_{ij}^{-}, \omega_{ij}^{+}\right]}$ $(i = 1, 2, \ldots, m, j = 1, 2, \ldots, n)$ be a collection of IV-CFSNs, an interval-valued complex fuzzy soft weighted arithmetic averaging (IV-CFSWAA) operator is a function $IV - CFSWAA : \alpha^{n} \to \alpha$, defined by

$$IV - CFSWAA\left(\alpha_{11}, \alpha_{12}, \ldots, \alpha_{mn}\right) = \underset{j=1}{\overset{n}{\oplus}}\left(\xi_j \underset{i=1}{\overset{m}{\oplus}}\left(\eta_i \alpha_{ij}\right)\right) \tag{4}$$

where $\eta_i$ and $\xi_j$ are the weights of expert and parameter, respectively, and $\sum_{i=1}^{m} \eta_i = 1$, $\sum_{j=1}^{n} \xi_j = 1$.

Based on IV-CFSWAA operator, we can get the following theorems.

*Theorem 2:* Let $\alpha_{ij} = \left[r_{ij}^{-}, r_{ij}^{+}\right] \cdot e^{i\left[\omega_{ij}^{-}, \omega_{ij}^{+}\right]}$ $(i = 1, 2, \ldots, m, j = 1, 2, \ldots, n)$ be a collection of IV-CFSNs, then aggregated value of IV-CFSWAA operator is also IV-CFSN and is given by

$$\begin{aligned} &IV - CFSWAA\left(\alpha_{11}, \alpha_{12}, \ldots, \alpha_{mn}\right) \\ &= \left[1 - \prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1 - r_{ij}^{-}\right)^{\eta_i}\right)^{\xi_j}, 1 - \prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1 - r_{ij}^{+}\right)^{\eta_i}\right)^{\xi_j}\right] \\ &\cdot e^{i\left[2\pi\left(1-\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-\frac{\omega_{ij}^{-}}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right), 2\pi\left(1-\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-\frac{\omega_{ij}^{+}}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right)\right]} \end{aligned} \tag{5}$$

*Proof:* For $m = 1$, we have $\eta_1 = 1$. According to Definition 7, we can get

$$\begin{aligned} &IV - CFSWAA\left(\alpha_{11}, \alpha_{12}, \ldots, \alpha_{mn}\right) = \underset{j=1}{\overset{n}{\oplus}}\left(\xi_j \alpha_{ij}\right) \\ &= \left[1 - \prod_{j=1}^{n}\left(1 - r_{ij}^{-}\right)^{\xi_j}, 1 - \prod_{j=1}^{n}\left(1 - r_{ij}^{+}\right)^{\zeta_j}\right] \cdot e^{i\left[2\pi\left(1-\prod_{j=1}^{n}\left(1-\frac{\omega_{ij}^{-}}{2\pi}\right)^{\zeta_j}\right), 2\pi\left(1-\prod_{j=1}^{n}\left(1-\frac{\omega_{ij}^{+}}{2\pi}\right)^{\zeta_j}\right)\right]} \\ &= \left[1 - \prod_{j=1}^{n}\left(\prod_{i=1}^{1}\left(1 - r_{ij}^{-}\right)^{\eta_i}\right)^{\xi_j}, 1 - \prod_{j=1}^{n}\left(\prod_{i=1}^{1}\left(1 - r_{ij}^{+}\right)^{\eta_i}\right)^{\xi_j}\right] \\ &\cdot e^{i\left[2\pi\left(1-\prod_{j=1}^{n}\left(\prod_{i=1}^{1}\left(1-\frac{\omega_{ij}^{-}}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right), 2\pi\left(1-\prod_{j=1}^{n}\left(\prod_{i=1}^{1}\left(1-\frac{\omega_{ij}^{+}}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right)\right]} \end{aligned}$$

And for $n = 1$, we have $\xi_1 = 1$, then

$$\begin{aligned} &IV - CFSWAA\left(\alpha_{11}, \alpha_{12}, \ldots, \alpha_{mn}\right) = \underset{i=1}{\overset{m}{\oplus}}\left(\eta_i \alpha_{ij}\right) \\ &= \left[1 - \prod_{i=1}^{m}\left(1 - r_{ij}^{-}\right)^{\eta_i}, 1 - \prod_{i=1}^{m}\left(1 - r_{ij}^{+}\right)^{\eta_i}\right] \cdot e^{i\left[2\pi\left(1-\prod_{i=1}^{m}\left(1-\frac{\omega_{ij}^{-}}{2\pi}\right)^{\eta_i}\right), 2\pi\left(1-\prod_{i=1}^{m}\left(1-\frac{\omega_{ij}^{+}}{2\pi}\right)^{\eta_i}\right)\right]} \\ &= \left[1 - \prod_{j=1}^{1}\left(\prod_{i=1}^{m}\left(1 - r_{ij}^{-}\right)^{\eta_i}\right)^{\xi_j}, 1 - \prod_{j=1}^{1}\left(\prod_{i=1}^{m}\left(1 - r_{ij}^{+}\right)^{\eta_i}\right)^{\xi_j}\right] \\ &\cdot e^{i\left[2\pi\left(1-\prod_{j=1}^{1}\left(\prod_{i=1}^{m}\left(1-\frac{\omega_{ij}^{-}}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right), 2\pi\left(1-\prod_{j=1}^{1}\left(\prod_{i=1}^{m}\left(1-\frac{\omega_{ij}^{+}}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right)\right]} \end{aligned}$$

For $m = p_1 - 1$, $n = p_2$, and $m = p_1$, $n = p_2 - 1$, we have

$$\begin{aligned} &IV - CFSWAA\left(\alpha_{11}, \alpha_{12}, \ldots, \alpha_{mn}\right) = \underset{j=1}{\overset{p_2}{\oplus}}\left(\xi_j \underset{i=1}{\overset{p_1-1}{\oplus}}\left(\eta_i \alpha_{ij}\right)\right) \\ &= \left[1 - \prod_{j=1}^{p_2}\left(\prod_{i=1}^{p_1-1}\left(1 - r_{ij}^{-}\right)^{\eta_i}\right)^{\xi_j}, 1 - \prod_{j=1}^{p_2}\left(\prod_{i=1}^{p_1-1}\left(1 - r_{ij}^{+}\right)^{\eta_i}\right)^{\xi_j}\right] \\ &\cdot e^{i\left[2\pi\left(1-\prod_{j=1}^{p_2}\left(\prod_{i=1}^{p_1-1}\left(1-\frac{\omega_{ij}^{-}}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right), 2\pi\left(1-\prod_{j=1}^{p_2}\left(\prod_{i=1}^{p_1-1}\left(1-\frac{\omega_{ij}^{+}}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right)\right]} \end{aligned}$$

$$IV-CFSWAA\left(\alpha_{11},\alpha_{12},\ldots,\alpha_{mn}\right)=\overset{p_2-1}{\underset{j=1}{\oplus}}\left(\xi_j\overset{p_1}{\underset{i=1}{\oplus}}\left(\eta_i\alpha_{ij}\right)\right)$$

$$=\left[1-\prod_{j=1}^{p_2-1}\left(\prod_{i=1}^{p_1}\left(1-r_{ij}^-\right)^{\eta_i}\right)^{\xi_j},\right.$$

$$\left.1-\prod_{j=1}^{p_2-1}\left(\prod_{i=1}^{p_1}\left(1-r_{ij}^+\right)^{\eta_i}\right)^{\xi_j}\right]$$

$$\cdot e^{i\left[2\pi\left(1-\prod_{j=1}^{p_2-1}\left(\prod_{i=1}^{p_1}\left(1-\frac{\omega_{ij}^-}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right),2\pi\left(1-\prod_{j=1}^{p_2-1}\left(\prod_{i=1}^{p_1}\left(1-\frac{\omega_{ij}^+}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right)\right]}$$

For $m=p_1$, $n=p_2$, we have

$$IV-CFSWAA\left(\alpha_{11},\alpha_{12},\ldots,\alpha_{mn}\right)=\overset{p_2}{\underset{j=1}{\oplus}}\left(\xi_j\overset{p_1}{\underset{i=1}{\oplus}}\left(\eta_i\alpha_{ij}\right)\right)$$

$$=\left[1-\prod_{j=1}^{p_2}\left(\prod_{i=1}^{p_1}\left(1-r_{ij}^-\right)^{\eta_i}\right)^{\xi_j},\right.$$

$$\left.1-\prod_{j=1}^{p_2}\left(\prod_{i=1}^{p_1}\left(1-r_{ij}^+\right)^{\eta_i}\right)^{\xi_j}\right]$$

$$\cdot e^{i\left[2\pi\left(1-\prod_{j=1}^{p_2}\left(\prod_{i=1}^{p_1}\left(1-\frac{\omega_{ij}^-}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right),2\pi\left(1-\prod_{j=1}^{p_2}\left(\prod_{i=1}^{p_1}\left(1-\frac{\omega_{ij}^+}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right)\right]}$$

So, the Theorem 2 is hold for all $m\geq 1$, $n\geq 1$.

*Example 1:* Let $(F,E)$ be an IV-CFSS, $K=\{k_1,k_2,k_3,k_4\}$ be the set of experts, $E=\{e_1,e_2,e_3\}$ be the set of parameters. let $\eta=(\eta_1=0.2,\eta_2=0.4,\eta_3=0.3,\eta_4=0.1)^T$ and $\xi=(\xi_1=0.2,\xi_2=0.3,\xi_3=0.5)^T$ be the weight vectors of experts and parameters, respectively. $(F,E)$ is shown in Table 1.

**TABLE 1.** Decision matrix $(F,E)$.

| | $e_1$ | $e_2$ | $e_3$ |
|---|---|---|---|
| $u_1$ | $[0.4,0.5].e^{i\left[\frac{\pi}{6},\frac{\pi}{3}\right]}$ | $[0.3,0.4].e^{i\left[\frac{5\pi}{6},\pi\right]}$ | $[0.6,0.7].e^{i\left[\frac{3\pi}{2},2\pi\right]}$ |
| $u_2$ | $[0.5,0.6].e^{i\left[\frac{\pi}{3},\pi\right]}$ | $[0.3,0.4].e^{i\left[\frac{\pi}{6},\pi\right]}$ | $[0.8,1].e^{i\left[\frac{3\pi}{2},\frac{5\pi}{3}\right]}$ |
| $u_3$ | $[0.2,0.3].e^{i\left[\frac{2\pi}{3},\frac{4\pi}{3}\right]}$ | $[0.2,0.3].e^{i\left[\frac{5\pi}{6},\pi\right]}$ | $[0.6,0.7].e^{i\left[\frac{\pi}{2},\pi\right]}$ |
| $u_4$ | $[0.7,0.8].e^{i\left[\frac{\pi}{6},\frac{\pi}{3}\right]}$ | $[0.1,0.2].e^{i\left[\pi,\frac{4\pi}{3}\right]}$ | $[0.1,0.2].e^{i\left[\frac{\pi}{2},\frac{2\pi}{3}\right]}$ |

The Theorem 9 satisfies the following properties.

*Theorem 3:* Let $\alpha_{ij}=\left[r_{ij}^-,r_{ij}^+\right]\cdot e^{i\left[\omega_{ij}^-,\omega_{ij}^+\right]}$ $(i=1,2,\ldots,m,$ $j=1,2,\ldots,n)$ be a collection of IV-CFSNs, and $\alpha_{ij}=\alpha$, then

$$IV-CFSWAA\left(\alpha_{11},\alpha_{12},\ldots,\alpha_{mn}\right)=\alpha \tag{6}$$

This property is called Idempotency Property.

*Proof:* Since $\alpha_{ij}=\alpha=\left[r^-,r^+\right]\cdot e^{i\left[\omega^-,\omega^+\right]}$, then we have

$$IV-CFSWAA\left(\alpha_{11},\alpha_{12},\ldots,\alpha_{mn}\right)$$

$$=\left[1-\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-r^-\right)^{\eta_i}\right)^{\xi_j},\right.$$

$$\left.1-\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-r^+\right)^{\eta_i}\right)^{\xi_j}\right]$$

$$\cdot e^{i\left[2\pi\left(1-\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-\frac{\omega^-}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right),2\pi\left(1-\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-\frac{\omega^+}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right)\right]}$$

$$=\left[1-\left(\left(1-r^-\right)^{\sum_{i=1}^{m}\eta_i}\right)^{\sum_{j=1}^{n}\xi_j},1-\left(\left(1-r^+\right)^{\sum_{i=1}^{m}\eta_i}\right)^{\sum_{j=1}^{n}\xi_j}\right]$$

$$\cdot e^{i\left[2\pi\left(1-\left(\left(1-\frac{\omega^-}{2\pi}\right)^{\sum_{i=1}^{m}\eta_i}\right)^{\sum_{j=1}^{n}\xi_j}\right),2\pi\left(1-\left(\left(1-\frac{\omega^+}{2\pi}\right)^{\sum_{i=1}^{m}\eta_i}\right)^{\sum_{j=1}^{n}\xi_j}\right)\right]}$$

$$=\left[1-\left(1-r^-\right),1-\left(1-r^+\right)\right]$$

$$\cdot e^{i\left[2\pi\left(1-\left(1-\frac{\omega^-}{2\pi}\right)\right),2\pi\left(1-\left(1-\frac{\omega^+}{2\pi}\right)\right)\right]}$$

$$=\left[r^-,r^+\right]\cdot e^{i\left[\omega^-,\omega^+\right]}$$

Therefore, $IV-CFSWAA\left(\alpha_{11},\alpha_{12},\ldots,\alpha_{mn}\right)=\alpha$.

*Theorem 4:* Let $\alpha_{ij}=\left[r_{ij}^-,r_{ij}^+\right]\cdot e^{i\left[\omega_{ij}^-,\omega_{ij}^+\right]}$ $(i=1,2,\ldots,m,$ $j=1,2,\ldots,n)$ be a collection of IV-CFSNs, and

$$\alpha_{ij}^-=\left[\min_m\min_n\left\{r_{ij}^-\right\},\min_m\min_n\left\{r_{ij}^+\right\}\right]$$

$$\cdot e^{i\left[\min_m\min_n\left\{\omega_{ij}^-\right\},\min_m\min_n\left\{\omega_{ij}^+\right\}\right]},$$

$$\alpha_{ij}^+=\left[\max_m\max_n\left\{r_{ij}^-\right\},\max_m\max_n\left\{r_{ij}^+\right\}\right]$$

$$\cdot e^{i\left[\max_m\max_n\left\{\omega_{ij}^-\right\},\max_m\max_n\left\{\omega_{ij}^+\right\}\right]}.$$

Then we can have

$$\alpha_{ij}^-\leq IV-CFSWAA\left(\alpha_{11},\alpha_{12},\ldots,\alpha_{mn}\right)\leq\alpha_{ij}^+ \tag{7}$$

This property is called Boundedness Property.

*Proof:* Since $\alpha_{ij}=\alpha=\left[r^-,r^+\right]\cdot e^{i\left[\omega^-,\omega^+\right]}$be IV-CFSN, then $\min_m\min_n\left\{r_{ij}^-\right\}\leq r_{ij}^-\leq\max_m\max_n\left\{r_{ij}^-\right\}$. we can get

$$1-\max_m\max_n\left\{r_{ij}^-\right\}$$

$$\leq 1-r_{ij}^-\leq 1-\min_m\min_n\left\{r_{ij}^-\right\}$$

$$\Rightarrow\prod_{i=1}^{m}\left(1-\max_m\max_n\left\{r_{ij}^-\right\}\right)^{\eta_i}\leq\prod_{i=1}^{m}\left(1-r_{ij}^-\right)^{\eta_i}$$

$$\leq\prod_{i=1}^{m}\left(1-\min_m\min_n\left\{r_{ij}^-\right\}\right)^{\eta_i}$$

$$
\begin{aligned}
& IV-CFSWAA\left(\alpha_{11},\alpha_{12},\ldots,\alpha_{43}\right) \\
&= \left[1-\prod_{j=1}^{3}\left(\prod_{i=1}^{4}\left(1-r_{ij}^{-}\right)^{\eta_i}\right)^{\xi_j},1-\prod_{j=1}^{3}\left(\prod_{i=1}^{4}\left(1-r_{ij}^{+}\right)^{\eta_i}\right)^{\xi_j}\right] \\
&\quad \cdot e^{i\left[2\pi\left(1-\prod_{j=1}^{3}\left(\prod_{i=1}^{4}\left(1-\frac{\omega_{ij}^{-}}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right),2\pi\left(1-\prod_{j=1}^{3}\left(\prod_{i=1}^{4}\left(1-\frac{\omega_{ij}^{+}}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right)\right]} \\
&= \left[\begin{array}{l}
1-\left((1-0.4)^{0.2}(1-0.5)^{0.4}(1-0.2)^{0.3}(1-0.7)^{0.1}\right)^{0.2} \\
\times\left((1-0.3)^{0.2}(1-0.3)^{0.4}(1-0.2)^{0.3}(1-0.1)^{0.1}\right)^{0.3} \\
\times\left((1-0.6)^{0.2}(1-0.8)^{0.4}(1-0.6)^{0.3}(1-0.1)^{0.1}\right)^{0.5}, \\
1-\left((1-0.5)^{0.2}(1-0.6)^{0.4}(1-0.3)^{0.3}(1-0.8)^{0.1}\right)^{0.2} \\
\times\left((1-0.4)^{0.2}(1-0.4)^{0.4}(1-0.3)^{0.3}(1-0.2)^{0.1}\right)^{0.3} \\
\times\left((1-0.7)^{0.2}(1-1)^{0.4}(1-0.7)^{0.3}(1-0.2)^{0.1}\right)^{0.5}
\end{array}\right] \\
&\quad \cdot e^{i\left[\begin{array}{l}
2\pi\left(\begin{array}{l}
1-\left(\left(1-(\pi/6)/(2\pi)\right)^{0.2}\left(1-(\pi/3)/(2\pi)\right)^{0.4}\left(1-(2\pi/3)/(2\pi)\right)^{0.3}\left(1-(\pi/6)/(2\pi)\right)^{0.1}\right)^{0.2} \\
\times\left(\left(1-(5\pi/6)/(2\pi)\right)^{0.2}\left(1-(\pi/6)/(2\pi)\right)^{0.4}\left(1-(5\pi/6)/(2\pi)\right)^{0.3}\left(1-(\pi)/(2\pi)\right)^{0.1}\right)^{0.3} \\
\times\left(\left(1-(3\pi/2)/(2\pi)\right)^{0.2}\left(1-(3\pi/2)/(2\pi)\right)^{0.4}\left(1-(\pi/2)/(2\pi)\right)^{0.3}\left(1-(\pi/2)/(2\pi)\right)^{0.1}\right)^{0.5}
\end{array}\right), \\
2\pi\left(\begin{array}{l}
1-\left(\left(1-(\pi/3)/(2\pi)\right)^{0.2}\left(1-(\pi)/(2\pi)\right)^{0.4}\left(1-(4\pi/3)/(2\pi)\right)^{0.3}\left(1-(\pi/3)/(2\pi)\right)^{0.1}\right)^{0.2} \\
\times\left(\left(1-(\pi)/(2\pi)\right)^{0.2}\left(1-(\pi)/(2\pi)\right)^{0.4}\left(1-(\pi)/(2\pi)\right)^{0.3}\left(1-(4\pi/3)/(2\pi)\right)^{0.1}\right)^{0.3} \\
\times\left(\left(1-(2\pi)/(2\pi)\right)^{0.2}\left(1-(5\pi/3)/(2\pi)\right)^{0.4}\left(1-(\pi)/(2\pi)\right)^{0.3}\left(1-(2\pi/3)/(2\pi)\right)^{0.1}\right)^{0.5}
\end{array}\right)
\end{array}\right]} \\
&= [0.53, 1]\cdot e^{i[2\pi(0.47),2\pi]}
\end{aligned}
$$

$$
\begin{aligned}
&\Rightarrow \left(1-\max_m\max_n\left\{r_{ij}^{-}\right\}\right)^{\sum_{i=1}^{m}\eta_i} \le \prod_{i=1}^{m}\left(1-r_{ij}^{-}\right)^{\eta_i} \\
&\le \left(1-\min_m\min_n\left\{r_{ij}^{-}\right\}\right)^{\sum_{i=1}^{m}\eta_i} \\
&\Rightarrow 1-\max_m\max_n\left\{r_{ij}^{-}\right\} \le \prod_{i=1}^{m}\left(1-r_{ij}^{-}\right)^{\eta_i} \\
&\le 1-\min_m\min_n\left\{r_{ij}^{-}\right\} \\
&\Rightarrow \left(1-\max_m\max_n\left\{r_{ij}^{-}\right\}\right)^{\xi_j} \le \left(\prod_{i=1}^{m}\left(1-r_{ij}^{-}\right)^{\eta_i}\right)^{\xi_j} \\
&\le \left(1-\min_m\min_n\left\{r_{ij}^{-}\right\}\right)^{\xi_j} \\
&\Rightarrow \prod_{j=1}^{n}\left(1-\max_m\max_n\left\{r_{ij}^{-}\right\}\right)^{\xi_j} \le \prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-r_{ij}^{-}\right)^{\eta_i}\right)^{\xi_j} \\
&\le \prod_{j=1}^{n}\left(1-\min_m\min_n\left\{r_{ij}^{-}\right\}\right)^{\xi_j} \\
&\Rightarrow \left(1-\max_m\max_n\left\{r_{ij}^{-}\right\}\right)^{\sum_{j=1}^{n}\xi_j} \le \prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-r_{ij}^{-}\right)^{\eta_i}\right)^{\xi_j} \\
&\le \left(1-\min_m\min_n\left\{r_{ij}^{-}\right\}\right)^{\sum_{j=1}^{n}\xi_j}
\end{aligned}
$$

$$
\begin{aligned}
&\Rightarrow 1-\max_m\max_n\left\{r_{ij}^{-}\right\} \le \prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-r_{ij}^{-}\right)^{\eta_i}\right)^{\xi_j} \\
&\le 1-\min_m\min_n\left\{r_{ij}^{-}\right\} \\
&\Rightarrow \min_m\min_n\left\{r_{ij}^{-}\right\} \le 1-\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-r_{ij}^{-}\right)^{\eta_i}\right)^{\xi_j} \\
&\le \max_m\max_n\left\{r_{ij}^{-}\right\}.
\end{aligned}
$$

Similarly, we can get

$$
\begin{aligned}
\min_m\min_n\left\{r_{ij}^{+}\right\} &\le 1-\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-r_{ij}^{+}\right)^{\eta_i}\right)^{\xi_j} \\
&\le \max_m\max_n\left\{r_{ij}^{+}\right\}, \\
\min_m\min_n\left\{\omega_{ij}^{-}\right\} &\le 1-\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-\omega_{ij}^{-}\right)^{\eta_i}\right)^{\xi_j} \\
&\le \max_m\max_n\left\{\omega_{ij}^{-}\right\}, \\
\min_m\min_n\left\{\omega_{ij}^{+}\right\} &\le 1-\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-\omega_{ij}^{+}\right)^{\eta_i}\right)^{\xi_j} \\
&\le \max_m\max_n\left\{\omega_{ij}^{+}\right\}.
\end{aligned}
$$

By the score function of IV-CFSNs, we have

$$\begin{aligned}
&S\left(IV-CFSWAA\left(\alpha_{11},\alpha_{12},\ldots,\alpha_{mn}\right)\right)\\
&=1-\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-r_{ij}^{-}\right)^{\eta_i}\right)^{\xi_j}\\
&\quad+1-\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-r_{ij}^{+}\right)^{\eta_i}\right)^{\xi_j}-1\\
&\quad+\frac{1}{2\pi}\left(\begin{array}{l}2\pi\left(1-\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-\frac{\omega_{ij}^{-}}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right)\\+2\pi\left(1-\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-\frac{\omega_{ij}^{+}}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right)-2\pi\end{array}\right)\\
&\geq\min_{m}\min_{n}\left\{r_{ij}^{-}\right\}+\min_{m}\min_{n}\left\{r_{ij}^{+}\right\}-1\\
&\quad+\frac{1}{2\pi}\left(\min_{m}\min_{n}\left\{\omega_{ij}^{-}\right\}+\min_{m}\min_{n}\left\{\omega_{ij}^{+}\right\}-2\pi\right)\\
&=S\left(\alpha_{ij}^{-}\right)
\end{aligned}$$

and

$$\begin{aligned}
&S\left(IV-CFSWAA\left(\alpha_{11},\alpha_{12},\ldots,\alpha_{mn}\right)\right)\\
&\quad\leq\max_{m}\max_{n}\left\{r_{ij}^{-}\right\}+\max_{m}\max_{n}\left\{r_{ij}^{+}\right\}-1\\
&\qquad+\frac{1}{2\pi}\left(\max_{m}\max_{n}\left\{\omega_{ij}^{-}\right\}+\max_{m}\max_{n}\left\{\omega_{ij}^{+}\right\}-2\pi\right)\\
&\quad=S\left(\alpha_{ij}^{+}\right)
\end{aligned}$$

So, we can obtain

$$\alpha_{ij}^{+}\leq IV-CFSWAA\left(\alpha_{11},\alpha_{12},\ldots,\alpha_{mn}\right)\leq\alpha_{ij}^{-}$$

*Theorem 5:* Let $\alpha_{ij}=\left[r_{ij}^{-},r_{ij}^{+}\right]\cdot e^{i\left[\omega_{ij}^{-},\omega_{ij}^{+}\right]}$ $(i=1,2,\ldots,m,j=1,2,\ldots,n)$ and $\alpha=\left[r^{-},r^{+}\right]\cdot e^{i\left[\omega^{-},\omega^{+}\right]}$ be IV-CFSNs. Then we can have

$$\begin{aligned}
&IV-CFSWAA\left(\alpha_{11}\oplus\alpha,\alpha_{12}\oplus\alpha,\ldots,\alpha_{mn}\oplus\alpha\right)\\
&\qquad=IV-CFSWAA\left(\alpha_{11},\alpha_{12},\ldots,\alpha_{mn}\right)\oplus\alpha
\end{aligned}\tag{8}$$

This property is called Shift Invariance Property.

*Proof:* Since

$$\begin{aligned}
\alpha_{ij}\oplus\alpha&=\left[1-\left(1-r_{ij}^{-}\right)\left(1-r^{-}\right),1-\left(1-r_{ij}^{+}\right)\left(1-r^{+}\right)\right]\\
&\quad\cdot e^{i[2\pi(1-(1-\frac{\omega_{ij}^{-}}{2\pi})(1-\frac{\omega^{-}}{2\pi})),2\pi(1-(1-\frac{\omega_{ij}^{+}}{2\pi})(1-\frac{\omega^{+}}{2\pi}))]},
\end{aligned}$$

then we have, $IV-CFSWAA(\alpha_{11}\oplus\alpha,\alpha_{12}\oplus\alpha,\ldots,\alpha_{mn}\oplus\alpha)$, as shown at the bottom of this page.

*Theorem 6:* Let $\alpha_{ij}=\left[r_{ij}^{-},r_{ij}^{+}\right]\cdot e^{i\left[\omega_{ij}^{-},\omega_{ij}^{+}\right]}$ $(i=1,2,\ldots,m,$ $j=1,2,\ldots,n)$ be a collection of IV-CFSNs and $\lambda>0$. Then we can have

$$\begin{aligned}
&IV-CFSWAA\left(\lambda\alpha_{11},\lambda\alpha_{12},\ldots,\lambda\alpha_{mn}\right)\\
&\qquad=\lambda IV-CFSWAA\left(\alpha_{11},\alpha_{12},\ldots,\alpha_{mn}\right)
\end{aligned}\tag{9}$$

This property is called Homogeneity Property.

*Proof:* Since $\lambda\alpha_{ij}=\left[1-\left(1-r_{ij}^{-}\right)^{\lambda},1-\left(1-r_{ij}^{+}\right)^{\lambda}\right]$ $\cdot e^{i[2\pi(1-(1-\frac{\omega_{ij}^{-}}{2\pi})^{\lambda}),2\pi(1-(1-\frac{\omega_{ij}^{+}}{2\pi})^{\lambda})]}$, then we have, $IV-CFSWAA\left(\lambda\alpha_{11},\lambda\alpha_{12},\ldots,\lambda\alpha_{mn}\right)$, as shown at the top of the next page.

### B. INTERVAL-VALUED COMPLEX FUZZY SOFT WEIGHTED GEOMETRIC AVERAGING OPERATOR

In this section, we defined the IV-CFSWGA operator and studied its properties.

*Definition 8:* Let $\alpha_{ij}=\left[r_{ij}^{-},r_{ij}^{+}\right]\cdot e^{i\left[\omega_{ij}^{-},\omega_{ij}^{+}\right]}$ $(i=1,2,\ldots,m,j=1,2,\ldots,n)$ be a collection of IV-CFSNs, an

$$\begin{aligned}
&IV-CFSWAA\left(\alpha_{11}\oplus\alpha,\alpha_{12}\oplus\alpha,\ldots,\alpha_{mn}\oplus\alpha\right)\\
&=\left[\begin{array}{l}1-\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(\left(1-r_{ij}^{-}\right)^{\eta_i}\left(1-r^{-}\right)^{\eta_i}\right)\right)^{\xi_j},\\1-\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(\left(1-r_{ij}^{+}\right)^{\eta_i}\left(1-r^{+}\right)^{\eta_i}\right)\right)^{\xi_j}\end{array}\right]\\
&\quad\cdot e^{i\left[2\pi\left(1-\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(\left(1-\frac{\omega_{ij}^{-}}{2\pi}\right)^{\eta_i}\left(1-\frac{\omega^{-}}{2\pi}\right)^{\eta_i}\right)\right)^{\xi_j}\right),2\pi\left(1-\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(\left(1-\frac{\omega_{ij}^{+}}{2\pi}\right)^{\eta_i}\left(1-\frac{\omega^{+}}{2\pi}\right)^{\eta_i}\right)\right)^{\xi_j}\right)\right]}\\
&=\left[\begin{array}{l}1-\left(\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-r_{ij}^{-}\right)^{\eta_i}\right)^{\xi_j}\right)\left(1-r^{-}\right),\\1-\left(\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-r_{ij}^{+}\right)^{\eta_i}\right)^{\xi_j}\right)\left(1-r^{+}\right)\end{array}\right]\\
&\quad\cdot e^{i\left[2\pi\left(1-\left(\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-\frac{\omega_{ij}^{-}}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right)\left(1-\omega^{-}\right)\right),2\pi\left(1-\left(\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-\frac{\omega_{ij}^{+}}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right)\left(1-\omega^{+}\right)\right)\right]}\\
&=IV-CFSWAA\left(\alpha_{11},\alpha_{12},\ldots,\alpha_{mn}\right)\oplus\alpha
\end{aligned}$$

$$
\begin{aligned}
&IV-CFSWAA\left(\lambda\alpha_{11},\lambda\alpha_{12},\ldots,\lambda\alpha_{mn}\right)\\
&=\left[1-\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-r_{ij}^{-}\right)^{\lambda\eta_i}\right)^{\xi_j},1-\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-r_{ij}^{+}\right)^{\lambda\eta_i}\right)^{\xi_j}\right]\\
&\quad\cdot e^{i\left[2\pi\left(1-\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-\frac{\omega_{ij}^{-}}{2\pi}\right)^{\lambda\eta_i}\right)^{\xi_j}\right),2\pi\left(1-\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-\frac{\omega_{ij}^{+}}{2\pi}\right)^{\lambda\eta_i}\right)^{\xi_j}\right)\right]}\\
&=\left[1-\left(\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-r_{ij}^{-}\right)^{\eta_i}\right)^{\xi_j}\right)^{\lambda},1-\left(\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-r_{ij}^{+}\right)^{\eta_i}\right)^{\xi_j}\right)^{\lambda}\right]\\
&\quad\cdot e^{i\left[2\pi\left(1-\left(\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-\frac{\omega_{ij}^{-}}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right)^{\lambda}\right),2\pi\left(1-\left(\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1-\frac{\omega_{ij}^{-}}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right)^{\lambda}\right)\right]}\\
&=\lambda IV-CFSWAA\left(\alpha_{11},\alpha_{12},\ldots,\alpha_{mn}\right)
\end{aligned}
$$

interval-valued complex fuzzy soft weighted geometric averaging (IV-CFSWGA) operator is a function $IV-CFSWGA:\alpha^n\rightarrow\alpha$ defined by

$$
IV-CFSWGA\left(a_{11},a_{12},\ldots,a_{mn}\right)=\bigotimes_{j=1}^{n}\left(\bigotimes_{i=1}^{m}\left(a_{ij}^{\eta_i}\right)\right)^{\xi_j}\tag{10}
$$

where $\eta_j$ and $\xi_i$ are the weights of expert and parameter, respectively, and $\sum_{j=1}^{n}\eta_j=1$, $\sum_{i=1}^{m}\xi_i=1$.

Based on IV-CFSWGA operator, we obtain the following theorems.

*Theorem 7:* Let $\alpha_{ij}=\left[r_{ij}^{-},r_{ij}^{+}\right]\cdot e^{i\left[\omega_{ij}^{-},\omega_{ij}^{+}\right]}$ $(i=1,2,\ldots,m,j=1,2,\ldots,n)$ be a collection of IV-CFSNs, then aggregated value of IV-CFSWGA operator is also IV-CFSN and is given by

$$
\begin{aligned}
&IV-CFSWGA\left(\alpha_{11},\alpha_{12},\ldots,\alpha_{mn}\right)\\
&=\left[\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(r_{ij}^{-}\right)^{\eta_i}\right)^{\xi_j},\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(r_{ij}^{+}\right)^{\eta_i}\right)^{\xi_j}\right]\\
&\quad\cdot e^{i\left[2\pi\left(\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(\frac{\omega_{ij}^{-}}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right),2\pi\left(\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(\frac{\omega_{ij}^{+}}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right)\right]}
\end{aligned}\tag{11}
$$

*Proof:* For $m=1$, we have $\eta_1=1$. According to Definition 8, we can get

$$
\begin{aligned}
&IV-CFSWGA\left(\alpha_{11},\alpha_{12},\ldots,\alpha_{mn}\right)=\bigotimes_{j=1}^{n}\left(a_{ij}^{\xi_j}\right)\\
&=\left[\prod_{j=1}^{n}\left(r_{ij}^{-}\right)^{\xi_j},\prod_{j=1}^{n}\left(r_{ij}^{+}\right)^{\zeta_j}\right]\\
&\quad\cdot e^{i\left[2\pi\left(\prod_{j=1}^{n}\left(\frac{\omega_{ij}^{-}}{2\pi}\right)^{\zeta_j}\right),2\pi\left(\prod_{j=1}^{n}\left(\frac{\omega_{ij}^{+}}{2\pi}\right)^{\zeta_j}\right)\right]}\\
&=\left[1-\prod_{j=1}^{n}\left(\prod_{i=1}^{1}\left(r_{ij}^{-}\right)^{\eta_i}\right)^{\xi_j},\prod_{j=1}^{n}\left(\prod_{i=1}^{1}\left(r_{ij}^{+}\right)^{\eta_i}\right)^{\xi_j}\right]\\
&\quad\cdot e^{i\left[2\pi\left(\prod_{j=1}^{n}\left(\prod_{i=1}^{1}\left(\frac{\omega_{ij}^{-}}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right),2\pi\left(\prod_{j=1}^{n}\left(\prod_{i=1}^{1}\left(\frac{\omega_{ij}^{+}}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right)\right]}
\end{aligned}
$$

And for $n=1$, we have $\xi_1=1$, then

$$
\begin{aligned}
&IV-CFSWGA\left(\alpha_{11},\alpha_{12},\ldots,\alpha_{mn}\right)=\bigotimes_{i=1}^{m}\left(\alpha_{ij}^{\eta_i}\right)\\
&=\left[\prod_{i=1}^{m}\left(r_{ij}^{-}\right)^{\eta_i},\prod_{i=1}^{m}\left(r_{ij}^{+}\right)^{\eta_i}\right]\\
&\quad\cdot e^{i\left[2\pi\left(\prod_{i=1}^{m}\left(\frac{\omega_{ij}^{-}}{2\pi}\right)^{\eta_i}\right),2\pi\left(\prod_{i=1}^{m}\left(\frac{\omega_{ij}^{+}}{2\pi}\right)^{\eta_i}\right)\right]}\\
&=\left[\prod_{j=1}^{1}\left(\prod_{i=1}^{m}\left(r_{ij}^{-}\right)^{\eta_i}\right)^{\xi_j},\prod_{j=1}^{1}\left(\prod_{i=1}^{m}\left(r_{ij}^{+}\right)^{\eta_i}\right)^{\xi_j}\right]\\
&\quad\cdot e^{i\left[2\pi\left(\prod_{j=1}^{1}\left(\prod_{i=1}^{m}\left(\frac{\omega_{ij}^{-}}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right),2\pi\left(\prod_{j=1}^{1}\left(\prod_{i=1}^{m}\left(\frac{\omega_{ij}^{+}}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right)\right]}
\end{aligned}
$$

For $m=p_1-1,n=p_2$, and $m=p_1,n=p_2-1$, we have

$$
\begin{aligned}
&IV-CFSWGA\left(\alpha_{11},\alpha_{12},\ldots,\alpha_{mn}\right)=\bigotimes_{j=1}^{p_2}\left(\bigotimes_{i=1}^{p_1-1}\left(\alpha_{ij}^{\eta_i}\right)\right)^{\xi_j}\\
&=\left[\prod_{j=1}^{p_2}\left(\prod_{i=1}^{p_1-1}\left(r_{ij}^{-}\right)^{\eta_i}\right)^{\xi_j},\prod_{j=1}^{p_2}\left(\prod_{i=1}^{p_1-1}\left(r_{ij}^{+}\right)^{\eta_i}\right)^{\xi_j}\right]\\
&\quad\cdot e^{i\left[2\pi\left(\prod_{j=1}^{p_2}\left(\prod_{i=1}^{p_1-1}\left(\frac{\omega_{ij}^{-}}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right),2\pi\left(\prod_{j=1}^{p_2}\left(\prod_{i=1}^{p_1-1}\left(\frac{\omega_{ij}^{+}}{2\pi}\right)^{\eta_i}\right)^{\xi_j}\right)\right]}
\end{aligned}
$$

$$
IV-CFSWGA\left(\alpha_{11},\alpha_{12},\ldots,\alpha_{mn}\right)=\bigotimes_{j=1}^{p_2-1}\left(\bigotimes_{i=1}^{p_1}\left(\alpha_{ij}^{\eta_i}\right)\right)^{\xi_j}
$$

$$= \left[ \prod_{j=1}^{p_2-1} \left( \prod_{i=1}^{p_1} \left( r_{ij}^{-} \right)^{\eta_i} \right)^{\xi_j}, \prod_{j=1}^{p_2-1} \left( \prod_{i=1}^{p_1} \left( r_{ij}^{+} \right)^{\eta_i} \right)^{\xi_j} \right] \cdot e^{i\left[ 2\pi \left( \prod_{j=1}^{p_2-1} \left( \prod_{i=1}^{p_1} \left( \frac{\omega_{ij}^{-}}{2\pi} \right)^{\eta_i} \right)^{\xi_j} \right), 2\pi \left( \prod_{j=1}^{p_2-1} \left( \prod_{i=1}^{p_1} \left( \frac{\omega_{ij}^{+}}{2\pi} \right)^{\eta_i} \right)^{\xi_j} \right) \right]}$$

For $m = p_1, n = p_2$, we have

$$IV - CFSWGA\left(\alpha_{11}, \alpha_{12}, \ldots, \alpha_{mn}\right) = \bigotimes_{j=1}^{p_2} \left( \bigotimes_{i=1}^{p_1} \left( \alpha_{ij}^{\eta_i} \right) \right)^{\xi_j}$$

$$= \left[ \prod_{j=1}^{p_2} \left( \prod_{i=1}^{p_1} \left( r_{ij}^{-} \right)^{\eta_i} \right)^{\xi_j}, \prod_{j=1}^{p_2} \left( \prod_{i=1}^{p_1} \left( r_{ij}^{+} \right)^{\eta_i} \right)^{\xi_j} \right] \cdot e^{i\left[ 2\pi \left( \prod_{j=1}^{p_2} \left( \prod_{i=1}^{p_1} \left( \frac{\omega_{ij}^{-}}{2\pi} \right)^{\eta_i} \right)^{\xi_j} \right), 2\pi \left( \prod_{j=1}^{p_2} \left( \prod_{i=1}^{p_1} \left( \frac{\omega_{ij}^{+}}{2\pi} \right)^{\eta_i} \right)^{\xi_j} \right) \right]}$$

So, the Theorem 7 is hold for all $m \geq 1, n \geq 1$.

*Example 2:* Let $(F, E)$ be an IV-CFSS, $K = \{k_1, k_2, k_3, k_4\}$ be the set of experts, $E = \{e_1, e_2, e_3\}$ be the set of parameters. let $\eta = (\eta_1 = 0.1, \eta_2 = 0.3, \eta_3 = 0.3, \eta_4 = 0.3)^T$ and $\xi = (\xi_1 = 0.2, \xi_2 = 0.4, \xi_3 = 0.4)^T$ be the weight vectors of experts and parameters, respectively. $(F, E)$ is shown in Table 2.

IV-CFSWGA operator satisfies the properties of IV-CFSWAA operator.

**TABLE 2.** Decision matrix $(F, E)$.

| | $e_1$ | $e_2$ | $e_3$ |
|---|---|---|---|
| $u_1$ | $[0.7,1].e^{i\left[\frac{\pi}{6},\frac{\pi}{3}\right]}$ | $[0.3,0.4].e^{i\left[\frac{5\pi}{6},\pi\right]}$ | $[0.5,0.7].e^{i\left[\frac{3\pi}{2},2\pi\right]}$ |
| $u_2$ | $[0.4,0.6].e^{i\left[\frac{\pi}{3},\pi\right]}$ | $[0.3,0.4].e^{i\left[\frac{\pi}{6},\pi\right]}$ | $[0.9,1].e^{i\left[\frac{3\pi}{2},\frac{5\pi}{3}\right]}$ |
| $u_3$ | $[0.2,0.3].e^{i\left[\frac{2\pi}{3},\frac{4\pi}{3}\right]}$ | $[0.3,0.4].e^{i\left[\frac{5\pi}{6},\pi\right]}$ | $[0.6,0.7].e^{i\left[\frac{\pi}{2},\pi\right]}$ |
| $u_4$ | $[0.9,1].e^{i\left[\frac{\pi}{6},\frac{\pi}{3}\right]}$ | $[0.2,0.3].e^{i\left[\pi,\frac{4\pi}{3}\right]}$ | $[0.1,0.2].e^{i\left[\frac{\pi}{2},\frac{2\pi}{3}\right]}$ |

*Theorem 8:* Let $\alpha_{ij} = \left[ r_{ij}^{-}, r_{ij}^{+} \right] \cdot e^{i\left[\omega_{ij}^{-}, \omega_{ij}^{+}\right]}$ $(i = 1, 2, \ldots, m, j = 1, 2, \ldots, n)$ be a collection of IV-CFSNs, and $\alpha_{ij} = \alpha$, then

$$IV - CFSWGA\left(\alpha_{11}, \alpha_{12}, \ldots, \alpha_{mn}\right) = \alpha \tag{12}$$

This property is called Idempotency Property.

*Theorem 9:* Let $\alpha_{ij} = \left[ r_{ij}^{-}, r_{ij}^{+} \right] \cdot e^{i\left[\omega_{ij}^{-}, \omega_{ij}^{+}\right]}$ $(i = 1, 2, \ldots, m, j = 1, 2, \ldots, n)$ be a collection of IV-CFSNs, and

$$\alpha_{ij}^{-} = \left[ \min_m \min_n \left\{ r_{ij}^{-} \right\}, \min_m \min_n \left\{ r_{ij}^{+} \right\} \right] \cdot e^{i\left[ \min_m \min_n \left\{ \omega_{ij}^{-} \right\}, \min_m \min_n \left\{ \omega_{ij}^{+} \right\} \right]},$$

$$IV - CFSWGA\left(\alpha_{11}, \alpha_{12}, \ldots, \alpha_{34}\right)$$

$$= \left[ \prod_{j=1}^{3} \left( \prod_{i=1}^{4} \left( r_{ij}^{-} \right)^{\eta_i} \right)^{\xi_j}, \prod_{j=1}^{3} \left( \prod_{i=1}^{4} \left( r_{ij}^{+} \right)^{\eta_i} \right)^{\xi_j} \right] \cdot e^{i\left[ 2\pi \left( \prod_{j=1}^{3} \left( \prod_{i=1}^{4} \left( \frac{\omega_{ij}^{-}}{2\pi} \right)^{\eta_i} \right)^{\xi_j} \right), 2\pi \left( \prod_{j=1}^{3} \left( \prod_{i=1}^{4} \left( \frac{\omega_{ij}^{+}}{2\pi} \right)^{\eta_i} \right)^{\xi_j} \right) \right]}$$

$$= \begin{bmatrix} \left(0.7^{0.1} \times 0.4^{0.3} \times 0.2^{0.3} \times 0.9^{0.3}\right)^{0.2} \\ \times \left(0.3^{0.1} \times 0.3^{0.3} \times 0.3^{0.3} \times 0.2^{0.3}\right)^{0.4} \\ \times \left(0.5^{0.1} \times 0.9^{0.3} \times 0.6^{0.3} \times 0.1^{0.3}\right)^{0.4}, \\ \left(1^{0.1} \times 0.6^{0.3} \times 0.3^{0.3} \times 1^{0.3}\right)^{0.2} \\ \times \left(0.4^{0.1} \times 0.4^{0.3} \times 0.4^{0.3} \times 0.3^{0.3}\right)^{0.4} \\ \times \left(0.7^{0.1} \times 1^{0.3} \times 0.7^{0.3} \times 0.2^{0.3}\right)^{0.4} \end{bmatrix}$$

$$\cdot e^{i\begin{bmatrix} 2\pi \begin{pmatrix} \left(\left((\pi/6)/(2\pi)\right)^{0.1} \left((\pi/3)/(2\pi)\right)^{0.3} \left((2\pi/3)/(2\pi)\right)^{0.3} \left((\pi/6)/(2\pi)\right)^{0.3}\right)^{0.2} \\ \times \left(\left((5\pi/6)/(2\pi)\right)^{0.1} \left((\pi/6)/(2\pi)\right)^{0.3} \left((5\pi/6)/(2\pi)\right)^{0.3} \left((\pi)/(2\pi)\right)^{0.3}\right)^{0.4} \\ \times \left(\left((3\pi/2)/(2\pi)\right)^{0.1} \left((3\pi/2)/(2\pi)\right)^{0.3} \left((\pi/2)/(2\pi)\right)^{0.3} \left((\pi/2)/(2\pi)\right)^{0.3}\right)^{0.4} \end{pmatrix}, \\ 2\pi \begin{pmatrix} \left(\left((\pi/3)/(2\pi)\right)^{0.1} \left((\pi)/(2\pi)\right)^{0.3} \left((4\pi/3)/(2\pi)\right)^{0.3} \left((\pi/3)/(2\pi)\right)^{0.3}\right)^{0.2} \\ \times \left(\left((\pi)/(2\pi)\right)^{0.1} \left((\pi)/(2\pi)\right)^{0.3} \left((\pi)/(2\pi)\right)^{0.3} \left((4\pi/3)/(2\pi)\right)^{0.3}\right)^{0.4} \\ \times \left(\left((2\pi)/(2\pi)\right)^{0.1} \left((5\pi/3)/(2\pi)\right)^{0.3} \left((\pi)/(2\pi)\right)^{0.3} \left((2\pi/3)/(2\pi)\right)^{0.3}\right)^{0.4} \end{pmatrix} \end{bmatrix}}$$

$$= [0.34, 0.47] \cdot e^{i[2\pi(0.28), 2\pi(0.50)]}$$

$$\alpha_{ij}^{+}=\left[\max_{m}\max_{n}\left\{r_{ij}^{-}\right\},\max_{m}\max_{n}\left\{r_{ij}^{+}\right\}\right]\cdot e^{i\left[\max_{m}\max_{n}\left\{\omega_{ij}^{-}\right\},\max_{m}\max_{n}\left\{\omega_{ij}^{+}\right\}\right]}.$$

Then we can have

$$\alpha_{ij}^{-}\leq IV-CFSWGA\left(\alpha_{11},\alpha_{12},\ldots,\alpha_{mn}\right)\leq\alpha_{ij}^{+}\quad(13)$$

This property is called Boundedness Property.

*Theorem 10:* Let $\alpha_{ij}=\left[r_{ij}^{-},r_{ij}^{+}\right]\cdot e^{i\left[\omega_{ij}^{-},\omega_{ij}^{+}\right]}$ $(i=1,2,\ldots,m,j=1,2,\ldots,n)$ and $\alpha=\left[r^{-},r^{+}\right]\cdot e^{i\left[\omega^{-},\omega^{+}\right]}$ be IV-CFSNs. Then we can have

$$IV-CFSWGA\left(\alpha_{11}\otimes\alpha,\alpha_{12}\otimes\alpha,\ldots,\alpha_{mn}\otimes\alpha\right)=IV-CFSWGA\left(\alpha_{11},\alpha_{12},\ldots,\alpha_{mn}\right)\otimes\alpha\quad(14)$$

This property is called Shift Invariance Property.

*Theorem 11:* Let $\alpha_{ij}=\left[r_{ij}^{-},r_{ij}^{+}\right]\cdot e^{i\left[\omega_{ij}^{-},\omega_{ij}^{+}\right]}$ $(i=1,2,\ldots,m,j=1,2,\ldots,n)$ be a collection of IV-CFSNs and $\lambda>0$. Then we can have

$$IV-CFSWGA\left(\alpha_{11}^{\lambda},\alpha_{12}^{\lambda},\ldots,\alpha_{mn}^{\lambda}\right)=\left(IV-CFSWGA\left(\alpha_{11},\alpha_{12},\ldots,\alpha_{mn}\right)\right)^{\lambda}\quad(15)$$

This property is called Homogeneity Property.

## IV. EDAS ALGORITHMS BASED ON AGGREGATION OPERATORS FOR IV-CFSS IN MCGDM ENVIRONMENT

In this section, we propose the EDAS method of interval-valued complex fuzzy soft set using IV-CFSWAA operator and IV-CFSWGA operator in MCGDM environment.

Suppose the set of alternatives is $U=\{u_1,u_2,\ldots,u_l\}$, the set of experts is $K=\{k_1,k_2,\ldots,k_m\}$, and the set of parameters is $E=\{e_1,e_2,\ldots,e_n\}$. The weight vector of experts $\eta=(\eta_1,\eta_2,\ldots,\eta_m)^T$ is known with $\sum_{i=1}^{m}\eta_i=1$, and the weight vector of parameters $\xi=(\xi_1,\xi_2,\ldots,\xi_n)^T$ is completely unknown. $(F,E)_s\ (s=1,2,\ldots,l)$ is IV-CFSS. For $\forall e_j\in E$, $F_s\left(e_j\right)=\left\{\left(k_1,\alpha_{1j}\right),\left(k_2,\alpha_{2j}\right),\ldots,\left(k_m,\alpha_{mj}\right)\right\}$, $j=1,2,\ldots,n$. $\alpha_{ij}=\left[r_{ij}^{-},r_{ij}^{+}\right]\cdot e^{i\left[\omega_{ij}^{-},\omega_{ij}^{+}\right]}$ is IV-CFSN, representing the evaluation value ith expert gives to jth parameter for the sth alternative. Then we present the EDAS algorithms of IV-CFSS in MCGDM environment.

### A. ALGORITHM 1: BY USING IV-CFSWAA OPERATOR

*Step 1:* Collect the required decision information in the form of IV-CFSS as Table 3. Interval-valued complex fuzzy soft decision matrix $(F,E)_s\ (s=1,2,\ldots,l)$corresponds to each alternative.

**TABLE 3.** IV-CFSS $(F,E)_S$.

| | $e_1$ | $e_2$ | $\cdots$ | $e_n$ |
|---|---|---|---|---|
| $k_1$ | $\alpha_{11}^s$ | $\alpha_{12}^s$ | $\cdots$ | $\alpha_{1n}^s$ |
| $k_2$ | $\alpha_{21}^s$ | $\alpha_{22}^s$ | $\cdots$ | $\alpha_{2n}^s$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| $k_m$ | $\alpha_{m1}^s$ | $\alpha_{m2}^s$ | $\cdots$ | $\alpha_{mn}^s$ |

*Step 2:* For each parameter, the evaluations of all experts are aggregated into a collective evaluation to construct an aggregate matrix $\beta=\left(\beta_{sj}\right)_{l\times n}$, which can be constructed by using the IV-CFSWAA operator and be shown in Table 4.

**TABLE 4.** Aggregation matrix using IV-CFSWAA operator.

| | $e_1$ | $e_2$ | $\cdots$ | $e_n$ |
|---|---|---|---|---|
| $u_1$ | $\beta_{11}$ | $\beta_{12}$ | $\cdots$ | $\beta_{1n}$ |
| $u_2$ | $\beta_{21}$ | $\beta_{22}$ | $\cdots$ | $\beta_{2n}$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| $u_l$ | $\beta_{l1}$ | $\beta_{l2}$ | $\cdots$ | $\beta_{ln}$ |

For each parameter, the evaluation values of all experts are aggregated into a collective evaluation value. So we can get $n=1$ and $\xi_1=1$ for IV-CFSWAA operator. Then,

$$\beta_{sj}=\mathop{\oplus}_{i=1}^{m}\left(\eta_i\alpha_{ij}\right)=\left[1-\prod_{i=1}^{m}\left(1-r_{ij}^{s-}\right)^{\eta_i},1-\prod_{i=1}^{m}\left(1-r_{ij}^{s+}\right)^{\eta_i}\right]\cdot e^{i\left[2\pi\left(1-\prod_{i=1}^{m}\left(1-\frac{\omega_{ij}^{s-}}{2\pi}\right)^{\eta_i}\right),2\pi\left(1-\prod_{i=1}^{m}\left(1-\frac{\omega_{ij}^{s+}}{2\pi}\right)^{\eta_i}\right)\right]}\quad(16)$$

*Step 3:* Determine the weight vector of parameters $\xi=(\xi_1,\xi_2,\ldots,\xi_n)^T$. The $\xi_j\ (j=1,2,\ldots,n)$ is determined by the following formula:

$$\xi_j=\frac{1-M_j}{n-\sum_{j=1}^{n}M_j},\quad(17)$$

where $M_j$ is defined as the generalized entropy $M\left(F,E\right)$, as shown at the bottom of this page, for IV-CFSS.

To simplify calculations, we take $\lambda=2$.

*Step 4:* Determine the average solution by the parameter value :

$$AV=\left[AV_j\right]_{1\times n}$$

$$M\left(F,E\right)=\frac{1}{n}\sum_{j=1}^{n}\left(1-\sqrt[\frac{1}{\lambda}]{\frac{1}{2m}\sum_{i=1}^{m}\left(\begin{array}{c}\left|r_{F\left(e_j\right)}^{-}\left(x_i\right)+r_{F\left(e_j\right)}^{+}\left(x_i\right)-1\right|^{\lambda}\\+\left(\frac{1}{2\pi}\left|\omega_{F\left(e_j\right)}^{-}\left(x_i\right)+\omega_{F\left(e_j\right)}^{+}\left(x_i\right)-2\pi\right|\right)^{\lambda}\end{array}\right)}\right),\quad\lambda>0$$

where

$$AV_j = \frac{1}{l}\overset{l}{\underset{s=1}{\oplus}}(\beta_{sj}) = \left[1-\left(\prod_{s=1}^{l}\left(1-r_{sj}^{-}\right)\right)^{\frac{1}{l}}, 1-\left(\prod_{s=1}^{l}\left(1-r_{sj}^{+}\right)\right)^{\frac{1}{l}}\right] \cdot e^{i\left[2\pi\left(1-\left(\prod_{s=1}^{l}\left(1-\frac{\omega_{sj}^{-}}{2\pi}\right)\right)^{\frac{1}{l}}\right), 2\pi\left(1-\left(\prod_{s=1}^{l}\left(1-\frac{\omega_{sj}^{+}}{2\pi}\right)\right)^{\frac{1}{l}}\right)\right]} \tag{18}$$

*Step 5:* Calculate the positive distance (PDA) matrix from the average solution and the negative distance (NDA) matrix from the average solution according to the type of the parameter (cost type, benefit type), as shown below:

$$PDA = \left[PDA_{sj}\right]_{l\times n}, \tag{19}$$

$$NDA = \left[NDA_{sj}\right]_{l\times n}. \tag{20}$$

If the jth parameter is a benefit indicator, then,

$$PDA_{sj} = \frac{\max\left(0, \left(S\left(\beta_{sj}\right) - S\left(AV_{sj}\right)\right)\right)}{S\left(AV_{sj}\right)}, \tag{21}$$

$$NDA_{sj} = \frac{\max\left(0, \left(S\left(AV_{sj}\right) - S\left(\beta_{sj}\right)\right)\right)}{S\left(AV_{sj}\right)}. \tag{22}$$

If the jth parameter is a cost indicator, then,

$$NDA_{sj} = \frac{\max\left(0, \left(S\left(\beta_{sj}\right) - S\left(AV_{sj}\right)\right)\right)}{S\left(AV_{sj}\right)}, \tag{23}$$

$$PDA_{sj} = \frac{\max\left(0, \left(S\left(AV_{sj}\right) - S\left(\beta_{sj}\right)\right)\right)}{S\left(AV_{sj}\right)}. \tag{24}$$

*Step 6:* Calculate the weighted sum of $PDA_{sj}$ and $NDA_{sj}$, and calculate the formula as follows:

$$SP_s = \sum_{j=1}^{n} w_j PDA_{sj}, \tag{25}$$

$$SN_s = \sum_{j=1}^{n} w_j NDA_{sj}. \tag{26}$$

*Step 7:* Standardize the values of $SP_s$ and $SN_s$. The standardization formula is as follows:

$$NSP_s = \frac{SP_s}{\max_s\left(SP_s\right)}, \tag{27}$$

$$NSN_s = 1 - \frac{SN_s}{\max_s\left(SN_s\right)}. \tag{28}$$

*Step 8:* Calculate the appraisal score (AS) for all alternatives, and calculate the formula is shown as follows:

$$AS_s = \frac{1}{2}\left(NSP_s + NSN_s\right) \tag{29}$$

*Step 9:* Rank the evaluation scores in descending order to get the ordering of the alternatives.

### B. ALGORITHM 2: BY USING IV-CFSWGA OPERATOR

Step 1 is the same as step 1 of Algorithm 1.

*Step 2:* For each parameter, the evaluations of all experts are aggregated into a collective evaluation to construct an aggregate matrix $\beta = \left(\beta_{sj}\right)_{l\times n}$, which can be constructed by using the IV-CFSWGA operator and be shown in Table 5.

**TABLE 5. Aggregation matrix using IV-CFSWGA operator.**

| | $e_1$ | $e_2$ | $\cdots$ | $e_n$ |
|---|---|---|---|---|
| $u_1$ | $\beta_{11}$ | $\beta_{12}$ | $\cdots$ | $\beta_{1n}$ |
| $u_2$ | $\beta_{21}$ | $\beta_{22}$ | $\cdots$ | $\beta_{2n}$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| $u_l$ | $\beta_{l1}$ | $\beta_{l2}$ | $\cdots$ | $\beta_{ln}$ |

For each parameter, the evaluation values of all experts are aggregated into a collective evaluation value. So we can get $n = 1$ and $\xi_1 = 1$ for IV-CFSWGA operator. Then,

$$\beta_{sj} = \overset{m}{\underset{i=1}{\otimes}}\left(\alpha_{ij}^{s}\right)^{\eta_i} = \left[\prod_{i=1}^{m}\left(r_{ij}^{s-}\right)^{\eta_i}, \prod_{i=1}^{m}\left(r_{ij}^{s+}\right)^{\eta_i}\right] \cdot e^{i\left[2\pi\left(\prod_{i=1}^{m}\left(\frac{\omega_{ij}^{s-}}{2\pi}\right)^{\eta_i}\right), 2\pi\left(\prod_{i=1}^{m}\left(\frac{\omega_{ij}^{s+}}{2\pi}\right)^{\eta_i}\right)\right]} \tag{30}$$

*Step 3:* Determine the weight vector of parameters $\xi = (\xi_1, \xi_2, \ldots, \xi_n)^T$. The $\xi_j$ $(j = 1, 2, \ldots, n)$ is determined by the following formula:

$$\xi_j = \frac{1 - M_j}{n - \sum_{j=1}^{n} M_j}, \tag{31}$$

where $M_j$ is defined as the generalized entropy $M(F, E)$, as shown at the bottom of this page. for IV-CFSS.

To simplify calculations, we take $\lambda = 2$.

*Step 4:* Determine the average solution by the parameter value :

$$AV = \left[AV_j\right]_{1\times n}$$

$$M(F,E) = \frac{1}{n}\sum_{j=1}^{n}\left(1 - \sqrt[\frac{1}{\lambda}]{\frac{1}{2m}\sum_{i=1}^{m}\left(\begin{array}{l}\left|r_{F(e_j)}^{-}(x_i) + r_{F(e_j)}^{+}(x_i) - 1\right|^{\lambda} \\ +\left(\frac{1}{2\pi}\left|\omega_{F(e_j)}^{-}(x_i) + \omega_{F(e_j)}^{+}(x_i) - 2\pi\right|\right)^{\lambda}\end{array}\right)}\right), \quad \lambda > 0$$

**TABLE 6.** Definition of parameter index.

| | Parameter index | Specific definition |
|---|---|---|
| $e_1$ | fiscal policy | It refers to the guiding principles and codes of conduct formulated by the government on fiscal work, and consists of fiscal revenue policies and fiscal expenditure policies. The main content of the fiscal revenue policy is the taxation policy consisting of taxes and tax rates. |
| $e_2$ | monetary policy | It refers to the guiding principles and codes of conduct established by the government to manage and regulate currency circulation in order to achieve certain macroeconomic goals. It consists of credit policies and interest rate policies. |
| $e_3$ | industrial policy | It refers to the sum of the policy measures and means adopted by the government in accordance with the needs of economic development to promote the balanced development of various industrial sectors. It consists of industrial layout policies, industrial structure policies, industrial technology policies and industrial organization policies. |
| $e_4$ | income policy | It refers to the mandatory or non-mandatory policy of restricting wages and prices adopted by the government to reduce the rate of increase in general price levels. |

**TABLE 7.** IV-CFSS $(F, E)_1$ for the province A.

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ |
|---|---|---|---|---|
| $k_1$ | $[0.6,0.7].e^{i[2\pi(0.5),2\pi(0.6)]}$ | $[0.6,0.7].e^{i[2\pi(0.4),2\pi(0.6)]}$ | $[0.3,0.5].e^{i[2\pi(0.6),2\pi(0.7)]}$ | $[0.4,0.7].e^{i[2\pi(0.5),2\pi(0.6)]}$ |
| $k_2$ | $[0.7,0.8].e^{i[2\pi(0.6),2\pi(0.8)]}$ | $[0.4,0.7].e^{i[2\pi(0.6),2\pi(0.7)]}$ | $[0.6,0.7].e^{i[2\pi(0.7),2\pi(0.9)]}$ | $[0.4,0.8].e^{i[2\pi(0.5),2\pi(0.7)]}$ |
| $k_3$ | $[0.5,0.8].e^{i[2\pi(0.5),2\pi(0.7)]}$ | $[0.4,0.8].e^{i[2\pi(0.6),2\pi(0.7)]}$ | $[0.6,0.7].e^{i[2\pi(0.5),2\pi(0.7)]}$ | $[0.5,0.6].e^{i[2\pi(0.6),2\pi(0.8)]}$ |

**TABLE 8.** IV-CFSS $(F, E)_2$ for the province B.

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ |
|---|---|---|---|---|
| $k_1$ | $[0.6,0.7].e^{i[2\pi(0.5),2\pi(0.6)]}$ | $[0.7,0.8].e^{i[2\pi(0.5),2\pi(0.7)]}$ | $[0.4,0.7].e^{i[2\pi(0.4),2\pi(0.5)]}$ | $[0.5,0.8].e^{i[2\pi(0.4),2\pi(0.7)]}$ |
| $k_2$ | $[0.4,0.6].e^{i[2\pi(0.3),2\pi(0.4)]}$ | $[0.6,0.8].e^{i[2\pi(0.5),2\pi(0.6)]}$ | $[0.5,0.6].e^{i[2\pi(0.6),2\pi(0.7)]}$ | $[0.5,0.7].e^{i[2\pi(0.5),2\pi(0.6)]}$ |
| $k_3$ | $[0.5,0.6].e^{i[2\pi(0.5),2\pi(0.6)]}$ | $[0.6,0.8].e^{i[2\pi(0.5),2\pi(0.7)]}$ | $[0.4,0.7].e^{i[2\pi(0.6),2\pi(0.8)]}$ | $[0.5,0.6].e^{i[2\pi(0.5),2\pi(0.7)]}$ |

**TABLE 9.** IV-CFSS $(F, E)_3$ for the province C.

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ |
|---|---|---|---|---|
| $k_1$ | $[0.8,0.9].e^{i[2\pi(0.6),2\pi(0.7)]}$ | $[0.4,0.6].e^{i[2\pi(0.5),2\pi(0.7)]}$ | $[0.3,0.5].e^{i[2\pi(0.4),2\pi(0.7)]}$ | $[0.6,0.7].e^{i[2\pi(0.3),2\pi(0.5)]}$ |
| $k_2$ | $[0.5,0.7].e^{i[2\pi(0.3),2\pi(0.6)]}$ | $[0.6,0.8].e^{i[2\pi(0.5),2\pi(0.8)]}$ | $[0.5,0.7].e^{i[2\pi(0.6),2\pi(0.7)]}$ | $[0.6,0.8].e^{i[2\pi(0.5),2\pi(0.6)]}$ |
| $k_3$ | $[0.7,0.9].e^{i[2\pi(0.6),2\pi(0.7)]}$ | $[0.5,0.6].e^{i[2\pi(0.5),2\pi(0.6)]}$ | $[0.3,0.6].e^{i[2\pi(0.4),2\pi(0.5)]}$ | $[0.4,0.7].e^{i[2\pi(0.4),2\pi(0.6)]}$ |

where,

$$AV_j = \frac{1}{l} \underset{s=1}{\overset{l}{\otimes}} (\beta_{sj}) = \left[ \left( \prod_{s=1}^{l} \left( r_{sj}^{-} \right) \right)^{\frac{1}{l}}, \left( \prod_{s=1}^{l} \left( r_{sj}^{+} \right) \right)^{\frac{1}{l}} \right] \cdot e^{i\left[ 2\pi \left( \left( \prod_{s=1}^{l} \left( \frac{\omega_{sj}^{-}}{2\pi} \right) \right)^{\frac{1}{l}} \right), 2\pi \left( \left( \prod_{s=1}^{l} \left( \frac{\omega_{sj}^{+}}{2\pi} \right) \right)^{\frac{1}{l}} \right) \right]} \tag{32}$$

The remaining steps are the same as Algorithm 1.

## V. NUMBERICAL EXAMPLE

In this section, an example to illustrate the validity and effectiveness of our proposed EDAS algorithms in MCGDM environment in section IV is presented. The proposed algorithms can rank the degree of the impact of economic policies on certain provinces and select the province with the most significant economic policy impact.

In economic development, the economic development strategies of different historical periods and different countries are different. Under the guidance of economic strategy, national economic regulation and control policies are the dominant factors in regional economic development, affecting the development pattern, development speed and development of regional economies quality. A country's economic regulation and control policies are multifaceted. The most influential factors are fiscal policy, monetary policy, industrial policy, and income policy. The specific interpretation of each policy is shown in Table 6. Every economic policy has a lagging effect on the economy, and lag time will affect the effectiveness of economic policies. So obviously this problem is two-dimensional, IV-CFSS provides a new idea

**TABLE 10.** IV-CFSS $(F, E)_4$ for the province D.

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ |
|---|---|---|---|---|
| $k_1$ | $[0.4,0.7].e^{i[2\pi(0.3),2\pi(0.5)]}$ | $[0.7,0.8].e^{i[2\pi(0.5),2\pi(0.6)]}$ | $[0.5,0.7].e^{i[2\pi(0.5),2\pi(0.7)]}$ | $[0.6,0.8].e^{i[2\pi(0.3),2\pi(0.6)]}$ |
| $k_2$ | $[0.4,0.5].e^{i[2\pi(0.4),2\pi(0.8)]}$ | $[0.6,0.8].e^{i[2\pi(0.5),2\pi(0.6)]}$ | $[0.6,0.7].e^{i[2\pi(0.5),2\pi(0.6)]}$ | $[0.4,0.6].e^{i[2\pi(0.5),2\pi(0.6)]}$ |
| $k_3$ | $[0.5,0.8].e^{i[2\pi(0.5),2\pi(0.6)]}$ | $[0.3,0.6].e^{i[2\pi(0.4),2\pi(0.7)]}$ | $[0.5,0.7].e^{i[2\pi(0.5),2\pi(0.8)]}$ | $[0.4,0.5].e^{i[2\pi(0.4),2\pi(0.6)]}$ |

**TABLE 11.** IV-CFSS $(F, E)_5$for the province E.

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ |
|---|---|---|---|---|
| $k_1$ | $[0.4,0.7].e^{i[2\pi(0.3),2\pi(0.5)]}$ | $[0.7,0.9].e^{i[2\pi(0.6),2\pi(0.7)]}$ | $[0.5,0.7].e^{i[2\pi(0.5),2\pi(0.7)]}$ | $[0.6,0.8].e^{i[2\pi(0.5),2\pi(0.6)]}$ |
| $k_2$ | $[0.6,0.7].e^{i[2\pi(0.5),2\pi(0.6)]}$ | $[0.4,0.7].e^{i[2\pi(0.6),2\pi(0.8)]}$ | $[0.3,0.5].e^{i[2\pi(0.4),2\pi(0.7)]}$ | $[0.4,0.6].e^{i[2\pi(0.5),2\pi(0.6)]}$ |
| $k_3$ | $[0.5,0.7].e^{i[2\pi(0.5),2\pi(0.7)]}$ | $[0.5,0.7].e^{i[2\pi(0.5),2\pi(0.8)]}$ | $[0.3,0.6].e^{i[2\pi(0.4),2\pi(0.7)]}$ | $[0.8,0.9].e^{i[2\pi(0.6),2\pi(0.7)]}$ |

**TABLE 12.** Aggregation matrix using IV-CFSWAA operator.

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ |
|---|---|---|---|---|
| $u_1$ | $[0.587,0.764].e^{i[2\pi(0.522),2\pi(0.690)]}$ | $[0.490,0.745].e^{i[2\pi(0.530),2\pi(0.663)]}$ | $[0.500,0.632].e^{i[2\pi(0.587),2\pi(0.759)]}$ | $[0.442,0.690].e^{i[2\pi(0.543),2\pi(0.714)]}$ |
| $u_2$ | $[0.526,0.643].e^{i[2\pi(0.465),2\pi(0.566)]}$ | $[0.643,0.800].e^{i[2\pi(0.500),2\pi(0.682)]}$ | $[0.421,0.682].e^{i[2\pi(0.530),2\pi(0.687)]}$ | $[0.500,0.714].e^{i[2\pi(0.462),2\pi(0.682)]}$ |
| $u_3$ | $[0.717,0.875].e^{i[2\pi(0.553),2\pi(0.682)]}$ | $[0.486,0.652].e^{i[2\pi(0.500),2\pi(0.690)]}$ | $[0.346,0.587].e^{i[2\pi(0.447),2\pi(0.632)]}$ | $[0.530,0.723].e^{i[2\pi(0.385),2\pi(0.563)]}$ |
| $u_4$ | $[0.442,0.717].e^{i[2\pi(0.217),2\pi(0.451)]}$ | $[0.554,0.736].e^{i[2\pi(0.340),2\pi(0.423)]}$ | $[0.522,0.700].e^{i[2\pi(0.340),2\pi(0.486)]}$ | $[0.490,0.610].e^{i[2\pi(0.245),2\pi(0.423)]}$ |
| $u_5$ | $[0.486,0.700].e^{i[2\pi(0.428),2\pi(0.610)]}$ | $[0.577,0.807].e^{i[2\pi(0.563),2\pi(0.765)]}$ | $[0.388,0.627].e^{i[2\pi(0.442),2\pi(0.700)]}$ | $[0.671,0.795].e^{i[2\pi(0.543),2\pi(0.643)]}$ |

**TABLE 13.** PDA matrix.

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ |
|---|---|---|---|---|
| $u_1$ | 0.5020 | 0 | 1.1028 | 0.2531 |
| $u_2$ | 0 | 0.3602 | 0.4096 | 0.1559 |
| $u_3$ | 1.2071 | 0 | 0 | 0 |
| $u_4$ | 0 | 0 | 0 | 0 |
| $u_5$ | 0 | 0.5465 | 0 | 1.1061 |

to describe the problem. IV-CFSS has the advantage of IVFS, which can overcome the personal preferences of experts given information in GDM. At the same time, IV-CFSNs can also use magnitude term to describe the information of economic policy impact, and use the phase term to describe the relevant information of economic policy lag time. How to analyze the comprehensive impact of a country's economic policies on the regional economy. We give the analysis method as follows.

Suppose $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a collection of five provinces in country M, representing provinces A, B, C, D, and E, respectively, $K = \{k_1, k_2, k_3\}$ is a collection of three experts, and $E = \{e_1, e_2, e_3, e_4\}$ is a set of four

**TABLE 14.** NDA matrix.

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ |
|---|---|---|---|---|
| $u_1$ | 0 | 0.0702 | 0 | 0 |
| $u_2$ | 0.4652 | 0 | 0 | 0 |
| $u_3$ | 0 | 0.2890 | 0.9499 | 0.3537 |
| $u_4$ | 1.4598 | 0.8841 | 0.7900 | 1.7482 |
| $u_5$ | 0.4033 | 0 | 0.3068 | 0 |

**TABLE 15.** Calculated results.

| | SP | NP | NSP | NSP | AS | Ranking |
|---|---|---|---|---|---|---|
| $u_1$ | 0.4046 | 0.0200 | 0.9378 | 0.9839 | 0.9609 | 1 |
| $u_2$ | 0.2151 | 0.1331 | 0.4986 | 0.8933 | 0.6959 | 4 |
| $u_3$ | 0.3453 | 0.3411 | 0.8004 | 0.7265 | 0.7635 | 3 |
| $u_4$ | 0 | 1.2472 | 0 | 0 | 0 | 5 |
| $u_5$ | 0.4315 | 0.1704 | 1 | 0.8633 | 0.9317 | 2 |

parameter indicators, representing fiscal policy, monetary policy, industrial policy, and income policy, respectively. Let $\eta = (\eta_1 = 0.4, \eta_2 = 0.2, \eta_3 = 0.4)^T$ be the weight vector of experts. Then we can get the best alternative according to the algorithms in section IV.

$$AV = \begin{bmatrix} [0.5314, 0.7249] \cdot e^{i[0.4547,0.6164]} & [0.5190, 0.7299] \cdot e^{i[0.4941,0.6851]} \\ [0.4133, 0.6346] \cdot e^{i[0.4941,0.6851]} & [0.5042, 0.6845] \cdot e^{i[0.4497,0.6318]} \end{bmatrix}_{1\times 4}$$

**TABLE 16.** **Aggregation matrix using IV-CFSWGA operator.**

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ |
|---|---|---|---|---|
| $u_1$ | $[0.575,0.758].e^{i[2\pi(0.519),2\pi(0.676)]}$ | $[0.470,0.738].e^{i[2\pi(0.510),2\pi(0.658)]}$ | $[0.455,0.612].e^{i[2\pi(0.575),2\pi(0.736)]}$ | $[0.437,0.676].e^{i[2\pi(0.538),2\pi(0.694)]}$ |
| $u_2$ | $[0.514,0.638].e^{i[2\pi(0.451),2\pi(0.553)]}$ | $[0.638,0.800].e^{i[2\pi(0.500),2\pi(0.658)]}$ | $[0.418,0.679].e^{i[2\pi(0.510),2\pi(0.645)]}$ | $[0.500,0.694].e^{i[2\pi(0.457),2\pi(0.679)]}$ |
| $u_3$ | $[0.690,0.856].e^{i[2\pi(0.522),2\pi(0.679)]}$ | $[0.474,0.636].e^{i[2\pi(0.500),2\pi(0.676)]}$ | $[0.332,0.575].e^{i[2\pi(0.434),2\pi(0.612)]}$ | $[0.510,0.719].e^{i[2\pi(0.373),2\pi(0.558)]}$ |
| $u_4$ | $[0.437,0.690].e^{i[2\pi(0.390),2\pi(0.591)]}$ | $[0.484,0.713].e^{i[2\pi(0.457),2\pi(0.638)]}$ | $[0.519,0.700].e^{i[2\pi(0.500),2\pi(0.716)]}$ | $[0.470,0.593].e^{i[2\pi(0.373),2\pi(0.600)]}$ |
| $u_5$ | $[0.474,0.700].e^{i[2\pi(0.408),2\pi(0.593)]}$ | $[0.547,0.774].e^{i[2\pi(0.557),2\pi(0.758)]}$ | $[0.368,0.615].e^{i[2\pi(0.437),2\pi(0.700)]}$ | $[0.621,0.751].e^{i[2\pi(0.538),2\pi(0.638)]}$ |

### A. BY USING IV-CFSWAA OPERATOR

*Step 1:* The information of the impact of economic policy of country M on the regional economy is given by three experts in the form of IV-CFSS, and decision matrix $(F, E)_s$ $(s = 1, 2, 3, 4, 5)$ in Table 7-11 is for the information of five provinces A, B, C, D, E, respectively. For IV-CFSN $[0.6, 0.7]\cdot e^{i[2\pi(0.5),2\pi(0.6)]}$in Table 7, the magnitude term [0.6, 0.7] indicates that expert $k_1$ agreed 60%-70% with the impact of policy $e_1$ on province A and the phase term $[2\pi\ (0.5)\ , 2\pi\ (0.6)]$ indicates that expert $k_1$ agreed 50%-70% with the lag time for impact of policy $e_1$ on province A.

*Step 2:* Aggregate evaluation values of all experts for each parameter using IV-CFSWAA operator and aggregation matrix is as shown in Table 12.

*Step 3:* Calculate the weight vector of parameters and we can get

$$\xi = (\xi_1 = 0.2853,\ \xi_2 = 0.2932,\ \xi_3 = 0.1944,\ \xi_4 = 0.2271)^T .$$

*Step 4:* Determine the average solution by the parameter value, $AV$, as shown at the bottom of the previous page.

*Step 5:* Calculated PDA matrix from the average solution and NDA matrix from the average solution are shown in Table 13 and Table 14, respectively.

*Step 6:* Calculate the weighted sum of $PDA_{sj}$ and $NDA_{sj}$, and the calculated results are shown in Table 15.

*Step 7:* Standardize the values of $SP_s$ and $SN_s$. The results after standardization are shown in Table 15.

*Step 8:* Calculate the $AS_s$, and the results are shown in Table 15.

*Step 9:* Rank the $AS_s$ in descending order, and we can get $u_1 \succ u_5 \succ u_2 \succ u_3 \succ u_4$. So the $u_1$ is the best alternative, that is, economic policies of country M has the greatest impact on province A.

### B. BY USING IV-CFSWGA OPERATOR

*Step 2:* Aggregate evaluation values of all experts for each parameter using IV-CFSWGA operator and aggregation matrix is as shown in Table 16.

*Step 3:* Calculate the weight vector of parameters and we can get

$$\xi = (\xi_1 = 0.2861,\ \xi_2 = 0.2853,\ \xi_3 = 0.1795,\ \xi_4 = 0.2491)^T .$$

**TABLE 17.** **PDA matrix.**

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ |
|---|---|---|---|---|
| $u_1$ | 0.6135 | 0 | 0.7437 | 0.2788 |
| $u_2$ | 0 | 0.4224 | 0.1655 | 0.2230 |
| $u_3$ | 1.2829 | 0 | 0 | 0 |
| $u_4$ | 0 | 0 | 1.0051 | 0 |
| $u_5$ | 0 | 0.46937 | 0 | 1.0263 |

**TABLE 18.** **NDA matrix.**

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ |
|---|---|---|---|---|
| $u_1$ | 0 | 0.1304 | 0 | 0 |
| $u_2$ | 0.5197 | 0 | 0 | 0 |
| $u_3$ | 0 | 0.3411 | 0.1260 | 0.4086 |
| $u_4$ | 0.6690 | 0.3264 | 0 | 0.8648 |
| $u_5$ | 0.4649 | 0 | 0.4432 | 0 |

**TABLE 19.** **Calculated results.**

| | SP | NP | NSP | NSP | AS | Ranking |
|---|---|---|---|---|---|---|
| $u_1$ | 0.3829 | 0.0382 | 1 | 0.9208 | 0.9604 | 1 |
| $u_2$ | 0.2067 | 0.1483 | 0.5397 | 0.6930 | 0.6164 | 3 |
| $u_3$ | 0.3660 | 0.4292 | 0.9558 | 0.1114 | 0.5336 | 4 |
| $u_4$ | 0.1954 | 0.4830 | 0.5102 | 0 | 0.2551 | 5 |
| $u_5$ | 0.3707 | 0.2188 | 0.9681 | 0.5470 | 0.7576 | 2 |

*Step 4:* Determine the average solution by the parameter value, $AV$, as shown at the bottom of this page.

*Step 5:* Calculated positive distance (PDA) matrix from the average solution and negative distance (NDA) matrix from the average solution are shown in Table 17 and Table 18, respectively.

*Step 6:* Calculate the weighted sum of $PDA_{sj}$ and $NDA_{sj}$, and the calculation results are shown in Table 19.

*Step 7:* Standardize the values of $SP_s$ and $SN_s$. The results after standardization are shown in Table 19.

*Step 8:* Calculate the $AS_s$, and the results are shown in Table 19.

$$AV = \begin{bmatrix} [0.5633, 0.7549]\cdot e^{i[0.4480,0.6088]} & [0.5541, 0.7537]\cdot e^{i[0.4917,0.6606]} \\ [0.4393, 0.6481]\cdot e^{i[0.4759,0.6641]} & [0.5339, 0.7126]\cdot e^{i[0.4459,0.6175]} \end{bmatrix}_{1\times 4}$$

**TABLE 20. Aggregation matrix.**

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ |
|---|---|---|---|---|
| $u_1$ | $[0.5871, 0.7648]$ | $[0.4898, 0.7449]$ | $[0.4996, 0.6320]$ | $[0.4422, 0.6896]$ |
| $u_2$ | $[0.5257, 0.6435]$ | $[0.6435, 0.8000]$ | $[0.4215, 0.6822]$ | $[0.5000, 0.7138]$ |
| $u_3$ | $[0.7175, 0.8754]$ | $[0.4856, 0.6518]$ | $[0.3456, 0.5871]$ | $[0.5296, 0.7234]$ |
| $u_4$ | $[0.4422, 0.7175]$ | $[0.5540, 0.7361]$ | $[0.5218, 0.7000]$ | $[0.4898, 0.6102]$ |
| $u_5$ | $[0.4856, 0.7000]$ | $[0.5773, 0.8067]$ | $[0.3881, 0.6272]$ | $[0.6712, 0.7952]$ |

**TABLE 21. Results.**

| Methods | Results | Ranking | Optimal alternative |
|---|---|---|---|
| Algorithm1 | $AS_1 = 0.9609, AS_2 = 0.6959, AS_3 = 0.7635, AS_4 = 0, AS_5 = 0.9317$ | $u_1 \succ u_5 \succ u_2 \succ u_3 \succ u_4$ | $u_1$ |
| Algorithm1 | $AS_1 = 0.9604, AS_2 = 0.6164, AS_3 = 0.5336, AS_4 = 0.2551, AS_5 = 0.7576$ | $u_1 \succ u_5 \succ u_3 \succ u_2 \succ u_4$ | $u_1$ |
| MABAC[26] | $Q_1 = -0.0060, Q_2 = 0.0047, Q_3 = 0.0215, Q_4 = -0.0278, Q_5 = 0.0258$ | $u_5 \succ u_3 \succ u_2 \succ u_1 \succ u_4$ | $u_5$ |
| Similarity measure[26] | $S_1 = 0.1168, S_2 = 0.1278, S_3 = 0.1439, S_4 = 0.0958, S_5 = 0.1049$ | $u_3 \succ u_2 \succ u_1 \succ u_5 \succ u_4$ | $u_3$ |
| WDBA[42] | $SI_1 = 0.3241, SI_2 = 0.4772, SI_3 = 0.4509, SI_4 = 0.3395, SI_5 = 0.4989$ | $u_5 \succ u_2 \succ u_3 \succ u_4 \succ u_1$ | $u_5$ |
| CODAS[42] | $RA_1 = -0.1285, RA_2 = -0.0103, RA_3 = 0.4559, RA_4 = -0.2924, RA_5 = 0.1060$ | $u_3 \succ u_5 \succ u_2 \succ u_1 \succ u_4$ | $u_3$ |
| Similarity measure[42] | $S_1 = 0.8062, S_2 = 0.8106, S_3 = 0.8124, S_4 = 0.7963, S_5 = 0.8192$ | $u_5 \succ u_3 \succ u_2 \succ u_1 \succ u_4$ | $u_5$ |

*Step 9:* Rank the $AS_s$ in descending order, and we can get $u_1 \succ u_5 \succ u_3 \succ u_2 \succ u_4$. So the $u_1$ is the best alternative, that is, economic policies of country M has the greatest impact on province A.

## VI. COMPARATIVE ANALYSIS AND FURTHER DISCUSSION

There is no research on the MCGDM method for interval-valued complex fuzzy soft information. So, in this section, we compare the proposed methods with the existing methods in the interval-valued fuzzy soft environment. First, the expert's evaluation values need to be converted into the form of interval-valued fuzzy soft sets (IVFSSs) by taking the phase term of the IV-CFSNs to 0. Then, the interval-valued fuzzy soft numbers of different experts are aggregated by weighted averaging operator corresponding to the weight vector of experts $\eta = (\eta_1 = 0.4, \eta_2 = 0.2, \eta_3 = 0.4)^T$. The aggregated interval-valued fuzzy soft matrix for different alternatives can be computed in Table 20. Based on aggregated interval-valued fuzzy soft matrix, we apply the existing methods including MABAC method [26], similarity measure method [26], Weighted Distance Based Approximation (WDBA) method [42], Combinative Distance-based Assessment (CODAS) method [42], and similarity measure method [42] to obtain the assessment results. The computed results are shown in Table 21. From the Table 21, we can see that the optimal alternative calculated by the algorithms proposed in this paper is $u_5$, which is different from the results calculated by the existing methods. The reason for this result is that this paper converts IV-CFSSs to IVFSSs by taking the phase term of IV-CFSNs as zero at the very beginning of this section. In this way, when assessing the impact of national economic policies on regional economies, the impact of policy time lags on the economy cannot be considered. This is not comprehensive and sufficient in the description of the information. Therefore, in the MCGDM environment, the EDAS algorithms for interval-valued complex fuzzy soft information proposed in this paper are advantageous.

## VII. CONCLUSION

The EDAS methods are useful decision-making technique that can solve the MCGDM problem. In the MCGDM environment, interval-valued complex fuzzy soft EDAS methods are developed for economic analysis problems. On the one hand, this paper extends the application of the EDAS methods to interval-valued complex fuzzy soft environment. On the other hand, we have improved the EDAS methods for more complex MCGDM environment. In addition, we define the aggregation operators for IV-CFSS, namely the IV-CFSWAA operator and the IV-CFSWGA operator. In this study, we successfully introduce the proposed methods to assess the impact of national economic policies on the region. In future research, the focus is on the promotion and application of complex fuzzy soft sets in other fuzzy sets.

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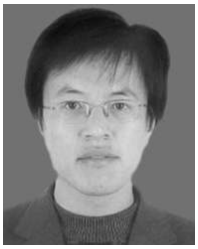

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