Extension of Poroelastic Analysis to Double-Porosity Materials: New Technique in Microgeomechanics

James G. Berryman[1]

Abstract: Double-porosity materials were introduced as models for oil and gas reservoirs having both storage and transport porosities, and were at first usually treated as static mechanical systems in order to study the flow patterns of fluids during reservoir pump down. Because fluid withdrawal normally increases the effective stress acting on the reservoir, it also turns out to be important to study the geomechanics of the reservoir and how changing fluid pressure affects the solid compaction and fluid permeability of these systems. At the microscale, the mechanical properties of the solid constituents and their distribution in space determine the overall macromechanics of the reservoir system. For systems containing two porosities and two types of solid constituents, exact results for all but one (which may be taken as the overall drained bulk modulus of the system) of the mechanical constants can be derived when the constitutents' properties are known using methods developed in this paper. For multiporosity systems, closure of the system of equations remains an open question, although it is clear that the system can always be closed by the addition of further macroscale measurements.

DOI: 10.1061/(ASCE)0733-9399(2002)128:8(840)

CE Database keywords: Porous materials; Poroelasticity; Micromechanics.

Introduction

The subject of ''geomechanics'' includes such topics as the study of rock mechanics, soil mechanics, and engineering geology, and has overlaping interests in some cases with ''hydrogeology'' when the mechanical behavior of the earth system of interest is strongly affected by the presence of water. We will use the term ''microgeomechanics'' to mean the study of the effects of micromechanics on earth systems. Our main interest here will be in the interaction of fluid pressure changes (usually induced by reservoir depletion) with the mechanical properties of the reservoir.

Terzaghi's (1925) early work on effective stress—accounting for the observed fact that pore pressure tends to counteract the effects of confining pressure in a porous fluid-saturated medium—was expanded into a theory of consolidation, both by himself and through the work of Biot (1941); Gassmann (1951); Skempton (1954); and many others. Biot (1941) is usually given credit for the first comprehensive theory of consolidation, at least in the case of simple, single porosity systems. Gassmann (1951) was the first to obtain one of the fundamental results of the theory—sometimes called the fluid-substitution formula, relating the dry or drained bulk modulus K^* to the undrained (or saturated) modulus K_u by $K_u = K^*/(1-\alpha B)$, where the pore-pressure buildup coefficient B is Skempton's second coefficient (Skempton 1954; Carroll 1980). Early laboratory measurements (Biot and Willis 1957; Fatt 1958, 1959) of the constants in Biot's equations helped to establish the theory. Early engineering solutions of the equations of poroelasticity were given by Rice and Cleary (1976) and Cleary (1977) which helped to make it a standard tool in civil engineering. Some fundamental extensions of the theory to systems having multiple solid constituents have been given by Brown and Korringa (1975); Rice (1975); Berryman and Milton (1991); Berryman (1992); Norris (1992). Poroelasticity is now a well-established subject having recent technical reviews by Detournay and Cheng (1993); Wang (1993); Pride and Berryman (1998); Berryman (1999), and books by Bourbié et al. (1987) and Wang (2000) describing the current state of our understanding.

Biot's original single-porosity, microhomogeneous theory of poroelasticity has significant limitations when the porous medium of interest is very heterogeneous. One important generalization of poroelasticity that has been studied extensively started with the work on double-porosity dual-permeability systems by Barneblatt and Zheltov (1960) and Warren and Root (1963). These papers take explicit note of the fact that real reservoirs tend to be very heterogeneous in both their porosity and permeability characteristics. In particular, the two types of porosity normally treated are storage and transport porosities. Storage porosity holds most of the volume of the fluid underground but may have rather a low permeability, while the transport porosity is low volume but high permeability. The transport porosity is usually treated as being in the form of fractures in the reservoir, or joints in the rock mass. The theory of double-porosity dual-permeability media has been expanding in both volume and scope during the last 20 years, and now includes work by Wilson and Aifantis (1982); Elksworth and Bai (1992); Bai et al. (1993a,b); Berryman and Wang (1995); Tuncay and Corapciaglu (1995); Bai (1999); and Berryman and Pride (2002). Computations of transport and subsidence in double-porosity dual-permeability media include work by Khaled et al. (1984); Nilson and Lie (1990); Cho et al. (1991); Lewallen and Wang (1998); and Bai et al. (1999).

Some technical details follow on the single-porosity poroelasticity needed in the main arguments of the paper. Then equations are formulated for double-porosity systems, and finally multi-

[1]Physicist, Univ. of California, Computational Physics, Lawrence Livermore National Laboratory, P.O. Box 808 L-200, Livermore, CA 94551-9900. E-mail: berryman1@llnl.gov

Note. Associate Editor: Franz-Josef Ulm. Discussion open until January 1, 2003. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on March 25, 2002; approved on March 25, 2002. This paper is part of the ***Journal of Engineering Mechanics***, Vol. 128, No. 8, August 1, 2002. ©ASCE, ISSN 0733-9399/2002/8-840–847/$8.00+$.50 per page.

porosity systems are discussed. The focus will be on determining how the coefficients of the resulting equations depend on the physical properties of the microstructural constituents' of these complex geomechanical systems. The main results are obtained using new techniques in micromechanics that permit a rather elementary analysis of these complex systems to be carried through exactly. For systems containing two porosities and two types of solid constituents, exact results for all but one (which may be taken as the overall drained bulk modulus of the system) of the macroscopic geomechanical constants are derived.

Single-Porosity Geomechanics

In the absence of external driving forces that can maintain fluid-pressure differentials over long time periods, double-porosity and multiporosity models must all reduce to single-porosity models. This reduction occurs in the long-time limit when the matrix fluid pressure and joint fluid pressure become equal. It is therefore necessary to remind ourselves of the basic results for single-porosity models in poroelasticity (Biot 1941; Detournay and Cheng 1993; Wang 2000), as the long-time behavior may be viewed as providing limiting temporal boundary conditions (for $t\rightarrow\infty$) on the analysis of multiporosity coefficients. Further, in the specific models we adopt for the geomechanical constants in the multiporosity theory, extensive use of the single-porosity results will be made.

The volume changes of any isothermal, isotropic material can only be created by hydrostatic pressure changes. The two fundamental pressures of single-porosity poroelasticity are the confining (external) pressure p_c and the fluid (pore) pressure p_f. The differential pressure (or Terzaghi effective stress) $p_d \equiv p_c - p_f$ is often used instead of the confining pressure. The volumetric response of a sample due to small changes in p_d and p_f take the form [e.g., Brown and Korringa (1975)]

$$-\frac{\delta V}{V}=\frac{\delta p_d}{K^*}+\frac{\delta p_f}{K_s} \tag{1}$$

for the total volume V,

$$-\frac{\delta V_\phi}{V_\phi}=\frac{\delta p_d}{K_p}+\frac{\delta p_f}{K_\phi} \tag{2}$$

for the pore volume $V_\phi = \phi V$ (where ϕ is the porosity), and

$$-\frac{\delta V_f}{V_f}=\frac{\delta p_f}{K_f} \tag{3}$$

or the fluid volume V_f. Eq. (1) serves to define the drained (or ''jacketed'') frame bulk modulus K^* and the unjacketed bulk modulus K_s for the composite frame. Eq. (2) defines the jacketed pore modulus K_p and the unjacketed pore modulus K_ϕ. Similarly, Eq. (3) defines the bulk modulus K_f of the pore fluid.

Treating δp_c and δp_f as the independent variables, we define the dependent variables to be $\delta e \equiv \delta V/V$ and $\delta\zeta \equiv (\delta V_\phi - \delta V_f)/V$, which are termed, respectively, the total volume dilatation (positive when a sample expands) and the increment of fluid content (positive when the net fluid mass flow is into the sample during deformation). Then, it follows directly from these definitions and from Eqs. (1), (2), and (3) that

$$\begin{pmatrix} \delta e \\ -\delta\zeta \end{pmatrix} = \begin{pmatrix} 1/K^* & 1/K_s - 1/K^* \\ -\phi/K_p & \phi(1/K_p + 1/K_f - 1/K_\phi) \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f \end{pmatrix} \tag{4}$$

Now we consider two well-known thought experiments: the drained test and the untrained test (Gassmann 1951; Biot and Willis 1957; Geertsma 1957; Wang 2000). In the drained test, the porous material is surrounded by an impermeable jacket and the fluid is allowed to escape through a conduit penetrating the jacket. Then, in a long duration experiment, the fluid pressure remains in equilibrium with the external fluid pressure (e.g., atmospheric) and so $\delta p_f = 0$. Hence, $\delta p_c = \delta p_d$. So changes of total volume and pore volume are given by the drained constants $1/K^*$ and $1/K_p$ as defined in Eqs. (1) and (2). In contrast, for the undrained test, the jacketed sample has no connection to the outside world, so pore pressure responds only to the confining pressure changes. With no way out, the total fluid content cannot change, so the increment $\delta\zeta = 0$. Then, the second equation in Eq. (4) shows that

$$0 = -\phi/K_p(\delta p_c - \delta p_f/B) \tag{5}$$

where Skempton's pore pressure buildup coefficient B (Skempton 1954) is defined by

$$B \equiv \left.\frac{\delta p_f}{\delta p_c}\right|_{\delta\zeta=0} = \frac{1}{1+K_p(1/K_f - 1/K_\phi)} \tag{6}$$

It follows immediately from this definition that the undrained modulus K_u is determined by [also see Carroll (1980)]

$$K_u = \frac{K^*}{1-\alpha B} \tag{7}$$

where α = combination of moduli known as the Biot-Willis parameter, or the total volume effective-stress coefficient. The precise definition of α follows immediately from the form of Eq. (1), by substituting $\delta p_d = \delta p_c - \delta p_f$ and rearranging the equation into the form

$$-\frac{\delta V}{V} = \frac{\delta p_c - \alpha\delta p_f}{K^*} \tag{8}$$

with $\alpha = 1 - K^*/K_s$. The result (7) was apparently first obtained by Gassmann (1951) (though not in this form) for the case of microhomogeneous porous media (i.e., $K_s = K_\phi = K_m$, the bulk modulus of the single mineral present) and by Brown and Korringa (1975) and Rice (1975) for general porous media with multiple minerals as constituents. We will sometimes use the term ''Gassmann material'' when making reference to a microhomogeneous porous medium.

Next, to clarify the structure of Eq. (4) further, note that Betti's reciprocal theorem (Love 1927), shows that the drained and undrained pressures and strains satisfy a reciprocal relation, from which it follows that

$$\frac{1}{K_u} = \frac{1}{K^*} - \frac{\phi B}{K_p} \tag{9}$$

Comparing Eq. (7) with Eq. (9), we obtain the general reciprocity relation (Brown and Korringa 1975)

$$\frac{\phi}{K_p} = \frac{\alpha}{K^*} \tag{10}$$

This reciprocity relation and the form of the compressibility laws (4) also follow directly from general thermodynamic arguments [e.g., Pride and Berryman (1998)]. Then, Skempton's pore-pressure buildup coefficient (Skempton 1954) may be written alternatively as

$$B = \frac{1/K^* - 1/K_s}{1/K^* - 1/K_s + \phi(1/K_f - 1/K_\phi)} \tag{11}$$

Finally, the condensed form of Eq. (4)—incorporating the reciprocity relations—is

$$\begin{pmatrix} \delta e \\ -\delta\zeta \end{pmatrix} = \frac{1}{K^*}\begin{pmatrix} 1 & -\alpha \\ -\alpha & \alpha/B \end{pmatrix}\begin{pmatrix} -\delta p_c \\ -\delta p_f \end{pmatrix} \tag{12}$$

where the Biot-Willis (1957) parameter α can now be expressed as $\alpha=(1-K^*/K_u)/B$. The parameter α is also known as the total volume effective-stress coefficient [see Berryman (1992) for elaboration]. This form of the compressibility laws is especially convenient because all the coefficients are simply related to the three moduli K^*, K_u, and B that have the clearest physical interpretations. This now completes our review of the standard results concerning the single-porosity compressibility laws.

Double-Porosity Geomechanics

In this section, we present the fundamental governing equations controlling the low-frequency (inertial effects being neglected) response of a double-porosity geomechanical system. See Berryman and Wang (1995) for details left out of the following brief summary.

Macroscopic Governing Equations

In the double-porosity formulation, two distinct phases are assumed to exist at the macroscopic level: (1) a porous matrix phase with the effective properties $K^{(1)}, K_m^{(1)}, \phi^{(1)}$ occupying volume fraction $V^{(1)}/V=v^{(1)}$ of the total volume and (2) a macroscopic crack or joint phase occupying the remaining fraction of the volume $V^{(2)}/V=v^{(2)}=1-v^{(1)}$. In earlier work (Berryman and Pride 2002), methods were developed to determine the coefficients of this system within a set of specific modeling assumptions. But the general laws presented in this section are independent of all such modeling assumptions, and the analysis to be presented in later sections is also independent of them as well.

The main difference between the single-porosity and double-porosity formulations is that we allow the average fluid pressure in the matrix phase to differ from that in the joint phase (thus the term ‘‘double porosity’’) over relatively long time scales. Altogether we have three distinct pressures: confining (external) pressure δp_c, pore-fluid pressure $\delta p_f^{(1)}$, and joint-fluid pressure $\delta p_f^{(2)}$. (See Fig. 1.) Treating $\delta p_c, \delta p_f^{(1)}$, and $\delta p_f^{(2)}$ as the independent variables in the double-porosity theory, we define the dependent variables to be $\delta e \equiv \delta V/V$, $\delta\zeta^{(1)}=(\delta V_\phi^{(1)}-\delta V_f^{(1)})/V$, and $\delta\zeta^{(2)}=(\delta V_\phi^{(2)}-\delta V_f^{(2)})/V$, which are, respectively, the total volume dilatation, the increment of fluid content in the matrix phase, and the increment of fluid content in the joints. Finally, we assume that the fluid in the matrix is the same kind of fluid as that in the joints.

Linear relations among strain, fluid content, and pressure then take the general form

$$\begin{pmatrix} \delta e \\ -\delta\zeta^{(1)} \\ -\delta\zeta^{(2)} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} -\delta p_c \\ -\delta p_f^{(1)} \\ -\delta p_f^{(2)} \end{pmatrix} \tag{13}$$

By analogy with the single-porosity result (12), it is easy to see that $a_{12}=a_{21}$ and $a_{13}=a_{31}$. The symmetry of the new off-diagonal coefficients may be demonstrated using Betti's reciprocal theorem in the form

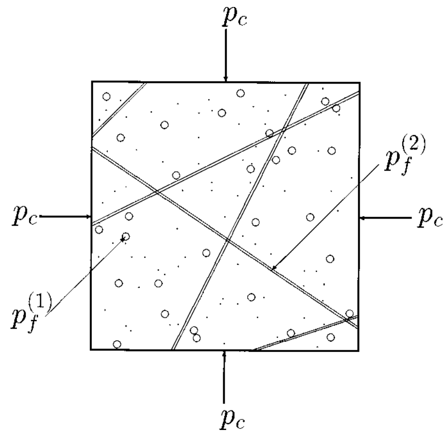

Fig. 1. Elements of double porosity model are: porous rock matrix intersected by fractures. Three types of macroscopic pressure are pertinent in such a model: external confining pressure p_c; internal pressure of the matrix pore fluid $p_f^{(1)}$; and internal pressure of the fracture pore fluid $p_f^{(2)}$.

$$(\delta e \quad -\delta\zeta^{(1)} \quad -\delta\zeta^{(2)})\begin{pmatrix} 0 \\ -\delta\overline{p}_f^{(1)} \\ 0 \end{pmatrix} = (\delta\overline{e} \quad -\delta\overline{\zeta}^{(1)} \quad -\delta\overline{\zeta}^{(2)}) \times \begin{pmatrix} 0 \\ 0 \\ -\delta p_f^{(2)} \end{pmatrix} \tag{14}$$

where nonoverlined quantities refer to one experiment and overlined to another experiment to show that

$$\delta\zeta^{(1)}\delta\overline{p}_f^{(1)} = a_{23}\delta p_f^{(2)}\delta\overline{p}_f^{(1)} = a_{32}\delta\overline{p}_f^{(1)}\delta p_f^{(2)} = \delta\overline{\zeta}^{(2)}\delta p_f^{(2)} \tag{15}$$

Hence, $a_{23}=a_{32}$. Thus, we have established that the matrix in Eq. (13) is completely symmetric, so we need to determine only six independent coefficients.

Constraints on the a_{ij} Coming From Long-Time Limit

Before passing on to the specific models for the various coefficients, we state here several general constraints (independent of any modeling assumptions) on the geomechanical constants a_{ij}. Note that in order to measure the a_{ij}'s in the laboratory, we need only consider an isolated sample immersed in a ‘‘reservoir’’ characterized by three control parameters: p_c, $p_f^{(1)}$, and $p_f^{(2)}$; i.e., gradients in these quantities and the subsequent flow induced by those gradients do not enter the definition of the a_{ij}'s.

The constraints are obtained from the limiting case in which the rate at which p_c, $p_f^{(1)}$, and $p_f^{(2)}$ are all changing is much slower than the rate at which internal fluid equilibration can take place. In this ‘‘long-time limit,’’ we are always in the quasistatic state where $p_f^{(1)}=p_f^{(2)}$. Left to itself, any system having finite permeability will achieve this state as $t\to\infty$.

Drained Test, Long Time

The long-time drained (or ‘‘jacketed’’) test for a double-porosity system should thus correspond to the condition $\delta p_f^{(1)}=\delta p_f^{(2)}=0$ so that the total volume obeys $\delta e=-a_{11}\delta p_c$. It follows therefore that

$$a_{11} \equiv \frac{1}{K^*}. \tag{16}$$

Undrained Test, Long Time

The long-time undrained test for a double-porosity system should also produce the same physical results as a single-porosity system (assuming only that it makes sense at some appropriate larger scale to view the medium as homogeneous). The conditions for this test are that

$$\begin{aligned} \delta p_f^{(1)} &= \delta p_f^{(2)} = \delta p_f \\ \delta\zeta &\equiv \delta\zeta^{(1)} + \delta\zeta^{(2)} = 0 \end{aligned} \tag{17}$$

from which follow

$$\begin{aligned} \delta e &= -a_{11}\delta p_c - (a_{12}+a_{13})\delta p_f \\ 0 &= -(a_{21}+a_{31})\delta p_c - (a_{22}+2a_{23}+a_{33})\delta p_f \end{aligned} \tag{18}$$

These require that the overall pore-pressure buildup coefficient be given by

$$B \equiv \left.\frac{\partial p_f}{\partial p_c}\right|_{\delta\zeta=0} = -\frac{a_{21}+a_{31}}{a_{22}+2a_{23}+a_{33}} \tag{19}$$

and that the undrained bulk modulus be given by

$$\frac{1}{K_u} \equiv \left.\frac{\delta e}{\delta p_c}\right|_{\delta\zeta=0} = a_{11} + (a_{12}+a_{13})B \tag{20}$$

Fluid Injection Test, Long Time

The conditions required to measure the three-dimensional storage coefficient R in the long-time limit are that $\delta p_f^{(1)} = \delta p_f^{(2)} = \delta p_f$, while $\delta p_c = 0$. It follows therefore from Eqs. (4) and (21) that

$$R \equiv \left.\frac{\partial\zeta}{\partial p_f}\right|_{\delta p_c=0} = a_{22} + 2a_{23} + a_{33} = \frac{\alpha}{K^*} + \phi\left(\frac{1}{K_f} - \frac{1}{K_\phi}\right) \tag{21}$$

Generalized Biot-Willis Parameters

Eq. (16) has already determined the coefficient a_{11}. Thus, Eq. (20) shows that

$$a_{12} + a_{13} = -\frac{1/K^* - 1/K_u}{B} = -\alpha/K^* \tag{22}$$

This relation provides a constraint on the sum of the two generalized Biot-Willis parameters for the double-porosity problem.

Not all of these long-time results are independent. In fact, there are only three independent equations among the five given above expressing the a_{ij} in terms of the single-porosity (long-time) moduli.

Double-Porosity Thought Experiment

Several of the main results obtained previously can be derived in a more elegant fashion by using a new self-similar (uniform expansion) thought experiment. The basic idea we are going to introduce here is analogous to, but nevertheless distinct from, other thought experiments used in thermoelasticity (Cribb 1968) and in single-porosity poroelasticity by Berryman and Milton (1991) and Berryman and Pride (1998). Cribb's method provided an independent and simpler derivation of Levin's (1967) results on thermoelastic expansion coefficients. The present results also provide an independent and simpler derivation of results obtained recently

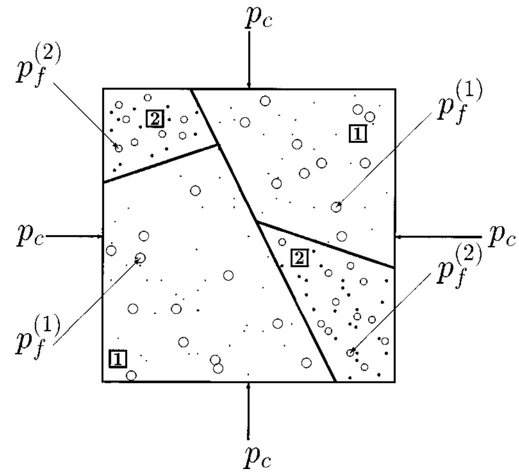

Fig. 2. Composite porous medium is composed of two distinct types of porous solid (1,2). In one version of this model, fractures may result from presence of misfit porosity between different types of solid, since they are assumed to be only weakly bonded at points of contact. In another version, two types of materials are well bonded but themselves have very different porosity types, one being a storage porosity and the other being a transport porosity (and therefore fracturelike or tubelike).

by Berryman and Pride (2002) for the double-porosity coefficients. Related methods in micromechanics are called "the method of uniform fields" by some authors (Dvorak and Benveniste 1997).

We have already shown that $a_{11} = 1/K^*$. We will now show how to determine the remaining five constants in the case of a binary composite system, such as that illustrated in Fig. 2. The components of the system are themselves porous materials 1 and 2, but each is assumed to be what we call a "Gassmann material" satisfying [in analogy to Eq. (12)]

$$\begin{pmatrix} \delta e^{(1)} \\ -\delta\zeta^{(1)}/v^{(1)} \end{pmatrix} = \frac{1}{K^{(1)}}\begin{pmatrix} 1 & -\alpha^{(1)} \\ -\alpha^{(1)} & \alpha^{(1)}/B^{(1)} \end{pmatrix}\begin{pmatrix} -\delta p_c^{(1)} \\ -\delta p_f^{(1)} \end{pmatrix} \tag{23}$$

for material 1 and a similar expression for material 2. The new constants appearing on the right are the drained bulk modulus $K^{(1)}$ of material 1, the corresponding Biot-Willis parameter $\alpha^{(1)}$, and the Skempton coefficient $B^{(1)}$. The volume fraction $v^{(1)}$ appears here to correct the difference between a global fluid content and the corresponding local variable for material 1. The main special characteristic of a Gassmann porous material is that it is composed of only one type of solid constituent, so it is "microhomogeneous" in its solid component, and in addition, the porosity is randomly, but fairly uniformly, distributed so there is a well-defined constant porosity $\phi^{(1)}$ associated with material 1, etc.

For our new thought experiment, we ask the question: Is it possible to find combinations of $\delta p_c = \delta p_c^{(1)} = \delta p_c^{(2)}, \delta p_f^{(1)}$, and $\delta p_f^{(2)}$ such that the expansion or contraction of the system is spatially uniform or self similar? This is the same as asking if we can find uniform confining pressure δp_c, and pore-fluid pressures $\delta p_f^{(1)}$ and $\delta p_f^{(2)}$, such that $\delta e = \delta e^{(1)} = \delta e^{(2)}$. If these conditions can all be met simultaneously, then results for system constants can be obtained purely algebraically without ever having to solve the equilibrium equations for nonconstant stress and strain. We have initially set $\delta p_c = \delta p_c^{(1)} = \delta p_c^{(2)}$, as the condition of uniform confining pressure is clearly necessary for this self-similar thought experiment to achieve a valid solution of the equilibrium equations.

So, the first condition to be considered is the equality of the strains of the two constituents

$$\delta e^{(1)} = -\frac{1}{K^{(1)}}(\delta p_c - \alpha^{(1)}\delta p_f^{(1)})$$

$$= \delta e^{(2)} = -\frac{1}{K^{(2)}}(\delta p_c - \alpha^{(2)}\delta p_f^{(2)}) \tag{24}$$

If this condition can be satisfied, then the two constituents are expanding or contracting at the same rate and it is clear that self similarity will prevail. If we imagine that δp_c and $\delta p_f^{(1)}$ have been chosen, then we only need to choose an appropriate value of $\delta p_f^{(2)}$, so that Eq. (24) is satisfied. This requires that

$$\delta p_f^{(2)} = \delta p_f^{(2)}(\delta p_c, \delta p_f^{(1)}) = \frac{1 - K^{(2)}/K^{(1)}}{\alpha^{(2)}}\delta p_c + \frac{\alpha^{(1)}K^{(2)}}{\alpha^{(2)}K^{(1)}}\delta p_f^{(1)} \tag{25}$$

which shows that, except for some very special choices of the material parameters (such as $\alpha^{(2)} = 0$), $\delta p_f^{(2)}$ can in fact always be chosen so the uniform expansion takes place. (We are not considering long-term effects here. Clearly, if the pressures are left to themselves, they will tend to equilibrate over time so that $\delta p_f^{(1)} = \delta p_f^{(2)}$. We are considering only the "instantaneous" behavior of the material permitted by our system of equations and finding what internal consistency of this system of equations implies must be true.)

Using formula (25), we can now eliminate $\delta p_f^{(2)}$ from the remaining equality so that

$$\delta e = -[a_{11}\delta p_c + a_{12}\delta p_f^{(1)} + a_{13}\delta p_f^{(2)}(\delta p_c, \delta p_f^{(1)})]$$

$$= \delta e^{(1)} = -\frac{1}{K^{(1)}}(\delta p_c - \alpha^{(1)}\delta p_f^{(1)}) \tag{26}$$

where $\delta p_f^{(2)}(\delta p_c, \delta p_f^{(1)})$ is given by Eq. (25). Making the substitution and then noting that δp_c and $\delta p_f^{(1)}$ were chosen independently and arbitrarily, we see that the resulting coefficients of these two variables must each vanish. The equations we obtain in this way are

$$a_{11} + a_{13}[1 - K^{(2)}/K^{(1)}]/\alpha^{(2)} = 1/K^{(1)} \tag{27}$$

and

$$a_{12} + a_{13}[\alpha^{(1)}K^{(2)}/\alpha^{(2)}K^{(1)}] = -\alpha^{(1)}/K^{(1)} \tag{28}$$

Since a_{11} is known, Eq. (27) can be solved directly for a_{13}, giving

$$a_{13} = -\frac{\alpha^{(2)}}{K^{(2)}}\frac{1 - K^{(1)}/K^*}{1 - K^{(1)}/K^{(2)}} \tag{29}$$

Similarly, since a_{13} is now known, substituting into Eq. (28) gives

$$a_{12} = -\frac{\alpha^{(1)}}{K^{(1)}}\frac{1 - K^{(2)}/K^*}{1 - K^{(2)}/K^{(1)}} \tag{30}$$

Thus, three of the six coefficients have been determined.

To evaluate the remaining three coefficients, we must consider what happens to the fluid increments during the same self-similar expansion thought experiment. We will treat only material 1, but the equations for material 2 are completely analogous. From the preceding equations, it follows that

$$\delta\zeta^{(1)} = a_{12}\delta p_c + a_{22}\delta p_f^{(1)} + a_{23}\delta p_f^{(2)}(\delta p_c, \delta p_f^{(1)})$$

$$= \frac{v^{(1)}}{K^{(1)}}[-\alpha^{(1)}\delta p_c + (\alpha^{(1)}/B^{(1)})\delta p_f^{(1)}] \tag{31}$$

Again substituting for $\delta p_f^{(2)}(\delta p_c, \delta p_f^{(1)})$ from Eq. (25) and noting once more that the resulting equation contains arbitrary values of δp_c and $\delta p_f^{(1)}$, so that the coefficients of the terms must vanish separately, gives two equations $a_{12} + a_{23}(1 - K^{(2)}/K^{(1)})/\alpha^{(2)} = -\alpha^{(1)}v^{(1)}/K^{(1)}$, and $a_{22} + a_{23}(\alpha^{(1)}K^{(2)}/\alpha^{(2)}K^{(1)}) = \alpha^{(1)}v^{(1)}/B^{(1)}K^{(1)}$. Solving these equations in sequence as before, we obtain

$$a_{23} = \frac{K^{(1)}K^{(2)}\alpha^{(1)}\alpha^{(2)}}{(K^{(2)} - K^{(1)})^2}\left[\frac{v^{(1)}}{K^{(1)}} + \frac{v^{(2)}}{K^{(2)}} - \frac{1}{K^*}\right] \tag{32}$$

and

$$a_{22} = \frac{v^{(1)}\alpha^{(1)}}{B^{(1)}K^{(1)}} - \left(\frac{\alpha^{(1)}}{1 - K^{(1)}/K^{(2)}}\right)^2\left[\frac{v^{(1)}}{K^{(1)}} + \frac{v^{(2)}}{K^{(2)}} - \frac{1}{K^*}\right] \tag{33}$$

Performing the corresponding calculation for $\delta\zeta^{(2)}$ produces formulas for a_{32} and a_{33}. Since the formula in Eq. (32) is already symmetric in the component indices, the formula for a_{32} provides nothing new. The formula for a_{33} is easily seen to be identical in form to a_{22}, but with the 1 and 2 indices interchanged everywhere.

This completes the derivation of all five of the needed coefficients of double porosity for the two constituent model.

These results can now be used to show how the constituent properties K, α, and B average at the macrolevel for a two-constituent composite. We find

$$\alpha = -\frac{a_{12} + a_{13}}{a_{11}} = \frac{\alpha^{(1)}(K^* - K^{(2)}) + \alpha^{(2)}(K^{(1)} - K^*)}{K^{(1)} - K^{(2)}} \tag{34}$$

and

$$\frac{1}{B} = -\frac{a_{22} + 2a_{23} + a_{33}}{a_{12} + a_{13}}$$

$$= \frac{K^*}{\alpha}\left(\frac{v^{(1)}\alpha^{(1)}}{B^{(1)}K^{(1)}} + \frac{v^{(2)}\alpha^{(2)}}{B^{(2)}K^{(2)}} - \left(\frac{\alpha^{(1)}K^{(2)} - \alpha^{(2)}K^{(1)}}{K^{(2)} - K^{(1)}}\right)^2\left[\frac{v^{(1)}}{K^{(1)}} + \frac{v^{(2)}}{K^{(2)}} - \frac{1}{K^*}\right]\right) \tag{35}$$

It should also be clear that parts of the preceding analysis generalize easily to the multiporosity problem. We discuss some of these remaining issues in the final section.

Example

To illustrate the use of the formulas derived for the coefficients of the double-porosity system, we will now compute and plot the coefficients for a realistic system. We will use data of Coyner (1984) for Navajo sandstone, and modify it somewhat to produce a plot that will highlight the results obtained from the equations. The first problem we encounter in doing so is that, although we

Table 1. Input Parameters for Navajo Sandstone Model of Double-Porosity System.

K_s (GPa)	$K_s^{(1)}$ (GPa)	$K^{(1)}$ (GPa)	$\nu^{(1)}$	$\phi^{(1)}$	$K_s^{(2)}$ (GPa)	$K^{(2)}$ (GPa)	$\nu^{(2)}$	$\phi^{(2)}$
34.0	34.5	16.5	0.15	0.118	34.5	1.65	0.017	0.354

Note: Poisson's ratio ν and porosity ϕ are dimensionless.

can make reasonable direct estimates of the bulk and shear moduli of the constituents, we also must have an estimate of the overall bulk modulus K^* of the composite double-porosity medium. And more than that, we need it as a function of the volume fractions of the two constituents. Our analysis has assumed that K^* was given or measured independently. For present purposes, it is sensible to use an effective medium theory such as the symmetric self-consistent method [or CPA=coherent potential approximation—see Berryman and Berge (1996) for a discussion and references therein for elaboration] to estimate K^*. The CPA has the advantage that it treats both constituents equally (i.e., symmetrically) and therefore does not assume that one constituent always surrounds the other—so there is no host material [see Berge et al. (1993) for further discussion]. With this addition to the theory, we can proceed to the calculations.

The parameters used for Navajo sandstone are listed in Table 1. Although Poisson's ratio ν does not appear explicitly in the equations here, it is required in the CPA (or any but the most elementary) effective medium calculation for the overall bulk modulus K^*. The results are shown in Fig. 3.

Note that the off-diagonal coefficient a_{23}, which couples the fluid in the storage porosity to the fluid in the transport porosity, is very close to zero for all values of storage material volume fraction. This behavior has been observed previously (Berryman and Wang 1995), and is believed to be a strong indication that the double-porosity approach is appropriate for the system studied. If this coefficient is not small, then the fluids in the two types of porosity are strongly coupled and therefore should not be treated as a double-porosity system.

The behavior of the other coefficients is as one would expect: All the coefficients for the transport porosity tend to vanish as the volume fraction of this phase vanishes, and the medium again reduces to a single-porosity system in this limit.

Discussion of Multiporosity Systems

Micromechanical analysis provides definite answers to the question of how the coefficients in double-porosity systems are to be computed from knowledge of the constituents' properties. The question then naturally arises whether this analysis can be generalized to multiporosity systems. Certainly, multiporosity systems are the ones most likely to represent realistic systems occurring in nature, for example, oil and gas reservoirs. And, therefore, we need to address these issues. Transport in triple-porosity and multiporosity systems have already been studied by some authors (Bai et al. 1993b; Bai and Roegiers 1997), hence, it is timely to consider the geomechanical aspects of these problems. We will set up the problem and describe its general characteristics here, but the full solution will be left to future work.

The resulting coefficient matrices will clearly take a form analogous to the ones already studied. For example, in a triple-porosity system, the macroscopic governing equations are

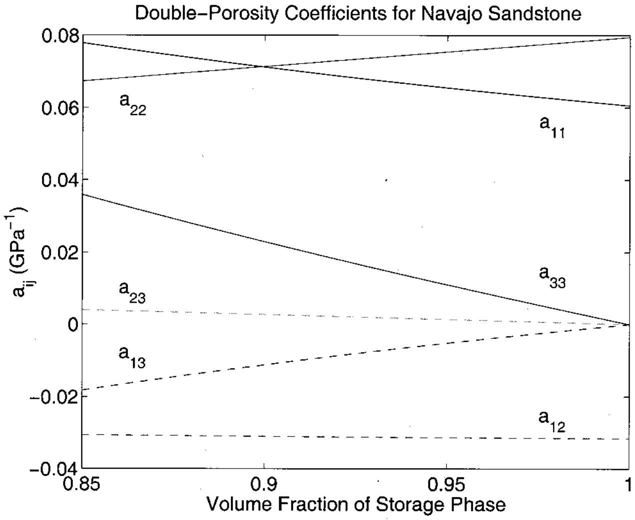

Fig. 3. Values of double-porosity coefficients a_{ij} for a system similar to Navajo sandstone. Values used for the input parameters are listed in Table 1.

$$\begin{pmatrix} \delta e \\ -\delta\zeta^{(1)} \\ -\delta\zeta^{(2)} \\ -\delta\zeta^{(3)} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f^{(1)} \\ -\delta p_f^{(2)} \\ -\delta p_f^{(3)} \end{pmatrix} \quad (36)$$

The meanings of all the coefficients follow immediately from the discussion of Eq. (13). The matrix is again symmetric, so there are four diagonal and six off-diagonal coefficients to be determined, for a total of ten unique coefficients. The leading coefficient $a_{11}=1/K^*$ as before, but the remaining coefficients require further analysis.

In general, for an N-porosity system of the form considered here, the total number of coefficients to be determined in the $(N+1)\times(N+1)$ system of equations is $N+1$ diagonal and $N(N+1)/2$ unique off-diagonal coefficients, for a total of $G=(N+1)(N+2)/2$ coefficients. And the nature of a_{11} remains unchanged for any N. If we assume that each of the unique porosities can be associated with a Gassmann (microhomogeneous) material, then we have equations of the same form as Eq. (23) for each of these constituents, and therefore three mechanical coefficients plus the porosity of each constituent is assumed to be known, at least approximately, in order for this analysis to proceed. The uniform expansion/contraction scenario carries over to the multiporosity system, but does not supply enough equations to close the system by itself for $N>2$. To see this, note that once δp_c and $\delta p_f^{(1)}$ are chosen, then all the remaining δp_f's are determined by the uniform strain condition and Gassmann's relations. Then, substituting these values into the multiporosity system [e.g., Eq. (36)], we see there are always two equations for each row of the matrix. This results in $S=2(N+1)$ equations just from this self-similar thought experiment. These two sets of numbers are compared in Table 2. In addition to these equations, we always have the three conditions from the long-time limits, and we can also find other equations as needed by considering other experiments on the system [e.g., see Berryman and Wang (1995)]. However, it is important to remember that it is the number of linearly independent equations that is pertinent, and determining

Table 2. Growth of Number $G=(N+1)(N+2)/2$ of Geomechanical Coefficients and Number $S=2(N+1)$ of Equations from Self-Similar Thought Experiment as Number N of Distinct Porosities within the System Increases

N	1	2	3	4
G	3	6	10	15
S	—	6	8	10

this number has so far not proven to be an easy task for the general case. At the present writing, closure of the system of equations for the multiporosity coefficients when $N>2$ is an open question.

The analysis presented here has been strictly for isotropic constituents, and an isotropic overall multiporosity system. Generalization to anisotropic systems is both possible and desirable, but the analysis obviously becomes more complex because of the proliferation of coefficients that results.

Conclusions and New Directions

The preceding results show how a micromechanical analysis based on poroelasticity and Gassmann's equations can be used to compute the geomechanical double-porosity coefficients in a very elegant manner. This makes use of all the information available and produces reasonable estimates of all the coefficients needed in reservoirs modeled by double-porosity geomechanics. Triple- and multiporosity geomechanics can also be studied using similar methods, but some work remains to be done on closure of the increasingly larger systems of equations involved. For multiporosity systems, closure of the system of equations can nevertheless always be achieved by the addition of further macroscale measurements. Analysis and solution of these systems of equations to eliminate the need for such additional measurements is therefore one subject of future work in this area of research.

Extension of this work in other directions is also possible. In particular, the applications presented here have been restricted for the sake simplicity to isotropic macroscopic systems. But it is known that the methods employed are not restricted to isotropic systems—as has already been shown in other micromechanical studies by Dvorak and Benveniste (1997). So careful extensions of these ideas to anisotropy due to oriented fractures will permit us to provide more realistic models of reservoir geomechanics, including effects of overburden, tectonic stresses, hydrofracing, etc.

Acknowledgments

The writer thanks Steve Pride and Herb Wang for their insightful collaborations on the phenomenology and micromechanics of double-porosity systems. The writer also thanks Patricia A. Berge and Franz-Josef Ulm for helpful comments that improved the manuscript. Work performed under the auspices of the U.S. Department of Energy by the University of California Lawrence Livermore National Laboratory under Contract No. W-7405-ENG-48 and supported specifically by the Engineering and Geosciences Research Program of the DOE Office of Energy Research within the Office of Basic Energy Sciences, Division of Chemical Sciences, Geosciences, and Biosciences.

Notation

The following symbols are used in this paper:

a_{ij} = double-porosity or N-porosity coefficients;
B = Skempton's coefficient;
e = volume strain;
G = number of distinct a_{ij} coefficients;
K^* = bulk modulus of drained porous frame (jacketed):
K_f = fluid bulk modulus;
K_m = material (or grain) bulk modulus;
K_p = effective pore bulk modulus (jacketed), equal to $\phi K/\alpha$;
K_s = effective solid bulk modulus (unjacketed);
K_u = bulk modulus of undrained (confined) porous frame;
K_ϕ = effective pore bulk modulus (unjacketed);
N = number of distinct porosity types;
p_c = confining pressure;
p_d = differential pressure, equal to $p_c - p_f$;
p_f = fluid pressure;
$p_f^{(1)}, p_f^{(2)}$ = matrix and fracture fluid pressures;
R = storage coefficient;
S = number of distinct equations obtained from self-similar thought experiment;
V = total volume;
$V^{(1)}, V^{(2)}$ = total matrix and fracture material volumes:
V_f = fluid volume;
V_s = solid volume, equal to $(1-\phi)V$;
V_ϕ = pore volume, equal to ϕV;
$v^{(1)}, v^{(2)}$ = volume fractions occupied by matrix and fractures with $v^{(1)}+v^{(2)}=1$;
α = Biot-Willis parameter;
ζ = increment of total fluid content;
$\zeta^{(1)}, \zeta^{(2)}$ = increments of matrix and fracture fluid content;
$\nu^{(1)}, \nu^{(2)}$ = Poisson's ratios for matrix and fracture phases;
ϕ = total porosity, equal to $v^{(1)}\phi^{(1)}+v^{(2)}\phi^{(2)}$; and
$\phi^{(1)}, \phi^{(2)}$ = matrix and fracture phase porosities.

References

Bai, M. (1999). "On equivalence of dual-porosity poroelastic parameters." *J. Geophys. Res.*, 104, 10461–10466.

Bai, M., Elsworth, D., and Roegiers, J.-C. (1993a). "Modeling of naturally fractured reservoirs using deformation dependent flow mechanism." *Int. J. Rock Mech. Min. Sci. Geomech. Abstr.*, 30, 1185–1191.

Bai, M., Elsworth, D., and Roegiers, J.-C. (1993b). "Multiporosity/multipermeability approach to the simulation of naturally fractured reservoirs." *Water Resour. Res.*, 29, 1621–1633.

Bai, M., Meng, F., Elsworth, D., Abousleiman, Y., and Roegiers, J. C. (1999). "Numerical modelling of coupled flow and deformation in fractured rock specimens." *Int. J. Numer. Analyt. Meth. Geomech.*, 23, 141–160.

Bai, M., and Roegiers, J.-C. (1997). "Triple-porosity analysis of solute transport." *J. Contam. Hydrol.*, 28, 247–266.

Barenblatt, G. I., and Zheltov, Yu. P. (1960). "Fundamental equations of filtration of homogeneous liquids in fissured rocks." *Sov. Phys. Dokl.*, **5**, 522–525 [Engish translation of *Dok. Akad. Nauk (SSSR)*, 132, 545–548.

Berge, P. A., Berryman, J. G., and Bonner, B. P. (1993). "Influence of microstructure on rock elastic properties." *Geophys. Res. Lett.*, 20, 2619–2622.

Berryman, J. G. (1992). "Effective stress for transport properties of inhomogeneous porous rock." *J. Geophys. Res.*, 97, 17409–17424.

Berryman, J. G. (1999). ''Origin of Gassmann's equations.'' *Geophysics,* 64, 1627–1629.

Berryman, J. G., and Berge, P. A. (1996). ''Critique of explicit schemes for estimating elastic properties of multiphase composites.'' *Mech. Mater.,* 22, 149–164.

Berryman, J. G., and Milton, G. W. (1991). ''Exact results for generalized Gassmann's equations in composite porous media with two constituents.'' *Geophysics,* 56, 1950–1960.

Berryman, J. G., and Pride, S. R. (1998). ''Volume averaging, effective stress rules, and inversion for microstructural response of multicomponent porous media.'' *Int. J. Solids Struct.,* 35, 4811–4843.

Berryman, J. G., and Pride, S. R. (2002). ''Models for computing geomechanical constants of double-porosity materials from the constituent's properties.'' *J. Geophys. Res.*, in press.

Berryman, J. G., and Wang, H. F. (1995). ''The elastic coefficients of double-porosity models for fluid transport in jointed rock.'' *J. Geophys. Res.,* 100, 24611–24627.

Biot, M. A. (1941). ''General theory of three-dimensional consolidation.'' *J. Appl. Phys.,* 12, 155–164.

Biot, M. A., and Willis, D. G. (1957). ''The elastic coefficient of the theory of consolidation.'' *J. Appl. Mech.,* 24, 594–601.

Bourbié, T., Coussy, O., and Zinszner, B. (1987). *Acoustics of porous media,* Gulf Publishing, Houston.

Brown, R. J. S., and Korringa, J. (1975). ''On the dependence of the elastic properties of a porous rock on the compressibility of a pore fluid.'' *Geophysics,* 40, 608–616.

Carroll, M. M. (1980). ''Mechanical response of fluid-saturated porous materials.'' *Theoretical and Applied Mechanics, Proc. 15th Int. Congress of Theoretical and Applied Mechanics,* Toronto, August 17–23, F. P. J. Rimrott and B. Tabarrok, eds., North-Holland, Amsterdam, 251–262.

Cho, T. F., Plesha, M. E., and Haimson, B. C. (1991). ''Continuum modeling of jointed porous rock.'' *Int. J. Numer. Analyt. Meth. Geomech.,* 15, 333–353.

Cleary, M. P. (1977). ''Fundamental solutions for a fluid-saturated porous solid.'' *Int. J. Solids Struct.,* 13, 785–806.

Coyner, K. B. (1984). ''Effects of stress, pore pressure, and pore fluid on bulk strain, velocity and permeability of rocks.'' PhD thesis, Massachusetts Institute of Technology, Cambridge, Mass.

Cribb, J. L. (1968). ''Shrinkage and thermal expansion of a two phase material.'' *Nature London,* 220, 576–577.

Detournay, E., and Cheng, A. H.-D. (1993). ''Fundamentals of poroelasticity.'' *Comprehensive rock engineering,* J. A. Hudson, ed., Vol. 2, Chap. 5, Pergamon, Oxford.

Dvorak, G. J., and Benveniste, Y. (1997). ''On micromechanics of inelastic and piezoelectric composites.'' *Theoretical and applied mechanics,* T. Tatsumi, E. Watanabe, and T. Kambe, eds., Elsevier, Amsterdam, The Netherlands.

Elsworth, D., and Bai, M. (1992). ''Flow-deformation response of dual-porosity media.'' *J. Geotech. Eng.,* 118, 107–124.

Fatt, I. (1958). ''Pore volume compressibilities of sandstone reservoir rocks.'' *Soc. Pet. Eng. Trans. AIME,* 213, 362–364.

Fatt, I. (1959). ''The Biot-Willis elastic coefficients for a sandstone.'' *J. Appl. Mech.*, 26, 296–297.

Gassmann, F. (1951). ''Über die elastizität poröser medien.'' *Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich,* 96, 1–23 (in German).

Geertsma, J. (1957). ''The effect of fluid pressure decline on volumetric changes of porous rocks.'' *Trans. AIME*, 210, 331–340.

Khaled, M. Y., Beskos, D. E., and Aifantis, E. C. (1984). ''On the theory of consolidation with double porosity. 3. A finite-element formulation.'' *Int. J. Numer. Analyt. Meth. Geomech.,* 8, 101–123.

Levin, V. M. (1967). ''Thermal expansion coefficients of heterogeneous materials.'' *Mech. Solids,* 2, 58–61.

Lewallen, K. T., and Wang, H. F. (1998). ''Consolidation of a double-porosity medium.'' *Int. J. Solids Struct.,* 35, 4845–4867.

Love, A. E. H. (1927). *A treatise on the mathematical theory of elasticity*, Dover, New York, pp. 173–174.

Nilson, R. H., and Lie, K. H. (1990). ''Double-porosity modelling of oscillatory gas motion and contaminant transport in a fractured porous medium.'' *Int. J. Numer. Analyt. Meth. Geomech.,* 14, 565–585.

Norris, A. N. (1992). ''On the correspondence between poroelasticity and thermoelasticity.'' *J. Appl. Phys.,* 71, 1138–1141.

Pride, S. R., and Berryman, J. G., (1998). ''Connecting theory to experiment in poroelasticity.'' *J. Mech. Phys. Solids,* 46, 719–747.

Rice, J. R. (1975). ''On the stability of dilatant hardening for saturated rock masses.'' *J. Geophys. Res.,* 80, 1531–1536.

Rice, J. R., and Cleary, M. P. (1976). ''Some basic stress diffusion solutions for fluid-saturated elastic porous media with compressible constituents.'' *Rev. Geophys. Space Phys.,* 14, 227–241.

Skempton, A. W. (1954). ''The pore-pressure coefficients A and B.'' *Geotechnique,* 4, 143–147.

Terzaghi, K. (1925). *Erdbaumechanik auf bodenphysikalischer grundlage,* Deuticke, Leipzig.

Tuncay, K., and Corapciaglu, M. Y. (1995). ''Effective stress principle for saturated fractured porous media.'' *Water Resour. Res.,* 31, 3103–3106.

Wang, H. F. (1993). ''Quasi-static poroelastic parameters in rock and their geophysical applications.'' *PAGEOPH,* 141, 269–286.

Wang, H. F. (2000). *Theory of linear poroelasticity with applications to geomechanics and hydrogeology,* Princeton Univ. Press, Princeton, N.J.

Warren, J. E., and Root, P. J. (1963). ''The behavior of naturally fractured reservoirs.'' *Soc. Pet. Eng. J.,* 3, 245–255.

Wilson, R. K., and Aifantis, E. C. (1982). ''On the theory of consolidation with double porosity.'' *Int. J. Eng. Sci.,* 20, 1009–1035.