Extended proton-neutron quasiparticle random-phase approximation in a boson expansion method

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The proton-neutron quasiparticle random phase approximation (pn-QRPA) is extended to include next to leading order terms of the QRPA harmonic expansion. The procedure is tested for the case of a separable Hamiltonian in the SO(5) symmetry representation. The pn-QRPA equation of motion is solved by using a boson expansion technique adapted to the treatment of proton-neutron correlations. The resulting wave functions are used to calculate the matrix elements of double-Fermi transitions. [S0556-2813(99)03107-6]

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I. INTRODUCTION

The use of the proton-neutron quasiparticle random phase approximation (pn-QRPA) in the treatment of proton-neutron excitations and in the description of charge-exchange and beta-decay observables is by now a well-known technique [1]. Among the various versions of the procedure, which was originally proposed by Baranger [2] and by Hableib and Sorensen [3], one can mention the introduction of renormalized particle-particle interactions by Vogel and others [4,5]. The agreement between earlier shell model and QRPA results, particularly in the field of double-beta-decay studies [6], was improved by the addition of the particle-particle terms of the proton-neutron interaction in the pn-QRPA equations and their use has motivated a considerable amount of work on the extension of the pn-QRPA method itself. We shall avoid the task of going into details about the several variations to the pn-QRPA approach existing in the literature, which have been reviewed recently [7], and we shall rather start the present discussion by referring to the work of Muto [8]. The work of Ref. [8] belongs to the group of theoretical works where the emphasis is put on the validity of the pn-QRPA beyond the level of the quasiboson approximation [9,10]. In Ref. [8] occupation numbers of the single-quasiparticle proton and neutron states are introduced in the equations of motion to account for density fluctuations different from the pure particle-hole ones. We think that the validity of this approximation, and the subsequent self-consistency requirement imposed at the quasiparticle level of approximation can be further investigated. We want to compare it with results obtained by performing a boson expansion method. In order to assess the validity of the method of Ref. [8] we have adopted a schematic but nontrivial Hamiltonian which belongs to the SO(5) representation and which has been shown to produce good results when compared with realistic interactions [11]. We have performed a boson expansion of this Hamiltonian by introducing a generalization of the boson expansion method developed by Evans and Krauss [12]. The use of boson expansions, in the context of proton-neutron interactions, is a relatively unexplored domain of the known applications of the boson expansion methods [13]. We have found that it is also a very useful technique in dealing with charge-dependent interactions, as we shall show next. The formalism is explained in Sec. II. The use of the present formalism to calculate the pn-QRPA eigenfunctions and eigenvectors and the behavior of the pn-QRPA near the phase transition point [14] is shown in Sec. III and the results are discussed in terms of the expectation value of the boson number operator. The pn-QRPA wave functions are used to calculate the matrix elements corresponding to double-beta-decay Fermi transitions. Conclusions about the effects due to the inclusion of the next-to-leading order terms in the proton-neutron QRPA are drawn in the last section.

II. FORMALISM

Since we are interested in the treatment of proton-neutron interactions we have adopted for the present study the schematic Hamiltonian which has been proposed by Kuz'min and Soloviev [15] and lately used in Refs. [16] and [11] in dealing with double-beta-decay calculations. The Hamiltonian includes a single-particle term, a separable monopole pairing interaction for protons and neutrons and a schematic charge-dependent residual interaction with both particle-hole and particle-particle proton(p)-neutron(n) channels [16]. It is written as

$$H=e_pN_p-G_pS_p^\dagger S_p+e_nN_n -G_nS_n^\dagger S_n+2\chi\beta^-\beta^+-2\kappa P^-P^+, \tag{1}$$

where

$$N_i=\sum_{m_i} a^\dagger_{m_i}a_{m_i},\quad S_i=\sum_{m_i} a^\dagger_{m_i}a^\dagger_{\bar{m}_i},\quad i=p,n,$$

$$\beta^-=\sum_{m_p=m_n} a^\dagger_{m_p}a_{m_n},\quad P^-=\sum_{m_p=m_n} a^\dagger_{m_p}a^\dagger_{\bar{m}_n}, \tag{2}$$

are the number operator, the monopole pair operator, the particle-hole and the particle-particle creation operators, respectively. These definitions are restricted to the same single-j state for protons and neutrons (single-shell limit) and the summations run over the m projection of the shell angular momentum j. Proton and neutron single particle orbits are denoted by the subindexes (p) and (n) and $a^\dagger_p$

$=a^{\dagger}_{j_p m_p}$ is a particle creation operator and $a^{\dagger}_{\bar{p}} =(-)^{j_p-m_p}a^{\dagger}_{j_p-m_p}$ its time reversal.

By performing the transformation of the particle creation and annihilation operators of the Hamiltonian (1) to the quasiparticle representation, by using the Bogoliubov transformations for protons and neutrons separately [1], the resulting Hamiltonian can be written

$$H_{\rm qp}=E_pN_p+E_nN_n+\lambda_1A^\dagger A+\lambda_2(A^\dagger A^\dagger+AA)-\lambda_3(A^\dagger B+B^\dagger A)-\lambda_4(A^\dagger B^\dagger+BA)+\lambda_5B^\dagger B+\lambda_6(B^\dagger B^\dagger+BB), \qquad (3)$$

where E_p ,E_n are the quasiparticle energies and the operators and matrix elements of the above equation are defined by

$$A^\dagger=[\alpha_p^\dagger\otimes\alpha_n^\dagger]^{J=0}_{M=0}, \quad B^\dagger=[\alpha_p^\dagger\otimes\alpha_{\bar{n}}]^{J=0}_{M=0},$$

$$N_i=\sum_{m_i}\alpha^\dagger_{m_i}\alpha_{m_i}, \quad m_i=n,p,$$

$$\lambda_1=4\Omega[\chi(u_p^2v_n^2+v_p^2u_n^2)-\kappa(u_p^2u_n^2+v_p^2v_n^2)],$$

$$\lambda_2=4\Omega(\chi+\kappa)u_pv_pu_nv_n,$$

$$\lambda_3=4\Omega(\chi+\kappa)u_nv_n(u_p^2-v_p^2),$$

$$\lambda_4=4\Omega(\chi+\kappa)u_pv_p(u_n^2-v_n^2),$$

$$\lambda_5=4\Omega[\chi(u_p^2u_n^2+v_p^2v_n^2)-\kappa(u_p^2v_n^2+v_p^2u_n^2)],$$

$$\lambda_6=-\lambda_2, \qquad (4)$$

following the notation of [11], with $\Omega=j+1/2$. The quasiparticle energies E_q and the occupation probabilities v_p^2 and v_n^2 are determined from the gap equation and the particle number conservation conditions.

The operator $A^\dagger(A)$ which creates (annihilates) a pair of unlike (proton-neutron)-quasiparticles and the ones corresponding to pairs of identical quasiparticles together with the charge-exchange operators $B^\dagger$, B, and the number operators N_p, N_n are the generators of the SO(5) algebra. The details about the solutions of this Hamiltonian in the SO(5) representation have been discussed in [11].

An alternative to the approximate diagonalization performed in [11], where the reference state is known to have an undetermined number of quasiparticles, consists on the boson expansion of the generators of the SO(5) algebra by applying a transformation which preserves the algebra of this group. The boson mapping proposed by Evans and Krauss [12] can be adapted to describe proton-neutron operators. We have obtained the following expressions for each of the operators defined above in terms of bosons, namely:

$$A_p^\dagger=b_p^\dagger(\Omega-n_p-n_f)^{1/2},$$

$$A_p=(A_p^\dagger)^\dagger,$$

$$A_{0p}=\left(n_p+\frac{1}{2}n_f-\frac{1}{2}\Omega\right),$$

$$A_n^\dagger=b_n^\dagger(\Omega-n_n-n_f)^{1/2},$$

$$A_n=(A_n^\dagger)^\dagger,$$

$$A_{0n}=\left(n_n+\frac{1}{2}n_f-\frac{1}{2}\Omega\right),$$

$$A^\dagger=b_f^\dagger(\Omega-n_n-n_f)^{1/2}(\Omega-n_p-n_f)^{1/2}\Phi(n_f)-\Phi(n_f)b_p^\dagger b_n^\dagger b_f,$$

$$A=(A^\dagger)^\dagger,$$

$$A_0=(n_p+n_n+n_f-\Omega),$$

$$B^\dagger=b_f^\dagger\Phi(n_f)(\Omega-n_p-n_f)^{1/2}b_n+b_p^\dagger(\Omega-n_n-n_f)^{1/2}\Phi(n_f)b_f,$$

$$B=(B^\dagger)^\dagger,$$

$$B_0=n_p-n_n, \qquad (5)$$

with

$$\Phi(n_f)=\left[\frac{(2\Omega+2-n_f)}{(\Omega+1-n_f)(\Omega-n_f)}\right]^{1/2}. \qquad (6)$$

To leading order in the previous mapping the Hamiltonian (3) reads

$$H(B)=\left(2E_p+\frac{\lambda_5}{2\Omega}\right)n_p+\left(2E_n+\frac{\lambda_5}{2\Omega}\right)n_n+\left(E_p+E_n+\frac{\lambda_5}{2\Omega}+\lambda_1\right)n_f+\frac{\lambda_6}{\Omega}(b_p^\dagger b_n+b_n^\dagger b_p)+\lambda_2(b_f^\dagger b_f^\dagger+b_fb_f)-\frac{\lambda_2}{\Omega}(b_p^\dagger b_n^\dagger+b_nb_p)-\frac{\lambda_3}{\sqrt{\Omega}}(b_n^\dagger+b_n)n_f-\frac{\lambda_4}{\sqrt{\Omega}}(b_p^\dagger+b_p)n_f. \qquad (7)$$

The linearized version of this Hamiltonian is obtained by introducing the phonon operator

$$\Gamma^\dagger=X_fb_f^\dagger-Y_fb_f, \qquad (8)$$

and by diagonalizing the Hamiltonian in the QRPA phonon basis. The QRPA equation of motion is written

$$[H(B),\Gamma^\dagger]=\omega\Gamma^\dagger, \qquad (9)$$

and the amplitudes X and Y and the energy ω are readily determined by solving the dispersion relation

$$(E_f-\omega)X+2\lambda_2Y=0,$$

$$2\lambda_2 X+(E_f+\omega)Y=0, \tag{10}$$

with solutions

$$\omega=\sqrt{E_f^2-(2\lambda_2)^2}, \tag{11}$$

for the energy and

$$X=-\sqrt{\frac{E_f+\omega}{2\omega}}, \quad Y=\sqrt{\frac{E_f-\omega}{2\omega}}, \tag{12}$$

for the amplitudes. In the above equations

$$E_f=E_p+E_n+\frac{\lambda_5}{2\Omega}+\lambda_1, \tag{13}$$

and it shows that the effect of the $1/\Omega$ terms is basically reflected upon the unperturbed quasiparticle-pair energy and it is quite similar to the exchange term of the QRPA ladder diagram [9], as expected. The addition of terms proportional to the couplings λ_3 and λ_4, which are of the order $1/\sqrt{\Omega}$ and which are usually referred to as scattering terms, would require a nonzero number of f bosons in the ground state and then it is not allowed by the present approach [17,18]. In the following section we shall present and discuss the results of the calculations which we have performed using the above introduced formalism.

III. RESULTS AND DISCUSSIONS

We have solved the pn-QRPA equations in the boson mapping representation and for two sets of model parameters. They are $\Omega=10$, $N_p=2$ protons, and $N_n=8$ neutrons (set 1) and $\Omega=20$, $N_p=6$, and $N_n=14$ (set 2). These values are taken from Ref. [11] in order to allow for a comparison between present results and the ones obtained in the SO(5) representation. Excited proton-neutron two-quasiparticle states in these model spaces represent states of the double-odd mass system built upon the initial double-even mass one. In order to determine the effects due to the inclusion of terms of the order of $1/\Omega$ in the pn-QRPA solutions we have diagonalized the pn-QRPA equations in the boson basis. The results corresponding to the energy of the one-phonon state are shown in Fig. 1, for the two sets of parameters described in the text. The results are given as functions of the coupling constant of the attractive proton-neutron particle-particle channels, κ, which is measured in units of the strength of the pairing interaction G.

It is evident from the results shown in Fig. 1 that the collapse of the energy of the first excited state is not avoided by the inclusion of terms of the order of $1/\Omega$ and that the relative contribution of these terms decreases for larger values of the shell degeneracy, as expected. In this respect, the present results are at variance with the results of Ref. [8] since the inclusion of the new terms does not shift the point of collapse to larger values of the coupling constant associated to attractive particle-particle channels but to smaller values, instead. The difference can be explained by saying that in the present approach the order of the contributions, in terms of the expansion in powers of $1/\Omega$, is controlled by the boson expansion while in Ref. [8] it should be a strong mixing of orders due to the diagonalization.

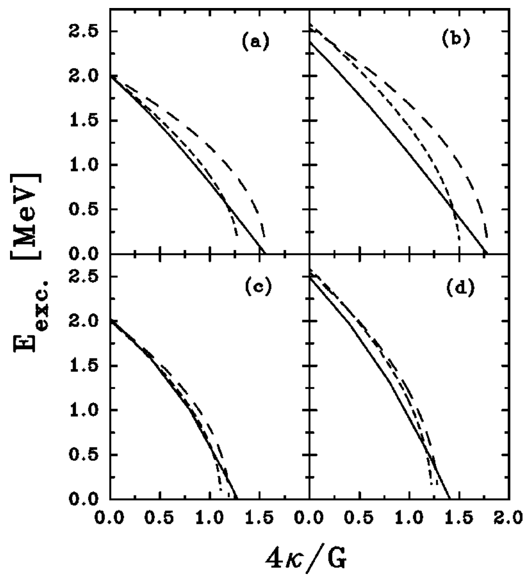

FIG. 1. Excitation energy as a function of $4\kappa/G$. In insets (a) and (b) are displayed results for $\Omega=10$, $N_p=2$, $N_n=8$, and for $\chi=0.0$ and $\chi=0.04$, respectively. Results shown in insets (c) and (d) correspond to $\Omega=20$, $N_p=6$, $N_n=14$, for $\chi=0.0$ and $\chi=0.025$, respectively. Solid lines correspond to the exact solution of the Hamiltonian in the SO(5) representation (see Ref. [11]), long-dashed lines represent the usual pn-QRPA and short-dashed lines correspond to the results obtained by including corrections of order $1/\Omega$ in the boson expansion of the pn-QRPA.

The overgrowing contribution of ground state correlations near the point of collapse, which in Ref. [8] is demostrated by the behavior of the number of quasiparticle in the final state, is shown here by the number of pn bosons in the ground state, which diverges at the point of collapse, as is shown by the curves of Fig. 2.

Finally, in order to demostrate the effects due to the treatment of the Hamiltonian beyond the leading QRPA terms, we have calculated the matrix elements

$$M_{2\nu}=\sum_n \frac{\langle f||\tau^-||n\rangle\langle n||\tau^-||i\rangle}{E_n+E_0}, \tag{14}$$

corresponding to double-Fermi transitions connecting the initial state (N,Z) with a final state $(N-2,Z+2)$, as is done in nuclear structure calculations of double-beta-decay transitions [7]. The results are shown in Fig. 3. As is well known [4,5,7,8], the second order matrix element $M_{2\nu}$ vanishes, as a function of the coupling strength κ and the inclusion of terms of the order of $1/\Omega$ does not prevent this trend. Similar results are obtained in Ref. [8] in spite of the fact that the corrections added to the pn-QRPA matrix have for the present case and for the case of Ref. [8] different origin. For the sake of completeness we have included in Figs. 1 and 3 the exact results obtained in the SO(5) representation, as done in Ref. [11].

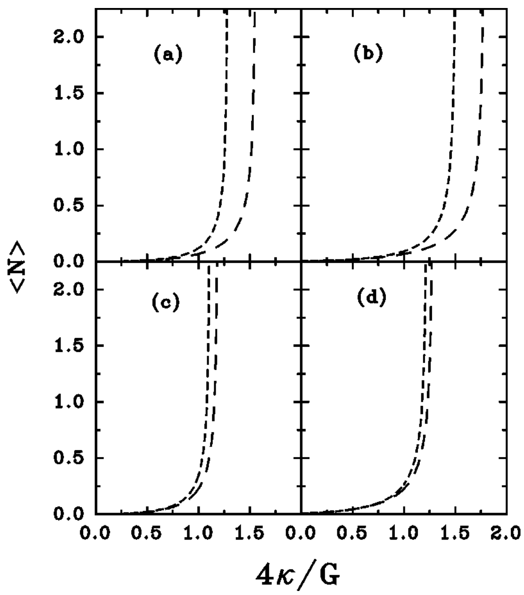

FIG. 2. Average number of quasiparticles as a function of $4\kappa/G$. The notation is the same as the one given in the caption to Fig. 1.

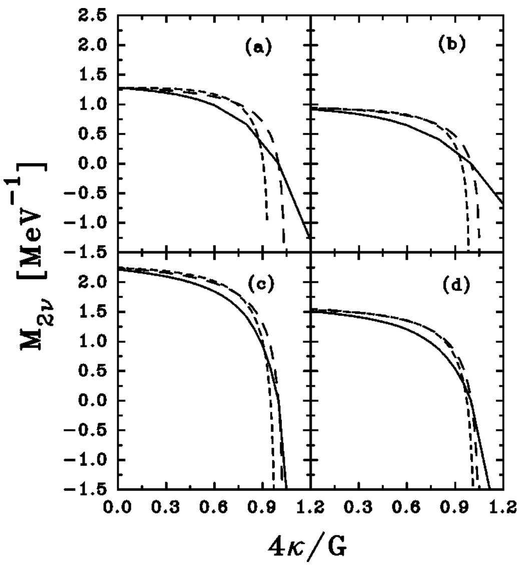

FIG. 3. Matrix element $M_{2\nu}$ as a function of $4\kappa/G$. The notation is explained in the caption to Fig. 1.

IV. CONCLUSIONS

In the present work we have adapted the boson mapping of Ref. [12] to the case of proton-neutron correlations. We have solved proton-neutron QRPA equations in the boson basis and shown that the inclusion of terms of the order of $1/\Omega$ does not prevent the collapse of the pn-QRPA induced by attractive proton-neutron, isospin dependent, interactions. The case is demostrated for a Hamiltonian belonging to the SO(5) group, which includes monopole isovector pairing interactions and isospin dependent two body interactions. The results obtained by using the proposed boson expansion method have been compared with the ones obtained by including density dependent corrections in the QRPA equations. It is found that both methods lead to basically the same conclusion about the collapse of the pn-QRPA, although the boson expansion method has the advantage of controlling the expansion in terms of the shell degeneracy. As for previously reported studies these conclusions, which are based on a schematic Hamiltonian, can be of some use in understanding the overall trend of more realistic calculations, as the ones reported in [7].

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