# EXTENDING THE PROFIT ELASTICITY MEASURE OF OPERATING LEVERAGE IN MANAGERIAL ECONOMICS TEXTS

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## ABSTRACT

*The authors suggest recasting operating leverage (DOL) treatments in managerial textbooks. They extend the profit elasticity form of DOL used by firms to other than competitive markets by introducing nonlinear cost and revenue functions. From their results, the authors urge text writers to highlight four key issues: the role and limitations of 1) management--long run versus short run operating leverage decisions, 2) engineering--variable cost changes associated with fixed cost changes, 3) economic forces—competitive versus non-competitive markets and 4) mathematical results--DOL equals zero at the maximum profit output level, regardless of the level of fixed cost.*

## INTRODUCTION

Operating leverage is important to firm management for one reason, additions to operating fixed costs affect a firm's value by increasing risk as measured by the variability of returns (Lev, 1974, and Berner, 2002). Operating leverage discussions often follow as a natural extension to linear breakeven analysis in managerial economics textbooks. Application of the generally received profit sensitivity formula, the degree of operating leverage (DOL), is limited both theoretically and practically.

Most textual treatments ignore the role DOL variables other than fixed costs play. Most authors assume overly restrictive linear cost and revenue functions, while subsequent chapters develop standard non-linear economic cost and revenue functions.

The roles of management, engineering and economic markets along with the measure's inherent mathematical limitations are left unstated. This article reviews unstated aspects of the DOL measure and offers a more theoretically complete framework for operating leverage textual discussions aimed at the practicing corporate managerial specialist.

Aspects of DOL we consider important include a) consistency with orthodox economic theory, b) recognizing the larger business risk context within which the DOL measure is applied, c) a clearer view of management's role in influencing certain DOL parameters as business risk components and d) some important analytical limitations inherent to the measure's form.

Business risk is a central determinant of a firm's value, the risk-adjusted present value of future profit. Several important parameters affect a firm's business risk position. Among them are price, variable costs, operating fixed costs, the output rate and the stability of demand. The DOL measure contains variables that capture four of these parameters. The fifth, demand stability, is a through-time assessment while DOL is a point in time measure. The level of operating fixed cost, the parameter of greatest attention in textual DOL discussions, is only one business risk parameter. A change in a single business risk parameter in the DOL expression also affects the remaining parameters. For example, increases in operating fixed cost without a compensating reduction in unit variable cost may require increases in output to sustain a desired profit level. A meaningful discussion of DOL should at least mention the distinction between management-led choices addressing the firm's business risk posture versus market forces and engineering-based limits. Finally, other than for expository simplicity, we question the use of restrictive linear cost functions in the DOL formula application, when textual narrative in the same text stresses non-linear relationships.

Discussion in Section 2 confirms the mathematical equivalence between various DOL measures and presents works by Dran (1991), Long (1992) and shows that DOL is sensitive not only to changes in the firm's operating fixed cost but also to short run output. That section suggests that narrative treatments indicating which DOL parameters management can directly influence would help place the measure into a useful operational context. Section 3 extends the DOL expression to include a cubic variable (and total) cost function and parabolic total revenue function to demonstrate that DOL equals zero at the theoretically optimized output, regardless of the level of fixed cost. Section 4 narrative reviews each of nine managerial text treatments on the DOL concept and measure. Section 5 provides the authors' suggestions on how to coalesce DOL treatments, given the arguments made.

## THEORETICAL REVIEW

As derived from short run linear revenue and cost functions for a profit maximizing firm producing a single product, in a purely competitive industry, the DOL measure is essentially a profit cum output sensitivity ratio. Managerial text DOL expressions take one of several algebraic forms such as those listed below.

$$\begin{aligned} DOL &= (\%\ \text{Change in Profit}) / (\%\ \text{Change in Q}) \\ &= Q(p-v) / [Q(p-v) - FC] \\ &= (TR - TVC) / (TR - TVC - FC) = (\Pi + FC) / \text{Profit} \end{aligned} \quad (1)$$

From the development in the Appendix, these mathematically equivalent *linear* DOL expressions above reduce to:

$$DOL = 1 + FC/(p{\cdot}Q - v{\cdot}Q - FC), \text{ where} \quad (2)$$

FC = operating fixed cost
p = unit price
v = unit variable cost
Q = quantity of output
Profit = earnings before interest and taxes = EBIT = $p{\cdot}Q - v{\cdot}Q - FC$

Notice that the profit sensitivity version for DOL (1) used by many managerial text authors is a stylized but very general form suitable for theoretical and practical applications. Yet, this generality goes unused by the very authors who, in subsequent chapters take the reader through nonlinear cost theory and nonlinear revenue generation in imperfect markets. We think it is important for the student and the professional manager that authors provide a more, rather than less, complete DOL discussion.

Dran (1991) and Long (1992) provided the economics literature a theoretical treatment that demonstrated how proximity to breakeven output influences DOL, independent of the level of operating fixed cost. Dran did so by defining the firm's output as a percentage of breakeven quantity. Separating the traditional DOL measure from the firm's cost structure revealed that DOL was also sensitive to the firm's output level, rising or falling asymptotically toward positive or negative infinity as breakeven output was approached either from above or below.

In a reply to Dran's original contribution, Long (1992) showed that there was no a priori fixed-in-proportion economic relationship between increases in fixed operating expenses and commensurate reductions in unit variable cost sufficient to maintain the prior breakeven output. There logically exists a lower unit variable cost value that could compensate for increased operating fixed costs sufficient to leave both breakeven output and DOL unchanged, but engineering and economic relationships determine that value more than does management. Empirical investigators (Li, 1991) offer evidence that management recognizes and considers such a tradeoff, though text writers uniformly avoid discussing such limits.

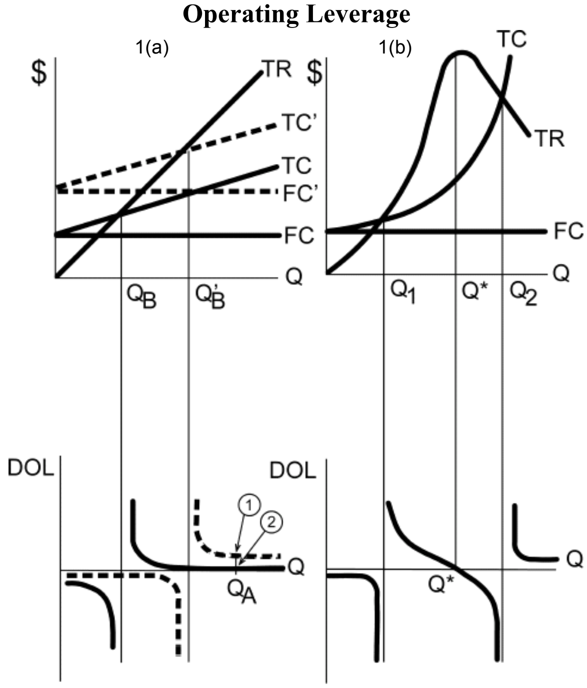
Given the assumed linear revenue and cost functions and regardless of the operating fixed cost level, as operating profit gets close to zero, DOL approaches negative or positive infinity in the vicinity of breakeven quantity, depending on whether breakeven output is approached from below or above. At the breakeven quantity, DOL provides no useful value. For output levels above the breakeven quantity, DOL falls asymptotically toward zero as output increases because profit in the denominator continues to increase while operating fixed cost is constant in the numerator. So, DOL varies for two reasons, the level of operating fixed cost and output, both of which management determines.

From equation (2), if FC = 0, then DOL = 1, indicating that there is no operating leverage. As FC assumes any positive value and, for simplicity, if profit remains positive and there is no change in unit variable cost, then DOL must rise above 1 for two analytical reasons. With an increased operating FC level, if price and unit variable cost remain the same, the denominator in the second part of the expression is smaller and the numerator is larger. This is precisely the point where textual treatments begin to get murky by confusing what is mathematically possible with what is economically plausible. Logically, a firm's management seeking to maximize profits will not voluntarily permit operating FC to rise without a commensurate fall in unit variable costs or a possible increase in price, if price-setting is within their power and strategically desired.

Readings of the DOL result after a fixed cost change are most meaningful when compared to the same output level. Looking at Figure 1(a) and output level $Q_A$ shows the result of a higher FC on the DOL at that output level. The measured DOL, (1), after the increase in FC is clearly greater than the originally measured DOL, (2). Depending on the prior output level, even a small increase in FC, depending on the slope of the TR function, with no change in unit variable costs for simplicity, would require a management decision to increase output, demand permitting, to avoid losses or to maintain desired profits. Failure to do so could lead to short run operating losses and disappointing profit reports as shown in Figure 1(a) and comparing breakeven quantities $Q_B$ and $Q_{B'}$ that differ only due to a greater FC.

Even in a perfectly competitive market, price plays a role in determining the DOL magnitude. Assume the firm is currently operating with a positive profit. When market equilibrium price rises, the denominator in (1) rises reducing the DOL with no change in output, operating fixed cost or unit variable cost. Consequently, breakeven output falls since the contribution margin is larger. DOL can vary due to changes in any of the variables appearing in equation (1). These variables include management-determined choices—operating fixed cost and output levels; market determined parameters—price in a competitive market as time passes; and economic and engineering relationships—unit variable costs, given operating fixed cost increases due to new capital integration.

**Figure 1: Linear and Non-linear Break-even and Operating Leverage**




## A THEORETICAL EXTENSION

A theoretical extension provides additional insight into DOL measurement. Relax the assumption of linear total variable cost and revenue functions in favor of a twice-differentiable cubic cost function and a parabolic revenue function derived from a downward sloping demand (applicable to less than competitive markets). The resulting profit elasticity expression from the development in the appendix appears below.

$$E_{\pi} = [(e - 2f{\cdot}Q) - (3a{\cdot}Q^2 + 2b{\cdot}Q + c)]\ Q/\ [(-a{\cdot}Q^3 - (f + b){\cdot}Q^2 + (e - c){\cdot}Q - d)] \quad (2)$$

Standard economic theory is useful here to help explain a counterintuitive result. From (2), DOL must be zero when profit is maximized or when losses are minimized, because at that point positive marginal revenue, $(e - 2f{\cdot}Q)$, must equal the negative marginal cost, $(3a{\cdot}Q^2 + 2b{\cdot}Q + c)$, in the numerator. It is also true that as TR approaches TC, DOL will approach positive or negative infinity depending on the direction from which quantity approaches breakeven. These analytical results are obtained regardless of the level of fixed cost. Hence, there exists two points where the DOL measure provides no useful information: breakeven quantity and profit maximizing/loss minimizing quantity. The irony is that if management were able to guide the firm to the profit maximizing output level as fixed costs rose through time, DOL would remain equal to zero!

Figure 1 shows the ranges of DOL values, given linear 1(a) and curvilinear 1(b) cost functions. Notice in Figure 1(a) when fixed costs rise from FC to FC', with no compensatory reduction in unit variable costs, two things occur. First, the breakeven quantity of output rises. Second, the DOL magnitude for any quantity above the new breakeven point is greater than before the addition of fixed cost. This is precisely what the DOL expression should show as a firm's business risk indicator.

Figure 1(b) reveals the influence on DOL from the more general curvilinear cost and revenue functions, other assumptions the same. Just as in the linear case, DOL approaches infinity at the breakeven quantity levels of output. At the profit maximizing output level, DOL also equals zero. Such a result poses a conflict, especially for empirical studies, between an important concept in economic theory and an important risk measure in finance theory. DOL for the firm operating at the theoretically optimal output in economics becomes impossible to directly measure.

## BASIC MANAGERIAL TEXT TREATMENTS

Managerial textbook treatments uniformly incorporate linear cost and revenue functions to motivate discussions on breakeven analysis. Applied linear breakeven analysis can serve as a first approximation to real-life business settings. Given that the hurdle to be exceeded by the margin of unit price over unit variable cost times unit volume in breakeven analysis is operating fixed cost, textual narrative regarding the degree of operating leverage follows in many instances.

Of the nine textbooks reviewed, six equate operating leverage with the level of operating fixed cost in a firm's operation. The degree of operating leverage, DOL, is most widely defined as the sensitivity of profit to changes in sales revenue or quantity. This interpretation of DOL is based on the notion that, in the presence of operating fixed costs, a small percentage change in sales may result in a larger percentage change in earnings—greater business risk, something about which managerial corporate stewards should be aware.

As with all indicators, DOL manifests useful characteristics and limitations. Its usefulness centers on its simplicity. As operating fixed costs rise, the DOL magnitude typically will rise. Its limitations relate to its sensitivity from changes in other of its parameters and with measurement discontinuities. A change in the magnitude of any variable in the DOL expression, including quantity, results in an altered DOL magnitude. Only two of nine text writers in our search directly mention this fact. (Salvatore, 2004; Keat and Young, 2003)

**Table 1 : Managerial Economics Text Treatments of Operating Leverage and Definitions***

| Authors | Operating Leverage | DOL | How to measure DOL | C-V-P Analysis |
|---|---|---|---|---|
| Salvatore 2004 | The ratio of the firm's total fixed costs to total variable costs. | The responsive-ness or sensitiv-ity of the firm's total profits to a change in its output or sales. | $DOL = \frac{Q(P-AVC)}{Q(P-AVC)-FC}$ | Linear |
| Hirshey 2003, 7th Ed. | The extent to fixed production facilities versus variable production facilities are employed. | The percentage change in profit from a 1 percent change in output. | $DOL = \frac{\partial\pi/\pi}{\partial Q/Q}$ | Linear, notes limitations |

| Table 1 : Managerial Economics Text Treatments of Operating Leverage and Definitions* | | | | |
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| Authors | Operating Leverage | DOL | How to measure DOL | C-V-P Analysis |
| Hirshey 2003, 10th Ed. | The extent to which production facilities versus variable production facilities are employed. | The percentage change in profit from a 1 percent change in output. | $DOL = \frac{\partial\pi/\pi}{\partial Q/Q}$ | Linear, notes limitations |
| Keat & Young 2003 | | A coefficient that measures the effects of a percentage change in quantity on the percentage change in profit. | $DOL = \frac{Q(P-AVC)}{Q(P-AVC)-FC}$ | Linear, notes limitations |
| McGuigan, Moyer, and Harris, 2002 | The use of assets having fixed cost in an effort to increase expected return. | The percentage change in a firm's EBIT resulting from a 1 percent change in sales or output. | $DOL = \frac{Q\,(P-V)}{Q\,(P-V)-FC}$ | Linear and non-linear, notes limitations |
| Mansfield, Allen, Doherty, and Weigelt, 2002 | The use of fixed cost in operation. | The percentage change in profit resulting from a 1 percent change in the number of units of product sold. | $DOL = \frac{\partial\pi/\pi}{\partial Q/Q}$ | Linear |
| *DOL discussion excluded in: Brickley, Smith, and Zimmerman 2004; Baye, 2002; Maurice and Thomas, 2002 | | | | |

Six of the nine managerial texts written for the college and university market examined introduce the degree of operating leverage measure in algebraic, graphical or elasticity form or some combination of these forms after discussing breakeven analysis. Text authors, apparently for simplicity and instructive purposes, assume linear revenue and cost relationships to motivate the DOL discussion.

Other textual discussions imply that positive net present value options to acquire new capital, while increasing fixed operating costs may also reduce unit variable cost as a trade-off benefit, but pay little attention to the fact that the resulting breakeven output may rise, fall or remain the same. It is not necessarily true that a given increase in operating fixed costs, due perhaps to new technology introduction, will automatically reduce variable unit costs sufficiently to maintain the original breakeven output.

Managerial students draw the lesson that increasing operating fixed costs in the firm's operating cost structure adds to business risk. The lesson seems obvious and, perhaps, that is sufficient introduction at the elementary level. The higher the operating fixed cost hurdle for a firm in the short run, the smaller the chance that the margin of unit price above unit variable cost times the count of units sold will be sufficient to generate a profit. Our view is that a bit more framing, theoretical generality and acknowledgement of measurement limits would better serve the learner and the professional manager with a relatively a small commitment of valuable page space.

## A SUGGESTED REVISION

We suggest a more complete framing and discussion on DOL limitations to form a more cohesive picture in the student's mind. We emphasize that the most useful DOL changes are those showing the expected consequences of management-led decisions on the DOL magnitude *at a given level of output*. It is the change in DOL, output constant, brought about by management's decisions, where DOL is most compelling as a business risk measure.

We separate the DOL expression variables into three related categories: management decision variables, economic market-determined variables and engineering variables. Management must assess the effect of any change in operating fixed cost on unit variable costs prior to committing to the decision. Unit variable cost is partly determined by the market, i.e. factor inputs at their market rate per time and partly by the engineering relationships that exist between old versus new capital equipment and related labor support requirements. Whether the newly adopted technology is labor saving or capital saving directly affects the relationship between fixed cost changes and unit variable cost changes to determine the new required minimum output for the firm to breakeven in the short run.

| Table 2: DOL Parameters and Business Risk Factors | | |
|---|---|---|
| Time Frame | Influencing Factor | DOL Parameter |
| Long-run per plant capacity | Economic Market Forces: | |
| | All markets | Selling price |
| | Management Decision: | |
| | All markets | Fixed cost |
| | Engineering and Economic Relationships: | |
| | All markets | Variable cost structure |
| Short-run per plant utilization | Engineering and Economic Relationships: | |
| | All markets | Unit variable cost |
| | Management Decision: | |
| | All markets | Output |

As firm management evaluates production cost reduction strategies, they have three options: increase production efficiency, outsource an operating fixed cost component to make it a variable cost, or acquire new technology that reduces unit variable cost. Once management acts on the commitment to increase operating fixed cost, reversing the decision is neither easy nor quick, giving it long run implications.

The information presented in Table 2 summarizes the DOL expression variables into short-run and long run time periods and the three influencing factors: management, economic markets and engineering. Management determines the optimal output rate given price and cost in the short run. Management assesses the effect of any change in operating fixed cost on unit variable costs and operating risk prior to committing to the decision in the long run. Unit variable cost is determined partly by economic market forces, i.e. factor inputs at their market rate per time and partly by the engineering relationships that exist between old versus new capital equipment and related labor support requirements. The relationship between fixed cost changes and unit variable cost changes determines the new required output for the firm. From the arguments presented above, the essential points worth emphasizing in textual discussions on DOL reduce to the following:

- Short-run output rate is a management decision that does not reflect a change in the firm's risk posture.
- In the long run, the effect of changes in operating fixed cost and per unit variable cost structure should be evaluated at the same output level.
- In the long run, engineering and economic market relationships largely dictate the trade-off between changes in operating fixed cost and unit variable cost, and management must assess each case on its own merit.
- If unit variable costs are non-linear, no useful leverage value is produced when the firm is operating at or very near operating breakeven output level or profit maximizing (or loss minimizing) output level.

## SUMMARY AND CONCLUSION

A review of operating leverage discussions from a selection of nine current managerial economics textbooks reveals that relevant aspects of DOL are absent from many textual discussions. The authors suggest that useful aspects include a) standard economic cost theory consistency, b) a clearer view of management's role in influencing DOL parameters as business risk components and c) mention of some important limitations inherent in nonlinear versions of the measure.

Articles by Dran and Long in the economics literature, using the profit sensitivity version of DOL, demonstrate the dual influence on profit sensitivity from operating fixed cost changes as well as from output changes. Variables in the DOL expression are economically interdependent. The authors suggest the usefulness of separating DOL parameters into those that management can influence, those that the market influences and those determined by engineering relationships.

By changing two assumptions used in elementary models of the firm, from linear to cubic variable cost and parabolic revenue functions, keeping other assumptions intact—single product, short run, and certainty, it was shown that DOL equals zero when the firm's output is optimized, regardless of the level of operating fixed cost. Hence, two measurement discontinuities for DOL exist, the quantity breakeven output and the profit maximizing or loss minimizing output.

The authors suggest that including these points in DOL narrative discussions will enhance both managerial student's and professional manager's understanding of the larger business risk context, sources of all formulaic DOL variability and more about its measurement limitations.

A more complete DOL discussion would include the following points -- a) the DOL profit sensitivity expression contains several business risk parameters, b) DOL changes due to operating fixed cost changes should be measured at the same output level, c) the DOL measure varies with changes in output in the short run but not its risk posture, a management decision, d) DOL provides no useful information at the firm's operating breakeven output level or profit maximizing (loss minimizing) output level, and e) engineering relationships affect the relation between changes in operating fixed cost and unit variable cost and must be weighed a priori by management.

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## APPENDIX

Assume a single product firm in the short run under certainty operating in competitive input and output markets. Total revenue and total cost functions are linear. The degree of operating leverage (DOL) function in standard managerial text treatments in profit elasticity form:

$$DOL = \%\Delta Profit / \%\Delta Q \quad (1)$$

$$= [( \Delta p \cdot Q - \Delta v \cdot Q - \Delta FC) / (p \cdot Q - v \cdot Q - FC)] [Q / \Delta Q]$$

where $\Delta FC = 0$

$$= [(p \cdot Q - v \cdot Q) / (p \cdot Q - v \cdot Q - FC)]$$

$$DOL = Q(p-v) / [Q(p-v) - FC] \quad (2)$$

$$DOL = (TR - TVC) / (TR - TVC - FC), \text{ where} \quad (3)$$

Q = quantity output per time
p = selling price per unit of output
v = variable cost per unit of output
FC = total operating fixed cost
TR = total revenue
TVC = total variable cost

Economic analysis defines Profit as,

$$Profit = TR - TVC - FC \quad (4)$$

$$Profit = p \cdot Q - v \cdot Q - FC.$$

If profit = 0, then

$Q = FC/(p - v)$, to solve for breakeven quantity of output. (5)

For any Profit value other than zero,

$$Profit - TR + TVC = - FC$$

$$TR - TVC = FC + Profit \quad (6)$$

Substituting (4) and (6) into (3) gives

$$DOL = (Profit + FC) / Profit, \text{ or} \quad (7)$$

$$DOL = 1 + (FC / Profit) = 1 + FC/(p \cdot Q - v \cdot Q - FC) \quad (8)$$

and the DOL becomes discontinuous at Profit = 0.

Now assume non-linear total cost and total revenue functions that are twice differentiable. Standard economic optimization theory confirms the parent profit maximization as:

$$Profit = TR(Q) - TC(Q) \quad (9)$$

Take the first derivative as the necessary condition, to determine candidate values for Q,

$$Profit' = R'(Q) - C'(Q) = 0, \quad (10)$$

and the second derivative sufficient condition to test the candidate values from (10)

$$Profit''(Q) = R''(Q) - C''(Q) < 0. \quad (11)$$

Now allow the single product firm in the short run under certainty operating in less than competitive output markets to make total revenue quadratic, i.e. demand is negatively sloping, and total costs are cubic, the general approach taken in microeconomic theory.

$P = e - f{\cdot}Q$

$TR = e{\cdot}Q - f{\cdot}Q^2$ (12)

$MR = e - 2f{\cdot}Q$ (13)

$TC = a{\cdot}Q^3 + b{\cdot}Q^2 + c{\cdot}Q + d$ (14)

To achieve the idealized total cost shape that economists prefer for all ranges of short run variable proportions, the coefficients in (14) must be restricted as follows:

$a,c,d > 0, \; b < 0, \; b^2 < 3ac$

$MC = 3a{\cdot}Q^2 + 2b{\cdot}Q + c$ (15)

$d = FC$,

$a{\cdot}Q^3 + b{\cdot}Q^2 + c{\cdot}Q = TVC$, and

$a{\cdot}Q^2 + b{\cdot}Q + c = AVC$

Substituting (13) and (15) into the profit function,

$Profit = TR - TC = (e{\cdot}Q - f{\cdot}Q^2) - (a{\cdot}Q^3 + b{\cdot}Q^2 + c{\cdot}Q + d)$ (16)

$= -a{\cdot}Q^3 - (f + b){\cdot}Q^2 + (e - c){\cdot}Q - d$

where,

$dProfit/dQ = -3a{\cdot}Q^2 - 2f{\cdot}Q - 2b{\cdot}Q + e - c$, and rearranging terms,

$dProfit/dQ = (e - 2f{\cdot}Q) - (3a{\cdot}Q^2 + 2b{\cdot}Q + c) = 0$ (17)

In words, marginal revenue less marginal cost equals zero as a necessary condition for an optimum.

Recall the quantity elasticity of profit, DOL, is written as:

Profit Elasticity = (dProfit/dQ) (Q/Profit)

Substituting (16) and (17) into the $E_\pi$ expression above

Profit Elasticity = $[(e - 2f{\cdot}Q) - (3a{\cdot}Q^2 + 2b{\cdot}Q + c)]\, Q /$
$[(-a{\cdot}Q^3 - (f + b){\cdot}Q^2 + (e - c){\cdot}Q - d)]$ (18)

Notice, from the bracketed term in the numerator for (18), that DOL must equal zero when the firm maximizes profit or minimizes losses. This is true because MR = MC at that output level, which means the numerator must be zero. If profit is positive, then at any output level above profit maximization DOL must be negative (MC > MR) and at any output level below profit maximization DOL must be positive (MR > MC). If profit is negative, and still minimized, DOL is negative below the loss minimizing output and positive above it.

# ECONOMICS ARTICLES