EXTENSION OF PENMAN'S FORMULAE TO MULTI-LAYER MODELS

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Abstract. In this paper, the well-established multi-layer model originally devised by Waggoner and Reifsnyder (1968) is used. This steady-state model based on an electrical analogue simulates the energy exchange between the vegetation and the atmosphere. A purely mathematical development of the basic equations of this model yields explicit expressions of the total fluxes of sensible and latent heat at the top of the canopy as a function of the net radiation absorbed in each layer, the soil heat flux, the water vapour pressure deficit at a reference height and the whole set of elementary conductances (stomatal, boundary-layer and aerodynamic). These new equations can be considered as a generalization of the familiar Penman's formulae to a multi-layer model.

List of Symbols

C	Sensible heat flux density in vertical direction (W m^{-2});
c_p	specific heat of air at constant pressure (J kg^{-1} K^{-1});
D_a	saturation deficit of air (Pa);
d_i	defined by Equation (19) (s m^{-1});
e_a	air water vapour pressure (Pa);
$e_s(T)$	saturated vapour pressure at temperature T (Pa);
ga	aerodynamic conductance in vertical direction (m s^{-1});
gb	boundary-layer conductance of leaves (m s^{-1});
gs	stomatal conductance (m s^{-1});
ge_c	equivalent conductance for horizontal heat transfer (m s^{-1})
ge_v	equivalent conductance for horizontal vapour transfer (m s^{-1})
J_0	defined by Equation (35) (W m^{-2});
K	eddy diffusivity for heat and vapour (m^2 s^{-1});
LAI	leaf area index (m^2 m^{-2});
Rn	net radiation flux density (W m^{-2});
S	soil heat flux density (W m^{-2});
T_a	air temperature (°C);
T_L	leaf temperature (°C);
γ	psychrometric constant (66 Pa K^{-1});
Δ	slope of the saturated vapour pressure curve (Pa K^{-1});
δz_i	thickness of layer i (m);
$\delta \mathrm{LAI}_i$	leaf area index of layer i (m^2 m^{-2});
δC_i	sensible heat flux density emanating from layer i (W m^{-2});
$\delta \lambda E_i$	latent heat flux density emanating from layer i (W m^{-2});
δRn_i	net radiation flux density absorbed in layer i (W m^{-2});
λE	latent heat flux density in vertical direction (W m^{-2});
ρ	air density (kg m^{-3});

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Subscripts:

a for air;
c for sensible heat;
i for layer i;
L for leaves;
n for soil surface;
0 for above canopy parameters;
v for latent heat (water vapour).

1. Introduction

For natural surfaces, the partitioning of available radiative energy $(Rn - S)$ into sensible heat flux (C) and latent heat flux (λE) is commonly expressed by means of the familiar Penman's formulae (Penman, 1948, 1953) based on the energy balance approach

$$C = \frac{\gamma^*(Rn - S) - \rho c_p GaD_a}{\Delta + \gamma^*}, \tag{1}$$

$$\lambda E = \frac{\Delta(Rn - S) + \rho c_p GaD_a}{\Delta + \gamma^*}, \tag{2}$$

in which D_a is the vapour pressure deficit of the air at a reference height, Ga the aerodynamic conductance calculated between the surface level and the reference height, Δ the slope of the saturated vapour pressure versus temperature curve and γ^* the apparent psychrometric constant defined as:

$$\gamma^* = \gamma(1 + Ga/Gs), \tag{3}$$

Gs being the surface conductance for water vapour transfer. This approach allows one to describe and analyse convective transfers from a surface acting as a single source. When applied to a vegetation stand (Monteith, 1981), this approach is often referred to as the single-layer approach because the stand is treated as a single equivalent surface absorbing radiative energy and transferring sensible and latent heat to the air.

In the multi-layer approach (Waggoner and Reifsnyder, 1968), the canopy is divided into several layers. They are all characterized by a given thickness and a mean value of relevant variables; leaves in all layers are treated as an equivalent surface exchanging sensible and latent heat with its environment. The multi-layer approach describes fairly well the transfers within the whole canopy but unfortunately does not yield simple and explicit expressions of total fluxes above the canopy as in the single-layer approach. Nevertheless, Shuttleworth (1976) succeeded in deriving a so-called combination equation from a continuous model (in which all the variables are continuous functions of height z) but the relevant resistances were redefined in an uncommon way. Chen (1984), using a discrete model, gives explicit expressions of total fluxes, the resistances being retained in their ordinary sense. In his calculation algorithm, he has to define a new flux linked with a fictitious variable called 'saturation heat'. The purpose of this paper is to derive, from a discrete approach, general expressions for sensible and latent heat fluxes like Chen's by means of a more direct algorithm which does not require the introduction of a fictitious flux.

2. The Basic Equations of Multi-Layer Models

The basic equations are those of the model originally devised by Waggoner and Reifsnyder (1968) and used by Waggoner and Turner (1972), and Halldin and Lindroth (1986). The crop canopy is assumed to be horizontally homogeneous and is divided into several parallel layers (1 to n). Air within layer i is specified by its mean temperature $T_{a,i}$, its mean water vapour pressure $e_{a,i}$. $\delta\mathrm{LAI}_i$ is the leaf area index of layer i and $T_{L,i}$ is the mean temperature of the leaves. $e_{L,i}$ is the water vapour pressure inside the substomatal cavity assumed to be saturated at the leaf temperature

$$e_{L,i} = e_s(T_{L,i}) . \tag{4}$$

The whole stand is visualized as an electrical analogue where sensible and latent heat fluxes replace current; corresponding driving potentials are $\rho c_p T$ for sensible heat and $(\rho c_p/\gamma)e$ for latent heat (Figure 1). The latent heat flux experiences two kinds of conductance: a stomatal conductance and a boundary-layer conductance denoted, respectively, by gs_i and gb_i. Sensible heat experiences only a boundary-layer conductance gb_i which is assumed to be the same as for latent heat.

The elementary fluxes of sensible and latent heat diffusing inside layer i from the leaves to the air are written as:

$$\delta C_i = \rho c_p ge_{c,i}(T_{L,i} - T_{a,i}) , \tag{5}$$

$$\delta\lambda E_i = (\rho c_p/\gamma) ge_{v,i}(e_s(T_{L,i}) - e_{a,i}) \tag{6}$$

with

$$ge_{c,i} = 2\delta\mathrm{LAI}_i gb_i , \tag{7}$$

$$ge_{v,i} = 2\delta\mathrm{LAI}_i \left(\frac{gb_i gs_i}{gb_i + gs_i} \right) . \tag{8}$$

Vertical fluxes denoted by C_i and λE_i experience diffusive conductance ga_i when crossing layer i. This diffusive conductance is linked with turbulent diffusivity K by the following relation:

$$ga_i = 1 \int_{z_i}^{z_{i-1}} [1/K(z)]\,\mathrm{d}z \approx K_i/\delta z_i , \tag{9}$$

where z_i is the height of layer i, K_i is a mean diffusivity for the given layer and δz_i is the layer thickness.

The vertical fluxes leaving layer i are written as:

$$C_i = \rho c_p ga_{i-1}(T_{a,i} - T_{a,i-1}) , \tag{10}$$

$$\lambda E_i = (\rho c_p/\gamma) ga_{i-1}(e_{a,i} - e_{a,i-1}) . \tag{11}$$

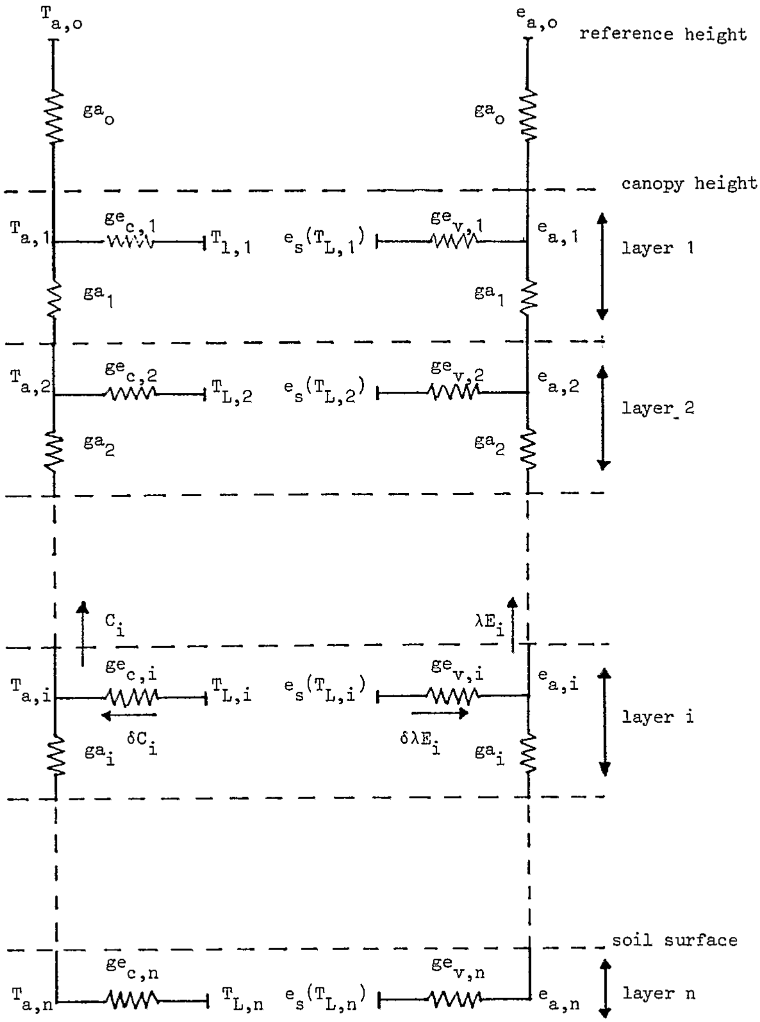

Fig.1 - Electrical analogue of exchange processes within the crop canopy

We have to point out that the model allows one to take into account the transfers at the ground surface, considered as the last layer (denoted by subscript n). For the latent heat exchange a surface conductance has to be defined in the same way as for leaves. Therefore, the total fluxes at the top of the canopy can be expressed as the algebraic sum of the contributions of each layer:

$$C_0 = \sum_{i=1}^{n} \delta C_i \quad \text{and} \quad \lambda E_0 = \sum_{i=1}^{n} \delta \lambda E_i . \tag{12}$$

3. Expressions of Fluxes as a Function of Net Radiation and Vapour Pressure Deficit

The net radiation absorbed in each layer δRn_i balances convective fluxes of sensible and latent heat:

$$\delta Rn_i = \delta C_i + \delta \lambda E_i . \tag{13}$$

This equation is still valid for layer n (soil surface) if δRn_n is replaced by $\delta Rn_n - S$, S being the soil heat flux. By summing Equation (13) from 1 to n we get:

$$\sum_{i=1}^{n} \delta Rn_i - S = Rn_0 - S = C_0 + \lambda E_0 . \tag{14}$$

Linearizing the saturated vapour pressure versus temperature curve between $T_{L,i}$ and $T_{a,i}$ by the slope Δ of the curve determined at the air-temperature $T_{a,0}$ at the reference height yields:

$$\Delta = [e_s(T_{L,i}) - e_s(T_{a,i})]/(T_{L,i} - T_{a,i}) . \tag{15}$$

Introducing the vapour pressure deficit in each layer,

$$D_{a,i} = e_s(T_{a,i}) - e_{a,i} . \tag{16}$$

Equation (6) can be rewritten as:

$$\delta \lambda E_i = (\rho c_p/\gamma) ge_{v,i}(\Delta(T_{L,i} - T_{a,i}) + D_{a,i}) . \tag{17}$$

From Equations (5), (13), and (17), it is possible to infer this expression:

$$T_{L,i} - T_{a,i} = d_i \, \delta Rn_i/\rho c_p - ge_{v,i} d_i D_{a,i}/\gamma \tag{18}$$

with

$$d_i = 1/(ge_{c,i} + (\Delta/\gamma) ge_{v,i}) . \tag{19}$$

After combining Equation (5) with Equation (18), expression (12) for C_0 can be rewritten as:

$$C_0 = \sum_{i=1}^{n} ge_{c,i} d_i \, \delta Rn_i - (\rho c_p/\gamma) \sum_{i=1}^{n} ge_{c,i} ge_{v,i} d_i D_{a,i} . \tag{20}$$

Taking into account Equation (13) yields:

$$\lambda E_0 = (\Delta/\gamma) \sum_{i=1}^{n} ge_{v,i} d_i \, \delta Rn_i + (\rho c_p/\gamma) \sum_{i=1}^{n} ge_{c,i} ge_{v,i} d_i D_{a,i} . \tag{21}$$

In these equations, the only unknown variables are the $D_{a,i}$.

4. Recurrent Formulae for the Calculation of $D_{a,i}$

At each node of the circuit characterized by potentials $T_{a,i}$ and $e_{a,i}$ elementary horizontal fluxes δC_i and $\delta \lambda E_i$ emanating from layer i are mixed with main vertical fluxes emanating from lower layers C_{i+1} and λE_{i+1}. For each node, it is possible to write the following conservation equation:

$$C_i = C_{i+1} + \delta C_i , \tag{22}$$

$$\lambda E_i = \lambda E_{i+1} + \delta \lambda E_i . \tag{23}$$

Developing these equations yields, respectively:

$$ga_{i-1}(T_{a,i} - T_{a,i-1}) = ga_i(T_{a,i+1} - T_{a,i}) + ge_{c,i}(T_{L,i} - T_{a,i}) , \tag{24}$$

$$ga_{i-1}(e_{a,i} - e_{a,i-1}) = ga_i(e_{a,i+1} - e_{a,i}) + ge_{v,i}(e_s(T_{L,i}) - e_{a,i}) . \tag{25}$$

These relations can be rewritten as:

$$T_{a,i+1} = (1 - b_i)T_{a,i} + b_i T_{a,i-1} + c_{c,i}(T_{L,i} - T_{a,i}) , \tag{26}$$

$$e_{a,i+1} = (1 - b_i)e_{a,i} + b_i e_{a,i-1} + c_{v,i}(e_s(T_{L,i}) - e_{a,i}) \tag{27}$$

with

$$\begin{aligned} b_i &= -ga_{i-1}/ga_i , \\ c_{c,i} &= -ge_{c,i}/ga_i , \\ c_{v,i} &= -ge_{v,i}/ga_i . \end{aligned} \tag{28}$$

Multiplying Equations (18) and (26) by Δ gives:

$$e_s(T_{L,i}) - e_s(T_{a,i}) = \Delta d_i \, \delta Rn_i/\rho c_p - (\Delta/\gamma) ge_{vi} d_i D_{a,i} , \tag{29}$$

$$e_s(T_{a,i+1}) = (1 - b_i)e_s(T_{a,i}) + b_i e_s(T_{a,i-1}) + c_{c,i}(e_s(T_{L,i}) - e_s(T_{a,i})) . \tag{30}$$

Combining Equations (27), (29), and (30) yields the following recurrent relation for saturation deficit:

$$D_{a,i+1} = a_i D_{a,i} + b_i D_{a,i-1} + c_i \, \delta Rn_i/\rho c_p \tag{31}$$

with

$$\begin{aligned} a_i &= 1 - b_i - c_{v,i} + (\Delta/\gamma)(c_{v,i} - c_{c,i}) ge_{v,i} d_i , \\ c_i &= \Delta(c_{c,i} - c_{v,i}) d_i . \end{aligned} \tag{32}$$

The first term of the recurrent process $D_{a,1}$ is the vapour pressure deficit at the top of the canopy. The second term $D_{a,2}$ is written as:

$$D_{a,2} = a_1 D_{a,1} + \Delta J_0/\rho c_p ga_1 + c_1 \, \delta Rn_1/\rho c_p \tag{33}$$

with

$$\begin{aligned} a_1 &= 1 - c_{v,1} + (\Delta/\gamma)(c_{v,1} - c_{c,1}) ge_{v,1} d_1 , \\ c_1 &= \Delta(c_{c,1} - c_{v,1}) d_1 , \end{aligned} \tag{34}$$

$$J_0 = C_0 - (\gamma/\Delta)\lambda E_0 . \tag{35}$$

For any subscript i, the vapour pressure deficit of layer i can be put in the form

$$D_{a,i} = \alpha_i D_{a,1} + \beta_i \Delta J_0/\rho c_p g a_1 + \sum_{j=1}^{i-1} \varepsilon_i^j \, \delta R n_j/\rho c_p , \tag{36}$$

coefficients α_i, β_i, and ε_i^j being calculated by means of the following recurrent formulae:

$$\begin{aligned} \alpha_{i+1} &= a_i \alpha_i + b_i \alpha_{i-1} , \\ \beta_{i+1} &= a_i \beta_i + b_i \beta_{i-1} , \\ \varepsilon_{i+1}^{j<i-1} &= a_i \varepsilon_i^j + b_i \varepsilon_{i-1}^j , \\ \varepsilon_{i+1}^{i-1} &= a_i \varepsilon_i^{i-1} = a_i c_{i-1} , \\ \varepsilon_{i+1}^{i} &= c_i , \end{aligned} \tag{37}$$

with the first coefficients defined as:

$$\alpha_1 = 1 , \qquad \beta_1 = 0 , \qquad \alpha_2 = a_1 , \qquad \beta_2 = 1 , \qquad \varepsilon_2^1 = c_1 . \tag{38}$$

5. Solutions for the Total Flux Densities C_0 and λE_0

The total flux density of latent heat at the top of the canopy can be written as (Slatyer and McIlroy, 1961; Monteith, 1981):

$$\lambda E_0 = [\Delta(Rn_0 - S) + \rho c_p g a_0 (D_{a,0} - D_{a,1})]/(\Delta + \gamma) , \tag{39}$$

where $D_{a,0}$ is the saturation deficit of the air at a reference height above the canopy and ga_0 is the aerodynamic conductance calculated between the top of the canopy and this reference height.

Expressing $D_{a,1}$ as a function of $D_{a,0}$ from Equation (39) and taking into account the energy balance equation,

$$Rn_0 - S = C_0 + \lambda E_0 , \tag{40}$$

expression (36) of $D_{a,i}$ becomes

$$D_{a,i} = \alpha_i D_{a,0} + (\alpha_i/ga_0 + \beta_i/ga_1)\Delta J_0/\rho c_p + \sum_{j=1}^{i-1} \varepsilon_i^j \, \delta R n_j/\rho c_p . \tag{41}$$

Substituting relation (41) into relations (20) and (21) and defining,

$$A = \sum_{i=1}^{n} ge_{c,i} ge_{v,i} d_i \alpha_i/ga_0 , \tag{42}$$

$$B = \sum_{i=1}^{n} ge_{c,i} ge_{v,i} d_i \beta_i/ga_1 , \tag{43}$$

we obtain, respectively:

$$C_0[\gamma + (\Delta + \gamma)(A + B)] - [\gamma(A + B)(Rn_0 - S) - \rho c_p g a_0 A D_{a,0}] = \\ = \sum_{i=1}^{n} \Big(\gamma ge_{c,i} d_i \, \delta R n_i - ge_{c,i} ge_{v,i} d_i \sum_{j=1}^{i-1} \varepsilon_i^j \, \delta R n_j\Big) \tag{44}$$

$$\lambda E_0[\gamma + (\Delta + \gamma)(A + B)] - [\Delta(A + B)(Rn_0 - S) + \rho c_p ga_0 AD_{a,0}] =$$
$$= \sum_{i=1}^{n} (\Delta ge_{vi} d_i\, \delta Rn_i + ge_{ci} ge_{vi} d_i \sum_{j=1}^{i-1} \varepsilon_i^j\, \delta Rn_j)\,. \tag{45}$$

Putting

$$\varepsilon_i^i = \Delta / ge_{c,i}\,, \tag{46}$$

the right-hand terms of Equations (44) and (45) can be, respectively, rewritten as:

$$\sum_{i=1}^{n} (\gamma ge_{c,i} d_i\, \delta Rn_i + \Delta ge_{v,i} d_i\, \delta Rn_i - ge_{c,i} ge_{v,i} d_i \sum_{j=1}^{i} \varepsilon_i^j\, \delta Rn_j)\,, \tag{47}$$

$$\sum_{i=1}^{n} ge_{c,i} ge_{v,i} d_i \sum_{j=1}^{i} \varepsilon_i^j\, \delta Rn_j\,. \tag{48}$$

Noticing that:

$$\sum_{i=1}^{n} (\gamma ge_{c,i} + \Delta ge_{v,i}) d_i\, \delta Rn_i = \gamma(Rn_0 - S)\,, \tag{49}$$

$$\sum_{i=1}^{n} ge_{c,i} ge_{v,i} d_i \sum_{j=1}^{i} \varepsilon_i^j\, \delta Rn_j = \sum_{i=1}^{n} \left(\sum_{j=i}^{n} ge_{c,j} ge_{v,j} d_j \varepsilon_j^i \right) \delta Rn_i\,, \tag{50}$$

and defining,

$$E_i = \sum_{j=i}^{n} ge_{c,j} ge_{v,j} d_j \varepsilon_j^i\,, \tag{51}$$

Equations (44) and (45) become:

$$C_0 = \frac{\gamma(1 + A + B)(Rn_0 - S) - \sum_{i=1}^{n} E_i\, \delta Rn_i - \rho c_p ga_0 AD_{a,0}}{\gamma + (\Delta + \gamma)(A + B)}\,, \tag{52}$$

$$\lambda E_0 = \frac{\Delta(A + B)(Rn_0 - S) + \sum_{i=1}^{n} E_i\, \delta Rn_i + \rho c_p ga_0 AD_{a,0}}{\gamma + (\Delta + \gamma)(A + B)}\,. \tag{53}$$

Parameters A, B, and E_i are functions only of elementary conductances ge_{ci}, ge_{vi}, and ga_i. Thus, this set of formulae can be considered as an extension of Penman's original formulae to a multi-layer system.

6. Particular Case of Completely Wet Canopies

If the canopy is completely wet, stomatal conductances gs_i are to be considered as infinite. Assuming that boundary-layer conductances for sensible and latent heat are the same allows one to write:

$$ge_{ci} = ge_{vi} = ge_i = 2\delta \mathrm{LAI}_i gb_i \,. \tag{54}$$

The general formulae given above simplify to:

$$C_0 = [\gamma(Rn_0 - S) - \rho c_p GaD_{a,0}]/(\Delta + \gamma)\,, \tag{55}$$

$$\lambda E_0 = [\Delta(Rn_0 - S) + \rho c_p GaD_{a,0}]/(\Delta + \gamma)\,, \tag{56}$$

in which the total conductance Ga is expressed as:

$$1/Ga = 1/ga_0 + 1/ga_c\,, \tag{57}$$

with

$$ga_c = A'/(1 + B'/ga_1)\,, \tag{58}$$

$$A' = \sum_{i=1}^{n} \alpha_i ge_i\,, \tag{59}$$

$$B' = \sum_{i=1}^{n} \beta_i ge_i\,. \tag{60}$$

Equations (55) and (56) are identical to Penman's formulae derived for a saturated surface (Penman, 1948). But, as is seen in relation (57), the aerodynamic conductance, in the case of a multi-layer system, must include conductance ga_c which represents the supplementary aerodynamic conductance experienced within the crop canopy by sensible and latent heat fluxes.

7. Discussion and Conclusion

Penman's formulae represent the basic equations derived from the energy balance approach for calculating the sensible and latent heat flux densities emanating from natural surfaces which can be considered as acting like a single source (or sink) of sensible and latent heat (open water, bare soil, short canopies like grass). In this paper, it has been shown how these basic equations can be mathematically extended to multi-layer models in which sensible and latent heat are transferred from a set of horizontal planes at different heights. This kind of model constitutes a sound analogue of energy exchange within tall crops. The classical Penman formulae (1) and (2) appear as a particular case (obtained by putting $n = 1$) of more general equations (52) and (53), provided the bulk boundary-layer conductance (2 LAI gb) is included together with the aerodynamic conductance above the canopy ga_0 in the bulk conductance Ga. Therefore, Equations (52) and (53) constitute the general expressions for partitioning of available radiative energy into sensible and latent heat fluxes. They are valid for both single-layer and multi-layer models. The fictitious flux of saturated heat introduced by Chen (1984) for establishing a bridge between single-layer and multi-layer approaches is no longer necessary.

The basic equations of multi-layer models form a closed set of equations which allows one to calculate the total convective fluxes at the top of the canopy and the profiles of temperature and humidity within the canopy. One of the main practical interests of the calculation algorithm presented in this paper is to simplify the mathematical procedure to solve this set of equations. In the discrete approach, the basic linear equations are solved by means of matrix methods (Waggoner *et al.*, 1969; Furnival *et al.*, 1975), whereas in the continuous approach (Goudriaan and Waggoner, 1972; Furnival *et al.*, 1975; Perrier, 1976), the authors derive differential equations which do not have analytical solutions and are solved by numerical methods. Although software for solving matrix problems or for integrating differential equations is available, it is more straightforward to derive the total fluxes above the canopy directly from Equations (52) and (53).

The assumptions used for deriving these formulae are basically the same as those used in the models cited as references. The similarity between the exchange coefficients (boundary-layer conductance and eddy diffusivity) for heat and water vapour is a rather good approximation which has been extensively discussed (Monteith, 1973). The linearization of the saturated vapour pressure curve is performed for each layer over the interval defined by the difference between the leaf temperature and the air temperature (Equation 15); Δ is then calculated at the temperature of the air at the reference height $T_{a,0}$ which is the only temperature introduced as input in the model. Chen (1984) showed that within a $10°$ temperature interval, the error caused by the linearization is rather small. But certainly the most difficult and problematic point of this kind of model is the practical calculation of diffusive conductances based on turbulent diffusivity. As a matter of fact, the flux-gradient relationship within the canopy is questionable and turbulent diffusivity must be handled with care (Waggoner and Turner, 1972).

We have also to point out that it is impossible to infer, from Equations (52) and (53), bulk conductances which would be combinations of elementary conductances and would play the same role in the multi-layer systems as the aerodynamic and surface conductances in the familiar Penman formulae. Only in the case of completely wet canopies is the form of multi-layer formulae (55) and (56) identical to that of the single-layer formulae; bulk conductance *Ga* defined by Equation (57) is the equivalent of the aerodynamic conductance of the familiar Penman formulae.

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