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# Extending the Philosophical Significance of the Idea of Complementarity

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**Summary.** We discuss a specific way in which the notion of complementarity can be based on the dynamics of the system considered. This approach rests on an epistemic representation of system states, reflecting our knowledge about a system in terms of coarse grainings (partitions) of its phase space. Within such an epistemic quantization of classical systems, compatible, comparable, commensurable, and complementary descriptions can be precisely characterized and distinguished from each other. Some tentative examples are indicated that, we suppose, would have been of interest to Pauli.

## 1 Introduction

In 1949 Pauli delivered a lecture on complementarity to the Philosophical Society in Zurich, which was then published (in German) under the title "The Philosophical Significance of the Idea of Complementarity" in the journal *Experientia* (Pauli, 1950). His article followed an earlier paper by Bernays (1948) "On the Extension of the Notion of Complementarity into Philosophy" (also in German). Pauli (1950) emphasized that the

> "situation in regard to complementarity within physics leads naturally beyond the narrow field of physics to analogous situations in connection with the general conditions of human knowledge."

Pauli's paper addressed a number of pertinent topics still unresolved today where the idea of complementarity might be of relevance, such as "the experimenter's free choice between mutually exclusive experimental arrangements", "the idea of the cut between observer or instrument of observation and the system observed" (nowadays dubbed the *Heisenberg cut*), "considerations of purposefulness" concerning the actual location of the cut, and eventually the "paradoxical" relationship between consciousness and the unconscious (Pauli, 1950):

> "On one hand, modern psychology demonstrates a largely objective reality of the unconscious psyche; on the other hand every bringing into consciousness, i.e. observation, constitutes an interference with the unconscious contents that is in principle uncontrollable; this limits the objective character of the reality of the unconscious and invests reality with a certain subjectivity."

The concept of complementarity was introduced into physics by Bohr (cf. Bohr, 1948), but he was familiar with it from psychological texts by William James and from his psychologist friend Arthur Rubin, who studied the perception of ambiguous stimuli. In simple words, two descriptions of a situation are complementary if they exclude each other and yet are both necessary for an exhaustive description of that situation. In quantum theory, this vague characterization was made much more precise in the mathematical framework of non-commutative algebras or non-Boolean lattices of quantum observables. The price to be paid for this precision is the restriction of the concept of complementarity to quantum physics.

However, there are many more candidates for complementary relationships in other sciences, e.g. in psychology and philosophy. The present article intends to reconsider the foundations of the notion of complementarity not only with respect to quantum systems but with a broader domain of applications.[3] It builds essentially on a recent paper by beim Graben and Atmanspacher (2006) which describes in technical detail how complementary observables can be defined in classical physical systems if their dynamics is taken into account properly. In the present paper we give a simplified exposition for a more general readership and address some issues that were in the focus of Pauli's interest for many years.

Section 2 contains a compact reminder of how complementarity and compatibility are defined in quantum theory. Section 3 introduces the concept of partitions for an epistemic treatment of classical dynamical systems. Section 4 illustrates how complementary observables can be introduced for epistemic states defined on the basis of particular phase space partitions. Only if such partitions are generating (or, more specifically, Markov), they define epistemic states that are stable under the dynamics and provide compatible epistemic descriptions. Partitions chosen more or less *ad hoc* generally lead to incompatible or complementary descriptions. Section 5 characterizes and delineates compatible, comparable, and commensurable theories (and their opposites) from each other. Some examples are outlined in Sect. 6.

[3] Compare Atmanspacher *et al.* (2002) and Primas (2007) for formally rigorous approaches in this direction.

## 2 Compatibility and Complementarity in Quantum Theory

In quantum theory, measurements of observables $A$ and $B$ with pure point spectra which produce dispersion-free values as results depend in general on the sequence in which the measurements are carried out. In this case, the observables $A, B$ are called *incompatible*. If, on the other hand, the order of measuring $A$ and $B$ does not play a role, the observables $A, B$ are called *compatible*. Therefore, compatibility can be formally expressed by the equation

$$AB = BA \quad , \tag{1}$$

while incompatibility means that $A$ and $B$ do not commute:

$$AB \neq BA \quad . \tag{2}$$

In a Hilbert space representation, (1) has the consequence that compatible observables are simultaneously diagonalizable, i.e. all eigenstates of $A$ are also eigenstates of $B$ (and vice versa), and these common eigenstates span the whole Hilbert space of (pure) quantum states. Since $A\psi = a\psi$ for eigenstates $\psi$ with eigenvalue $a$ of $A$, observable $A$ assumes the sharp, *dispersion-free* value $a$ in eigenstate $\psi$.

Compatible observables with pure point spectra are therefore dispersion-free in their common eigenstates which span the whole Hilbert space. Incompatible observables do not share all eigenstates, although they may share some of them. *Complementary* observables can be characterized as being *maximally incompatible*; they do not have any eigenstate in common (beim Graben and Atmanspacher, 2006). These results will be used for generalizing the concepts of complementarity and compatibility to classical systems, i.e. beyond quantum systems, in the next sections.

## 3 Epistemic Descriptions of Classical Dynamical Systems

Measurements (or observations) require the preparation of a state of the system to be measured (or observed), choices of initial and boundary conditions for this state, and the selection of particular measurement setups. They refer to operationally defined *observables* which can be deliberately chosen by the experimenter (Pauli, 1950; Primas, 2007).

### 3.1 Observables and Partitions

A classical dynamical system is characterized by the fact that all observables are compatible with each other. However, in general this holds only for a so-called *ontic description* (Atmanspacher, 2000) where the state of a system is considered as if it could be characterized precisely as it is (relative to

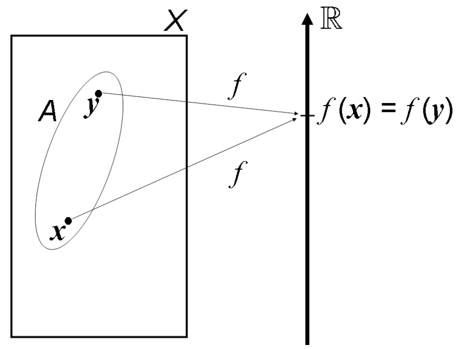



**Fig. 1.** States $\boldsymbol{x}, \boldsymbol{y}$ in a phase space $X$ of a classical system (left) and the real numbers as the range of a classical observable $f : X \to \mathbb{R}$ (right). Epistemically equivalent states $\boldsymbol{x}, \boldsymbol{y} \in X$ belong to the same equivalence class $A \subset X$.

a chosen ontology (Quine, 1969; Atmanspacher and Kronz, 1999; Dale and Spivey, 2005). On such an account, the *ontic state* of the system is given by a point $\boldsymbol{x}$ in phase space $X$. The associated observables are real-valued functions $f : X \to \mathbb{R}$, such that $a = f(\boldsymbol{x})$ is the value of $f$ in state $\boldsymbol{x}$. By contrast, *epistemic descriptions* refer to the "knowledge that can be obtained about an ontic state" (Atmanspacher, 2000). For the sake of simplicity we shall identify *epistemic states* with subsets $S \subset X$ in phase space, thus expressing that they can be specified only with limited accuracy.

Figure 1 displays a situation in which the observable $f$ is not injective, such that different states $\boldsymbol{x} \neq \boldsymbol{y} \in A \subset X$ lead to the same measurement result

$$f(\boldsymbol{x}) = f(\boldsymbol{y}) \quad . \tag{3}$$

In this case, the states $\boldsymbol{x}$ and $\boldsymbol{y}$ are *epistemically* indistinguishable by means of the observable $f$ (Shalizi and Moore, 2003; beim Graben and Atmanspacher, 2006). Measuring $f$ cannot tell us whether the system is in state $\boldsymbol{x}$ or $\boldsymbol{y}$. The two states are therefore *epistemically equivalent* with respect to $f$ (beim Graben and Atmanspacher, 2006).

In this way, the observable $f$ induces an equivalence relation "$\sim_f$" on the phase space $X$: $\boldsymbol{x} \sim_f \boldsymbol{y}$ if $f(\boldsymbol{x}) = f(\boldsymbol{y})$. The resulting equivalence classes of ontic states partition the phase space into mutually exclusive and jointly exhaustive sets $A_1, A_2, \ldots$ such that $A_i \cap A_j = \emptyset$ for all $i \neq j$ and $\bigcup_i A_i = X$. These sets are the epistemic states that are induced by the observable $f$. The collection $\mathcal{F} = \{A_1, A_2, \ldots\}$ of epistemic states is a phase space *partition*.

We call $f$ an *epistemic observable* if the partition $\mathcal{F}$ is not the *identity partition* $\mathcal{I}$ where every cell $A_k$ is a singleton set containing exactly one element $A_k = \{\boldsymbol{x}_k\}$ (Shalizi and Moore, 2003). In this limiting case, $f$ is injective and can be called an *ontic observable*. In the opposite limit, epistemic observables are constant over the whole phase space: $f(\boldsymbol{x}) = \text{const}$ for all $\boldsymbol{x} \in X$. In this

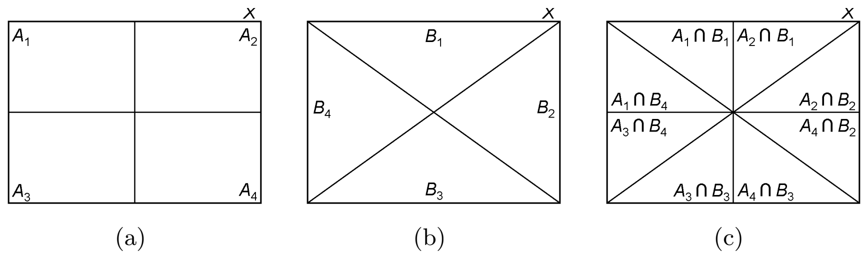



**Fig. 2.** Examples for finite partitions of the phase space $X$: (a) "rectangular" partition $\mathcal{F} = \{A_1, A_2, A_3, A_4\}$, (b) "triangular" partition $\mathcal{G} = \{B_1, B_2, B_3, B_4\}$, (c) product partition $\mathcal{F} \vee \mathcal{G}$.

case, all states are epistemically equivalent with each other and belong to the (same) equivalence class $X$ of the *trivial partition* $\mathcal{T}$.

Most interesting for our purposes are finite partitions $\mathcal{F} = \{A_1, A_2, \ldots A_n\}$ (where $n$ is a finite natural number) which are neither trivial nor identity. Figures 2(a,b) display two different finite partitions. From the partitions $\mathcal{F}$ and $\mathcal{G}$ shown in Figs. 2(a,b), a *product partition*, $\mathcal{P} = \mathcal{F} \vee \mathcal{G}$ can be constructed. This partition, depicted in Fig. 2(c), contains all possible intersections of sets in $\mathcal{F}$ with sets in $\mathcal{G}$:

$$\mathcal{P} \;=\; \mathcal{F} \vee \mathcal{G} \;=\; \{A_i \cap B_j | A_i \in \mathcal{F}, B_j \in \mathcal{G}\} \quad . \tag{4}$$

The product partition $\mathcal{P}$ is a *refinement* of both partitions $\mathcal{F}$ and $\mathcal{G}$. The refinement relation introduces a partial ordering relation "$\prec$" among partitions. If $\mathcal{G}$ is a refinement of $\mathcal{F}$, $\mathcal{G} \prec \mathcal{F}$, then there is a "factor partition" $\mathcal{H}$ such that $\mathcal{G} = \mathcal{F} \vee \mathcal{H}$. If neither $\mathcal{G}$ is a refinement of $\mathcal{F}$ nor *vice versa* (and $\mathcal{G} \neq \mathcal{F}$), the partitions $\mathcal{G}$ and $\mathcal{F}$ have been called *incomparable* (Shalizi and Moore, 2003).

### 3.2 Dynamics

A dynamical system evolves as a function of parameter time $t$. In other words, any present state (e.g. an initial condition) in phase space, $\boldsymbol{x}_0 \in X$, gives rise to future states $\boldsymbol{x}_t \in X$. This evolution is described by a flow map $\Phi : X \rightarrow X$. In the simple case of a deterministic dynamics in discrete time, $\Phi$ maps any state $\boldsymbol{x}_t$ onto a state $\boldsymbol{x}_{t+1}$, as illustrated in Fig. 3. Iterating the map $\Phi$, yields a *trajectory* of states

$$\boldsymbol{x}_{t+1} \;=\; \Phi^{t+1}(\boldsymbol{x}_0) \;=\; \Phi(\Phi^t(\boldsymbol{x}_0)) \;=\; \Phi(\boldsymbol{x}_t) \tag{5}$$

for integer positive times $t \in \mathbb{N}$. Likewise, the inverse map $\Phi^{-1}$ can be iterated if the dynamics is invertible: $\boldsymbol{x}_{-(t+1)} = \Phi^{-(t+1)}(\boldsymbol{x}_0) = \Phi^{-1}(\Phi^{-t}(\boldsymbol{x}_0)) = \Phi^{-1}(\boldsymbol{x}_{-t})$, again for integer positive times $t \in \mathbb{N}$. Therefore, the dynamics of an invertible discrete-time system is described by the one-parameter group of integer numbers $t \in \mathbb{Z}$.

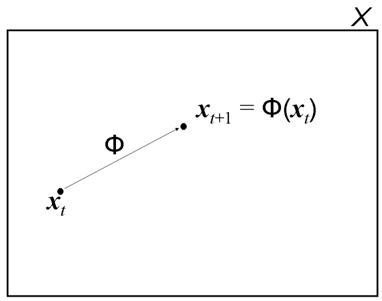



**Fig. 3.** A discrete-time dynamics of a classical system is given by a map $\Phi : X \to X$ which assigns to a state $\boldsymbol{x}_t$ at time $t$ its successor $\boldsymbol{x}_{t+1} = \Phi(\boldsymbol{x}_t)$ at time $t+1$.

### 3.3 Continuous Measurements

In Sect. 3.1, we have described instantaneous measurements by the action of an observable $f : X \to \mathbb{R}$ on an ontic state $\boldsymbol{x}$. Now we are able to describe *continuous measurements*[4] by combining the action of an observable $f$ with the dynamics $\Phi$. Let the system be in state $\boldsymbol{x}_0 \in X$ at time $t = 0$. Measuring $f(\boldsymbol{x}_0)$ tells us to which class of epistemically equivalent states in the partition $\mathcal{F}$, associated with $f$, the state $\boldsymbol{x}_0$ belongs. Suppose that this is the cell $A_{i_0} \in \mathcal{F}$. Suppose further that measuring $f$ in the subsequent state $\boldsymbol{x}_1 = \Phi(\boldsymbol{x}_0) \in X$ reveals that $\boldsymbol{x}_1$ is contained in another cell $A_{i_1} \in \mathcal{F}$.

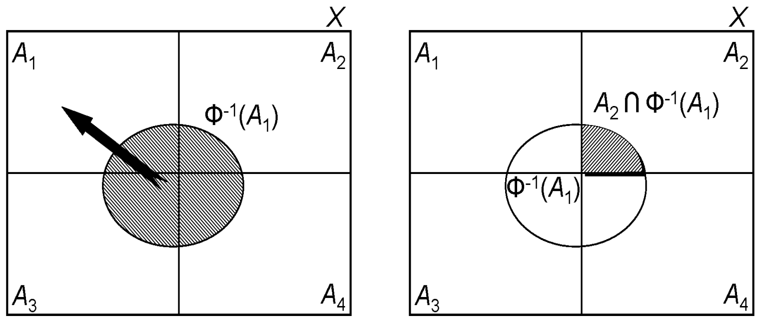
An alternative way to describe this situation is to say that the initial state $\boldsymbol{x}_0 = \Phi^{-1}(\boldsymbol{x}_1)$ belongs to the pre-image $\Phi^{-1}(A_{i_1})$ of $A_{i_1}$. The information about $\boldsymbol{x}_0$ that is gained by measuring $f(\boldsymbol{x}_1)$ is, then, that the initial state $\boldsymbol{x}_0$ was contained in the intersection $A_{i_0} \cap \Phi^{-1}(A_{i_1})$. Continuing the observation of the system over one more instant in time yields that the initial state $\boldsymbol{x}_0$ belonged to the set $A_{i_0} \cap \Phi^{-1}(A_{i_1}) \cap \Phi^{-2}(A_{i_2})$ if the third measurement result was $\boldsymbol{x}_2 = \Phi^2(\boldsymbol{x}_0) \in A_{i_2}$.

A systematic investigation of continuous measurements relies on the definition of the pre-image of a partition,

$$\Phi^{-1}(\mathcal{F}) \; = \; \{\Phi^{-1}(A_i) | A_i \in \mathcal{F}\} \quad , \tag{6}$$

which consists of all pre-images of the cells $A_i$ of the partition $\mathcal{F}$. Then, a continuous measurement over two successive time steps is defined by the product partition $\mathcal{F} \vee \Phi^{-1}(\mathcal{F})$, containing all intersections of cells of the original partition $\mathcal{F}$ with cells of its pre-image $\Phi^{-1}(\mathcal{F})$. The result of the measurement of $f$ over two time steps is $\boldsymbol{x}_0 \in A_{i_0} \cap \Phi^{-1}(A_{i_1}) \subset \mathcal{F} \vee \Phi^{-1}(\mathcal{F})$. This product partition is called the *dynamic refinement* of $\mathcal{F}$, illustrated in Fig. 4.

[4] The notion of a continuous measurement does not refer to continuous time $t \in \mathbb{R}$ but characterizes that a measurement extends over time. This can also be the case for discrete time $t \in \mathbb{Z}$.




(a) (b)

**Fig. 4.** Dynamic refinement of a partition. (a) For each cell $A_i$ of the partition $\mathcal{F}$ the pre-image $\Phi^{-1}(A_i)$ under the dynamics is determined. The bold arrow indicates that the shaded region in phase space is mapped onto cell $A_1$. (b) The shaded region in the product partition $\mathcal{F} \vee \Phi^{-1}(\mathcal{F})$ is the element $A_2 \cap \Phi^{-1}(A_1)$ of the dynamically refined partition.

Most information about the state of a system can be gained by an ideal, "ever-lasting" continuous measurement that began in the infinite past and terminates in the infinite future. This leads to the *finest dynamic refinement*

$$\mathbf{R}\mathcal{F} = \bigvee_{t=-\infty}^{\infty} \Phi^t(\mathcal{F}) \quad , \tag{7}$$

expressed by the action of the "finest-refinement operator" $\mathbf{R}$ upon a partition $\mathcal{F}$. It would be desirable that such an ever-lasting measurement yields complete information about the initial condition $\boldsymbol{x}_0$ in phase space. This is achieved if the refinement (7) entails the identity partition,

$$\mathbf{R}\mathcal{F} = \mathcal{I} \quad . \tag{8}$$

A partition $\mathcal{F}$ obeying (8) is called *generating*.

Given the ideal finest refinement $\mathbf{R}\mathcal{F} = \mathcal{P}$ of a (generating or non-generating) partition $\mathcal{F}$ that is induced by an epistemic observable $f$, we are able to regain a description of continuous measurements of arbitrary finite duration by joining subsets of $\mathcal{P}$ which are visited by the system's trajectory during measurement. Supplementing the "join" operation by the other Boolean set operations over $\mathcal{P}$ leads to a *partition algebra* $A(\mathcal{P})$ of $\mathcal{P}$. Then, every set in $A(\mathcal{P})$ is an epistemic state measurable by $f$.

Note that the concept of a generating partition in the ergodic theory of deterministic systems is related to the concept of a *Markov chain* in the theory of stochastic systems. Every deterministic system of first order gives rise to a Markov chain which is generally neither ergodic nor irreducible. Such Markov chains can be obtained by so-called *Markov partitions* that exist for expanding or hyperbolic dynamical systems (Sinai, 1968; Bowen, 1970; Ruelle, 1989). For non-hyperbolic systems no corresponding existence theorem is available, and the construction can be even more tedious than for hyperbolic systems (Viana *et al.*, 2003). For instance, both Markov and generating partitions for nonlinear systems are generally non-homogeneous. In contrast to Figure 2, their cells are typically of different size and form.

Note further that every Markov partition is generating, but the converse is not necessarily true (Crutchfield, 1983; Crutchfield and Packard, 1983). For the construction of "optimal" partitions from empirical data it is often more convenient to approximate them by Markov partitions (Froyland, 2001).

## 4 Compatibility and Complementarity in Classical Dynamical Systems

If a partition $\mathcal{F}$ is not generating, its finest refinement is not the identity partition. In this case, the refinement operator produces a partition $\mathcal{P} = \mathbf{R}\mathcal{F}$ with some residual coarse grain. Moreover, the cells of a non-generating partition are not stable under the dynamics $\Phi$, so that they become dynamically ill-defined – a disaster for any attempt to formulate a properly robust coarse-grained description (Atmanspacher and beim Graben, 2007).

Let $P \in \mathcal{P}$ be an epistemic state of the finest refinement of $\mathcal{F}$. Because $\mathcal{F}$ is induced by an observable $f$ whose epistemic equivalence classes are the cells of $\mathcal{F}$, all cells of $\mathcal{P}$ can be accessed by continuous measurements of $f$. However, as $\mathcal{P}$ is not the identity partition $\mathcal{I}$, the singleton sets $\{\boldsymbol{x}\}$ representing ontic states in $X$ are not accessible by measuring $f$. An arbitrary epistemic state $S \subset X$ induced by an observable $g$ is called *epistemically accessible with respect to $f$* (beim Graben and Atmanspacher, 2006) if $S$ belongs to the partition algebra $A(\mathcal{P})$ produced by the finest refinement of $\mathcal{F}$.

Measuring the observable $f$ in all ontic states $\boldsymbol{x} \in P$ belonging to an epistemic state $P \in \mathcal{P}$ always yields the same result $a = f(\boldsymbol{x})$ since $f$ is by construction constant over $P$. Therefore, the variance of $f(\boldsymbol{x})$ across $P$ vanishes such that $f$ is dispersion-free in the epistemic state $P$. In other words, $P$ is an eigenstate of $f$. One can now easily construct another observable $g$ that is not dispersion-free in $P$ such that $P$ is not a common eigenstate of $f$ and $g$. According to Sect. 2, the observables $f$ and $g$ are, thus, incompatible as they do not share all (epistemically accessible) eigenstates. Beim Graben and Atmanspacher (2006) refer to this construction as an *epistemic quantization* of a classical dynamical system.

In an ontic description of a classical system, ontic states are common eigenstates of all observables. Therefore, classical observables associated with ontic states are always compatible. By contrast, if the ontic states are not epistemically accessible by continuous measurements, the smallest epistemically accessible states are cells in the finest refinement of a partition $\mathcal{F}$ induced by an epistemic observable $f$. These epistemic states are not eigenstates of every observable, such that observables associated with them are incompatible. As in quantum theory, two observables $f$ and $g$ are complementary if they do not have any (epistemically accessible) eigenstate in common, i.e. if they are maximally incompatible.

Nevertheless, even in an epistemic description, classical observables $f$ and $g$ can be compatible with each other. This is the case if all ontic states $\boldsymbol{x} \in X$ are epistemically accessible with respect to both $f$ and $g$. The necessary and sufficient condition for this is that the partitions $\mathcal{F}$, $\mathcal{G}$ be generating (Eq. 8). This leads to a generalization of the concepts of compatibility and complementarity: Two partitions $\mathcal{F}, \mathcal{G}$ are called compatible if and only if they are both generating: $\mathbf{R}\,\mathcal{F} = \mathbf{R}\,\mathcal{G} = \mathcal{I}$. They are incompatible if $\mathbf{R}\,\mathcal{F} \neq \mathbf{R}\,\mathcal{G}$, which is always the case if at least one partition is not generating. They are complementary if their finest refinements are disjoint: $\mathbf{R}\,\mathcal{F} \cap \mathbf{R}\,\mathcal{G} = \emptyset$.[5]

These definitions give rise to three main corrolaries. (1) For compatible partitions, every ontic state $\boldsymbol{x}$ is epistemically accessible with respect to observables $f, g$ inducing the partitions $\mathcal{F}$, $\mathcal{G}$. Hence, every ontic state is a common eigenstate of $f$ and $g$ and all ontic states span the whole phase space $X = \bigcup_{\boldsymbol{x}} \{\boldsymbol{x}\}$. (2) For incompatible partitions, epistemically accessible eigenstates of one observable are not necessarily epistemically accessible eigenstates of another observable. (3) For complementary partitions, the observables do not have any eigenstates in common and are therefore maximally incompatible.

## 5 Compatible, Comparable, and Commensurable Theories

A proposition such as "the observable $f$ assumes the value $a$ in state $\boldsymbol{x} \in X$", or briefly "$a = f(\boldsymbol{x})$", induces a binary partition of the phase space $X$ of a classical dynamical system into two subsets,

$$\mathcal{F} = \{S,\, X \setminus S\} \quad , \tag{9}$$

where $S = \{\boldsymbol{x} \in X | a = f(\boldsymbol{x})\}$. Because propositions can be combined by the logical connectives "and", "or", and "not", the structure of a *classical theory* is that of a Boolean algebra of subsets of the phase space (Primas, 1977; Westmoreland and Schumacher, 1993; Primas, 2007). In the following we shall elucidate such theories with respect to the epistemic quantization discussed in Sect. 4.

Given a classical dynamical system with phase space $X$, dynamics $\Phi$, and a family of appropriately chosen epistemic observables $f_1, f_2, \ldots f_n$, these observables induce partitions $\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_n$ whose product $\mathcal{F} = \bigvee_{i=1}^{n} \mathcal{F}_i$ characterizes one particular setup for possible measurements. The partition algebra

[5] These concepts can also be defined by means of $\sigma$-algebras in measure theory (beim Graben and Atmanspacher, 2006). For the present simplified exposition, which captures very much the same idea, set-theoretical concepts are sufficient.

$A(\mathcal{F})$, comprising all subsets of $X$ that can be formed by the Boolean set operations "join", "intersection", and "difference", can be identified with a classical theory of propositions corresponding to instantaneous measurements of one of the observables $f_1, f_2, \dots f_n$ on $X$.

For continuous measurements, the dynamic refinement according to (7) has to be taken into account. In this case we have to consider the partition algebra $A(\mathbf{R}\,\mathcal{F})$ in order to form propositions about continuous measurements of arbitrary duration. Hence, a *classical theory* $T(\mathcal{F})$ refers to the Boolean set algebra $A(\mathbf{R}\,\mathcal{F})$ over the finest refinement $\mathbf{R}\,\mathcal{F}$.

Using the results of Sect. 4, two theories $T(\mathcal{F})$ and $T(\mathcal{G})$ are called compatible if their partitions $\mathcal{F}$ and $\mathcal{G}$ are compatible (i.e. if $\mathcal{F}$ and $\mathcal{G}$ are both generating). They are called incompatible if their partitions are incompatible, and they are called complementary if their partitions are complementary. The "experimenter's free choice between mutually exclusive experimental arrangements" (Pauli, 1950; Primas, 2007) corresponds to the choice of incompatible or complementary theories that are based upon non-generating partitions associated to epistemic observables. Insofar as classical ontic observables always induce the identity partition $\mathcal{I}$ on the phase space, ontic theories are always compatible with each other.

Following Shalizi and Moore (2003), we call two theories $T(\mathcal{F})$ and $T(\mathcal{G})$ *comparable* with each other if either $\mathbf{R}\,\mathcal{F}$ is a refinement of $\mathbf{R}\,\mathcal{G}$, or $\mathbf{R}\,\mathcal{G}$ is a refinement of $\mathbf{R}\,\mathcal{F}$, or $\mathbf{R}\,\mathcal{F} = \mathbf{R}\,\mathcal{G}$. Two theories are *incomparable* if they are not comparable. It is easy to realize that compatible theories are also comparable, as $\mathbf{R}\,\mathcal{F} = \mathbf{R}\,\mathcal{G} = \mathcal{I}$. However, even incompatible theories might be comparable, e.g. if one of them is based on a generating partition.

Another notion related to compatibility and comparability is that of *commensurability* (Kuhn, 1983; Hoyningen-Huene, 1990), which has gained some popularity in relativist accounts within the philosophy of science. Two theories are said to be commensurable if there is a common theoretical language that can be used to compare them. Following Primas (1977), this can be reformulated by saying that two theories $T(\mathcal{F}), T(\mathcal{G})$ are commensurable if they can be embedded into one *universal theory* $T(\mathcal{U})$ such that $T(\mathcal{F}), T(\mathcal{G})$ are sub-theories of $T(\mathcal{U})$.

More specifically, we call two theories $T(\mathcal{F}), T(\mathcal{G})$ commensurable if a theory $T(\mathcal{U})$ exists such that $\mathbf{R}\,\mathcal{U}$ is a refinement of $\mathbf{R}\,\mathcal{F}$ and $\mathbf{R}\,\mathcal{G}$. If $\mathcal{F}$ and $\mathcal{G}$ are both generating partitions, $\mathbf{R}\,\mathcal{F} = \mathbf{R}\,\mathcal{G} = \mathcal{I}$. Then $T(\mathcal{I})$ is a common refinement of $T(\mathcal{F})$ and $T(\mathcal{G})$, entailing that compatible theories are always commensurable. Comparable theories, whose partitions are refinements of each other, are trivially commensurable.

## 6 Examples

Let us finally give some selected examples for how the notions of compatibility, comparability, and complementarity can be useful for the discussion of topics within Pauli's lifelong interest.

A first illustrative example refers back to where Bohr became familiar with the notion of complementarity: the bistable perception of ambiguous stimuli. The involved processes can be described as (i) an oscillation between the two possible representations of the stimulus, and (ii) a projection into one of them, mimicking its observation. These two processes can indeed be shown to be complementary (Atmanspacher *et al.*, 2008) in basically the same sense as complementarity in quantum physics is due to non-commuting observables.

Along a different vein, beim Graben (2004) discussed three examples of implementations of symbol processors that are generically incompatible with respect to different partitions. This is due to the fact that, in these examples, the partitions are not generating. As an important consequence of this result, symbolic and subsymbolic (e.g. neural) descriptions of cognitive processes are incompatible in general. This confirms – though on different grounds – an assertion by Smolensky (1988, 2006) that an *integrated connectionist/symbolic architecture* is mandatory for cognitive science, where (Smolensky, 2006)

> "higher cognition must be formally characterized on two levels of description. At the microlevel, parallel distributed processing (PDP) characterizes mental processing; this PDP system has special organization in virtue of which it can be characterized at the macrolevel as a kind of symbolic computational system."

However, the apparent algorithmic behavior at the symbolic macrolevel is not implemented by algorithms performed at the microlevel. The microlevel dynamics only "approximates" the symbolic computations at the macrolevel, thus making both levels incompatible with each other (Smolensky, 1988). This shows any discomfort about the lack of a coherent unified framework for cognitive science to be misplaced. Incompatible descriptions are unavoidable and not an obstacle that one may hope to overcome some day. This applies also to incompatibilities and incommensurabilities in psychological theories (Yanchar and Slife, 1997; Slife, 2000; Dale and Spivey, 2005) as discussed by Atmanspacher and beim Graben (2007).

This relates to an issue raised by Pauli (1950) in terms of "considerations of purposefulness" for choosing between incompatible descriptions. An example is the notion of an *intended partition* for the dynamical systems approach to cognition (beim Graben, 2004). Among the many possible (and presumably incompatible) partitions of a dynamical system only a few, either explicitly constructed or evolutionarily optimized, give rise to a high-level interpretation of the system's low-level behavior in terms of symbol processing or cognitive computation. Such intended partitions are able to shed light onto the *symbol grounding problem* (Harnad, 1990; Atmanspacher and beim Graben, 2007).

Yet another incompatibility, maybe even complementarity, was proposed by Pauli between conscious and unconscious mental states (see Sect. 1 and Pauli, 1950). In an afterword to his essay "On the Nature of the Psyche", Jung (1969, §439, footnote 130) quotes Pauli with the statement that

> "the epistemological situation with regard to the concepts 'conscious' and 'unconscious' seems to offer a pretty close analogy to the ... situation in physics. ... From the standpoint of the psychologist, the 'observed system' would consist not of physical objects only, but would also include the unconscious, while consciousness would be assigned the role of 'observing medium'."

In other words: mental objects and their mental environments are conceived to be generated by the transformation of elements of the unconscious into consciously and, thus, epistemically accessible categories. As long as elements of the unconscious are not yet transformed into conscious categories, they remain unconscious, and whenever a category is generated and becomes consciously accessible, it leaves the domain of the unconscious. In this sense, conscious and unconscious domains are mutually disjoint, yet they are both together necessary to characterize the mental as a whole.

In such a framework of thinking, the unconscious is explicitly conceived as part of the mental. This is in contrast to many modern accounts, in which ongoing brain activity, i.e. the dynamics of subsystems of the *material brain*, is referred to by the notion of the unconscious. This brings us to the relation between mental and material states, or the mind-matter problem as the most general topic mentioned in this section. Among the many proposals that have been made to address this problem, Pauli (1952) emphasized the idea of an ontically monistic and epistemically dualistic, namely complementary, relationship between mind and matter:

> "The general problem of the relationship between psyche and physis, between inside and outside, will hardly be solved with the notion of a 'psychophysical parallelism', put forward in the past century. However, modern science has perhaps brought us closer to a more satisfying conception of this relationship insofar as it introduced the concept of *complementarity* within physics. It would be most satisfactory if physis and psyche could be conceived as complementary aspects of the same reality."

Recent publications (Walach and Römer, 2000; Atmanspacher, 2003; Römer, 2004; Primas, 2008) have tried to popularize this idea and elaborate on it.

The controversy of what constitutes the most basic aspects of reality accompanies the development of Western philosophy since its beginning. Countervailing positions favoring either *stasis*, and thus being (e.g. Parmenides), or *change*, and thus becoming (e.g. Heraclitus) followed and responded to each other time and again. It was recently shown by Römer (2006) that the corresponding distinction of *substance* and *process* can be considered as complementary in a formally anchored way.

The general scheme of thinking which such approaches follow is today called a dual-aspect or double-aspect framework, as discussed in more detail by Seager (2008). Chalmers (1995) advocated such a framework when he introduced the notion of the "hard problem of consciousness" as the problem of how to relate the first-person, phenomenal experience of a mental state to the third-person perspective characterizing the scientific (neural, cognitive, or otherwise) study of such a state. Velmans (2002, 2008) suggested to regard first-person and third-person accounts as incompatible or complementary.

Atmanspacher and beim Graben (2007) demonstrated how the *phenomenal families* introduced by Chalmers (2000), which partition the mental space of phenomenal experiences, induce a partition of the neural phase space. If this induced partition is not generating, the resulting description in terms of mental states will be incompatible with any other description. However, carefully constructed Markov partitions of the neural phase space of macroscopic brain activity, e.g. by means of EEG signals, can lead to descriptions that are compatible with mental (symbolic) descriptions.

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