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# Extended Look-Ahead Tracking Controller With Orientation-Error Observer for Vehicle Platooning

Anggera Bayuwindra<sup>ID</sup>, Erjen Lefeber<sup>ID</sup>, Jeroen Ploeg<sup>ID</sup>, and Henk Nijmeijer<sup>ID</sup>, *Fellow, IEEE*

**Abstract**—This paper presents a novel extended look-ahead concept of an integrated lateral and longitudinal vehicle following controller with an orientation-error observer. The control law is based on input-output feedback to address a local tracking problem. It is known that due to the position control in the look-ahead approach, the follower vehicle may cut corners. To address this problem, a reference-induced extended look-ahead tracking point is introduced such that the cutting-corner is compensated. Moreover, the stability of the internal dynamics is analyzed. To address the situation where the orientation tracking error is not measurable or corrupted by noise, an orientation-error observer, constructed from the position tracking error, is designed. The performance of the extended look-ahead controller and the orientation-error observer is investigated by means of a simulation study, and validated with experiments on a mobile robot platform.

**Index Terms**—Control algorithm, vehicle platooning, observer, cascaded system, longitudinal and lateral control.

## I. INTRODUCTION

IN RECENT years, the increasing needs for mobility has caused a high need of transportation. One solution to compensate for the increasing number of vehicles is to develop more infrastructure or to increase the capacity of existing infrastructure. Since the development of the infrastructure is time consuming, costly, and infeasible in some situations, increasing highway capacity is seen as the most effective solution. One of the methods to increase the highway capacity is vehicle platooning. With the concept of automated vehicles, platooning allows a vehicle to drive closer to its preceding vehicle by eliminating the reaction time of human driver. The concept of vehicle platooning in longitudinal movement is realized through Cooperative Adaptive Cruise Control (CACC). CACC, as an extension of ACC, utilizes wireless communications between vehicles (V2V communication) so that acceleration information of the preceding vehicle can be used as a feed-forward term to attenuate disturbances along the

platoon [1]. This is an advantage of CACC over ACC, whereas the disturbance in ACC may be amplified in the upstream direction [2]. For lateral movement, vehicle platooning control can be designed by two main approaches, i.e., a path following approach and a trajectory tracking approach. In path following, the control objective is to drive the vehicle over a desired path without any time constraint, i.e., there is no requirement of when the vehicle should arrive at a certain point. Since there is no time requirement, the vehicle's longitudinal velocity can be freely regulated, independent of the position on the spatial path [3]. On the other hand, in the trajectory tracking approach, the desired path is parameterized with respect to time, i.e., the vehicle is required to be at a specific position along the path at a specific time.

In [4], a lane keeping controller based on the path following approach, is designed such that the vehicle follows a reference path, e.g., the path composed of lane markings (either road surface or embedded magnetic markings) using a camera or magnetic sensor, known as a “look-down” technique. It should be noted that the term “look-down” is rather loose since the vehicle also requires to look for the lane markings in front of it. The control objective of a lane keeping is to design a steering input that brings the lateral error, i.e., distance from the vehicle's position to the path, to zero. Most path following methods address the control problem by assigning the motion along a path in a single coordinate. The single intrinsic coordinate system used in the path following itself is known as a Serret-Frenet reference frame, where the origin is determined by the projection of the vehicle [3]. The projection of the vehicle's position onto the path is then used as reference for the control problem. The research in [5] has shown that the orthogonal projection with respect to the path has a local character in the sense that the vehicle has at first to get to the desired path orthogonally before it can project itself on the path. Since in the Serret-Frenet frame the longitudinal distance has been transformed into a curvilinear distance, the longitudinal control then can be realized through CACC. The combination of a lane keeping controller and CACC becomes a trajectory control problem, since there is a time requirement to be fulfilled. With this combined approach, the follower vehicles in a platoon drive in the exact same path as the leader vehicle, and the spacing distance objective can be fulfilled with the CACC controller [6]. However, the lane keeping performance in this approach relies heavily on the reference markers and V2I (vehicle-to-infrastructure) communication to provide the platoon with information about

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road structure. From the viewpoint of vehicle platooning, the major disadvantage of the path following approach is when the inter-vehicle distance is getting small, the look-down system is unable to track the lane markings accurately as they are obstructed by the preceding vehicle [7].

As an alternative to lane keeping, a direct vehicle-following control is designed. A direct vehicle-following control uses the current preceding vehicle's position as a reference while keeping a desired distance. The vehicle-following control of vehicle platoons was developed from CACC and was first designed for a longitudinal control [8]–[10]. In this approach, the follower vehicle tracks the current position of the preceding vehicle by using a camera (or lidar) and determines the relative distance with respect to the follower vehicle, commonly known as a “look-ahead” technique. The vehicle-following control was then extended to both longitudinal and lateral control in [11]–[13]. The objective of this longitudinal and lateral vehicle-following control is to minimize the error between the measured relative distance and the desired distance (e.g., spacing policy in CACC), and to minimize the lateral error with respect to the preceding vehicle's path. One of the main challenges in this approach is to determine the path of the preceding vehicle. Since the follower vehicle can only measure the distance as a straight line (as opposed to the curvilinear distance in the Serret-Frenet frame), the follower vehicle can deviate from the path of the preceding vehicle during cornering, known as corner cutting. In [12], a reference virtual point, which is positioned at a desired known distance behind the lead vehicle, is proposed to compensate the corner cutting. The results shows that the proposed solution was able to compensate the corner cutting for the path with small curvatures, but was ineffective for the path with large curvatures. In [14], an extended look-ahead approach has been designed, based on dynamic feedback linearization, to compensate for the corner cutting. The extended look-ahead uses the velocity and heading information of the preceding vehicle (which are available from radar and V2V communication) to create a virtual reference-induced look-ahead point as a new tracking objective for the follower vehicle. The results are then elaborated in [15] with the formal stability analysis, where the stability of the internal dynamics is guaranteed under bounded curvatures, lateral jerk, and acceleration of the preceding vehicle. The error dynamics in [14], [15] are defined as a global tracking problem, in which the position and orientation of each vehicle is assumed to be measurable with respect to a global (i.e., fixed) coordinate frame. The shortcoming of this method is that the global position and orientation of vehicles are commonly not available in practical situations.

The main contribution of this paper is the design of the extended look-ahead controller as a local tracking problem, where the error dynamics are defined with respect to the target position of the follower vehicle. The advantage of our proposed controller to the path-following control (e.g., [3], [5]) is that it does not need lane markings and utilizes the already available information from CACC setup, thus providing benefits for a practical implementation and cost-efficiency. To study the internal dynamics of the resulting system, a formal stability

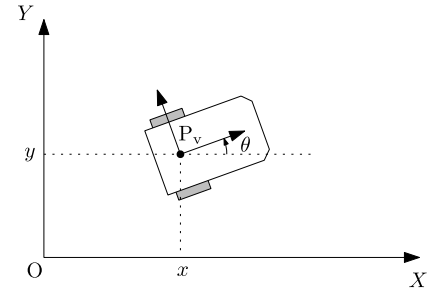


Fig. 1. A unicycle mobile robot in Cartesian coordinates.

analysis is provided. Moreover, the control strategy is then further extended with an orientation-error observer, addressing the situation where the relative orientation between vehicles is not measurable, or corrupted by noise. The effectiveness of the extended look-ahead controller against corner-cutting is demonstrated by a simulation case study, and further validated by an experiment with mobile robots.

The organization of this paper is as follows: Section II presents the concept of the extended look-ahead control design for vehicle platoons, starting with the problem formulation. The extended look-ahead controller is proposed, and a stability analysis is subsequently provided. Section III presents the design of the orientation-error observer. The results of the simulation study are presented in Section IV. For further validation, the extended look-ahead controller and the orientation-error observer are implemented in a mobile robot platform in Section V. Finally, the concluding remarks are discussed in Section VI.

## II. CONTROL OF VEHICLE PLATOONING WITH EXTENDED LOOK-AHEAD

### A. Problem Formulation

Consider a unicycle-type vehicle with the posture  $[x(t), y(t), \theta(t)]^T$  (see Fig. 1) that can be described by following differential equations

$$\dot{x} = v \cos \theta \quad (1a)$$

$$\dot{y} = v \sin \theta \quad (1b)$$

$$\dot{\theta} = \omega, \quad (1c)$$

where  $P_v = (x, y)$  are the Cartesian coordinates of the axle center of the vehicle,  $\theta$  is the orientation of the vehicle with respect to the global  $X$  axis,  $v$  is the linear velocity input and  $\omega$  is the angular velocity input of the vehicle.

Consider a reference vehicle with the posture  $[x_r(t), y_r(t), \theta_r(t)]^T$  and the kinematics given by

$$\dot{x}_r = v_r \cos \theta_r \quad (2a)$$

$$\dot{y}_r = v_r \sin \theta_r \quad (2b)$$

$$\dot{\theta}_r = \omega_r, \quad (2c)$$

where  $(x_r, y_r)$  are the Cartesian coordinates of the axle center of the reference vehicle,  $\theta_r$  is the orientation of the reference vehicle with respect to the global  $X$  axis,  $v_r$  and  $\omega_r$  are the reference velocity and angular velocity

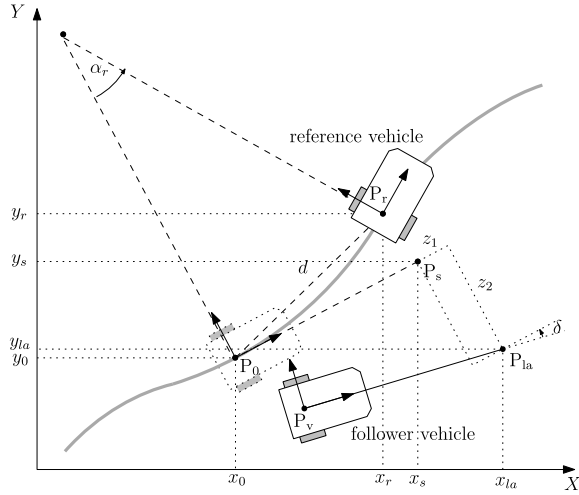


Fig. 2. Trajectory tracking problem with extended look-ahead, where  $|P_0 P_r| = |P_0 P_s| = d$ .

input, respectively. The trajectory tracking problem is typically solved by stabilizing the position of  $P_v$  with respect to the reference  $P_r$  and orientation of  $\theta$  with respect to the reference orientation  $\theta_r$ . The relative kinematics between these points can be determined with respect to the follower vehicle frame (e.g., see [16]–[19]), the reference vehicle frame (e.g., see [3], [12]), or any moving frame, which results in different error dynamics. In our approach, we choose the relative kinematics with respect to the frame of the desired posture of the follower vehicle. This choice makes the mathematical development easier than other frame choices, as explained in the next section.

Now consider a trajectory tracking problem with a look-ahead distance (referred as a vehicle-following control problem), in which the objective of the follower vehicle is to follow the reference vehicle at a desired distance  $d$ . We define  $P_0$  as a target point of the follower vehicle, and  $P_{la}$  as a look-ahead point attached to the follower vehicle. The coordinates of  $P_{la}$  are defined as

$$x_{la} = x + d \cos \theta \quad (3a)$$

$$y_{la} = y + d \sin \theta, \quad (3b)$$

where  $(x, y)$  are the Cartesian coordinates of the follower vehicle,  $|P_0 P_r| = d$ , and the distance  $d \in \mathbb{R}_+$  (see Fig. 2). With this look-ahead point, the control objective of the vehicle following problem could then be to stabilize at zero the tracking errors  $(x_{la} - x_r, y_{la} - y_r)$  of that point  $P_{la}$  with respect to the reference point  $P_r$ . However, in a curve, the leader-follower vehicle system has a unique instantaneous center of rotation (ICR), such that the line through the axle of each unicycle goes through this ICR. Consequently, when  $(x_{la}, y_{la})$  have converged to  $(x_r, y_r)$ , the follower vehicle will drive at a shorter distance to the ICR, i.e., it will cut corners [12], [14]. It is interesting to remark that this problem is analogous to a truck-trailer combination, see Fig. 3. On cornering maneuvers, a trailer coupled to the truck will also have the cutting-corner problem and human drivers solve this problem by letting the truck turn at the point in front of the cornering point, denoted

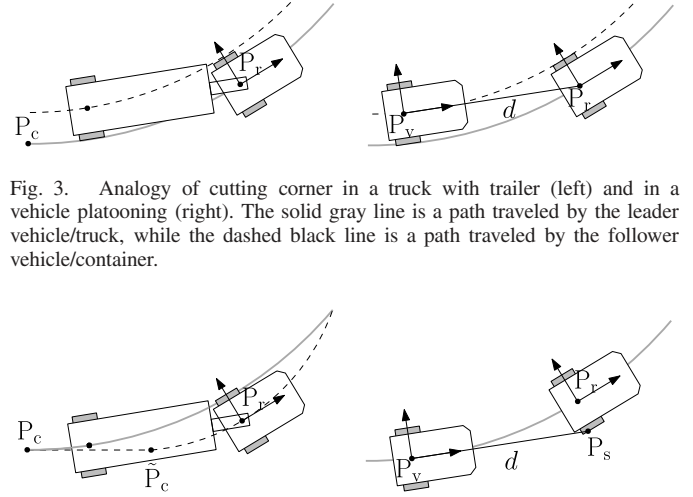


Fig. 3. Analogy of cutting corner in a truck with trailer (left) and in a vehicle platooning (right). The solid gray line is a path traveled by the leader vehicle/truck, while the dashed black line is a path traveled by the follower vehicle/container.

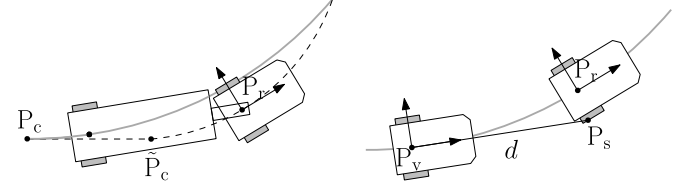


Fig. 4. Compensation of the cutting corner problem in a truck with container (left), and in a vehicle platooning (right).

by  $\tilde{P}_c$ , such that the trailer will travel on the desired arc, see Fig. 4(left).

Based on the same approach, we extend the look-ahead point in our error dynamics, thus creating a “reference-induced look-ahead point” as the new tracking point objective (denoted by  $P_s$ ) for the follower vehicle such that cutting corner can be compensated. The position of  $P_s$  in the Cartesian coordinate system is defined by  $(x_s, y_s)$  and formulated such that the distance of  $|P_v P_s|$  equals the look-ahead distance  $d$  (Fig. 4(right)). In other words, the “reference-induced look-ahead point”  $P_s$  can be regarded as the position of where the look-ahead point  $P_{la}$  should be. With this new look-ahead point  $P_s$ , our control objective is then to stabilize at zero the tracking errors  $(x_{la} - x_s, y_{la} - y_s)$ , see Fig. 2. Before we define the tracking error, first we shall derive the position of  $P_s$  geometrically based on the position of reference vehicle  $P_r$ .

#### B. Derivation of the Reference-Induced Look-Ahead Point $P_s$

To derive the position of  $P_s$ , let us first denote  $P_0$  as a moving origin point, where  $(x_0, y_0)$  is the position of  $P_0$  in the Cartesian coordinate system,  $\theta_0$  is the angle with respect to the global  $X$  axis, and  $|P_0 P_r| = d$ . This point  $P_0$  can also be considered as the position of where the follower vehicle  $P_v$  should be. Define  $\alpha_r$  as the angle of the circular arc formed by  $P_0$  and  $P_r$ , see Fig. 5. To derive the angle  $\alpha_r$ , let us denote  $\kappa_r$  as the curvature of the reference vehicle, which is defined as

$$\kappa_r := \frac{d\theta_r}{ds_r} = \left( \frac{d\theta_r}{dt} \right) / \left( \frac{ds_r}{dt} \right) = \frac{\omega_r}{v_r}, \quad (4)$$

where  $v_r \neq 0$ ,  $\theta_r$  is the orientation of the reference vehicle (which can also be considered as the angle of the tangent to the curve or path), and  $s_r$  is the curvilinear coordinate. On a curved path, it can be observed that  $d$  characterizes the chord length of a circular segment formed by  $P_0$  and  $P_r$ , and the angle  $\alpha_r$  can be defined as

$$\alpha_r = 2 \arcsin \left( \frac{1}{2} d \kappa_r \right), \quad (5)$$



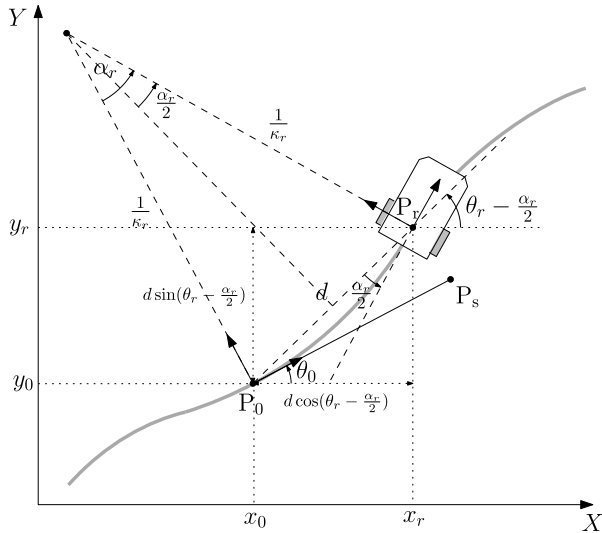


Fig. 5. Projection of the reference-induced look-ahead point  $P_s$ .

where  $|\kappa_r| \leq \kappa^{\max} < 1/d$ , and  $\kappa^{\max}$  is a constant, maximum value of the curvature of the reference vehicle. It should be noted that  $\alpha_r$  is defined as a function of the curvature  $\kappa_r$  of the reference vehicle, which fully determines the circular arc. Thus, it is not a circular arc through any two arbitrary points, but it is an arc through the position of  $P_r$  with a known curvature. By noting that (see Fig. (5))

$$\sin \frac{\alpha_r}{2} = \frac{1}{2} d\kappa_r, \quad \cos \frac{\alpha_r}{2} = \frac{\sqrt{4 - d^2 \kappa_r^2}}{2}, \quad (6)$$

the derivative of  $\alpha_r$  with respect to time is obtained as

$$\dot{\alpha}_r = \frac{2d}{\sqrt{4 - d^2\kappa_r^2}} \dot{\kappa}_r. \quad (7)$$

Since the length of  $P_0P_r$  equals the desired inter-vehicle distance  $d$ , the position of  $P_0$  in a global Cartesian coordinate system can be defined as

$$x_0 = x_r - d \cos \left( \theta_r - \frac{\alpha_r}{2} \right) \quad (8a)$$

$$y_0 = y_r - d \sin \left( \theta_r - \frac{\alpha_r}{2} \right). \quad (8b)$$

First we define the rotation matrix as

$$R(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}. \quad (9)$$

From Fig. 5, it can be observed that the coordinate of  $P_s$  can be obtained by a rotation of  $-\frac{\alpha_r}{2}$  from the coordinate  $P_r$  around  $(x_0, y_0)$  as

$$\begin{bmatrix} x_s \\ y_s \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + R^T \left( \frac{\alpha_r}{2} \right) \begin{bmatrix} x_r - x_0 \\ y_r - y_0 \end{bmatrix}. \quad (10)$$

By substituting (8) into (10), applying the angle sum formula, and noting that  $R(\theta_r - \alpha_r) = R(\theta_r)R^T(\alpha_r)$ , we can eventually

rewrite (10) as

$$\begin{aligned} \begin{bmatrix} x_s \\ y_s \end{bmatrix} &= \begin{bmatrix} x_r \\ y_r \end{bmatrix} + d \begin{bmatrix} \cos(\theta_r - \alpha_r) - \cos\left(\theta_r - \frac{\alpha_r}{2}\right) \\ \sin(\theta_r - \alpha_r) - \sin\left(\theta_r - \frac{\alpha_r}{2}\right) \end{bmatrix} \\ \begin{bmatrix} x_s \\ y_s \end{bmatrix} &= \begin{bmatrix} x_r \\ y_r \end{bmatrix} + dR(\theta_r - \alpha_r) \begin{bmatrix} 1 - \cos\frac{\alpha_r}{2} \\ -\sin\frac{\alpha_r}{2} \end{bmatrix}, \end{aligned} \quad (11)$$

where  $R(\theta_r - \alpha_r)$  as the rotation matrix through an angle  $\theta_r - \alpha_r$ . It can be observed that the position of the reference-induced look-ahead point  $P_s$  depends on the position of the reference vehicle  $P_r$ , the angle  $\alpha_r$ , and the angle  $\theta_r - \alpha_r$ , which in fact, is the desired orientation of the follower vehicle. On a straight path,  $\alpha_r = 0$ , thus  $P_s$  will be equal to  $P_r$ . In the following section, we define the error dynamics with the extended look-ahead approach.

### C. Error Dynamics and Controller Design of the Extended Look-Ahead

We consider the trajectory tracking problem between  $P_{la}$  and  $P_s$ , expressed in the frame of the desired posture of the follower vehicle, with origin  $P_0$ . It should be noted that we define the relative kinematics with respect to this particular frame because we want to cancel the rotation matrix  $R(\theta_r - \alpha_r)$  factor in (11). Hence, the error state components are defined as

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = R^T (\theta_r - \alpha_r) \begin{bmatrix} x_{la} - x_s \\ y_{la} - y_s \end{bmatrix}, \quad (12)$$

with  $x_{la} = x + d \cos \theta$ ,  $y_{la} = y + d \sin \theta$ , and  $[x_s, y_s]^T$  as described in (11). It can be seen directly that  $[z_1, z_2]^T$  denotes the relative position error. To obtain the error dynamics, we start first by differentiating  $[z_1, z_2]^T$  with respect to time and taking (11) and (5) into account, resulting in

$$\begin{aligned} \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} &= (\omega_r - \dot{\alpha}_r) \begin{bmatrix} z_2 \\ -z_1 \end{bmatrix} + \begin{bmatrix} \cos \delta & -d \sin \delta \\ \sin \delta & d \cos \delta \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \\ &\quad - \begin{bmatrix} \cos \alpha_r \\ \sin \alpha_r \end{bmatrix} v_r - \begin{bmatrix} \sin \frac{\alpha_r}{2} \\ 1 - \cos \frac{\alpha_r}{2} \end{bmatrix} d \omega_r \\ &\quad + d \dot{\alpha}_r \begin{bmatrix} \frac{1}{2} \sin \frac{\alpha_r}{2} \\ 1 - \frac{1}{2} \cos \frac{\alpha_r}{2} \end{bmatrix}, \end{aligned} \quad (13)$$

where

$$\delta = \theta - \theta_r + \alpha_r. \quad (14)$$

By applying the double angle formula on  $\sin \alpha_r$  and  $\cos \alpha_r$ , substituting (6) and (4) into (13), we eventually obtain the error dynamics as

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = (\omega_r - \dot{\alpha}_r) \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} - \begin{bmatrix} v_r \\ d\omega_r \end{bmatrix} + \begin{bmatrix} \cos \delta & -d \sin \delta \\ \sin \delta & d \cos \delta \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} + \begin{bmatrix} h_{\kappa,1} \\ h_{\kappa,2} \end{bmatrix} \dot{\kappa}_r \quad (15)$$

with

$$h_{\kappa,1} = \frac{d^3 \kappa_r}{2\sqrt{4 - d^2 \kappa_r^2}}, \quad h_{\kappa,2} = \frac{4d^2 - d^2 \sqrt{4 - d^2 \kappa_r^2}}{2\sqrt{4 - d^2 \kappa_r^2}}. \quad (16)$$

If the follower vehicle converges to its desired position,  $\theta$  converges to  $\theta_r - \alpha_r$ . Hence,  $\delta$  in (14) is, in fact, the angular error of the follower vehicle. It can be observed that error dynamics (15) consist of: a linear time-varying term multiplying  $[z_1, z_2]^T$ , since  $\omega_r$  and  $\dot{\alpha}_r$  are external time-varying parameters; and a nonlinear term multiplying inputs  $[v, \omega]^T$ , since  $\delta$  is a state of the system. The objective now is to design control laws  $[v, \omega]^T$  that asymptotically stabilize the system (15) at the origin, based on input-output feedback linearization in [20, Chapter 13], [21]. Since the matrix multiplying  $[v, \omega]^T$  is invertible, by choosing the control inputs

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} \cos \delta & \sin \delta \\ -\frac{1}{d} \sin \delta & \frac{1}{d} \cos \delta \end{bmatrix} \begin{bmatrix} -k_1 z_1 + v_r - h_{\kappa,1} \dot{\kappa}_r \\ -k_2 z_2 + d\omega_r - h_{\kappa,2} \dot{\kappa}_r \end{bmatrix}, \quad (17)$$

where  $d > 0$ , we obtain the closed-loop system as follows

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} -k_1 & \omega_r - \dot{\alpha}_r \\ -\omega_r + \dot{\alpha}_r & -k_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}. \quad (18)$$

Noted that by the input-output feedback linearization, we obtain a closed-loop system which is a linear time-varying (LTV) system. Hence, the Lyapunov stability criterion is used to prove the stability of the closed-loop system. By the choice of  $k_1, k_2 > 0$ , it can be directly verified that the origin of subsystem  $[z_1, z_2]^T$  is exponentially stable by the Lyapunov function  $V_{12}(z_1, z_2) = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2$  such that

$$\dot{V}_{12}(z_1, z_2) = -k_1 z_1^2 - k_2 z_2^2 < 0, \quad (19)$$

for  $(z_1, z_2) \neq 0$ , which means that  $\dot{V}_{12}$  is negative definite in  $(z_1, z_2)$ . Since, however, the model (1) is of third order and the error dynamics (15) are of second order, first-order internal dynamics are present. The internal dynamics are the unobservable part of the system dynamics that comply with the desired output, while the zero dynamics is the internal dynamics of the system when the system output is kept at zero by the input [20]. It should be noted that analyzing the (global) stability of the internal dynamics has a more generic meaning than only analyzing the stability of the zero dynamics. Therefore, in the next section we analyze the stability of the internal dynamics.

#### D. Stability Analysis of the Internal Dynamics

From Section II.C, it has been shown that the control law (17) exponentially stabilizes the second-order error dynamics, which leaves us with the first-order internal dynamics since the original model (1) is of third order. The first obvious choice for the internal state would be  $\delta$ , since  $\delta$  resembles the orientation error between the actual and the desired orientation of the follower vehicle. However, in a steady state condition, which implies that  $z_1 = z_2 = 0$  and  $\dot{\kappa}_r = 0$ , this choice leads to two equilibrium points  $\delta = 0$  and  $\delta = \arctan[(-2d\kappa_r)/(d^2\kappa_r^2 - 1)]$ , where the physical interpretation of these points is depicted in Fig. 6 (see Appendix A for the derivation). The posture  $(x_1, y_1, \theta_1)$  is the stable equilibrium point, and can be considered as the correct posture of where the follower vehicle should be. On the other hand,  $(x_2, y_2, \theta_2)$  is the unstable equilibrium point, and depends on the curvature of the preceding vehicle. Due to

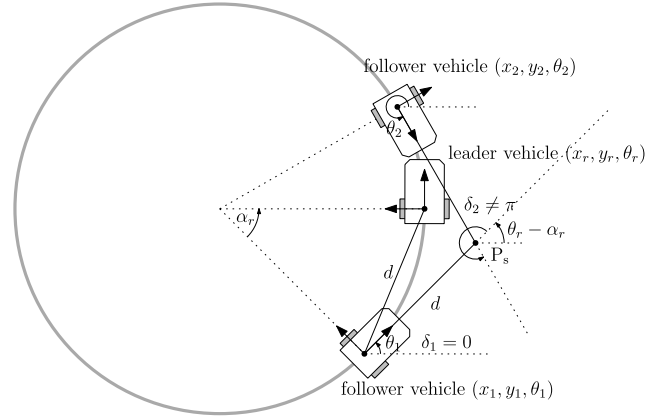


Fig. 6. Posture of the follower vehicle,  $(x_1, y_1, \theta_1)$  and  $(x_2, y_2, \theta_2)$ , on two equilibrium points. In the stable equilibrium point  $(x_1, y_1, \theta_1)$ ,  $\delta_1 = 0$ ; and in the unstable equilibrium point  $(x_2, y_2, \theta_2)$ ,  $\delta_2 \neq \pi$ .

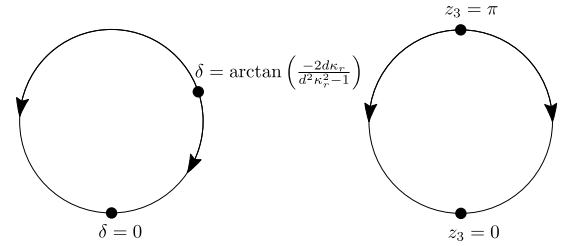


Fig. 7. The mapping of the equilibrium points from  $\delta$  (left) to  $z_3$  (right). By this mapping, the stable equilibrium point is at  $z_3 = 0$ ; and the unstable equilibrium point maps to  $z_3 = \pi$ .

the curvature-dependence of this unstable equilibrium point, we decided to define  $z_3$  such that the stable equilibrium point corresponds with  $z_3 = 0$  and the unstable equilibrium point with  $z_3 = \pi$  (see Fig. 7). To that end, we define

$$z_3 = \delta + \beta, \quad (20)$$

where the angle  $\beta$  is characterized by (see Appendix B for the derivation)

$$\sin \beta = \frac{d\kappa_r \cos \delta - d\kappa_r}{\sqrt{d^2\kappa_r^2 (1 - \cos \delta)^2 + (1 + d\kappa_r \sin \delta)^2}}, \quad (21a)$$

$$\cos \beta = \frac{1 + d\kappa_r \sin \delta}{\sqrt{d^2\kappa_r^2 (1 - \cos \delta)^2 + (1 + d\kappa_r \sin \delta)^2}}. \quad (21b)$$

where  $|\kappa_r| \leq \kappa_r^{\max} < 1/d$ . Note that from (20) and straightforward application of the trigonometric rules for the sum of angles, we have

$$\sin z_3 = \frac{\sin \delta + d\kappa_r (1 - \cos \delta)}{\sqrt{d^2\kappa_r^2 (1 - \cos \delta)^2 + (1 + d\kappa_r \sin \delta)^2}}, \quad (22a)$$

$$\cos z_3 = \frac{\cos \delta + d\kappa_r \sin \delta}{\sqrt{d^2\kappa_r^2 (1 - \cos \delta)^2 + (1 + d\kappa_r \sin \delta)^2}}. \quad (22b)$$

Moreover, the derivative of  $\beta$  with respect to time follows from the inverse tangent function, derived from (21a) and (21b),

yielding

$$\dot{\beta} = \frac{d\dot{\kappa}_r \cos \delta - d\dot{\kappa}_r}{d^2\kappa_r^2 (1 - \cos \delta)^2 + (1 + d\kappa_r \sin \delta)^2} - \left( \frac{d\kappa_r \sin \delta + d^2\kappa_r^2 (1 - \cos \delta)}{d^2\kappa_r^2 (1 - \cos \delta)^2 + (1 + d\kappa_r \sin \delta)^2} \right) \dot{\delta}. \quad (23)$$

Using (21), (22), (17), and noting the fact that  $\dot{\delta} = \omega - \omega_r + \dot{\alpha}_r$ , we obtain the derivative of  $z_3$  with respect to time as

$$\begin{aligned} \dot{z}_3 &= \frac{d\dot{\kappa}_r \cos \delta - d\dot{\kappa}_r}{d^2\kappa_r^2 (1 - \cos \delta)^2 + (1 + d\kappa_r \sin \delta)^2} \\ &+ \left( \frac{d^2\kappa_r^2 (1 - \cos \delta) + d\kappa_r \sin \delta + 1}{d^2\kappa_r^2 (1 - \cos \delta)^2 + (1 + d\kappa_r \sin \delta)^2} \right) \dot{\delta} \\ &= f_3(z), \end{aligned} \quad (24)$$

where

$$f_3(z) = -\frac{\bar{v}_r}{d} \sin z_3 + \zeta_r \quad (25)$$

$$\bar{v}_r = \frac{N}{\sqrt{\Delta}} v_r \quad (26)$$

$$\zeta_r = \frac{N}{\Delta} \left( \frac{k_1 \sin \delta}{d} z_1 - \frac{k_2 \cos \delta}{d} z_2 \right) + g_\kappa \dot{\kappa}_r \quad (27)$$

$$N = d^2\kappa_r^2 (1 - \cos \delta) + d\kappa_r \sin \delta + 1 \quad (28)$$

$$\Delta = d^2\kappa_r^2 (1 - \cos \delta)^2 + (1 + d\kappa_r \sin \delta)^2 \quad (29)$$

$$g_\kappa = \frac{N}{\Delta} f_r(\delta, d, \kappa_r) - \frac{d(1 - \cos \delta)}{\Delta} \quad (30)$$

$$\begin{aligned} f_r(\delta, d, \kappa_r) &= \frac{4d}{2\sqrt{4 - d^2\kappa_r^2}} + \frac{d^2\kappa_r}{2\sqrt{4 - d^2\kappa_r^2}} \sin \delta \\ &- \frac{4d - d\sqrt{4 - d^2\kappa_r^2}}{2\sqrt{4 - d^2\kappa_r^2}} \cos \delta, \end{aligned} \quad (31)$$

$z = (z_1, z_2, z_3)$ ,  $(N, \Delta) > 0$  (see (60) and (61) in Appendix C), and  $\delta$  as in (14). It should be noted that (24) is a closed-loop system since the inputs  $(v, \omega)$  have been taken into account. Thus, the overall closed-loop system is composed of (18) and (24), which is a third order system.

*Remark 1:* Note that since  $|\kappa_r| \leq \kappa_r^{\max} < 1/d$ ,  $\bar{v}_r$  is lower- and upper-bounded by

$$|v_r| \leq |\bar{v}_r| < \sqrt{2} |v_r|, \quad (32)$$

and  $|\zeta_r|$  is bounded by

$$|\zeta_r| \leq \frac{2+\sqrt{2}}{d} k_1 |z_1| + \frac{2+\sqrt{2}}{d} k_2 |z_2| + \frac{7d}{9} |\dot{\kappa}_r|. \quad (33)$$

*Proof of (32) and (33):* See Appendix C. ■

Using these bounds on  $|\bar{v}_r|$  and  $|\zeta_r|$ , asymptotic stability of the internal dynamics (24) can be concluded by the following proposition.

*Proposition 2:* Consider the dynamics (24) where  $\bar{v}_r$  and  $\zeta_r$  are given in (26) and (27), respectively. Let  $z_{12} = [z_1, z_2]^T$ , and assume for all  $t \geq 0$  that  $0 < v_r^{\min} \leq v_r(t)$ ,  $|\kappa_r(t)| \leq \kappa_r^{\max} < 1/d$ , and  $|\dot{\kappa}_r(t)| \leq K$ , where  $d, K \in \mathbb{R}^+$ .

1) For  $\varepsilon > 0$ , if

$$\|z_{12}(0)\| \leq \frac{v_r^{\min} \varepsilon}{(2 + \sqrt{2}) \sqrt{k_1^2 + k_2^2}}, \quad (34)$$

where  $k_1, k_2 > 0$ , then there exists  $t^*$  such that for  $t \geq t^*$ ,

$$|\sin z_3(t)| \leq \frac{7d^2}{9v_r^{\min}} K + \varepsilon. \quad (35)$$

2) Moreover, if additionally

$$\lim_{t \rightarrow \infty} \dot{\kappa}_r(t) = 0, \quad (36)$$

then  $\lim_{t \rightarrow \infty} \sin z_3(t) = 0$ .

3) Finally, for  $0 < \varepsilon < \frac{3}{10}$ , if (34) holds,

$$\cos z_3(0) \geq \sqrt{1 - \left(\frac{7}{18} - \frac{8}{27}\varepsilon\right)^2}, \quad (37)$$

and

$$|\dot{\kappa}_r(t)| \leq K = \frac{v_r^{\min}}{d^2} \left( \frac{1}{2} - \frac{5}{3}\varepsilon \right), \quad (38)$$

we have  $v(t) \geq \varepsilon v_r^{\min} > 0$  and  $\lim_{t \rightarrow \infty} z_3(t) = 0$ , rendering the internal dynamics (24) stable.

*Proof:* See Appendix D. ■

Therefore, we can conclude that the internal dynamics (24), which correspond to the orientation of the vehicle, are stable under these conditions: the initial position error is not too large (bounded by (34)), the initial orientation error is bounded by (37), and the curvature derivative of the preceding vehicle is bounded by (38). Moreover, it is important to note that for a platoon with more than 2 vehicles,  $v(t)$  will become the reference for the next vehicle. By Proposition 2(c), the requirement of  $v(t) > 0$  is fulfilled for the initial condition  $z_3(0)$  being bounded by (37) and  $\dot{\kappa}_r(t)$  satisfying (38).

### III. ORIENTATION-ERROR OBSERVER DESIGN

From the previous section, it has been proven that the proposed controller design (17) guarantees that all error states  $(z_1, z_2, z_3)$  converge to zero, under the assumption that all states of the kinematic model are available and measurable for control. Here, the relative position  $(z_1, z_2)$  can be obtained from the camera or lidar, the preceding vehicle states  $(v_r, \omega_r)$  can be obtained through wireless communication with the preceding vehicle,  $\kappa_r$  can be determined from  $v_r$  and  $\omega_r$ ,  $\dot{\kappa}_r$  can be approximated by the backward Euler method, and  $\delta$  is determined from the relative orientation  $\theta - \theta_r$  and  $\alpha_r$ , which may be measured using the camera. It is assumed that the relative position  $(z_1, z_2)$  can be measured accurately, and there is no delay involved in the wireless communication. In practical situations, there is often a case where the orientation of vehicles  $(\theta, \theta_r)$ , or both, are not available, or disturbed by noise due to inherent limitations of the vision system. To address this problem, a state feedback controller combined with an observer that estimates the orientation was designed in [22], [23]. However, these approaches result in a combined observer-controller design which is different than the proposed tracking controller (17). Therefore, we adapt the observer designed in [23] by determining the orientation angle  $\theta$  from the available states  $(z_1, z_2, v, \omega)$  and design an observer such that the estimated angle (denoted by  $\hat{\theta}$ ) converges to the actual orientation angle  $\theta$ .

Consider the kinematic model of the unicycle as given in (1), and the available outputs as  $[x, y]^T$ . We extend the

dimension of the system (1) by defining new variables  $s$  and  $c$  as

$$s = \sin \theta, \quad c = \cos \theta, \quad (39)$$

which replace the orientation angle  $\theta$ . As a result, we obtain the extended model of the unicycle as

$$\dot{x} = vc, \quad \dot{y} = vs, \quad (40a)$$

$$\dot{s} = \omega c, \quad \dot{c} = -\omega s, \quad (40b)$$

where  $[x, y]^T$  are the available outputs,  $[v, \omega]^T$  are inputs,  $s$  and  $c$  as defined in (39). It should be noted that the transformation from the three-dimensional system (1) to four-dimensional system (40) introduces a constraint of the form  $s^2 + c^2 = 1$ .

Based on (40), an observer for  $x$ ,  $y$ ,  $c$ , and  $s$  can be defined as follows

$$\dot{\hat{x}} = v\hat{c} + l_1\zeta_x \quad (41a)$$

$$\dot{\hat{y}} = v\hat{s} + l_2\zeta_y \quad (41b)$$

$$\dot{\hat{c}} = -\omega\hat{s} + l_3v\zeta_x \quad (41c)$$

$$\dot{\hat{s}} = \omega\hat{c} + l_4v\zeta_y, \quad (41d)$$

where  $l_1, l_2, l_3, l_4 > 0$ , and  $\zeta_x = x - \hat{x}$ ,  $\zeta_y = y - \hat{y}$ ,  $\zeta_c = c - \hat{c}$ , and  $\zeta_s = s - \hat{s}$  are the observer errors. Thus, we obtain the following observer error dynamics

$$\dot{\zeta}_x = \dot{x} - \dot{\hat{x}} = v\zeta_c - l_1\zeta_x \quad (42a)$$

$$\dot{\zeta}_y = \dot{y} - \dot{\hat{y}} = v\zeta_s - l_2\zeta_y \quad (42b)$$

$$\dot{\zeta}_c = \dot{c} - \dot{\hat{c}} = -\omega\zeta_s - l_3v\zeta_x \quad (42c)$$

$$\dot{\zeta}_s = \dot{s} - \dot{\hat{s}} = \omega\zeta_c - l_4v\zeta_y. \quad (42d)$$

It can be observed directly that if (42) converges to zero, then the estimated states  $(\hat{x}, \hat{y}, \hat{c}, \hat{s})$  converge to  $(x, y, c, s)$ . To prove stability of (42), the following proposition can be used.

**Proposition 3** ([24], [25]): *Consider the dynamics (42) with  $l_1, l_2, l_3, l_4 > 0$ . If  $v, \omega$  are bounded differentiable functions,  $\dot{v}$  is bounded, and  $0 < v_{\min} \leq v(t)$ , then (42) is uniformly globally asymptotically stable (UGAS) at the origin.*

*Proof:* See Appendix E. ■

Using Proposition 3, we have that the origin of (42) is uniformly globally asymptotically stable (UGAS) and  $\zeta_x(t)$ ,  $\zeta_y(t)$ ,  $\zeta_c(t)$ ,  $\zeta_s(t)$  converge to zero as  $t \rightarrow \infty$ , subject to the necessary and sufficient condition of  $v(t) > 0$  for all  $t$  [see Proposition 2(c)]. It remains to prove the convergence of the estimated orientation angle to the actual orientation angle  $\theta$ . Define the estimated orientation angle  $\hat{\theta}$  as

$$\hat{\theta} := \text{atan2}(\hat{s}, \hat{c}), \quad (43)$$

where  $\hat{c}$  and  $\hat{s}$  are generated by the observer (41). Note also that

$$\sin \hat{\theta} = \frac{\hat{s}}{\sqrt{\hat{c}^2 + \hat{s}^2}}, \quad \cos \hat{\theta} = \frac{\hat{c}}{\sqrt{\hat{c}^2 + \hat{s}^2}}, \quad \tan \hat{\theta} = \frac{\hat{s}}{\hat{c}}. \quad (44)$$

Let us define  $\zeta_{\theta} = \tan(\theta - \hat{\theta})$ . By noting that  $\zeta_c = c - \hat{c}$ ,  $\zeta_s = s - \hat{s}$ , and using the fact that  $\tan \theta = \sin \theta / \cos \theta$ , we have

$$\begin{aligned} \zeta_{\theta} &= \frac{\tan \theta - \tan \hat{\theta}}{1 + \tan \theta \tan \hat{\theta}} = \frac{\hat{c} \sin \theta - \hat{s} \cos \theta}{\hat{c} \cos \theta + \hat{s} \sin \theta} \\ &= \frac{(c - \zeta_c) \sin \theta - (s - \zeta_s) \cos \theta}{(c - \zeta_c) \cos \theta + (s - \zeta_s) \sin \theta} \\ &= \frac{\zeta_s \cos \theta - \zeta_c \sin \theta}{1 - \zeta_c \cos \theta - \zeta_s \sin \theta}. \end{aligned} \quad (45)$$

Since  $\zeta_c(t)$  and  $\zeta_s(t)$  converge to zero, we have  $\zeta_{\theta}(t)$  converge to zero as  $t \rightarrow \infty$ , which directly implies the convergence of  $\hat{\theta}$  to  $\theta$  for the initial estimated orientation error satisfying  $|\theta(0) - \hat{\theta}(0)| < \pi/2$ .

#### IV. SIMULATIONS

In order to illustrate the effectiveness of the extended look-ahead controller and the observer, a number of simulations are performed. Additionally, the purpose of this simulation is to properly determine the control parameters for the experimental setup. First, we consider a scenario of 4 vehicles platoon, where all states (position and orientation) can be measured accurately and are not disturbed by noise. This allows us to investigate the optimal gains  $k_1$  and  $k_2$  and the effectiveness of the extended look-ahead controller against corner-cutting. Second, we consider a scenario of a 2 vehicles platoon where the second vehicle is controlled by extended look-ahead controller with the orientation-observer, in the presence and absence of noise. In this scenario, the performance of the observer is evaluated.

We consider a platoon of 4 vehicles, with the first vehicle controlled by the tracking controller of [16] to track a predefined eight-shaped trajectory, while the other vehicles controlled by the extended look-ahead controller to track their respective preceding vehicle. It should be noted that the first vehicle can also be directly controlled (simulating a driving scenario where the first vehicle is driven by a human), or controlled by other trajectory-tracking (e.g., [17]) or path-following controllers (e.g., [5]). The eight-shaped trajectory is generated by two-half circles with the radius 0.3 m and quintic polynomial functions. The controller performance to track a circular trajectory (constant curvature) as in [15] is also performed, but the eight-shaped trajectory is chosen since it also represents a combination of constant and varying curvatures. The reference curvature of the eight-shaped path is given in Fig. 8. The dimensions of the track are chosen in accordance with the experimental setup, which is presented in the next section. The first vehicle starts at initial position  $(x, y) = (0.7, 0.5)$  m, maneuvering along the eight-shaped path. The other vehicles start at  $(0.625, 0.425)$ ,  $(0.55, 0.35)$ , and  $(0.475, 0.275)$  for vehicle 2, 3, and 4, respectively. All vehicles are initiated with  $v = 0.06$  m/s and  $\theta = 0.9707$  rad/s, and  $d = 0.1$  m is chosen. The extended look-ahead controller gain is determined by an iterative manner and is equal for vehicle 2, 3, and 4. It should be noted that the choice for the proper gain is also determined by the available experimental arena and the reference trajectory. It is also worth noting that the higher gain value results in a faster convergence towards



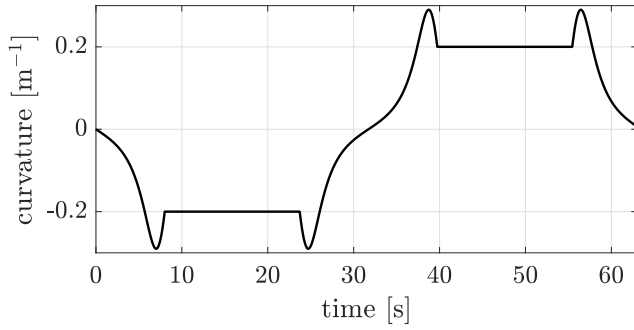


Fig. 8. Reference curvature for an eight-shaped path with  $v = 0.06$  m/s.

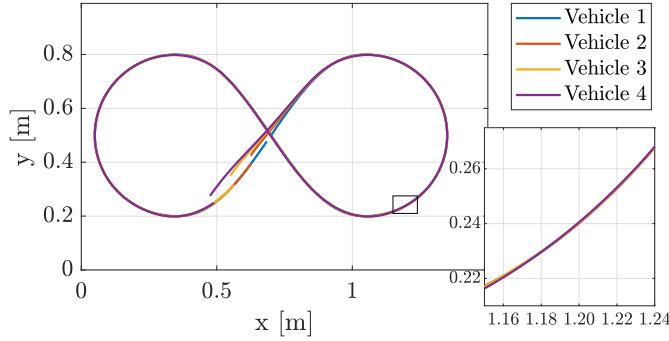


Fig. 9. Trajectory of 4-vehicles platoon tracking an eight-shaped path, with  $k_1 = k_2 = 0.75$ .

the desired path. However, the higher gain value also results in a more sensitive response to the curvature change. In practical situation, this is undesirable since a slight change in states measurement (e.g., due to noise, sensor inaccuracy) could result in an over compensation. The trajectory of all vehicles with  $k_1 = k_2 = 0.75$  is shown in Fig. 9. It can be observed that all vehicles in the platoon converges to the reference path and corners are not cut. This shows the advantage of our controller in comparison to the controller in [12], where corners with  $\dot{\kappa}_r \neq 0$  are still cut. From Fig. 10, it can be observed that  $z_1$  and  $z_2$  converge to zero. On the other hand, the orientation error  $z_3$  converges to zero if  $\dot{\kappa}_r = 0$ , which can be seen from  $t = 11$  s until  $t = 22$  s and from  $t = 42$  s until  $t = 55$  s. On the transition state when  $\dot{\kappa}_r \neq 0$ ,  $z_3$  is bounded given the condition that  $\dot{\kappa}_r$  is small enough.

In the second scenario, we consider a platoon of 2 vehicles, with identical parameters as vehicle 1 and vehicle 2 in the previous simulation. A two-vehicles setting is used since we want to study the convergence of the estimated orientation to the true orientation of the follower vehicle. The first vehicle is controlled by the tracking controller [16], while the second vehicle is controlled by the extended look-ahead controller (17) with the observer (41). The initial condition of the observer states are set as  $\hat{x}(0) = 0.625$ ,  $\hat{y}(0) = 0.425$ , and  $\hat{\theta} = 0.8$  rad/s. It is assumed that the position can be measured accurately without any noise, while the orientation measurement is disturbed by a noise. The noise of the orientation sensor is simulated as a white noise with a power spectral density of  $5 \times 10^{-5}$ . For the observer gains, we select  $l_1 = 10$ ,  $l_2 = 10$ ,  $l_3 = 1000$ , and  $l_4 = 1000$ .

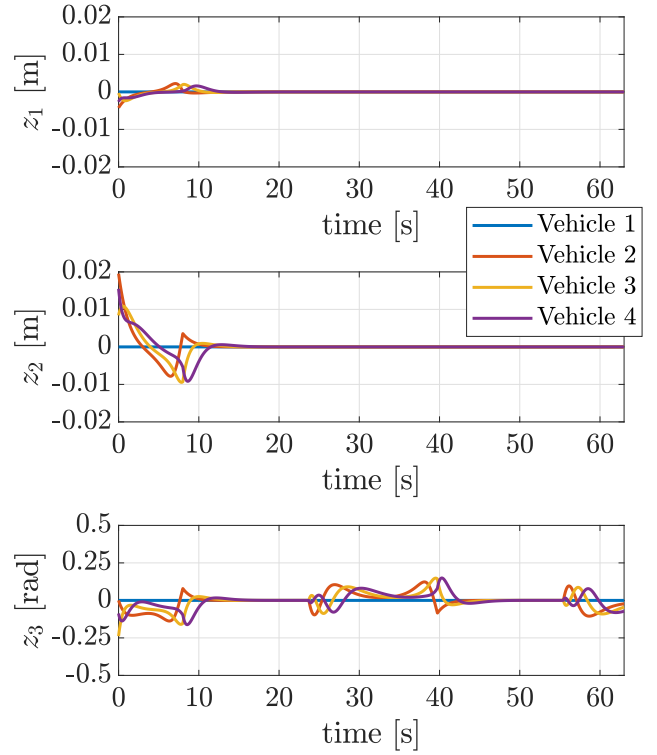


Fig. 10. Errors of a platoon with the extended look-ahead controller on an eight-shaped path.

First, we simulate the system without the observer, i.e., the control laws (17) are calculated using the orientation measured from the sensor with noise. Second, we simulate the system with the orientation-error observer, where the orientation is estimated based on the position sensor. We denote  $\theta$  as the true orientation,  $\hat{\theta}$  as the estimated orientation, and  $\bar{\theta}$  as the orientation obtained from the sensor. The error plots of  $\theta - \bar{\theta}$  (for the scenario without the observer) and  $\theta - \hat{\theta}$  (for the scenario with the observer) are depicted in Fig. 11. It can be observed that for the scenario without the observer, the measured orientation (shown in gray line) is heavily disturbed by noise. On the other hand, the error  $\theta - \hat{\theta}$  (shown in black line) is not disturbed by noise and converges to zero, which means that the estimated orientation  $\hat{\theta}$  converges to the true orientation  $\theta$ .

## V. EXPERIMENTS

In this section, we conduct an experiment to confirm the theoretical analyses and subsequent to the simulation results. This practical experiment is conducted also to provide an insight in how the parameters for our controller can be chosen to accommodate the slave controller in mobile robots. Furthermore, the purpose of this experiment is to verify the performance of the orientation-error observer in an experimental environment, where the orientation measurement is disturbed by noise due to inherent limitation of the sensor/vision system. The main components of this experimental setup are: four mobile robots (E-puck [26]), a PC, and a camera. A unique marker (2D barcode) is attached to each E-puck for identification, such that the orientation and the position of each E-puck can

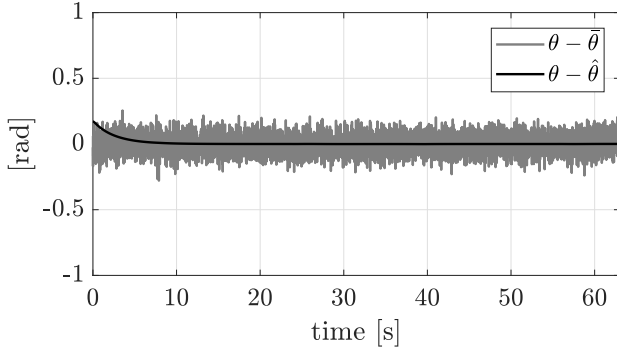


Fig. 11. Errors of  $\theta - \hat{\theta}$  and  $\theta - \bar{\theta}$ , where  $\theta$  is the true orientation,  $\hat{\theta}$  is the estimated orientation, and  $\bar{\theta}$  is the orientation obtained from the sensor with noise.

TABLE I  
E-PUCK SPECIFICATION

Specification	Values
Robot radius	37 mm
Wheel radius ( $r$ )	20.5 mm
Axle length ( $L$ )	52 mm
Speed unit	0.00628 rad/s
Encoder resolution	159.23
Maximum angular speed	1000 units
Maximum velocity	13 cm/s
Maximum angular velocity	5 rad/s



Fig. 12. The E-puck mobile robot and markers used for identification.

be determined (see Fig. 12). The PC is used to generate a reference trajectory and to determine the absolute coordinates and orientations of the robots from the camera. The control algorithm is also computed in the PC, and is send directly to each E-puck via a Bluetooth protocol. The experimental setup is shown in Fig. 13. Since the E-puck is a differential-drive mobile robot, its motion is controlled by providing velocity inputs of the left and right wheels, denoted by  $v_l$  and  $v_r$ , respectively. The transformation of the linear and angular velocity,  $v$  and  $\omega$ , to the individual wheels velocity can be determined using the following relation

$$v_l = v - \frac{\omega L}{2}, \quad v_r = v + \frac{\omega L}{2}, \quad (46)$$

where  $L$  is the length between the left and right wheel of E-puck, as given in Table I.

In order to support comparison of the simulation results presented in the previous section, for this experiment we use the identical eight-shaped reference trajectory as in the simulation. In the first experiment, we use the extended look-ahead controller without the observer, thus, the orien-



Fig. 13. The mobile robot experimental setup with E-pucks in the  $1.75 \times 1.28$  m arena. The camera is attached to the frame to measure the position and orientation of the robots.

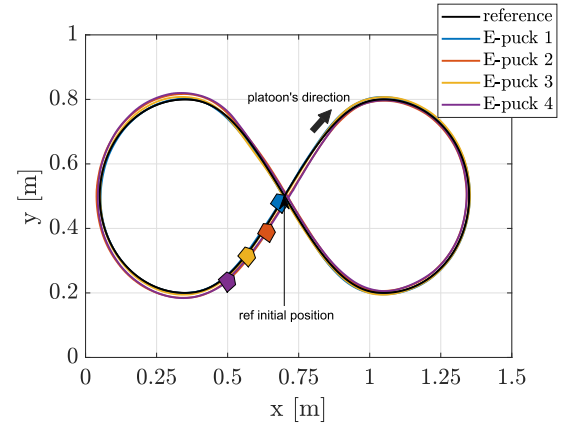


Fig. 14. Trajectory of E-pucks with extended look-ahead controller, where the orientation is measured directly from the camera.

tation of the mobile robots are obtained directly from the camera. The objectives of the first experiment are to study the effectiveness of the extended look-ahead controller, compared to the theoretical results presented in the previous section, to verify the suitable gain for the experiment, and to study the behavior of the system under the presence of measurement noises. In the second experiment, we apply the observer to estimate the orientation of the mobile robots. The objective of this second experiment is to study the effectiveness of the observer in practice, and to confirm the simulation results. All E-pucks are initiated with  $v = 0.06$  m/s, with the controller gains  $k_1 = k_2 = 0.75$  for both experiments, and with the observer gains  $l_1 = 10$ ,  $l_2 = 10$ ,  $l_3 = 1000$ , and  $l_4 = 1000$  for the second experiment.

Fig. 14 shows the trajectory of robots with the extended look-ahead controller. It can be observed that the trajectory of all follower robots converge to the reference trajectory. Clearly, this experiment shows that the extended look-ahead controller effectively avoids corner-cutting. However, it can be seen that the follower vehicles start to deviate on the left side of the eight-shaped path, due to the inaccuracy (due to noise, or displacement of the 2D marker) in the orientation

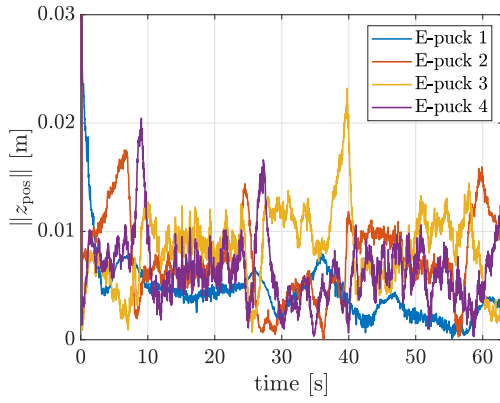


Fig. 15. 2-norm of the position error from experiments incorporating extended look-ahead controller without observer.

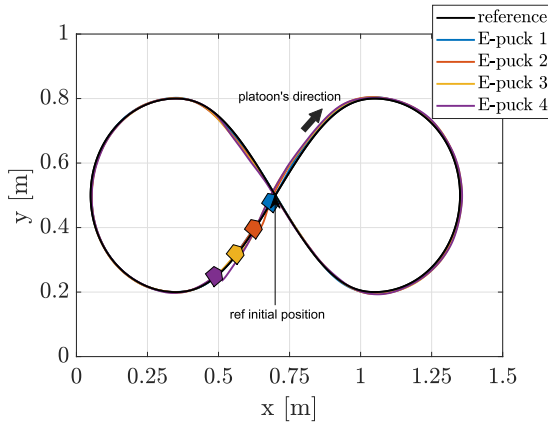


Fig. 16. Trajectory of E-pucks with extended look-ahead controller and an orientation observer. The orientation of each E-puck is estimated from the position.

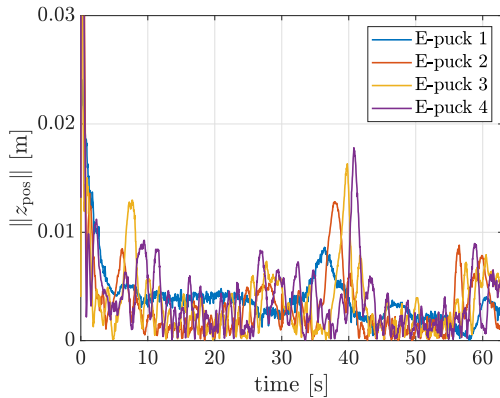


Fig. 17. 2-norm of the position error from experiments incorporating extended look-ahead controller with observer.

measurement. The 2-norm of the position error, define as  $\sqrt{z_1^2 + z_2^2}$ , is depicted in Fig. 15. It can be observed from this plot that E-puck 3 has the largest deviation, peaking at  $t = 40$  s. To address this problem, the orientation-error observer is integrated to the extended-look-ahead controller in the second experiment. In Fig. 16, the trajectory of all robots with the extended look-ahead controller and orientation-error observer is shown. It can be seen directly that the tracking accuracy of all follower robots is improved, thus showing the effectiveness of the orientation-error observer. From Fig. 17,

where the 2-norm of the position error is depicted, it can also be verified that the observer reduces the deviation caused by the inaccuracy of the orientation measurement. We may also notice some measurement noises on the position measurement, but they are small enough (less than 0.02 m) and can be safely neglected.

## VI. CONCLUSION

This paper presents a novel extended look-ahead controller in vehicle platooning. The look-ahead target point is extended to a virtual point induced from the position and the curvature of the reference vehicle, thus ensuring a better tracking performance at cornering, preventing corner-cutting behavior. A stability result on the internal dynamics is presented, showing that the closed-loop system is stable under the given bound of the reference curvature and the initial relative position and orientation of the vehicles. The simulation results show that the proposed approach improves the tracking performance at cornering, ensuring that the corners are not cut. To address the orientation measurement noise in the experiment, an orientation-error observer is also designed. The effectiveness of the integrated extended look-ahead controller and orientation-error observer is further validated by means of experiment with a platoon consisting of four E-pucks. The experimental results confirm that the application of the extended look-ahead controller in vehicle platooning compensates for corner-cutting, and also confirm that the observer reduces the noise presents in the orientation measurement. The continuation of this paper is to extend the approach to the single-track model, as an important step towards the application of our controller in a real vehicle. To adapt our controller to a single-track dynamic model with tire forces, we can design a slave controller that controls acceleration or speed, internally compensating for the vehicle mass. The stability of the real vehicle can be guaranteed by other controllers (Electronic Stability Control (ESC), or Anti-Lock Braking System (ABS), for instance), and our controller can be used in conjunction with those other controllers. In conclusion, the application of our controller to the real vehicle may poses additional conditions, but is feasible.

## APPENDIX A

In this appendix we show the derivation in obtaining the equilibrium point of  $\delta$ . Consider  $\delta = \theta - \theta_r + a_r$ , as in (14). By differentiating it with respect to time, we obtain

$$\dot{\delta} = \omega - \omega_r + \dot{a}_r. \quad (47)$$

Substituting  $\omega$  as in (17) and (7) into (47) eventually yields

$$\dot{\delta} = -(1 - \cos \delta) \omega_r - \frac{v_r}{d} \sin \delta + \zeta_r, \quad (48)$$

where

$$\zeta_r = \frac{k_1 \sin \delta}{d} z_1 - \frac{k_2 \cos \delta}{d} z_2 + \left( \frac{h_{\kappa,1}}{d} \sin \delta - \frac{h_{\kappa,2}}{d} \cos \delta + \frac{2d}{\sqrt{4 - d^2 \kappa_r^2}} \right) \dot{\kappa}_r, \quad (49)$$

$(z_1, z_2)$  as in (12), and  $(h_{\kappa,1}, h_{\kappa,2})$  as in (16). In a steady state condition, which implies that  $z_1 = z_2 = 0$  and  $\dot{\kappa}_r = 0$ , we have

$$\dot{\delta} = -(1 - \cos \delta) \omega_r - \frac{v_r}{d} \sin \delta. \quad (50)$$

By noting that  $\kappa_r = \omega_r/v_r$  and  $\sin^2 \delta = 1 - \cos^2 \delta$ , the equilibrium points of (50) are determined by

$$(1 + d^2 \kappa_r^2) \cos^2 \delta - 2d^2 \kappa_r^2 \cos \delta + (d^2 \kappa_r^2 - 1) = 0,$$

and given by

$$\delta^* = 2n\pi \quad (51)$$

$$\delta^* = \arctan\left(\frac{-2d\kappa_r}{d^2\kappa_r^2 - 1}\right) + 2n\pi, \quad (52)$$

where  $n = 0, \pm 1, \pm 2, \dots$

#### APPENDIX B

In this appendix we show how  $\beta$  is derived mathematically, such that  $z_3$  has equilibrium points at  $[0, \pi]$ . Consider

$$z_3 = \theta - \theta_r + \alpha_r + \beta, \quad (53)$$

where  $\beta : [-2\pi, 2\pi] \rightarrow [-1, 1]$ . Taking the input  $\omega$  as in (17) into account, the derivative of  $z_3$  is given by

$$\begin{aligned} \dot{z}_3 = & -\frac{1}{d} \sin(\theta - \theta_r + \alpha) (-k_1 z_1 + v_r - \dot{\kappa}_r h_{\kappa,1}) \\ & + \frac{1}{d} \cos(\theta - \theta_r + \alpha) (-k_2 z_2 + d\omega_r - \dot{\kappa}_r h_{\kappa,2}) \\ & - \omega_r + \dot{\alpha}_r + \dot{\beta}. \end{aligned} \quad (54)$$

In the equilibrium we have

$$\dot{z}_3 = -\frac{v_r}{d} \sin(z_3 - \beta) + \omega_r \cos(z_3 - \beta) - \omega_r, \quad (55)$$

and we want  $\dot{z}_3 = 0$  for  $z_3 = 0$  and  $z_3 = \pi$ , i.e.,

$$\begin{aligned} z_3 = 0 & \Rightarrow \sin \beta = -d\kappa_r \cos \beta + d\kappa_r \\ z_3 = \pi & \Rightarrow \sin \beta = -d\kappa_r \cos \beta - d\kappa_r \end{aligned}$$

which can be rewritten as

$$\sin \beta = d\kappa_r \cos z_3 - d\kappa_r \cos \beta. \quad (56)$$

By substituting (53) into (56), and noting that  $\delta = \theta - \theta_r + \alpha_r$ , we have

$$\begin{aligned} \sin \beta &= d\kappa_r (\cos \delta \cos \beta - \sin \delta \sin \beta - \cos \beta) \\ \sin \beta (1 + d\kappa_r \sin \delta) &= \cos \beta (d\kappa_r \cos \delta - d\kappa_r) \\ \frac{\sin \beta}{\cos \beta} &= \frac{d\kappa_r \cos \delta - d\kappa_r}{1 + d\kappa_r \sin \delta}, \end{aligned} \quad (57)$$

resulting in  $\sin \beta$  and  $\cos \beta$  as in (21a) and (21b).

#### APPENDIX C

In this section the claim on the boundedness of  $\bar{v}_r$  and  $\xi_r$  is proven.

*Proof:* First we want to show the lower and upper bound of  $\bar{v}_r$ . We can rewrite (29) as

$$\begin{aligned} \Delta &= d^2 \kappa_r^2 (1 - \cos \delta)^2 + (1 + d\kappa_r \sin \delta)^2 \\ &= 2d^2 \kappa_r^2 (1 - \cos \delta) + 2d\kappa_r \sin \delta + 1. \end{aligned} \quad (58)$$

$$= 2d^2 \kappa_r^2 + 1 + 2(d\kappa_r \sin \delta - d^2 \kappa_r^2 \cos \delta). \quad (59)$$

To obtain the lower- and upper-bound of  $\Delta$ , let us define an angle  $\gamma$ , characterized by  $\sin \gamma = d\kappa_r / \sqrt{d^2 \kappa_r^2 + 1}$  and  $\cos \gamma = 1 / \sqrt{d^2 \kappa_r^2 + 1}$  such that we can write (59) as

$$\begin{aligned} \Delta &= 2d^2 \kappa_r^2 + 1 + 2d\kappa_r \sqrt{d^2 \kappa_r^2 + 1} (\cos \gamma \sin \delta - \sin \gamma \cos \delta) \\ &= 2d^2 \kappa_r^2 + 1 + 2d\kappa_r \sqrt{d^2 \kappa_r^2 + 1} (\sin(\delta - \gamma)). \end{aligned}$$

Since  $|\sin(\delta - \gamma)| \leq 1$ , we have

$$\begin{aligned} \Delta &\geq 2d^2 \kappa_r^2 + 1 - 2 \left| d\kappa_r \sqrt{d^2 \kappa_r^2 + 1} \right| \text{ and} \\ \Delta &\leq 2d^2 \kappa_r^2 + 1 + 2 \left| d\kappa_r \sqrt{d^2 \kappa_r^2 + 1} \right| \\ &\Rightarrow 3 - 2\sqrt{2} \leq \Delta \leq 3 + 2\sqrt{2}, \end{aligned} \quad (60)$$

as the lower- and upper-bound of  $\Delta$ , where the extreme value is obtained for  $|d\kappa_r| = 1$ . To show that  $N > 0$ , note that we can rewrite (28) as

$$N = \frac{1}{4} d^2 \kappa_r^2 (3 - \cos \delta) (1 - \cos \delta) + \left(1 + \frac{1}{2} d\kappa_r \sin \delta\right)^2. \quad (61)$$

Since  $N > 0$ , by taking (58) into account, we can also rewrite (28) as

$$\begin{aligned} N &= \sqrt{(d^2 \kappa_r^2 (1 - \cos \delta) + d\kappa_r \sin \delta + 1)^2} \\ &= \sqrt{\Delta + (d^2 \kappa_r^2 (1 - \cos \delta) + d\kappa_r \sin \delta)^2}. \end{aligned} \quad (62)$$

Moreover, since  $\Delta > 0$  (which follows directly from (29)), by substituting (62) into (26) we obtain

$$\begin{aligned} \bar{v}_r &= v_r \sqrt{1 + \frac{1}{\Delta} (d^2 \kappa_r^2 (1 - \cos \delta) + d\kappa_r \sin \delta)^2} \\ &= v_r \sqrt{1 + \frac{(N - 1)^2}{\Delta}} \geq v_r, \end{aligned} \quad (63)$$

which is the lower bound of  $\bar{v}_r$ . Note also that by using (58), we can rewrite  $N$  as

$$N = \frac{1}{2} (\Delta + 1). \quad (64)$$

By substituting (64) into (63), and taking the upper bound of  $\Delta$  in (60) into account, we eventually obtain

$$\bar{v}_r = v_r \sqrt{\frac{(\Delta + 1)^2}{4\Delta}} \leq v_r \sqrt{2}, \quad (65)$$

which is the upper bound of  $\bar{v}_r$ .



To show the upper bound of  $\xi_r$ , note that we can rewrite  $N/\Delta$  as

$$\begin{aligned} \frac{N}{\Delta} &= \frac{d^2 \kappa_r^2 (1 - \cos \delta) + d \kappa_r \sin \delta + 1}{2d^2 \kappa_r^2 (1 - \cos \delta) + 2d \kappa_r \sin \delta + 1} \\ &= \frac{1}{2} \left( 1 + \frac{1}{\Delta} \right) \leq \frac{1}{2} \left( 1 + \frac{1}{\Delta_{\min}} \right) \\ &\leq \frac{1}{2} \left( 1 + \frac{1}{3-2\sqrt{2}} \right) = \frac{2-\sqrt{2}}{3-2\sqrt{2}} = 2 + \sqrt{2}, \end{aligned} \quad (66)$$

where we use the lower bound of  $\Delta$  in (60). Moreover, we also have

$$|g_\kappa| = \left| \frac{N}{\Delta} f_r(\delta, d, \kappa_r) - \frac{d(1 - \cos \delta)}{\Delta} \right| < \frac{7}{9}d, \quad (67)$$

where the bound on  $|g_\kappa|$  is obtained by evaluating the function and the maximum value is obtained for  $\kappa_r = 1/d$  and  $\delta = \frac{3}{4}\pi$ . By using the triangle inequality and substituting (66), (67) into (27), we have

$$\begin{aligned} \xi_r &= \frac{N}{\Delta d} [\sin \delta \quad -\cos \delta] \begin{bmatrix} k_1 z_1 \\ k_2 z_2 \end{bmatrix} + g_\kappa \dot{\kappa}_r \\ |\xi_r| &\leq \frac{2+\sqrt{2}}{d} (k_1 |z_1| + k_2 |z_2|) + \frac{7}{9}d |\dot{\kappa}_r|, \end{aligned} \quad (68)$$

which is the bound of  $|\xi_r|$ . ■

#### APPENDIX D

*Proof of Proposition 2:* From (19), we have  $\|z_{12}(t)\| \leq \|z_{12}(0)\|$ . Since  $|\dot{\kappa}_r(t)| \leq K$ , from (33) we have

$$\begin{aligned} |\xi_r(t)| &\leq \frac{2+\sqrt{2}}{d} \sqrt{k_1^2 + k_2^2} \|z_{12}(t)\| + \frac{7d}{9} K \\ &\leq \frac{2+\sqrt{2}}{d} \sqrt{k_1^2 + k_2^2} \|z_{12}(0)\| + \frac{7d}{9} K \\ &\leq \frac{v_r^{\min}}{d} \varepsilon + \frac{7d}{9} K =: \xi_r^{\max}, \end{aligned} \quad (69)$$

where we use (34). Consider a positive-definite function

$$V_3(z_3) = 1 - \cos z_3. \quad (70)$$

The time derivative of  $V_3(z_3)$  along the trajectory (24), by taking (32) into account, is given by

$$\begin{aligned} \dot{V}_3(z_3) &= -\frac{\bar{v}_r}{d} \sin^2 z_3 + \xi_r \sin z_3 \\ &\leq -\frac{v_r}{d} \sin^2 z_3 + |\xi_r| |\sin z_3| \\ &\leq -\frac{v_r^{\min}}{2d} \left[ \sin^2 z_3 + \left( |\sin z_3| - \frac{d}{v_r^{\min}} |\xi_r| \right)^2 - \left( \frac{d}{v_r^{\min}} |\xi_r| \right)^2 \right] \\ &\leq -\frac{v_r^{\min}}{2d} \left[ \sin^2 z_3 - \left( \frac{d}{v_r^{\min}} |\xi_r| \right)^2 \right]. \end{aligned} \quad (71)$$

Let us define  $\Omega_u = \{z_3 \in \mathbb{R} \mid |\sin z_3| \leq \frac{d}{v_r^{\min}} \xi_r^{\max}\}$ , where  $\xi_r^{\max}$  is as defined in (69). By noting that  $|\xi_r(t)| \leq \xi_r^{\max}$ , solutions starting outside  $\Omega_u$  move in the direction of decreasing  $V_3$ , since  $\dot{V}_3 < 0$  outside  $\Omega_u$ , and eventually will be inside and cannot leave  $\Omega_u$  as  $t \rightarrow \infty$ , which corresponds to (35) when substituting  $\xi_r^{\max}$  from (69) in the definition for  $\Omega_u$ . This proves claim (a).

Moreover, for

$$\lim_{t \rightarrow \infty} z_1(t) = 0, \quad \lim_{t \rightarrow \infty} z_2(t) = 0, \quad \lim_{t \rightarrow \infty} \dot{\kappa}_r(t) = 0,$$

we have  $\xi_r(t) \rightarrow 0$  as  $t \rightarrow \infty$ , according to (33). From claim (a), we have that any solution of  $z_3$  will be inside and cannot leave  $\Omega_u$  as  $t \rightarrow \infty$ , which means that  $\sin z_3(t) \rightarrow \frac{d}{v_r^{\min}} \xi_r(t)$  as  $t \rightarrow \infty$ . Since  $d > 0$ ,  $v_r^{\min} > 0$ , we have  $\sin z_3(t)$  converges to zero if and only if  $\xi_r(t)$  converges to zero, hence proving claim (b).

It is important to note that for a platoon with more than 2 vehicles,  $v(t)$  will become the reference for the next vehicle. Thus, we also need the condition of  $v(t) \geq v^{\min} > 0$ . From (37) we have  $|\sin z_3(0)| \leq \frac{7}{18} - \frac{8}{27}\varepsilon$ , so using (38), we start in the set  $\Omega_u$  and stay in the set  $\Omega_u$ , which implies  $|\sin z_3(t)| \leq \frac{7}{18} - \frac{8}{27}\varepsilon$  and  $\cos z_3(t) \geq \sqrt{1 - \left(\frac{7}{18} - \frac{8}{27}\varepsilon\right)^2} > 0$  for all  $t \geq 0$ .

From (17), we have

$$\begin{aligned} v &= v_r (\cos \delta + d \kappa_r \sin \delta) - (k_1 z_1 \cos \delta + k_2 z_2 \sin \delta) \\ &\quad - \dot{\kappa}_r (h_{\kappa,1} \cos \delta + h_{\kappa,2} \sin \delta). \end{aligned} \quad (72)$$

Let us denote  $\eta := \cos \delta + d \kappa_r \sin \delta$ . Note that by using (22) we have

$$\begin{aligned} \eta &= \cos z_3 \sqrt{d^2 \kappa_r^2 (1 - \cos \delta)^2 + (1 + d \kappa_r \sin \delta)^2} \\ &= \cos z_3 \sqrt{2d \kappa_r (d \kappa_r - d \kappa_r \cos \delta + \sin \delta) + 1} \\ &= \sqrt{2d \kappa_r \sin z_3 \cos z_3 \eta + \cos^2 z_3}. \end{aligned} \quad (73)$$

By noting that  $|\kappa_r| < 1/d$ , we solve (73) with respect to  $\eta$  as

$$\begin{aligned} \eta &= \cos z_3 \left( d \kappa_r \sin z_3 + \sqrt{d^2 \kappa_r^2 \sin^2 z_3 + 1} \right) \\ &\geq (-1 + \sqrt{2}) \cos z_3. \end{aligned} \quad (74)$$

Moreover, from (16) we have

$$|h_{\kappa,1}| \leq \frac{d^2}{2\sqrt{3}} =: h_{\kappa,1}^{\max} \quad (75a)$$

$$|h_{\kappa,2}| \leq d^2 \left( \frac{4-\sqrt{3}}{2\sqrt{3}} \right) =: h_{\kappa,2}^{\max}. \quad (75b)$$

Thus, from (72), by substituting (37), (38), and (75), we obtain

$$\begin{aligned} v(t) &\geq v_r \left( -1 + \sqrt{2} \right) \cos z_3(t) - \sqrt{k_1^2 + k_2^2} \|z_{12}(t)\| \\ &\quad - K \sqrt{|h_{\kappa,1}^{\max}|^2 + |h_{\kappa,2}^{\max}|^2} \\ &\geq v_r^{\min} \left( -1 + \sqrt{2} \right) \sqrt{1 - \left( \frac{7}{18} - \frac{8}{27}\varepsilon \right)^2} - \frac{v_r^{\min} \varepsilon}{2+\sqrt{2}} \\ &\quad - v_r^{\min} \left( \frac{1}{2} - \frac{5}{3}\varepsilon \right) \sqrt{\frac{5-2\sqrt{3}}{3}} \\ &\geq \varepsilon v_r^{\min}, \end{aligned} \quad (76)$$

i.e., for  $|\dot{\kappa}_r(t)| \leq K$ , where  $K$  is given in (38), we have  $v(t) \geq \varepsilon v_r^{\min} > 0$ . Moreover, due to  $\cos z_3(t) \geq \sqrt{1 - \left(\frac{7}{18} - \frac{8}{27}\varepsilon\right)^2} > 0$  for all  $t \geq 0$ , we can guarantee that  $z_3(t)$  does not converge to  $\pi$ . Thus, the claim of  $\sin z_3(t) \rightarrow 0$  also implies that  $z_3(t) \rightarrow 0$  (modulo  $2\pi$ ), as  $t \rightarrow \infty$ . This proves claim (c). ■

#### APPENDIX E

*Proof of Proposition 3:* Let  $\zeta = (\zeta_x, \zeta_y, \zeta_c, \zeta_s)$ . Differentiating the positive definite Lyapunov function candidate

$$V_1(\zeta) = \frac{l_3}{2} \zeta_x^2 + \frac{l_4}{2} \zeta_y^2 + \frac{1}{2} \zeta_c^2 + \frac{1}{2} \zeta_s^2, \quad (77)$$

along solutions of (42) results in

$$\begin{aligned}\dot{V}_1(\zeta) &= l_3 \zeta_x v \zeta_c - l_1 l_3 \zeta_x^2 + l_4 \zeta_y v \zeta_s - l_2 l_4 \zeta_y^2 \\ &\quad - \omega \zeta_c \zeta_s - l_3 v \zeta_x \zeta_c + \omega \zeta_c \zeta_s - l_4 v \zeta_y \zeta_s \\ &= -l_1 l_3 \zeta_x^2 - l_2 l_4 \zeta_y^2 \leq 0,\end{aligned}\quad (78)$$

which is negative semi-definite. We can conclude that the origin of (42) is uniformly globally stable (UGS). We can not only conclude that  $\zeta_x$ ,  $\zeta_y$ ,  $\zeta_c$ , and  $\zeta_s$  are bounded, but using (42) that also  $\dot{\zeta}_x$ ,  $\dot{\zeta}_y$ ,  $\dot{\zeta}_c$ , and  $\dot{\zeta}_s$  are bounded, and therefore also  $\ddot{\zeta}_x$ , and  $\ddot{\zeta}_y$  (which follows by differentiating (42), and by using the fact that  $\dot{v}$  and  $\dot{\omega}$  are bounded).

Differentiating the bounded function

$$V_2(\zeta) = -\dot{\zeta}_x \zeta_x - \dot{\zeta}_y \zeta_y \quad (79)$$

along the solutions of (42) results in

$$\begin{aligned}\dot{V}_2(\zeta) &= -\ddot{\zeta}_x \zeta_x - (v \zeta_c - l_1 \zeta_x)^2 - \ddot{\zeta}_y \zeta_y - (v \zeta_s - l_2 \zeta_y)^2 \\ &= -v^2 (\zeta_c^2 + \zeta_s^2) + 2l_1 v \zeta_c \zeta_x + 2l_2 v \zeta_s \zeta_y \\ &\quad - l_1^2 \zeta_x^2 - l_2^2 \zeta_y^2 - \ddot{\zeta}_x \zeta_x - \ddot{\zeta}_y \zeta_y \\ &\leq -v_{\min}^2 (\zeta_c^2 + \zeta_s^2) + M_x |\zeta_x| + M_y |\zeta_y|,\end{aligned}\quad (80)$$

for certain constants  $M_x$  and  $M_y$ , where we used the previously derived boundedness of signals. Using Matrosov's theorem ([27, Theorem 1], cf. [28, Theorem 2]), we can conclude that (42) is UGAS. ■

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longitudinal and lateral

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