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Extending the applicability of Newton's method for variational inequality problems under Smale-Wang-γ criteria

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Dedicated to the memory of Professor Ştefan Măruşter

Abstract. The $\gamma-$ theory for solving variational inequality problems using Newton's method is extended using restricted convergence domains and the center $\gamma-$ criterion.

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1 Introduction

Let i be a positive integer and D be a non-empty, closed and convex subset of $H = \mathbb{R}^i$. Let also $G : D \longrightarrow H$ be differentiable. The variational inequality problem, denoted by $A := VIP(D, G)$ involves finding a vector $p \in D$ such that

$$(y - p)^T G(p) \geq 0 \text{ for all } y \in D. \tag{1.1}$$

Many problems can be formulated as (1.1) using mathematical modeling [1–10]. Such problems involve but not limited to equilibrium problems, constrained optimization problems complementarity and other problems [1–9]. One wishes to find a vector p satisfying (1.1) in explicit form, but this is seldomly possible. That is why, one employs a numerical method to generate a sequence approximating p.

One of the most important methods is Newton's defined for all $n = 0, 1, \ldots$ by

$$(y - x_{n+1})^T (G(x_n) + G'(x_n)(x_{n+1} - x_n)) \geq 0 \text{ for all } y \in D, \tag{1.2}$$

where x_0 is an initial point. That is x_{n+1} solve $A_n = VIP(D, G^n)$, where for all $x \in D$, $G^n(x) := G(x_n) + G'(x_n)(x - x_n)$. The convergence region for Newton's method using the Kantorovich theory [1, 2, 7] or Smale's α theory [9] or Wang's $\gamma-$ theory [2, 3, 10] is small in general. It turns out that the convergence region can be extended using smaller Lipschitz, α, γ constants than before [1–3, 7, 9, 10] and without additional hypotheses. This is the novelty of the present study. We achieve this goal by considering a more precise convergence domain where the Newton iterates lie than in the aforementioned studies. This way the resulting new constants are at least as small leading to the following advantages for local convergence analysis:

(a) Larger ball of convergence.

(b) More precise distances $\|x_n - p\|$.

Therefore, the local convergence advantages lead to a greater choice of initial guesses x_0 and at least as few iterates to achiev a desired error tolerance. Semi-local convergence analysis:

(a) Larger convergence region.

(b) More precise distances $\|x_n - p\|$.

(c) Better information about the solution.

That is the convergence domain is extended and again the number of iterates is reduced.

These advantages are important in computational mathematics [1–10] (see also Remark 2.8 and Remark 3.2). Our idea of restricted convergence domains can be used to extend the applicability of other iterative methods along the same lines [1–10].

Let $p \in H$ and $\rho > 0$. Let $U(p, \rho)$ be the open metric ball in H and let $\bar{U}(p, \rho)$ be its closure. In the rest of the study $G : D \subseteq H \longrightarrow H$ is a C^2 mapping and by $\tilde{\mathrm{M}} := \frac{1}{2}(M + M^T)$ for all $M \in H \times H$.

The rest of the paper is structured as follows. Section 2 contains the local convergence whereas in Section 3 we study the semi-local convergence analysis of the method (1.2).

2 Local convergence

Some definitions and auxiliary results follow:

Definition 2.1. *Let $0 < r < \frac{1}{\gamma_0}$, for some $\gamma_0 > 0$. Let also $w \in H$ be such that $G'(w)$ is positive definite. It is said that G satisfies the center γ_0-criterion at w in $D \cap U(w,r)$, if*

$$\|\tilde{G}'(w)^{-1}\|\|G'(x) - G'(w)\| \leq \frac{1}{(1-\gamma_0\|x-w\|)^2} - 1 \text{ for all } x \in U(w,r). \tag{2.1}$$

We shall use the function g defined on the interval $[0, \frac{2-\sqrt{2}}{2})$ by $g(t) = 1-4t+2t^2$. Clearly, function g is monotonically decreasing on $[0, \frac{2-\sqrt{2}}{2})$). We need an auxiliary result on positive definite matrices.

Lemma 2.1. *Let $0 < r < r_0 := \frac{2-\sqrt{2}}{2\gamma_0}$. Let also $w \in H$ be such that $\tilde{G}'(w)$ is positive definite. Suppose that G satisfies the center γ_0- criterion at w in $D \cap U(w,r)$. Then, the following items hold $\tilde{G}(x)$ is positive definite and*

$$\|\tilde{G}'(x)^{-1}\|\|\tilde{G}'(w)\| \leq \frac{(1-\gamma_0\|x-w\|)^2}{2(1-\gamma_0\|x-w\|)^2 - 1} = \frac{(1-\gamma_0\|x-w\|)^2}{g(\gamma_0\|x-w\|)}. \tag{2.2}$$

for all $x \in D \cap U(w,r)$.

Proof. It follows from (2.1) that for all $x \in D \cap U(w,r)$

$$\|\tilde{G}'(x)^{-1}\|\|\tilde{G}'(x) - \tilde{G}'(w)\| \leq \frac{1}{(1-\gamma_0\|x-w\|)^2} - 1 < 1, \tag{2.3}$$

since the right hand side inequality in (2.3) is equivalent to $\|x-w\| < \frac{2-\sqrt{2}}{2\gamma_0}$ which is true by the definition of r and r_0. Hence, by [3, Lemma 2], $G'(x)$ is positive definite. Moreover, estimate (2.2) holds by (2.3) and the Banach Lemma [1, 3, 7]. □

Definition 2.2. *Let $0 < r < r_0$. Let also $w \in H$ be such that $\tilde{G}'(w)^{-1}$ exists. It is said that G satisfies the restricted $\bar{\gamma}$-criterion at w in $D \cap U(w,r)$, if*

$$\|\tilde{G}'(w)^{-1}\|\|G''(x)\| \leq \frac{2\bar{\gamma}}{(1-\bar{\gamma}\|x-w\|)^3} \text{ for all } x \in D \cap U(w,r). \tag{2.4}$$

Lemma 2.2. *Let $0 < r < r_0$ and $\gamma_0 \leq \bar{\gamma}$. Let also $w \in H$ be such that $\tilde{G}'(w)$ is positive definite. Suppose that G satisfies the restricted $\bar{\gamma}-$ criterion at w in $D \cap U(w,r)$. Then, $\tilde{G}(x)$ is positive definite and (2.2) holds for all $x \in D \cap U(w,r)$.*

Proof. We can write

$$G'(x) - G'(w) = \int_0^1 G''(w + \theta(x-w))(x-w)d\theta. \tag{2.5}$$

Using (2.4), (2.5) and the estimate connecting $\tilde{G}'$ with G' (see [1, 3, 7])

$$\|\tilde{G}'(x) - \tilde{G}'(w)\| \leq \|G'(x) - G'(w)\|, \tag{2.6}$$

we obtain in turn that

$$\begin{aligned} & \|\tilde{G}'(w)^{-1}\|\|\tilde{G}'(x) - \tilde{G}'(w)\| \\ \leq\ & \|\tilde{G}'(x)^{-1}\|\|G'(x) - G'(w)\| \\ \leq\ & \int_0^1 \|\tilde{G}'(w)^{-1}\|\|G''(w + \theta(x-w))\|\|x-w\|d\theta \\ \leq\ & \frac{\int_0^1 2\bar{\gamma}\|x-w\|}{(1-\bar{\gamma}\|x_\theta - w\|)^3}d\theta = \frac{1}{(1-\bar{\gamma}\|x-w\|)^2} - 1, \end{aligned} \tag{2.7}$$

where $x_\theta = w + \theta(x-w)$ for each $\theta \in [0,1]$. The preceding equality estimate in (2.7) is standard [2, 3, 10]. The result now follows from Lemma 2.1. $\square$

The $\gamma-$ criterion given in the literature [2, 3, 10] is:

Definition 2.3. *Let $0 < r < \frac{1}{\gamma}$ for some $\gamma > 0$. Let also $w \in H$ be such that $\tilde{G}'(w)^{-1}$ exists. It is said that G satisfies the γ-criterion at w in $D \cap U(w,r)$, if*

$$\|\tilde{G}'(w)^{-1}\|\|G'(x)\| \leq \frac{2\gamma}{(1-\gamma\|x-w\|)^3} \text{ for all } x \in U(w,r). \tag{2.8}$$

Remark 2.1. It follows from Definition 2.2, Lemma 2.2 and Definition 2.3 that

$$\gamma_0 \leq \gamma \tag{2.9}$$

and for $\frac{2-\sqrt{2}}{2\gamma_0} \leq \frac{1}{\gamma}$,

$$\bar{\gamma} \leq \gamma. \tag{2.10}$$

Notice that $\bar{\gamma}$ depends on γ_0. The definition of $\bar{\gamma}$ was not possible before using Definition 2.3. Therefore, if

$$\frac{2-\sqrt{2}}{2\gamma_0}\gamma \leq \gamma_0 \leq \gamma, \tag{2.11}$$

then $\bar{\gamma}$ can replace γ in the local convergence results in [3].

Let us define function φ on the interval $[0, \frac{1}{\gamma})$ by

$$\varphi(t) = \left(\frac{1-\gamma_0 t}{1-\bar{\gamma}t}\right)^2 \frac{\bar{\gamma}t}{g(\gamma_0 t)} - 1.$$

We have $\varphi(0) = -1$ and $\varphi(t) \longrightarrow +\infty$ as $t \longrightarrow \frac{1}{\bar{\gamma}}^{-}$. Function φ has zeros in the interval $(0, \frac{1}{\bar{\gamma}})$ by the intermediate value theorem. Let $r^* = \psi(\gamma_0, \bar{\gamma})$ be the smallest zero. Notice that if $\gamma_0 = \bar{\gamma} = \gamma$, then

$$\psi(\gamma, \gamma) = r_1 := \frac{5-\sqrt{17}}{4\gamma} \leq r^*. \tag{2.12}$$

Define function ω on $[0, \frac{1}{\bar{\gamma}}) \times (0, +\infty)^2$ by

$$\omega(t, \gamma_0, \bar{\gamma}) = \left(\frac{1-\gamma_0 t}{1-\bar{\gamma}t}\right)^2 \frac{\bar{\gamma}t}{g(\gamma_0 t)}.$$

Then, we have that

$$\omega(t, \gamma_0, \bar{\gamma}) \leq \omega(t, \gamma, \gamma). \tag{2.13}$$

Hence, we arrive at:

Theorem 2.3. *Suppose: p is a solution of A; G satisfies the restricted $\bar{\gamma}-$criterion at p in $D \cap U(p, r^*)$ and $G'(p)$ is positive definite. Then, the sequence $\{x_n\}$ generated for $x_0 \in U(p, r^*)$ by Newton's method converges to p so that*

$$\|x_{n+1} - p\| \leq \omega(\|x_n - p\|, \gamma_0, \bar{\gamma})\|x_n - p\| \text{ for all } n = 0, 1, 2, \ldots. \tag{2.14}$$

Proof. We follow the proof in Theorem 3.1 in [3] but with the important difference of using (2.1) instead of (2.8) to arrive at

$$|\tilde{G}'(x_k)^{-1}\|\|\tilde{G}'(p)\| \leq \frac{(1-\gamma_0\|x_k - p\|)^2}{g(\gamma_0\|x_k - p\|)} \tag{2.15}$$

(instead of the less precise used in [3], since $\gamma_0 \leq \gamma$)

$$\|\tilde{G}'(x_k)^{-1}\|\|\tilde{G}'(p)\| \leq \frac{(1-\gamma\|x_k - p\|)^2}{g(\gamma\|x_k - p\|)}. \tag{2.16}$$

Moreover, as in [3], we get

$$\begin{aligned}
&\|\tilde{G}'(p)^{-1}\|\|G(x_k) - G(p) + G'(x_k)(p - x_k)\| \\
\leq\ & \|\tilde{G}'(p)^{-1}\| \int_0^1 \int_0^\theta \|G''(p + \tau(x_k - p))\|\|x_k - p\| d\tau d\theta \\
\leq\ & \int_0^1 \int_\theta^1 \frac{2\bar{\gamma}}{(1-\bar{\gamma}\tau\|x_k - p\|)^3} \|\|x_k - p\|^2 d\tau d\theta. \qquad (2.17)
\end{aligned}$$

Then, from formula (3.8) in [3], (3.15) and (2.17) we obtain in turn that

$$\begin{aligned}
\|x_{k+1}-p\| &\leq \|\tilde{G}'(x_k)^{-1}\|\|G(x_k)-G(p)-G'(x_k)(x_k-p)\| \\
&\leq \frac{(1-\gamma_0\|x_k-p\|)^2}{g(\gamma_0\|x_k-p\|)}\int_0^1\int_0^\theta \frac{2\bar{\gamma}}{(1-\bar{\gamma}\tau\|x_k-p\|)^3}\|\|x_k-p\|^2 d\tau d\theta \\
&\leq \frac{(1-\gamma_0\|x_k-p\|)^2}{g(\gamma_0\|x_k-p\|)(1-\bar{\gamma}\|x_k-p\|)^2} \\
&\quad \times\left[(1-\gamma_0\|x_k-p\|)^2\int_0^1\int_0^\theta \frac{2\bar{\gamma}}{(1-\bar{\gamma}\tau\|x_k-p\|)^3}\|\|x_k-p\|^2 d\tau d\theta\right] \\
&= \frac{(1-\gamma_0\|x_k-p\|)^2\bar{\gamma}\|x_k-p\|^2}{(1-\bar{\gamma}\|x_k-p\|)^2 g(\gamma_0\|x_k-p\|)} \\
&= \omega(\|x_k-p\|,\gamma_0,\bar{\gamma})\|x_k-p\|, \qquad (2.18)
\end{aligned}$$

which completes the induction for (2.14). Then, from the estimate

$$\|x_{k+1}-p\| \leq c\|x_k-p\| < r^*, \qquad (2.19)$$

where $c=\omega(\|x_k-p\|,\gamma_0,\bar{\gamma})\in[0,1)$, we conclude that $\lim_{k\to\infty} x_k = p$ and $x_{k+1}\in U(p,r^*)$. □

Remark 2.2. It follows from Remark 2.6 and Theorem 2.3 that (2.12) holds and

$$\omega(\|x_k-p\|,\gamma_0,\bar{\gamma}) \leq \omega(\|x_k-p\|,\gamma,\gamma). \qquad (2.20)$$

Hence, Theorem 2.3 improves Theorem 3.1 in [3] with advantages as already stated previously.

3 Semi-local convergence

The semi-local convergence for Newton's method uses majorizing sequences and the majorizing function h_δ due to Wang [10] for $\delta>0$. Let also $b>0$. Let function h_δ on the interval $[0,\frac{1}{\delta})$ be defined by

$$h_\delta(t) = b - t + \frac{\delta t^2}{1-\delta t} \qquad (3.1)$$

and scalar sequences $\{r_n\},\{s_n\},\{t_n\}$ for all $n=0,1,2,\ldots$ by

$$\begin{aligned}
r_{n+1} &= r_n - h'_{\gamma_0}(r_n)^{-1}h_{\bar{\gamma}}(r_n) \qquad (3.2)\\
s_{n+1} &= s_n - h'_{\bar{\gamma}}(s_n)^{-1}h_{\bar{\gamma}}(s_n) \qquad (3.3)\\
t_{n+1} &= t_n - h'_{\gamma}(t_n)^{-1}h_{\gamma}(t_n). \qquad (3.4)
\end{aligned}$$

Sequence $\{t_n\}$ has appeared in the literature and is due to Wang [10]. Suppose that for $\gamma = \delta$

$$\alpha_0 := b\gamma \leq 3 - 2\sqrt{2}. \tag{3.5}$$

Then, sequence $\{t_n\}$ is monotonically increasing and converges to

$$R_\alpha := \frac{1 + \alpha - \sqrt{(1+\alpha)^2 - 8\alpha}}{4\gamma}. \tag{3.6}$$

Suppose that

$$\alpha := b\bar{\gamma} \leq 3 - 2\sqrt{2}. \tag{3.7}$$

Then, again sequences $\{r_n\}$ and $\{s_n\}$ are monotonically increasing and converge to

$$R_{\alpha_0} := \frac{1 + \alpha_0 - \sqrt{(1+\alpha_0)^2 - 8\alpha_0}}{4\bar{\gamma}}. \tag{3.8}$$

A simple induction argument shows that for $n = 0, 1, 2, \ldots,$ if $\gamma_0 \leq \bar{\gamma}$ then

$$r_n \leq s_n \leq t_n \tag{3.9}$$

$$r_{n+1} - r_n \leq s_{n+1} - s_n \leq t_{n+1} - t_n \tag{3.10}$$

and

$$R_{\alpha_0} = \lim_{n\to\infty} r_n = \lim_{n\to\infty} s_n = \lim_{n\to\infty} t_n = R_\alpha. \tag{3.11}$$

Estimated (3.9), (3.10) are strict if $\gamma_0 < \bar{\gamma} < \gamma$ for all $n = 2, 3, \ldots$ and all $n = 1, 2, \ldots,$ respectively. Moreover, notice that

$$\alpha \leq 3 - 2\sqrt{2} \Longrightarrow \alpha_0 \leq 3 - 2\sqrt{2}. \tag{3.12}$$

The reverse is not necessarily true, unless, if $\bar{\gamma} = \gamma$. Items (3.9)-(3.12) justify the claims made in the introduction of this study.

Next, we show that (3.7), $\{r_n\}$ (or$\{s_n\}$) can replace (3.5), $\{t_n\}$, respectively in the semi-local convergence analysis of Newton's method given in [3].

Theorem 3.1. *Suppose: $\tilde{G}'(x_0)^{-1}$ exists for some $x_0 \in D$; Let $\alpha_0 = b\bar{\gamma}$ for $b := \|\tilde{G}'(x_0)^{-1}\|\|G(x_0)\|$; G satisfies the restricted $\bar{\gamma}-$criterion at $x_0 \in D_0 := D \cap U(x_0, R\alpha_0)$; $G'(x_0)$ is positive definite; $\gamma_0 \leq \bar{\gamma}$ and $\alpha_0 \leq 3 - 2\sqrt{2}$. Then, the sequence $\{x_n\}$ generated by Newton's method (1.2) starting from x_0 exists in $U(x_0, R\alpha_0)$ remains in $U(x_0, R\alpha_0)$ and converges to a solution p of A in D_0. Moreover, the following assertions hold*

$$\|x_{n+1} - x_n\| \leq s_{n+1} - s_n \tag{3.13}$$

and

$$\|x_n - p\| \leq s^* - s_n. \tag{3.14}$$

Proof. We use induction as in Theorem 4.1 in [3]. The proof is similar to the proof of Theorem 4.1 in [3] and the proof of Theorem 2.3 in the present study. Using (3.15) (for $p = x_0$) instead of (2.16) (for $p = x_0$) used in [3], we get

$$\|\tilde{G}'(x_k)^{-1}\|\|\tilde{G}'(x_0)\| \leq \frac{(1-\gamma_0\|x_k - x_0\|)^2}{g(\gamma_0\|x_k - x_0\|)}. \tag{3.15}$$

Moreover, we get in turn that

$$\begin{aligned} &\|\tilde{G}'(x_0)^{-1}(G(x_k) - G(x_{k-1}) - G'(x_{k-1})(x_k - x_{k-1}))\| \\ \leq\ & \|\tilde{G}'(x_0)^{-1}\int_0^1\int_0^\theta G''(x_{k-1} + \tau(x_k - x_{k-1}))(x_k - x_{k-1})^2 d\tau d\theta\|. \end{aligned} \tag{3.16}$$

Then, by (3.2), (3.3) and (3.16), we obtain in turn that

$$\begin{aligned} \|x_{k+1} - x_k\| \ &\leq\ \|\tilde{G}'(x_k)^{-1}\|\|\|\tilde{G}'(x_0)\| \\ &\quad \times \|\int_0^1\int_0^\theta \tilde{G}'(x_0)^{-1}G''(x_{k-1} + \tau(x_k - x_{k-1}))(x_k - x_{k-1})^2 d\tau d\theta\| \\ &\leq\ -h'_{\gamma_0}(r_k)^{-1}\int_0^1\int_0^\theta \frac{2\bar{\gamma}(s_k - s_{k-1})^2 d\tau d\theta}{(1-\bar{\gamma}(\|x_{k-1} - x_0\|) + \tau\|x_k - x_{k-1}\|)^3} \\ &\leq\ -h'_{\bar{\gamma}}(r_k)^{-1}\int_0^1\int_0^\theta \frac{2\bar{\gamma}(s_k - s_{k-1})^2 d\tau d\theta}{(1-\bar{\gamma}(\|x_{k-1} - x_0\|) + \tau\|x_k - x_{k-1}\|)^3} \\ &\leq\ -h'_{\bar{\gamma}}(s_k)^{-1}h(s_k) = s_{k+1} - s_k, \end{aligned} \tag{3.17}$$

which completes the induction for (3.13). Estimate (3.13) implies that sequence $\{s_n\}$ is complete in $\mathbb{R}^i$, so $\lim_{k\longrightarrow\infty} x_k = p \in \bar{U}(x_0, R\alpha_0)$. G is a C^2 mapping and x_{n+1} solves A for each n, so finally, estimate (3.14) follows from (3.13) by standard arguments [2, 7]. □

Remark 3.1. (a) If $\gamma_0 = \bar{\gamma} = \gamma$, Theorem 3.1 becomes Theorem 4.1 in [3]. Otherwise it constitutes an improvement (see also (3.9)-(3.12)).

(b) Clearly, if $\bar{\gamma} \leq \gamma_0$, then the result of Theorem 3.1 hold with γ_0 replacing $\bar{\gamma}$. Examples where $\gamma_0 < \bar{\gamma} < \gamma$ can also be found in [2] in the more general setting of a Banach space.

(c) The improvements in this study are obtained using the same or less computational effort as in [3]. The evaluation of constant $\gamma-$ involves, in practice the evaluation of γ_0 and $\bar{\gamma}$ as special cases.

(d) The sufficient convergence criteria for the semi-local case can be improved even further, if D_0 is replaced by $D_1 := D \cap U(x_1, \frac{2-\sqrt{2}}{2\gamma_0} - \|x_1 - x_0\|)$. Then, the corresponding constant to $\bar{\gamma}$ denoted by $\bar{\bar{\gamma}}$ can replace $\bar{\gamma}$ in the preceding results. Notice that $\bar{\bar{\gamma}} \leq \bar{\gamma}$, since $D_1 \subseteq D_0$. In this case we are still using initial data, since x_1 depends on x_0 and G.

References

[1] **I. K. Argyros**, *Computational theory of iterative methods*, Studies in Computational Mathematics **15**, Elsevier Publ. Co., New York, U.S.A, 2007

[2] **I. K. Argyros and S. George**, Extending the applicability of Newton's method using Wang's– Smale's α-theory, *Carpathian Journal of Mathematics*, **33, No. 1**, (2017), 27-33

[3] **C. D. Chang, J. Wang, and J. C. Yao**, Newton's method for variational inequality problems. Smale's point estimate theory under the $\gamma-$ condition, *Applicable Analysis*, **94**, (2015), 44-55

[4] **A. L. Dontchev**, Uniform convergence of the Newton method for Aubin continuous maps, *Serdica Math. J.*, **22**, (1996), 385-398

[5] **M. C. Ferris and J. S. Pang**, Engineering and economics applications of complementary problems, *SIAM Rev.*, **39**, (1997), 669-713

[6] **P. T. Harker and J. S. Pang**, Finite-dimensional variational inequality and nonlinear complementary problems: a survey of theory, algorithms and applications, *Math. Program*, **48**, (1990), 161-220

[7] **L. V. Kantorovich and G. P. Akilov**, *Functional Analysis*, Pergamon Press, Oxford, 1982

[8] **M. Patriksson**, *Nonlinear programming and variational inequality problems: a unified approach*, Kluwer Academic Publication, Dordrecht, 1999

[9] **S. Smale**, *Newton's method estimates from data at one point*, In: Ewing R, Gross K, Martin C, editors. The merging of disciplines: new directions in pure, applied and computational mathematics, Springer, New York, 1986, 185-196

[10] **X. H. Wang**, Convergence of Newton's method and uniqueness of the solution of equations in Banach space, *IMA. J. Numer. Anal.*, **20**, (2000), 123-134

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