Extending Orthogonal Block Codes with Partial Feedback

Jabran Akhtar, David Gesbert
Department of Informatics, University of Oslo
P.O.Box 1080 Blindern, 0316 Oslo, Norway
E-mail: jabrana@ifi.uio.no, gesbert@ifi.uio.no

Abstract

During the last years a number of space-time block codes have been proposed for use in multiple transmit antennas systems. In this letter we propose a method to extend any space-time code constructed for m transmit antennas to $m\,p$ transmit antennas through Group-Coherent Codes (GCC). GCC make use of very limited feedback from the receiver (as low as one bit). In particular the scheme can be used to extend any orthogonal code (e.g. Alamouti code) to more than two antennas while preserving low decoding complexity, full diversity benefits and full data rate.

Index Terms

Diversity, transmit diversity, space-time block codes, antenna array processing.

I. Introduction

Since the work of Alamouti [1], space-time block coding (STBC) has been an intensive area of research, with several original design strategies having been put forward recently [2], [3], [4]. Because of the decoding simplicity they offer, orthogonal design approaches have been in particular focus lately. However, as was shown in [2], truly orthogonal full-rate designs offering full diversity (diversity order being equal to the number of independent fading transmit/receive antenna pairs) for any arbitrary complex symbol constellation are limited to the case of 2 transmit antennas. The data-rate or decoding simplicity must then be sacrificed if the number of antenna is increased.

As we show here, simple alternative design solutions are possible that can exploit the limited feedback data provided in an increasing number of standards (for example the WCDMA standard allows for roughly one feedback bit per slot [5]). The majority of orthogonal STBC codes are designed assuming the transmitter has no knowledge about the channels. Ideally, the transmitter could exploit channel state

This work was supported by a research contract with Telenor R&D. Parts of this paper were presented at VTC Spring 2003.

information (CSI) to improve the performance of the system substantially. For example, with complete channel knowledge at the emitter data can be transmitted on the eigenvector related to the largest eigenvalue [6] providing both a full diversity advantage combined with a transmit array (beamforming) gain. More realistically, the feedback channel may allow only partial CSI to be returned to the transmitter in order to save bandwidth. For instance quantized phases of the channels can be fed back, and in [7], [8] a method to provide transmit diversity by taking account of the relative phases was introduced. The same symbols are transmitted from all the transmit antennas but with different phase shifts. This technique in general requires $(M-1)\log_2 K$ bits of feedback where M is the number of transmit antennas and K denotes the number of points in the uniformly quantized channel phase space. Other construction methods with finite rate feedback are discussed in e.g. [9].

Despite the recent interest in orthogonal codes and the practically of feedback systems, research on block codes with partial feedback is only beginning to gain attention. In this letter we suggest a simple way to combine space-time block codes over groups of antennas to ensure the full diversity advantage from arbitrary number of transmit antenna at full rate. The idea is based on the concept of *group-coherent* code construction. If a full diversity code is available for m antennas, then group-coherent codes can be constructed for $m\ p$ $(p \geq 2)$ antennas exhibiting $m\ p$ orders of diversity and full rate by making use of as low as $p-1$ bits of feedback encoding certain channel-related phase information described in this paper. Importantly, we show that if we start from an orthogonal design, the extension fully preserves the simple decoding structure permitting a low decoder complexity. The key argument for using group-coherent codes is that an exact block design providing orthogonal decoding can be obtained at a loss of rate (in the form of feedback) largely inferior to the loss of rate incurred by the best available non-feedback orthogonal codes found in for example [2] [10] and [11].

II. System design

We first consider a system with m transmit antennas and a single receive antenna. The transmitting antennas are assumed to be placed sufficiently apart from each other so that symbols transmitted from each antenna follow different uncorrelated paths to the receiver. We assume that the receiver can estimate the channels from each transmit antenna $\mathbf{h} = [h_1\ h_2\ ...\ h_m]^T$ and has perfect knowledge of $\mathbf{h}$. The channel gains are modeled as complex Gaussian with zero mean and unit variance $h_i \sim C(0,1)$. For a setup with $m\ p$ transmit antennas we further assume that a limited feedback link is available from receiver to the transmitter in the form of a minimum of $p-1$ bits per frame/channel coherence period.

A. STBC for m transmit antennas

Consider a block code matrix $\mathbf{C}$ (of size T rows (time-slots) and m columns (transmitters)) and linear in the input modulation symbols (collected in vector $\mathbf{s}$) and their conjugates. Let us denote the received signal over T symbol durations by $\hat{\mathbf{y}} = \mathbf{C}\mathbf{h}+\mathbf{n}$, where $\mathbf{n}$ is the noise vector, which is usually conveniently denoted as $\mathbf{y} = \mathbf{H}\mathbf{s}+\mathbf{n}$. $\mathbf{H}$ is the equivalent channel matrix containing linear combinations of the channels and their conjugates. The transformation for orthogonal codes between $\hat{\mathbf{y}}$ and $\mathbf{y}$ would typically require possible conjugation operations on $\hat{\mathbf{y}}$ to obtain $\mathbf{y}$ and/or splitting of the symbol components in real and imaginary parts. A general discussion on this transformation and when the equivalence between these two representations holds is given in [12].

Assuming the code to be an orthogonal design, we have $\mathbf{H}^*\mathbf{H} = \frac{1}{m}\alpha\mathbf{I}$, $\alpha = \sum_{i=1}^{m} |h_i|^2$, exhibiting m-th order diversity. For instance decoding can be performed through simple matched filtering using $\mathbf{H}^*$, $\hat{\mathbf{s}} = \frac{1}{\mathbf{H}^*\mathbf{H}}\mathbf{H}^*\mathbf{y}$. In the case of orthogonal block codes and PSK modulation this has been shown to be equivalent with ML decoding. The terms on the diagonal reflect the SNR at the receiver if they are normalized by a constant factor $\frac{E}{P}$ where E is the total transmit energy and P the noise power.

Full-rate orthogonal codes providing full diversity for complex symbols only exist for $m = 2$ and extensions of these to a larger number of antennas is an important issue. Non full-rate orthogonal codes providing full diversity for any number of transmit antennas have been constructed in [2]. It is known for instance that the best available orthogonal design with 4 antennas can only be $3/4$ rate, which implies that a full fourth order diversity is obtained at the price of 25% reduction in rate.

B. Group-coherent codes for $m\ p$ antennas

We next build an extension method to utilize STBC on a larger number of antennas which replaces the bandwidth rate loss of orthogonal STBC by a marginal loss in the form of feedback, while still preserving full performance.

We start by presenting a simple design for group-coherent codes. Assuming we have an orthogonal space-time block code $\mathbf{C}$ for m transmit antennas, a code for $m\ p$ antennas, where p is an integer ≥ 2, may be constructed as:

$$\mathbf{D}_l = \frac{1}{\sqrt{p}}[\mathbf{C} \;\; b_1\mathbf{C} \;\; b_2\mathbf{C} \;\; ... \;\; b_{p-1}\mathbf{C}], \quad l = 1, 2, ..., 2^{p-1}. \tag{1}$$

The columns in the matrices represent the antennas while the rows in matrices still refer to the time slots. We assume $b_i \in \{-1, 1\}$ and each $b_i\mathbf{C}$, $i = 1, .., p-1$ can therefore take two forms, $\mathbf{C}$ or $-\mathbf{C}$, yielding a total of 2^{p-1} different possible realizations of $\mathbf{D}_l$. A discussion over possible further expansion is found in [13]. The factor $\frac{1}{\sqrt{p}}$ ensures proper normalization of $\mathbf{D}$. For convenience we also define $b_0 = +1$.

If a particular $\mathbf{D}_l$ structure is implemented then the channel vector $\mathbf{h}$ contains $m\ p$ coefficients and can be written in a split form of p subsets $\mathbf{h} = [\mathbf{h}_1^T\ ...\ \mathbf{h}_p^T]^T$, where the k'th ($k = 1, .., p$) group of channels is associated with the k'th subblock in $\mathbf{D}_l$.

The feedback information from the receiver is now defined as the index l in $\mathbf{D}_l$ which will determine which version of code to implement for transmission. Clearly, the original code $\mathbf{C}$ for m antennas consists of m columns while the new variants $\mathbf{D}_l$ contains p times as many columns. The same symbols as by the original code are now transmitted from p antennas simultaneously, with the possibility of different signs. Figure 1 displays a schematic construction of the system. Although the codes $\mathbf{D}_l$ do not satisfy the requirements for an orthogonal design as set in [2], we next show two properties of group-coherent codes addressing respectively the (low) complexity decoding and the diversity performance. Our first result demonstrates that a linear combination of linear orthogonal designs provides a design which inherits the simplified decoding structure of the orthogonal block code $\mathbf{C}$:

Proposition *1: For any combination of b_1, b_2, ..,b_{p-1} $\mathbf{D}_l$ is an orthogonal linear design in the sense that matched-filter-based decoding can be applied.*

Proof: Transmitting via $\mathbf{D}_l$, the receiver notices

$$\hat{\mathbf{y}} = \frac{1}{\sqrt{p}}\mathbf{D}_l\mathbf{h} + \mathbf{n}$$

which is equivalent to

$$\hat{\mathbf{y}} = \frac{1}{\sqrt{p}}[\mathbf{C}\ b_1\mathbf{C}\ ...\ b_{p-1}\mathbf{C}]\mathbf{h} + \mathbf{n}.$$

Writing out with the subsets of channel $\mathbf{h} = [\mathbf{h}_1^T\ ...\ \mathbf{h}_p^T]^T$ we arrive to

$$\hat{\mathbf{y}} = \frac{1}{\sqrt{p}}(\mathbf{C}\mathbf{h}_1 + b_1\mathbf{C}\mathbf{h}_2 + ... + b_{p-1}\mathbf{C}\mathbf{h}_p) + \mathbf{n}$$

$$= \frac{1}{\sqrt{p}}\mathbf{C}(\mathbf{h}_1 + b_1\mathbf{h}_2 + ... + b_{p-1}\mathbf{h}_p) + \mathbf{n} = \mathbf{C}\hat{\mathbf{h}} + \mathbf{n}. \quad \square$$

The transmission is thus equivalent to using the block code $\mathbf{C}$ over the virtual channel $\hat{\mathbf{h}}$ consisting of linear combinations of subsets $\mathbf{h}_1,\ ...,\ \mathbf{h}_p$. On average, using a group code with randomly selected b_i will not improve upon the performance offered by the original m-antenna code, however, using the feedback information from the receiver on which code matrix $\mathbf{D}_l$ to employ can ensure that the symbols repeated on groups of p antennas always add up coherently at the receiver.

More generally when the number of antennas is scaled from m to $m\ p$ and the coding strategy proposed in (1) is used then the receiving party observes:

$$\hat{\mathbf{y}} = \frac{1}{\sqrt{p}}\mathbf{D}_l\mathbf{h} + \mathbf{n} = \frac{1}{\sqrt{p}}(\mathbf{Ch}_1 + b_1\mathbf{Ch}_2 + b_2\mathbf{Ch}_3 + .. + b_{p-1}\mathbf{Ch}_p) + \mathbf{n}$$

which can be written as

$$\mathbf{y} = \frac{1}{\sqrt{p}}(\mathbf{H}_1 + b_1\mathbf{H}_2 + ... + b_{p-1}\mathbf{H}_p)\mathbf{s} + \mathbf{n} = \frac{1}{\sqrt{p}}\mathbf{Rs} + \mathbf{n} \tag{2}$$

where $\mathbf{H}_i$ contains channel coefficients corresponding to $\mathbf{h}_i$. From Proposition 1 we know that transmission with $\mathbf{D}_l$ is comparable to employment of code $\mathbf{C}$. The corresponding channel matrix $\mathbf{R}$ is therefore orthogonal (up to a scalar multiplication) assuming the conditions put forward in [12] are satisfied, and decoding can be based on using $\mathbf{R}^*$ on $\mathbf{y}$. The inner channel product $\mathbf{R}^*\mathbf{R}$ leads to:

$$\mathbf{R}^*\mathbf{R} = \sum_{i=1}^{p}\mathbf{H}_i^*\mathbf{H}_i + \sum_{i,j=1,i\neq j}^{p} b_{i-1}b_{j-1}\mathbf{H}_i^*\mathbf{H}_j. \tag{3}$$

p factors of form $\mathbf{H}_i^*\mathbf{H}_i = \frac{1}{m}\alpha_i\mathbf{I}, \alpha_i = \sum_{j=m(i-1)+1}^{mi}|h_j|^2$, offer the total $m\ p$ orders of diversity. Writing out the diagonal elements in full we come to:

$$(\mathbf{R}^*\mathbf{R})_{i,i} = \frac{1}{m}(\sum_{i=1}^{m\ p}|h_i|^2 + \beta). \tag{4}$$

where β originates from terms of form $\mathbf{H}_i^*\mathbf{H}_j$, introducing ”interference” between channels $h_{m(i-1)+1}, ..., h_{mi}$ and $h^*_{m(j-1)+1}, ..., h^*_{mj}$. By collecting the anti-symmetrical factors together, real valued β can be written out as:

$$\beta = \sum_{i=1}^{m(p-1)} b_{\llcorner\frac{i-1}{m(p-1)}\lrcorner}b_{\llcorner\frac{i+m-1}{m(p-1)}\lrcorner}(h_ih^*_{i+m} + h_{i+m}h^*_i)$$
$$+b_{\llcorner\frac{i-1}{m(p-1)}\lrcorner}b_{\llcorner\frac{i+2m-1}{m(p-1)}\lrcorner}(h_ih^*_{i+2m} + h_{i+2m}h^*_i)$$
$$+... + b_{\llcorner\frac{i-1}{m(p-1)}\lrcorner}b_{\llcorner\frac{i+(p-1)m-1}{m(p-1)}\lrcorner}(h_ih^*_{i+(p-1)m} + h_{i+(p-1)m}h^*_i). \tag{5}$$

$\llcorner\lrcorner$ represent rounding off downwards to the nearest integer.

The second key property now follows:

Proposition *2: Using the coding strategy from (1) on an $m\ p$ antenna system, then $p-1$ bits of feedback are sufficient to assure*

$$(\mathbf{R}^*\mathbf{R})_{i,i} \geq \frac{1}{m}\sum_{i=1}^{m\ p}|h_i|^2, \tag{6}$$

i.e. to guarantee a diversity order of $m\ p$.

Proof: For $p = 1$ (no feedback) the result clearly holds. We now follow an inductive approach and assume the group-coherent code for $m\ (p-1)$ transmitters offers full diversity with $p-2$ bits of feedback. The enlarged system (2) for $m\ p$ transmitters ($p \geq 2$) can then be written as

$$\begin{aligned} \sqrt{p}\ \mathbf{y} &= (\mathbf{H}_1 + b_1\mathbf{H}_2 + ... + b_{p-2}\mathbf{H}_{p-1})\mathbf{s} + b_{p-1}\mathbf{H}_p\mathbf{s} + \sqrt{p}\mathbf{n} \\ &= (\mathbf{R}_1 + b_{p-1}\mathbf{H}_p)\mathbf{s} + \sqrt{p}\mathbf{n} \end{aligned} \tag{7}$$

where $\mathbf{R}_1$ is self-defineable. The inner channel product gives:

$$\begin{aligned} (\mathbf{R}_1^* + b_{p-1}\mathbf{H}_p^*)(\mathbf{R}_1 + b_{p-1}\mathbf{H}_p) &= \\ \mathbf{R}_1^*\mathbf{R}_1 + \mathbf{H}_p^*\mathbf{H}_p + b_{p-1}(\mathbf{R}_1^*\mathbf{H}_p + \mathbf{H}_p^*\mathbf{R}_1). & \qquad \square \end{aligned} \tag{8}$$

From our assumption we already know $\mathbf{R}_1^*\mathbf{R}_1$ provides $m\ (p-1)$ orders of diversity and consequently selecting an appropriate value of either $+1$ or -1 for b_{p-1} is sufficient to make proposition 2 hold.

C. Feedback bit selection

The inductive argument of the second proposition also provides an efficient method to find suitable values of $b_1, ..., b_{p-1}$ which can offer full diversity. At each iteration, starting from b_1 one would select $b_{p-1} = -sign(\mathbf{R}_1^*\mathbf{H}_p + \mathbf{H}_p^*\mathbf{R}_1)$ and then proceed with the next block.

As an alternative approach, the $p-1$ bits for feedback b_i may be selected according to:

$$\mathbf{D}_l = \arg \max_{b_i | \mathbf{h}} \ \beta, \qquad l = 1, 2, ..., 2^{p-1} \tag{9}$$

i.e. an exhaustive search can be used. If $p \geq 3$ then for certain channel realizations, several selections of the coefficients may return positive values for β. In this case the particular selection giving the largest β through (9) may be preferable, as it would provide the largest array gain but at additional computational complexity than the inductive search.

D. Non-orthogonal block codes

The approach presented above may also be applied with full-rate non-orthogonal block codes [3], [12]. These codes typically introduce interference elements which appear on the non-diagonal elements of the corresponding inner channel matrix $\mathbf{H}^*\mathbf{H}$. The diagonal terms on the other hand still follow the identical structure as in orthogonal codes and the results presented in this article would therefore still apply. This design construction assures that the time-delay of non-orthogonal codes, which increases very rapidly with growing number of transmitters, is kept at a low level.

III. EXAMPLE AND SIMULATIONS

We next provide a simple example to demonstrate how these techniques may be applied to a four transmitter antenna system. An one bit feedback scheme may be constructed by extending any orthogonal space-time block for m antenna to $2m$ transmit antennas.

Starting from the Alamouti code we have:

$$\mathbf{C} = \frac{1}{\sqrt{2}} \begin{pmatrix} s_0 & s_1 \\ -s_1^* & s_0^* \end{pmatrix}. \tag{10}$$

Group-coherent codes for 4 transmitters are then directly obtained from (1) by setting $p = 2$, which gives

$$\mathbf{D}_1 = \frac{1}{\sqrt{2}}[\mathbf{C}\ \mathbf{C}] = \frac{1}{2} \begin{pmatrix} s_0 & s_1 & s_0 & s_1 \\ -s_1^* & s_0^* & -s_1^* & s_0^* \end{pmatrix}, \tag{11}$$

$$\mathbf{D}_2 = \frac{1}{\sqrt{2}}[\mathbf{C}\ -\mathbf{C}] = \frac{1}{2} \begin{pmatrix} s_0 & s_1 & -s_0 & -s_1 \\ -s_1^* & s_0^* & s_1^* & -s_0^* \end{pmatrix}. \tag{12}$$

With Alamouti the receiver will over two time frames observe $\hat{\mathbf{y}} = \mathbf{C}\mathbf{h} + \mathbf{n}$, or $\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}$ (where the second observation in $\hat{\mathbf{y}}$ has been conjugated to obtain $\mathbf{y}$) where

$$\mathbf{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{pmatrix}. \tag{13}$$

$\mathbf{D}_1$ or $\mathbf{D}_2$ is the code used over four transmitters, giving the equivalent channel matrices

$$\mathbf{R}_{\hat{1}} = \frac{1}{2}(\mathbf{H}_1 + \mathbf{H}_2) = \frac{1}{2} \begin{pmatrix} h_1 + h_3 & h_2 + h_4 \\ h_2^* + h_4^* & -h_1^* - h_3^* \end{pmatrix}, \tag{14}$$

$$\mathbf{R}_{\hat{2}} = \frac{1}{2}(\mathbf{H}_1 - \mathbf{H}_2) = \frac{1}{2} \begin{pmatrix} h_1 - h_3 & h_2 - h_4 \\ h_2^* - h_4^* & -h_1^* + h_3^* \end{pmatrix}. \tag{15}$$

Looking at the hermitian squares of the decoding matrices we observe that interference has been introduced on the diagonal:

$$\mathbf{R}_{\hat{1}}^*\mathbf{R}_{\hat{1}} = \frac{1}{4} \begin{pmatrix} \hat{\alpha}_1 & 0 \\ 0 & \hat{\alpha}_1 \end{pmatrix}, \quad \hat{\alpha}_1 = \sum_{i=1}^{4} |h_i|^2 \ + \beta \tag{16}$$

and the interference factor

$$\beta = h_1 h_3^* + h_1^* h_3 + h_2 h_4^* + h_2^* h_4. \tag{17}$$

Transmitting with $\mathbf{D}_2$, β in $\mathbf{R}_2^*\mathbf{R}_2$ will exhibit the opposite sign:

$$\mathbf{R}_{\hat{2}}^*\mathbf{R}_{\hat{2}} = \frac{1}{4}\begin{pmatrix} \hat{\alpha}_2 & 0 \\ 0 & \hat{\alpha}_2 \end{pmatrix}, \quad \hat{\alpha}_2 = \sum_{i=1}^{4} |h_i|^2 - \beta. \tag{18}$$

In the scheme, the receiver computes (17) and feeds back $+1$ if (17) ≥ 0 or -1 if (17) < 0, whichever maximizes/returns a positive value. If the feedback bit is then used to select the appropriate code at the receiver and the transmitter with a positive β, the resulting code permits orthogonal decoding and achieves fourth order diversity. In addition there is *some* array gain (about 1.5dB, see below) since $|\beta| > 0$ with probability one. Note that from a pure bandwidth point of view, the loss due to feedback is negligible compared to the loss (25%) in the best known orthogonal design for 4 antennas.

Figure 2 displays a simulation under a QPSK constellation with quasi-static flat Rayleigh fading. The cases considered include single transmitter and single receiver (full-rate), the Alamouti scheme with two transmitters (full-rate) and the curve for an ideal fourth order diversity system. These are compared against a feedback based group-coherent code for four transmitters offering full-rate. The one-bit feedback strategy, as detailed in example 1 is simulated, demonstrating the full diversity and limited array gain. The energy of the setup is kept at a constant level as the number of emitters grow. We also assume perfect knowledge of the channels at the receiver and also zero-delay in the feedback. The transmitter thus always picks the correct version of the code for transmission.

IV. Conclusions

This letter proposes a method to extend space-time block codes with the aid of limited feedback. Compared to traditional feedback based diversity methods the new technique requires fewer bits of feedback and provides significant rate efficiency than orthogonal block codes with no feedback.

References

[1] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Comm.*, vol. 16, pp. 1451–1458, Oct. 1998.

[2] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inf. Theory*, vol. 45, pp. 1456–1467, July 1999.

[3] O. Tirkkonen, A. Boariu, and A. Hottinen, "Minimal non-orthogonality rate 1 space-time block code for 3+ tx antennas," in *Proc. IEEE Int. Symp. Spread Spectrum Technology*, 2000.

[4] H. Jafarkhani, "A quasi orthogonal space time block code," *IEEE Trans. Comm.*, vol. 49, pp. 1–4, Jan. 2001.

[5] R. T. Derryberry, S. D. Gray, D. M. Ionescu, G. Mandyam, and B. Raghothaman, "Transmit diversity in 3G CDMA systems," *IEEE Communications Magazine*, vol. 40, April 2002.

[6] A. Narula, M. Lopez, M. Trott, and G. Wornell, "Efficient use of side information in multiple antenna data transmission over fading channels," *IEEE J. Sel. Areas Comm.*, vol. 16, pp. 1423–1436, Oct. 1998.

[7] R. W. Heath and A. J. Paulraj, "A simple scheme for transmit diversity using partial channel feedback," in *Proc. of the 32nd Asilomar Conf. on Signals, Systems, and Computers*, 1998.

[8] K. K. Mukkavilli, A. Sabharwal, M. Orchard, and B. Aazhang, "Transmit diversity with channel feedback," in *Proc. Intl. Symposium on Telecommunications*, 2001.

[9] K. K. Mukkavilli, A. Sabharwal, E. Erkip, and B. Aazhang, "On beamforming with finite rate feedback in multiple antenna systems," *IEEE Trans. on Info. Theory*, p. to appear, 2003.

[10] W. Su and X.-G. Xia, "Two generalized complex orthogonal space-time block codes of rates 7/11 and 3/5 for 5 and 6 transmit antennas," *IEEE Trans. Inf. Theory*, vol. 49, pp. 313–316, Jan. 2003.

[11] X.-B. Liang, "A high-rate orthogonal space-time block code," *IEEE Communications Letters*, vol. 7, pp. 222–223, May 2003.

[12] A. Boariu and M. Ionescu, "A class on non-orthogonal rate-one space-time block codes with controlled interference," *IEEE Trans. Wireless Communications*, vol. 2, pp. 270–276, Mar. 2003.

[13] J.Akhtar and D. Gesbert, "Partial feedback based orthogonal block coding," in *Proc. VTC Spring*, vol. 1, pp. 287–291, 2003.

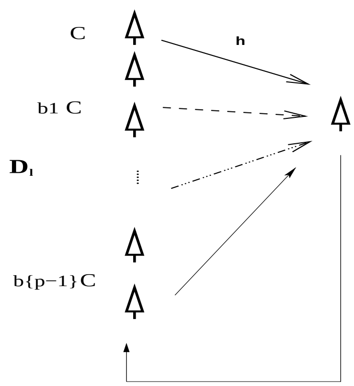

Fig. 1. Schematic description of group-coherent codes

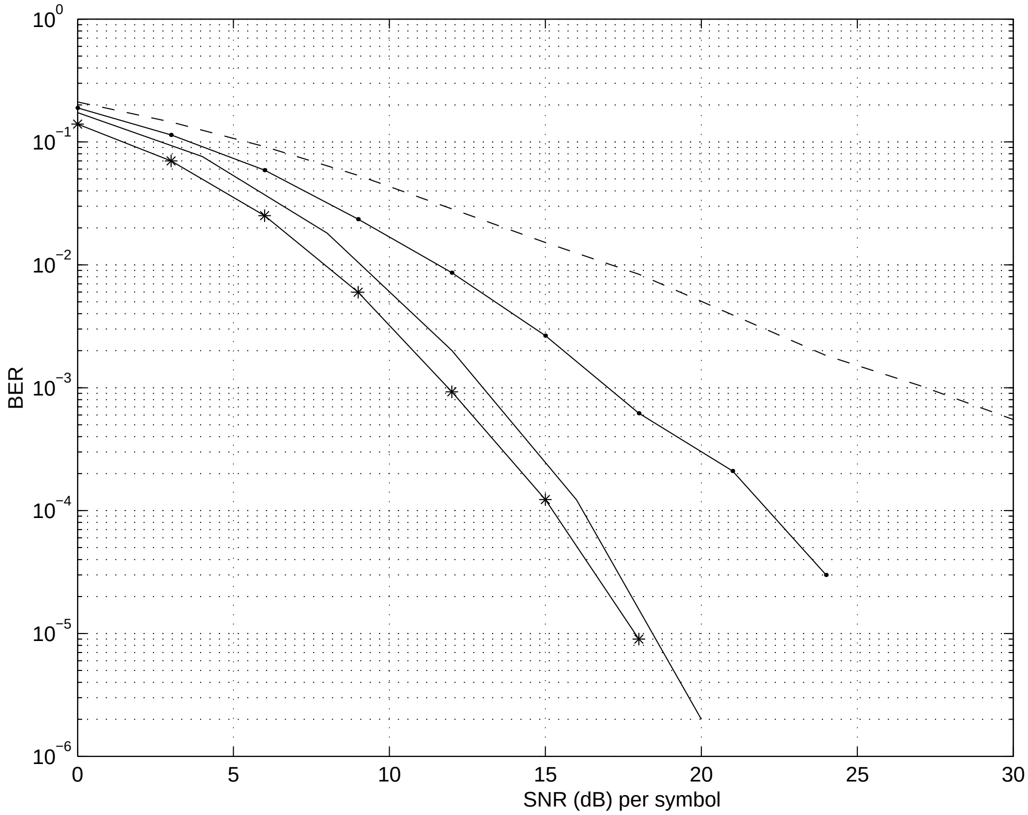

Fig. 2. From worst to best: No diversity (1tx, 1rx); Alamouti (2tx, 1rx); ideal fourth order diversity (4tx, 1rx); Proposed code with 1 bit feedback (4tx, 1rx)