# EXTENSION OF WEYL-HEISENBERG WAVE PACKET BESSEL SEQUENCES TO DUAL FRAMES IN $L^{2}(\mathbb{R})$ 

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#### Abstract

Christensen et al. proved in [Extensions of Bessel sequences to dual pairs of frames, Appl. Comput. Harmon. Anal., 34 (2013), 224-233] that in any separable Hilbert space, any pairs of Bessel sequences (even if the given Bessel system is Gabor system in $L^{2}(\mathbb{R})$ ) can be extended to a pair of dual frames. In this paper, we extend results by Christensen et al. to a pair of Bessel sequences in $L^{2}(\mathbb{R})$ having wave packet structure to a pair of dual frames such that extension have wave packet structure as well. We present sufficient conditions for the extension of a pair of Bessel sequences to wave packet type dual frames for $L^{2}(\mathbb{R})$. Several examples and counter-examples are given to illustrate our results.


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