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EXTENSIONS AND ADJUNCTS TO THE
BRL-COMGEOM PROGRAM
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| 20. Abstract (Continue an revorea aife tr neceqeary and feontity by block number) For several years the Ballistic Research Laboratori called, "combinatorial geometry (COM-GEOM) target d model the geometric properties of complex physical and humans. The model of the item is used as input which perform a simulation of the effects that "ene neutron and gamma radiation have upon the modeled it This report presents several developments that redu model items via the COM-GEOM description technique | es have employed a technique escription technique" to items such as tanks, aircraft to different computer progr rgy sources" such as projecti tem (target). <br> ce the effort required to and requires a familiarity |

of the concepts of the COM-GEOM technique. The developments reported are: (a) the development of two new solids: the general ellipsoid and the "arbitrary quadric surface". (b) the development of a geometry preprocessor program that aids in the generation of COM-GEOM description input data.
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Several independent tasks were performed under this contract, all of them related to the BRL-COMGEOM technique. This computer technique for geometry representation was originally embodied in the BRL-MAGIC code (written by MAGI) and is now, in a form developed by BRL, the basis of the BRL-GIFT code.

The tasks completed under this contract and described in the report are:
A. The addition to the list of available body types of two new bodies, the general ellipsoid and the "arbitrary quadric surface";
B. The development of a geometry preprocessor program capable of producing GIFT input from a more limited, user-oriented, set of descriptions;
C. Development of a BRL-COMGEOM model of the M109Al tank.

The first two tasks will be described in some detail。 The modeling task is embodied in an input deck for the GIFT code and will be described briefly in the last section.

## II. ADDITIONAL BODY TYPES

The body list of the BRI-COMGEOM (combinatorial geometry) package was extended to include the general ellipsoid and a quadratic surface generated from input points. The coding that was produced for each of these bodies covers input processing, ray tracing and generation of unit normals.
A. The General Ellipsoid (ELL)

This body type is a generalization of the ellipsoid of revolution which was included in the body list of the original BRL-COMGEOM package. In the following paragraphs the equations for tracking and for generating the local surface normal will be developed and the code implementation will be described. Details of the input requirements will also be specified.

1. A Vector Representation of the General Ellipsoid.

Let $\bar{x}$ be a point on the surface of a general ellipsoid, let the vertex be called $v$, and let the three semi-axes be given by

$$
\begin{align*}
& \bar{a}=a \bar{a}_{0} \\
& \bar{b}=b \bar{b}_{0}  \tag{1}\\
& \bar{c}=c \bar{c}_{0}
\end{align*}
$$

as in Figure 3.
Now we define three parameters for represent-
ing $\overline{\mathrm{x}}$ :

$$
\begin{align*}
& \alpha=(\bar{x}-\bar{v}) \cdot \bar{a}_{0} / a \\
& \beta=(\bar{x}-\bar{v}) \cdot \bar{b}_{0} / b  \tag{2}\\
& \gamma=(\bar{x}-\bar{v}) \cdot \bar{c}_{0} / c
\end{align*}
$$



FIGURE l: Ellipsoid of revolution, specified by blank in column 7 and by data $\bar{F}_{1}, \bar{F}_{2}, \ell$.


FIGURE 2: $\begin{aligned} & \text { Ellipsoid of revolution, specified by } 1 \text { in } \\ & \text { column } 7 \text { and by data } \overline{\mathrm{V}}, \mathrm{a},|\mathrm{b}| .\end{aligned}$


FIGURE 3: General ellipsoid, specified by $G$ in column 7 and by data $\bar{V}, \bar{a}, \bar{b}, c$.

These definitions head to the relationships:

$$
\begin{align*}
& \alpha^{2}+\beta^{2}+\gamma^{2}=1 \\
& \alpha d \alpha+\beta d \beta+\gamma d \gamma+0 \tag{3}
\end{align*}
$$

and $\bar{a}$, Thus $\bar{x}$ can be expressed in terms of $\bar{v}, \alpha, \beta, \gamma$ and $\overline{\mathrm{a}}, \overline{\mathrm{b}}$ and $\overline{\mathrm{c}}$ :

$$
\begin{equation*}
\overline{\mathrm{x}}=\overline{\mathrm{v}}+\alpha \overline{\mathrm{a}}+\beta \overline{\mathrm{b}}+\gamma \overline{\mathrm{C}} \tag{4}
\end{equation*}
$$

2. Ray Tracing to the General Ellipsoid

Let a ray emanate from a position $\overline{\mathrm{XB}}$ in the direction $\mathrm{WB}_{0}$ We wish to determine the point(s) of intersection $\bar{x}$ of the ray with the general ellipsoid and the distance(s) $S$ from $X B$ to $X$. We define

$$
\begin{equation*}
\overline{\mathrm{D}}=\overline{\mathrm{XB}}-\overline{\mathrm{V}} \tag{5}
\end{equation*}
$$

and note that

$$
\begin{equation*}
\overline{\mathrm{X}}-\overline{\mathrm{V}}=\overline{\mathrm{D}}+\overline{\mathrm{WB}} \cdot \mathrm{~S} \tag{6}
\end{equation*}
$$

Using equations (2) for $\alpha, \beta$ and $\gamma$ we find:
$\left\{(\bar{D}+\overline{W B} \cdot S) \cdot \bar{a}_{0} / a\right\}^{2}+\left\{(\bar{D}+\overline{W B} \cdot S): \bar{b}_{0} / b\right\}^{2}$

$$
\begin{equation*}
+\left\{(\bar{D}+\overline{W B} \cdot S) \cdot \bar{C}_{0} / C\right\}^{2} \equiv 1 \tag{7}
\end{equation*}
$$

Equation (7) is a quadratic in the distance S. The following definitions are helpful in writing the coefficients of the quadratic:

$$
\begin{array}{lll}
c_{1}=\bar{D} \cdot \bar{a}_{0} / a & ; & c_{2}=\overline{W B} \cdot \bar{a}_{0} / a \\
c_{3}=\overline{\mathrm{D}} \cdot \overline{\mathrm{~b}}_{0} / \mathrm{b} & ; & \mathrm{c}_{4}=\overline{\mathrm{WB}} \cdot \overline{\mathrm{~b}}_{0} / \mathrm{b} \\
\mathrm{c}_{5}=\overline{\mathrm{D}} \cdot \overline{\mathrm{c}}_{0} / \mathrm{c}: & ; & \mathrm{c}_{6}=\overline{\mathrm{WB}} \cdot \overline{\mathrm{c}}_{0} / \mathrm{c} \tag{8}
\end{array}
$$

In terms of (8) the coefficients of the quadratic term, the coefficient of the linear term and the constant are, respectively:

$$
\begin{align*}
& \mathrm{AA}=\mathrm{C}_{2}^{2}+\mathrm{C}_{4}^{2}+\mathrm{C}_{6}^{2} \\
& \mathrm{BB}=2\left(\mathrm{C}_{1} \mathrm{C}_{2}+\mathrm{C}_{3} \mathrm{C}_{4}+\mathrm{C}_{5} \mathrm{C}_{6}\right) \\
& \mathrm{CC}=\mathrm{C}_{1}^{2}+\mathrm{C}_{3}^{2}+\mathrm{C}_{5}^{2}-1.0 \tag{9}
\end{align*}
$$

If the discriminant is less than zero or vanishes, the ray misses the ellipsoid entirely or just grazes it. Two meaningful solutions are obtained if the discriminant is positive. The smaller value is identified with RIN, the distance to enter, and the larger with ROUT, the distance to emerge from the ellipsoid.

The numerical technique implemented in the ray tracing is one which avoids small differences of large numbers in the denominator.

The general solution

$$
\begin{equation*}
S=\left(-B B \pm \sqrt{B B^{2}-4 A A \cdot C C}\right) / 2 A A \tag{10}
\end{equation*}
$$

can also be written

$$
\begin{equation*}
S=2 C C /\left(-B B \mp \sqrt{B B^{2}-4 A A \cdot C C}\right) \tag{11}
\end{equation*}
$$

The choice between (10) and (11) is made by examining the sign of BB . Let

$$
\begin{array}{ll}
\mathrm{U}=\mathrm{BB}+\sqrt{\mathrm{BB}^{2}-4 \mathrm{AA} \cdot \mathrm{CC}}, & \mathrm{BB}<0 \\
\mathrm{U}=-\mathrm{BB}-\sqrt{\mathrm{BB}^{2}-4 \mathrm{AA} \cdot \mathrm{CC}}, & \mathrm{BB}>0
\end{array}
$$

Then

$$
\begin{align*}
& \mathrm{S}_{1}=\mathrm{U} /(2 \mathrm{AA})  \tag{12}\\
& \mathrm{S}_{2}=2 \mathrm{CC} / \mathrm{U}
\end{align*}
$$

is a numerically sound solution pair.
3. The Unit Normal for the General Ellipsoid

The unit normal to the ellipsoid surface at the point $\bar{X}$ can be obtained as the normalized cross product of two differential vectors, both tangent to the surface at $\overline{\mathrm{X}}$. The general tangent vector can be obtained from (4) and from (3):

$$
\begin{align*}
d \bar{X} & =\bar{a} d \alpha+\bar{b} d \beta+\bar{c} d \gamma \\
& =\bar{a} d \alpha+\bar{b} d \beta-\bar{c}(\alpha d \alpha / \gamma+\beta d \beta / \gamma) \tag{13}
\end{align*}
$$

If we choose $d \bar{x}_{1}$ such that $d \beta=0$ and $d \bar{x}_{2}$ such that $d \alpha=0$, then

$$
\begin{align*}
& d \bar{x}_{1}=d \alpha(\bar{a}-\alpha \bar{c} / \gamma) \\
& d \bar{x}_{2}=\alpha \beta(\bar{b}-\beta \bar{c} / \gamma) \tag{14}
\end{align*}
$$

and the unit normal, without regard to sign, is given by

$$
\begin{align*}
& \overline{\mathrm{n}}=d \overline{\mathrm{X}}_{1} \times d \overline{\mathrm{X}}_{2} / d \overline{\mathrm{X}}_{1} \times d \overline{\mathrm{X}}_{2} \\
&=\left(\alpha \cdot \bar{a}_{0} / a+\beta \cdot \bar{b}_{0} / b+\gamma \cdot \bar{c}_{0} / c\right) \\
& \sqrt{\alpha^{2} / a^{2}+\beta^{2} / b^{2}+\gamma^{2} / c^{2}} \tag{1.5}
\end{align*}
$$

4. Implementation of the General Ellipsoid

The coding provided for the implementation of the general ellipsoid consisted of an input routine ELLP a tracking routine ELL and a section of coding in the routine for computing normals.

Three properly distinguished forms of input are acceptable to the subroutine ELLP. Two of these describe the ellipsoid of revolution: l) the two-focus, length-of-string form distinguished on the leading data card by a blank in column 7 and 2) the vertex, semimajor axis (vector), semi-minor axis (magnitude) form, distinguished on the lead data card by any entry other than a blank or a "G" in column 7. The third form is the general form and is given by the vertex and the three (vector) semi-axes. The general form is signaled on the lead data card by a "G" in column 7. The various input forms are demonstrated in Figures l-3.

The function of ELLP is to recognize the data form and, if the form is not general, to convert to the general form. The data stored by ELLP in the ASTER array are the vertex and the three quantities $a_{0} / a$, $\bar{b}_{0} / b, \bar{c}_{0} / c$. ELLP also stores a pointer for locating these data.

Ray tracing as performed by the subroutine ELL consists in straightforward implementation of equations (5) through (12). A miss is recorded by returning the pair RIN=+PINF and ROUT=-PINF, where PINF is a very large number. For a hit the solution pair is assigned so that ROUT is greater than RIN.

## B. The Arbitrary Quadric Surface

This body, or surface, is a completely new type of geometrical unit and requires special input and tracking techniques which are discussed below. It is intended to permit modeling of elements.which are not approximated by any of the other body types, but which can be fitted in sections with paraboloids. The input for each section is a set of measured points to which a least squares fit is to be applied.

The choice of a paraboloid, as opposed to a general quadratic, is related to the region description technique: we need only a small section of the surface produced but we must insure that the rest of the surface does not re-enter the region being described.

## 1. General Considerations

As indicated above, the technique adopted here is not a completely general quadratic fit but one which anticipates the function to be performed by the surface description and also the uncertainty attendent on any fitting technique. Thiss is illustrated in Figure 4, in which the AQS forms one boundary of a region bounded elsewhere by an ellipsoid. The quadratic of Figure 4 a doesn't re-enter the ellipsoid and the region description is therefore meaningful. However, a general quadratic could result in a flat ellipsoid as in Figure 4b. Then the region description $R=F+G$ no longer describes the region of interest.

The paraboloid, which doesn't close on itself, represents our best means of avoiding an ambiguous region description. We can further insure against this catastrophe by creating the paraboloid $z=h(u, v)$ in a co-ordinate system $u, v, h$ such that the $u-v$ plane is the best plane through the set of defining points - in the least square sense。

This approach does, however, lead to a restriction on the use of the AQS. A section of surface to be fitted should meet the requirement that the maximum amount by which the section deviates from a plane is small compared with the radius of curvature of the section. As an example of this restriction, a paraboloidal fit to a section of a sphere


$$
\text { REGION } R=F+G
$$

FIGURE 4a: AQS (BODY G) does not reenter BODY F.


REGION R $=\mathrm{F}+\mathrm{G}$
$\begin{aligned} \text { FIGURE 4b: } & \text { AQS (BODY G) reenters BODY } F \text { and truncates } \\ & \text { REGION R. }\end{aligned}$
defined by a 45 degree polar angle would deviate from the sphere by approximately 0.05 of the radius at the apex if the edge is matched exactily. The ratio of the maximum height of this section to the radius (of curvature) is 0.3 , so the error in fitting is about $16 \%$ at the apex. In practice, the error would be divided between the apex and the edges. If the ratio mentioned above is limited to 0.13 (corresponding to a cone of $30^{\circ}$ half angle) the error in fitting is about a half of one percent.

## 2. Theory and Analysis

The general approach is to assume that the piece of surface represented by the data $\left\{\left(X_{p}, Y_{p}, Z_{p}\right), P=1,2,00 N P\right\}$ is not grossly different from a plane, and to seek first the "best" plane, i。e. that plane from which the mean square perpendicular distance of the points is a minimum. This is fruitful in several ways: if, for each point, the perpendicular distance from the plane is less than the input tolerance, $T$, the plane can be accepted as a proper representation. If, however, the plane is not sufficiently representative, it can be used as one of the co-ordinate planes of a co-ordinate system in which the paraboloidal representation of the surface is single-valued:

$$
\begin{align*}
h(u, v)= & \alpha_{1} u^{2}+\alpha_{2} v^{2}+\alpha_{3} u v+\alpha_{4} u \\
& +\alpha_{5} v+\alpha_{6} \tag{16}
\end{align*}
$$

where the $u-v$ plane is the "best" plane and the third co-ordinate $h$ is the actual height of the surface above the best plane. The difference $\left\{h\left(u_{i}, v_{i}\right)-h_{i}\right\}$ for the individual point will not be very different from the perpendicular distance of the point from the surface and can be used as a measure of the success of the fit.

The first step in processing is to perform a transformation into the center of mass co-ordinate system. This is simply a computational convenience and involves subtracting from each point: vector, $\bar{r} p$, the mean value of $\overline{r p}$ over all input points. In this system the best plane is sought.

The method is to minimize the mean square perpendicular distance of points to the plane

$$
\begin{equation*}
f\left(x^{\prime}, y^{\prime}, z^{\prime}\right)=\beta_{1} x^{\prime}+\beta_{2} y^{\prime}+\beta_{3} z^{\prime}+\beta_{4}=0 \tag{17}
\end{equation*}
$$

subject to the condition that $\beta_{1}^{2}+\beta_{2}^{2}+\beta_{3}^{2}=1$ or that these $\beta^{\prime}$ 's are direction cosines.

The perpendicular distance of a point $\mathrm{P}_{\mathrm{k}}$ from the plane is

$$
\begin{gather*}
\varepsilon_{k}=\beta_{3}\left(z_{k}-z\left(w_{k}\right)\right)=\beta_{1} x_{k}+\beta_{2} y_{k} \\
 \tag{19}\\
+\beta_{3} z_{k}+\beta_{4}
\end{gather*}
$$

and

$$
\begin{equation*}
\sum_{k} \varepsilon_{k}^{2}=\sum_{k}\left(\beta_{1} x_{k}+\beta_{2} y_{k}+\beta_{3} z_{k}+\beta_{4}\right)^{2} \tag{20}
\end{equation*}
$$

Since there is no constraint involving $\beta_{4}$ we can solve directly for it:
$\frac{\partial \sum_{k}^{2}}{\partial \beta_{4}}=0=2\left(\beta_{1} \sum_{k}^{2} x_{k}+\beta_{2} \sum_{k} Y_{k}+\beta_{3} \sum_{k} z_{k}+\beta_{4} \cdot N P\right)$
In the center of mass system the sums on the right hand side of equation (21) all vanish - whence $\beta_{4} \equiv 0$ and the plane passes through the center of mass (as one might have guessed to start with)..

Let us call the $i^{\text {th }}$ co-ordinate of the $k^{\text {th }}$ point $w_{i k}$ and write that

$$
\begin{align*}
& \varepsilon_{k}=\sum_{i} \beta_{i} W_{i k}  \tag{22}\\
& \frac{1}{N P} \sum_{k} \varepsilon_{k}^{2}=\overline{\varepsilon^{2}}=\frac{1}{N P} \sum_{k} \sum_{i} \sum_{j} \beta_{i} \beta_{j} W_{i k} W_{j k} \\
&=\sum_{i} \sum_{j} \beta_{i} A_{i j} \beta_{j} \tag{23}
\end{align*}
$$

where the $A_{i j}=\frac{1}{N P} \sum_{k} w_{i k} w_{j k}=\overline{w_{i} w_{j}}$
are the elements of a symmetric matrix $A$ 。 Then $\bar{\beta}$ can be thought of as a vector in 3-space and

$$
\begin{equation*}
\overline{\varepsilon^{2}}=(\bar{\beta} A) \bar{B} . \tag{25}
\end{equation*}
$$

The minimization of $\overline{\varepsilon^{2}}$ subject to the constraint $\beta^{2}=1$ requires that

$$
\begin{equation*}
\frac{\partial \overline{\varepsilon^{2}}}{\partial \beta_{\ell}}-\lambda \frac{\partial \beta^{2}}{\partial \beta_{\ell}}=0 \quad \text { for } \ell=1,2,3 . \tag{26}
\end{equation*}
$$

or

$$
\begin{equation*}
\underset{j}{2 \Sigma} A_{\ell j} \beta_{j}-2 \lambda \beta_{\ell}=0 \tag{27}
\end{equation*}
$$

or

$$
\begin{equation*}
(A-\lambda I) \bar{\beta}=0 \tag{28}
\end{equation*}
$$

where $I$ is the unit matrix.
$\lambda$ is then the solution of a cubic equation. Its physical significance is seen by multiplying the condition equation by $\beta_{\ell}$ and summing over $\ell$ :

$$
\begin{align*}
& \sum_{l} \sum_{j} \beta_{l} A_{l j} \beta_{j}-\lambda \sum \beta_{l}{ }^{2}=0=(\bar{\beta} A) \bar{\beta}-\lambda \\
\text { or } \quad \lambda & \lambda \bar{\varepsilon}{ }^{2} . \tag{29}
\end{align*}
$$

Thus the three values of $\frac{\lambda}{2}$ correspond to the local extrema of the quantity $\overline{\varepsilon^{2}}$ under rotation of the plane. The smallest value of $\lambda$ gives the desired plane.

After the plane has been determined the action of the processing subroutine AQSIN is to compare the perpendicular distance of each point with the tolerance value to determine whether the plane should be accepted. If it is accepted, the quadric fit section described in the next few paragraphs is by-passed and the coefficients are corrected to refer to the laboratory coordinates.

If the plane is not accepted the paraboloidal fit is pursued. A transformation is made (a rotation) to a coordinate system $u, v, h$ such that the "best" plane is the $u-v$ plane and $h(u, v)$ will describe the paraboloid:

$$
\begin{align*}
h(u, v) & =\alpha_{1} u^{2}+\alpha_{2} v^{2}+\alpha_{3} u v+\alpha_{4} u \\
& +\alpha_{5} v+\alpha 6=\sum_{j} \alpha_{j} w_{j} ; w_{6}=1 \tag{30}
\end{align*}
$$

The quantity to be minimized is the mean square of the point height above (or below) the paraboloid. Under the assumed conditions this differs little from a perpendicular distance and is much easier to handle:

$$
\begin{align*}
\varepsilon_{k} & =h_{k}-h\left(u_{k}, v_{k}\right)=h_{k}-\sum_{j}^{\sum \alpha_{j}} w_{j k}  \tag{31}\\
\overline{\varepsilon^{2}} & =\frac{1}{N} \sum_{k} \alpha_{j}^{\sum \alpha_{j} w_{j k} \sum \alpha_{i} w_{i k}} \\
& \left.-2 h_{k_{i}}^{\sum \alpha_{i} w_{i k}}+h_{k}^{2}\right] \tag{32}
\end{align*}
$$

Agajn we identify the symmetric matrix A whose ij element is

$$
\begin{equation*}
A_{i j}=\frac{1}{N} \sum_{k} w_{i k} w_{j k} ; \tag{33}
\end{equation*}
$$

and the vectors $\bar{\alpha}$ and $\bar{w}$ such that

$$
\begin{equation*}
\overline{\varepsilon^{2}}=(\bar{\alpha} A) \bar{\alpha}-\overline{2 h(\bar{\alpha} \bar{w})}+\overline{h^{2}} ; \tag{34}
\end{equation*}
$$

and

$$
\begin{align*}
& \frac{\partial \overline{\varepsilon^{2}}}{\partial \alpha_{\ell}}=2 \Sigma A_{\ell j} \alpha_{j}-2 \overline{h w}_{\ell}=00 \\
& \text { In vector notation: } \\
& \bar{\alpha}=A^{-1}(\overline{h w}) \tag{36}
\end{align*}
$$

Thus the code constructs the matrix A:
$A=\left(\begin{array}{l}\overline{u^{4}} \\ \end{array}\right.$
$\begin{array}{lllll}\overline{u^{2} v^{2}} & \overline{u^{3} v} & \bar{u} & \bar{u} & \overline{u^{2} v} \\ \overline{u^{2}} \\ \overline{v^{4}} & \overline{u v^{3}} & \overline{u v^{2}} & \overline{v^{3}} & \overline{v^{2}} \\ & \overline{u^{2} v^{2}} & \overline{u^{2} v} & \overline{u v^{2}} & \overline{u v}\end{array}$
)
and the vector $\overline{\mathrm{hw}}$ :
$\overline{h w}=\left(\begin{array}{l}\overline{h u^{2}} \\ \overline{h v^{2}} \\ \overline{h u v} \\ \overline{h u} \\ \overline{h v} \\ \bar{h}\end{array}\right)$

The matrix equation is solved by the subroutine SOLVE to produce the 6 -element vector $\alpha$.

The vector $\bar{\alpha}$ consists of the coefficients of the paraboloid in the rotated center of mass system. The inverse transformations to the laboratory system must be performed to produce the 10 element vector of coefficients in that system.

The 10 coefficients of the equation

$$
\begin{align*}
B_{0}+B_{1} x & +B_{2} y+B_{3} z+B_{4} x y+B_{5} x z+B_{6} y z \\
& +B_{7} x^{2}+B_{8} y^{2}+B_{9} z^{2}=0 \tag{39}
\end{align*}
$$

are the essential data stored for the representation of the AQS.
3. Ray Tracing to the AQS

Ray tracing to the AQS consists in substituting into (39) the following expressions for the coordinates of the point of intersection:

$$
\begin{equation*}
\overline{\mathrm{X}}=\overline{\mathrm{XB}}+\overline{\mathrm{WB}} \cdot \mathrm{~S}, \tag{40}
\end{equation*}
$$

where (40) is analogous to (6) for the general ellipsoid. This substitution results in a quadratic in $S$ which is solved in a manner entirely analogous to that described for the general ellipsoid (equations 10 through 12).

The roots of this quadratic, $S_{1}$ and $S_{2}$, must be interpreted with care because the AQS is not a closed convex body. Figure 5 demonstrates the various special cases to be considered and the interpretation of $S_{1}$ and $S_{2}\left(S_{2}>S_{1}\right)$ as RIN and ROUT.
a) An AQS which is concave toward the inside has a single pair specified for the intersections: (RIN,ROUT) $=\left(S_{1}, S_{2}\right)$
b) An AQS which is convex toward the inside has a pair associated with each intersection:
(RIN, ROUT) $=\left(-\right.$ PINF, $\left.\mathrm{S}_{1}\right)$ and
(RIN, ROUT) $=\left(S_{2}\right.$, PINF $)$
c) A plane AQS is covered by (a):

Entering: (RIN,ROUT) $=\left(S_{1}\right.$, PINF $)$
Leaving: $\quad($ RIN, ROUT $)=\left(-\right.$ PINF, $\left.S_{2}\right)$
4. The Normal to the AQS

The normal calculation is straight forward and consists in taking the gradient of the function given by equation (39):

$$
\begin{align*}
\nabla f(x, y, z) & =\bar{i}\left(B_{1}+B_{4} y+B_{5} z+B_{7} x\right) \\
& +\bar{j}\left(B_{2}+B_{4} x+B_{6} z+B_{8} y\right) \\
& +\bar{k}\left(B_{3}+B_{5} x+B_{6} y+B_{9} z\right) \tag{41}
\end{align*}
$$

and normalizing it. The signs of the coefficients in (39) have been chosen so that the normal points toward the "inside", as indicated by the point given on input.

a. A concave AQS. $(R I N, \operatorname{ROUT})=\left(S_{1}, S_{2}\right)$

b. A convex AQS.

$$
\begin{aligned}
& (\mathrm{RIN}, \text { ROUT })_{1}=\left(-\mathrm{PINF}, \mathrm{~S}_{1}\right) \\
& (\mathrm{RIN}, \text { ROUT })_{2}=\left(\mathrm{S}_{2}, \mathrm{PINF}\right)
\end{aligned}
$$

FIGURE 5: Tracking to the AQS
5. Implementation

Three subroutines were developed to handle the AQS in the GIFT code and two old routines were altered:
a. AQSIN(LBOT, DATA,FX) is a new subroutine called by GENI to read and process AQS data.
b. SOLVE ( $A, B, K S, N$ ) is an IBM subroutine called by AQSIN to invert a matrix and return coefficients of the quadratic in a special coordinate system.
c. AQS was added to implement the solution of the ray tracing equation.
a. GENI was altered to accept the AQS input and to call AQSIN.
e. CALC was altered to compute the normal for the AQS.

The input for the AQS consists of :
Card 1, read by GENI in format (3A1, A3, A4, 6F10.0)
Columns
$\left.\begin{array}{ll}1-3 & \text { (body number) } \\ 4-6 & \text { AQS } \\ 11-20 & N P, ~ n u m b e r ~ o f ~ p o i n t s ~(f i x e d ~ i n ~ A Q S I N) ~ \\ 21-30 & \text { TOL, tolerance } \\ 31-40 & \mathrm{X} \\ 41-50 & \mathrm{Y} \\ 51-60 & \mathrm{Z}\end{array}\right\} \quad$ coordinates of an "inside" point

Subsequent cards are read by AQSIN in format (10X, 6Flo.5) and contain coordinates of pairs of points:

Columns

| $11-20$ | $X_{p}$ |
| :--- | :--- |
| $21-30$ | $Y_{p}$ |
| $31-40$ | $Z_{p}$ |
| $41-50$ | $X_{p+1}$ |
| $51-60$ | $Y_{p+1}$ |
| $61-70$ | $Z_{p+1}$ |

The points are read into the ASTER array in which 6* (NP+8) temporary locations are reserved for processing.

The data stored in the ASTER array for the AQS (ITYPE=12) of body number NBO is the following:

| LOCATION | CONTENTS |
| :---: | :---: |
| LBODY+3* (NRPP+NBO-1) | $2^{15}$ *ITYPE+LDATA |
| LDATA | Location of first coefficient $=$ LOCDA |
| LDATA+1 | $2{ }^{15}$ * (LOCDA +10$)+($ LOCDA +13$)$ |
| LDATA+2 | $2^{15}$ * (LOCDA+16)+(LOCDA+19) |
| LOCDA | $\mathrm{B}_{9}$ coefficient of $\mathrm{x}^{2}$ |
| - |  |
| LOCDA +9 | $\mathrm{B}_{0}$, constant term |

## III. BRITL: A GEOMETRY PREPROCESSOR PROGRAM

FOR INPUT TO THE GIFT SYSTEM

## A. Introduction

BRITL (Body-Region Input Translator) was developed as an aid in preparing geometry data for the GIFT program. It's several functions are by no means exhaustive of the possibilities but represent an important beginning in the automation of input production.

The tasks of the preprocessor are listed below and are discussed in some detail in the subsequent sections:

1. Given a body (other than an ARS) from among the types acceptable to the GIFT program, BRITL produces an inner body of the same type by moving the surfaces (nearly) normal to themselves by distances given by input. Both bodies are recorded in the body table for use together as a "thick" body.
2. Given a series of vector positions and associated radii, BRITL forms a PIPE by joining successive positions with either an RCC or a TRC, as indicated by the comparison of radii。 A PIPE may be thickened.
3. BRITL produces region descriptions (as called for in the region table) corresponding to the "shell" of the "thick" body, the interior or the exterior of the "thick" body.
4. BRITL allows backward references to already defined regions and, by applying DeMorgan's rules, produces a region description in the required conjunctive form.
5. Given a series of regions, already described, BRITL produces a new series of regions by means of a transformation applied to each of the bodies in the description. In general, the transformation must consist of rotations, translations and reflections without distortion or scale change. For special bodies (those described entirely by vectors) distortion is allowed.
6. BRITL allows the stripping of "model" regions and unused bodies from the body and region tables. It produces properly sequenced and numbered bodies and regions and region identifications in the format required by the GIFT program. Body and region numbers may start at initial values provided on input.

Because new bodies are produced, and because region descriptions may refer to previous regions as well as to resequenced bodies, numerical sequencing of bodies and regions is not feasible.

Instead bodies and regions are "names" with distinct 3-character alphanumeric tags. The reference, within a region description, to a body or region is made with a signed four-character name, the last three characters of which is the tag mentioned above。 The first character is used in connection with the "thick" body to designate shell, interior or exterior.

Thus, the input to the geometry preprocessor is 1) a body table, with bodies designated by names, and with some bodies designated as "thick"; 2) a region table with references to bodies and previously described regions, and special regions designated as transformations of previously described regions and accompanied by transformation data and 3) a region identification table, which, in addition to the usual information, may carry a "model" tag. The input formats are as close as possible to those of the GIFT program and the output formats are precisely those of the GIFT program.

The basic formats are given here:

Title Card
Parameter Card
Column $\begin{array}{r}1-10 \\ 11-20 \\ 21-30\end{array}$
Enclosing RPP card (2A3, A4, 6El0.3, A4, 2A3)
Columns 1-10
71-80
11-70
Lead Body Card
Columns 1-3
Columns 4-6
7 Used to distinguish ARB types, otherwise blank
Blanks
T for a thick body, otherwise blank Standard body data Comments

Succeeding Body Cards (10X, 6F10.5, A4, 2A3)
Same as lead body card except that first ten columns are not read.
Thickness Card(s) (10X, 6Fl0.5, A4, 2A3)
Same format as body cards - six thicknesses per card. Read only if $T$ in column 10 of lead body card.
Region Card
(2X, A3, lX, 9 (A2, 2A1, A3))
Columns 3-5
Region name (alphanumeric)

For the $I^{\text {th }}$ body or region reference, first column is $5+7 .(I-1)+1=N I+1$

Columns ( $\mathrm{NI}+1$ )-(NI+2)
Column ( $\mathrm{NI}+3$ )
Column (NI+4)

Columns (NI+5)-(NI+7)

OR or blanks
Sign (+ or -)
Special symbol for reference to a "thick" body
3 character body or region name

For the special case of a region or series of regions, to be obtained by transformation from a previous region. series, the region card has the above format for the following information:

| Columns $3-5$ | Region name (one name fo <br> whole series) |
| :--- | :--- | :--- |
| Columns $7-8$ | DP |
| Columns 21-13 | Name of first region in |

For this case the region card is followed by a card bearing the number of regions (I5) and by a set of four cards with transformation data for four non-coplanar points, (l0X, 6Flo.5):
Columns $\left.\begin{array}{ll}11-20 \\ 21-30 \\ 31-40 & X \\ 41-50 & Z \\ 51-60 & X^{\prime} \\ 61-70 & Y^{\prime} \\ Z^{\prime}\end{array}\right\} \quad$ point in space of model regions $\quad$ point in space of new regions

End Card for Region Table: -1 in columns 4-5
Region Identification Cards
(A3, A2, 2X, A3, A3, A4, A3, I10, 10X, 10A4)

Column 1-3

Columns 4-5

Either blank
or MOD for a model region
or DUP for a region set obtained by transformation

Either blank
or ST if this is a region to be stripped out

Columns 8-10
Columns 11-20

Columns 21-30

Columns 41-80
Initial body number and Initial region number card
Columns 1-10
Columns 11-20

Region name
$\begin{aligned} & \text { Ident code - } \text { right justified } \\ & \text { or, for a region } \\ & \text { set obtained by } \\ & \text { transformation, } \\ & \text { name of first } \\ & \text { region that is to } \\ & \text { be transformed } \\ & \text { right justified } \\ & \text { Space code - right justified } \\ & \text { or for a region } \\ & \text { set obtained by } \\ & \text { transformation, } \\ & \text { number of regions } \\ & \text { to be transformed } \\ & \text { right justified }\end{aligned}$
Region comments

Integer indicating the number of the first output body
Integer indicating the number of the first output region

## B. The Thick Body

Any body type acceptable to the GIFT code, except the ARS, can be "thickened." In addition, the PIPE body can be "thickened." The number of thicknesses that can be specified for a given body type is peculiar to that body as described below. A "T" in column 7 of the lead data card for the body indicates to the code that the body is to be thickened, i.e., that another body is to be produced. Thickness card(s) follow the body data cards (up to 6. thicknesses per card in the same format as the body cards) except for the PIPE body. The order in which the thicknesses are given associates them with the proper dimensions.

1. BOX (also RPP)

Six distinct thicknesses can be specified, one for each side, in the order shown in Figure 6a. Thus $t_{1}, t_{3}$ and $t_{5}$ have the effect of moving the vertex, $\overline{\mathrm{V}}$, along each of the height vectors in order (and of shortening the height vectors) and $t_{2}$, $t_{4}$ and $t_{6}$ further shorten the height vectors $\bar{H}_{1}, \bar{H}_{2}$ and $\bar{H}_{3}$.

The body type RPP is converted to a BOX before thickening.

## 2. RAW

The right angle wedge has almost the same thickness format as the BOX except that the inputs $t_{2}$ and $t_{4}$ are equal to each other and to the distance that the inclined face is to be moved normal to itselfo The shortening of $\bar{H}_{1}$ and $\bar{H}_{2}$ are calculated internally. See Figure 6b.
3. SPH

The sphere can have one thickness along the radius.
4. RCC

The RCC can have three thicknesses: $t_{1}$ normal to the base, $t_{2}$ normal to the upper face, $t_{3}$ normal to the curved surface。
5. REC

The REC can have three thicknesses, just similar to those of the RCC. The 3rd thickness is interpreted as a shortening of the semi-axes.
6. TRC

The TRC has three possible thicknesses, as for the RCC and REC.
7. ELL

The ELL has one thickness, $t_{1}$, interpreted as a shortening of the semi-axes.
8. ARB

The ARB can have one thickness for each face, given in the same order as the face descriptions.

There is one pathological case, namely that of the five-sided pyramid. The problem arises because the apex is the intersection of four planes (rather than three). A general choice of thicknesses would lead to one of several different ARB types. This complication has been avoided. It has been assumed that the inner body is congruent with the original and the intersection of the first three displaced planes is taken as the apex. Thus the only meaningful input thicknesses are those that preserve the shape.
9. TEC

The TEC can have three thicknesses, described as for the RCC, REC and TRC.

a. BOX

b. RAW

FIGURE 6: Thicknesses for the BOX and the RAW
10. TOR

The TOR has one thickness corresponding to a shortening of the small radius.

## 11. ARS

No provision has been made for thickening this body.
12: AQS

The AQS may have either one thickness (indicated by a zero in columns 6l-70 of the lead card) or one thickness for each input point (indicated by a 1.0 in columns 61-70). The action of the code is to form an AQS through the original points, calculate a local normal (approximate) for each point, and then create a new point (belonging to the new AQS) at the proper distance. Thus the output AQS is just a set of points derived from the original body and its thickness set.

## C. Region Descriptions Based on the Thick Body

Let the body to be thickened have the name $A B C$. A reference to the shell (see Figure 7) in a region description is $\pm S A B C$. A reference to the interior is $\pm I A B C$.

If the translator gives the original body the number $N$ the inner body will have the number $\mathrm{N}+1$ and SABC will be replaced by $\{N-(N+1)\}$ while IABC will be replaced by $N+1$.

The former references are handled automatically in the code, but two other regions demonstrated in Figure 7b, can be referred to by the user in a region description in various ways:

1. For the region BRG between $A B C$ and an enclosing body BBC (numbered NBB by the code), the best (i。e. least redundant) input description is

$$
\mathrm{BRG}=\mathrm{BBC}-\mathrm{ABC}
$$

which translates to $\{\mathrm{NBB}-\mathrm{N}\}$
2. The union of BRG with IABC should be referenced as $O R(B C C-A B C) O R(I A B C)$
which translates to $O R(N B B-N) O R(N+1)$.
The other possible descriptions of these regions have geometrical redundancies which are not recognized by the translator.

a. The shell and the interior

b. Other regions of interest

FIGURE 7: Some regions based on the thick body

## D. The Pipe

The pipe consists of a series of truncated cones or cylinders connected by spheres, as shown in Figure 8. The options available for the pipe are as follows:

1. constant radius (cylindrical sections) or variable radius (conical sections);
2. walls of non-zero thickness (inner bodies are constructed with the normal separation constant in each section) or zero thickness (no inner bodies needed);
3. for a thick pipe the wall thickness may be the same or different for all sections;
4. the pipe may close on itself or it may be double-ended.

The essential input to the PIPE is a set of NP vector positions, radii and (possibly) thicknesses: ( $\left.{ }_{p}, r_{p}, t_{p}, p=1,2 \ldots N P\right)$ Additional input on the lead data card gives the common radius if the pipe is cylindrical, and the common thickness if the pipe walls have the same normal thickness in each section. Zero values of these parameters indicate that the radius and thickness are variable. Details of the input are given at the end of this section.

1) SNAME can refer only to a thick pipe and means the shell of the pipe. If the outer bodies are designated as $B_{I}$ and the inner bodies as
$C_{I}, I=1,2 \ldots N B D$ as in Figure 2, the region
SNAME is the union of the regions:

$$
B_{I}-C_{I-I}-C_{I}-c_{I+1}
$$

for all I such that $1<I<N B D$ and

$$
B_{1}-C_{N B D}-C_{1}-C_{2} \quad(I=1)
$$

and

$$
\mathrm{B}_{\mathrm{NBD}}-\mathrm{C}_{\mathrm{NBD}-1}-\mathrm{C}_{\mathrm{NBD}}-\mathrm{C}_{1} \quad(\mathrm{I}=\mathrm{NBD})
$$

This specification handles both the doubleended pipe and the closed pipe.
2) INAME is a valid reference for a thick pipe or a thin pipe. In either case it means the inner air region and is given by the union of all $\mathrm{C}_{\mathrm{I}}$ (or of all $\mathrm{B}_{\mathrm{I}}$ for a thin pipe).


FIGURE 8: Spheres join truncated cones to form a pipe. X's are common vertices for the 3 bodies. The sphere and the two bases have the same radius.


FIGURE 9: A thick pipe
3) NAME is a valid reference for either a thick pipe or a thin pipe, but is not very useful for a thick pipe. It is identical to INAME for a thin pipe. It is the union of all $B_{I}$.
4) -NAME is valid and meaningful for either a thick pipe or a thin pipe and refers to the outside region. It is the intersection of all negated ${ }^{\prime} \mathrm{B}_{\mathrm{I}}$.

The action of the preprocessor program is the following: the lead data card is read and its information stored in the ASTER array. When the body has been identified as a pipe, the pipe index LPIPE is increased by 1 , the starting body number is stored in the array NNPIPE (IPPIPE) and the subroutine PIPE (LI, L2) is called. Ll is the starting location in the ASTER array of the pipe data and $L 2$ will be returned as the last location. PIPE (L1,L2) reads the remaining data into temporary storage, constructs the bodies (RCC's or TRC's and SPH's) and updates all pointer arrays indexed on body number. In addition, it counts the new bodies and stores that number in the array LBD (LPIPE).

The rest of the coding related to the pipe is in the regionprocessing section of the main program. When a name in a region description has been identified as a body name, the body number, NN, is compared with the list NNPIPE (LPIPE). If there is an LPIPE such that NNPIPE equals NN, control is transferred to the pipe section where the prefix of the NAME is examined to determine the appropriate region description.

The lead data card for a pipe is formatted as follows:

| COLUMN | FORMAT |  |
| :---: | :---: | :---: |
| 1-3 | A3 | NAME of pipe body (alphanumeric) |
| 4-6 | A3 | "PIP", identifier |
| 7-9 | 3A1 | Blank |
| 10 | Al | Blank or "T" for a thick pipe |
| 11-20 | F10.0 | Number of points |
| 21-30 | F10.0 | Common radius for all points or 0.0 |
| 31-40 | F10.0 | Common wall thickness for all sections or 0.0 |
| 41-50 | F10.0 | 0.0 for double-ended pipe, any number $\neq 0.0$ for a closed pipe |
| 51-70 |  | Blank |
| 71-80 | A $4,2 \mathrm{~A} 3$ | Comments |

The rest of the data cards, one per point, specify the point data:

COLUMN FORMAT

1-10
11-20 F10.5
21-30 Fl0.5
31-40 F10.5
41-50 Fl0.5

51-60 Fl0.5

61-70
71-80
A4, 2A3

Not read
$X_{p}$
$Y_{p}$
$Z_{p}$
Radius-unless a common radius has been specified
Thickness - unless a common thickness has been specified or the body is thin Blank

Comments

## E. Region Transformations

A series of regions may be designated as "model" regions and a backward reference on a region card, together with a card giving the number of regions in the series and four cards giving transformation data for four non-coplanar points, will produce a series of new bodies and regions. The format for these data has been given in the introduction. Here we simply show a sample input (Figure l0) and the resulting output (Figure 11).

A model of a man (crew member) consists of 11 bodies and 11 regions, as shown in Figure 10. The model is to be rotated and translated into each of two new positions (regions RAP and RAQ, respectively). Following the region cards for RAP and RAQ is a card bearing the number 11 to indicate the number of regions to be transformed. The four following cards carry the 4 -point transformation ( $r_{p} \rightarrow r_{p}^{i}, p=1,2,3,4$ ).

The region identification cards for the model bear the tag MOD and the tag ST which indicates that the model is to be stripped out at the end. The region identification cards (one per series) for the copies bear the tag DUP, the name of the first region in the model (RAA) and the number of regions in the model set. Finally there is a card giving the initialization of body number (536) and region number (536) for the output.

The output, Figure 11, shows properly sequenced body, region and region ident cards for the two crew members copied from the model. The model itself has been stripped out.

The four-point transformations here represent translations and rotations only. One point is therefore redundant (and must be consistent). For regions involving only the AQS, the ARS, the ELLG and the TEC scale changes and distortion are allowed and these, in general, require the four point specification. Reflections in a plane can be specified for any body.

The output of BRITL appears on TAPE 7 and can be transferred to the PUNCH file or the print file via job control cards.

## F. Subroutines and Important Arrays

In this section we list and describe briefly the important arrays and the subprograms of BRITL.

1. Some important arrays:

ASTER-MASTER currently dimensioned at 10000
LOCDB(2000) starting locations in ASTER of body data, indexed on body number
LOCDR(1000) starting location in ASTER of region data, indexed on region number
NAMB(2000) array of body names (3 characters) versus body numbers

| CREW MEMBERS0 |  |  | 13 | 0.0 | 0.0 | 20.0 | HEAD 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AAAELL | 0.0 | 0.0 | 16.5 |  |  |  |  |
| AABRCC | 9.0 | 0.0 | 9.4 | 0.0 | 0.0 | 5.0 | HEAD 2 |
|  | 0.0 |  |  |  |  |  | HEAD 1 |
|  | 2.0 |  |  |  |  |  | NECK 2 |
| AACREC | 0.0 | -5.75 | -0.425 | 0.0 | 11.5 | 0.0 | BODY 1 |
|  | 0.0 | 0.0 | 11.075 | 4.0 | 0.0 | 0.0 | BODY 2 |
| AADRCC | 0.0 | -7.00 | -3.75 | 0.0 | 0.0 | 14.4 | UPRARM 1 |
|  | 1.25 |  |  |  |  |  | UPRARM 2 |
| AAERCC | 0.0 | 7.00 | -3.75 | 0.0 | 0.0 | 14.4 | UPLARM 1 |
|  | 1.25 |  |  |  |  |  | UPLARM 2 |
| AAFRCC | 1.25 | -7.00 | -2. 50 | 12.25 | 0.0 | 0.0 | LORARM 1 |
|  | 1.25 |  |  |  |  |  | LORARM 2 |
| AAGRCC | 1.25 | 7.00 | -2. 50 | 12.25 | 0.0 | 0.0 | LOLARM 1 |
|  | 1.25 |  |  |  |  |  | LOLARM 2 |
| AAHRCC | 0.0 | -3.50 | -9.0 | 16.5 | 0.0 | 0.0 | RTHIGH 1 |
|  | 2.5 |  |  |  |  |  | RTHIGH 2 |
| AAIRCC | 0.0 | 3.50 | -9.0 | 16.5 | 0.0 | 0.0 | LTHIGH 1 |
|  | 2.5 |  |  |  |  |  | LTHIGH 2 |
| AAJRCC | 15.0 | -3.50 | -9.0 | 0.0 | 0.0 | -15.0 | RT LEG 1 |
|  | 1.5 |  |  |  |  |  | RT LEG 2 |
| AAKRCC | 15.0 | 3.5 | -9.0 | 0.0 | 0.0 | -15.0 | LF LEG 1 |
|  | 1.5 |  |  |  |  |  | LF LEG 2 |



FIGURE 10, continued: Region table, region identification table and initial body and region card.

CREW MEMBERS

|  | 22 | 22 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 536ELL | 15.20070 | -52.00000 | 3.65000 | 15.19957 | -52.00000 | 7.15000 | AAA |
| 536 | 9.00000 |  |  |  |  |  | AAA |
| 537REC | 20.95614 | -52.00000 | -13.27500 | -11. 50000 | . 00000 | . 00000 | AAC |
| 537 | -. 00356 | -. 00000 | 11.07500 | -. 00024 | 4.00000 | -. 00000 | AAC |
| 538RCC | 15.20298 | -52.00000 | -3.45000 | -. 00161 | -. 00000 | 5.00000 | AAB |
| 538 | 2.00000 |  |  |  |  |  | AAB |
| 539RCC | 22.20721 | -52.00000 | -16.60000 | -. 00463 | -. 00000 | 14.40000 | AAD |
| 539 | 1.25000 |  |  |  |  |  | AAD |
| 540RCC | 8.20721 | -52.00000 | -16.60000 | -. 00463 | -. 00000 | 14.40000 | AAE |
| 540 | 1.25000 |  |  |  |  |  | AAE |
| 541RCC | 22.20673 | -50.75000 | -15.35000 | -. 00073 | 12.25000 | -. 00000 | AAF |
| 541 | 1.25000 |  |  |  |  |  | AAF |
| 542 RCC | 8.20673 | -50.75000 | -15.35000 | -. 00073 | 12.25000 | -. 00000 | AAG |
| 542 | 1.25000 |  |  |  |  |  | AAG |
| 543RCC | 18.708889 | -52.00000 | -21.85000 | -. 00098 | 16.50000 | -. 00000 | AAH |
| 543 | 2.50000 |  |  |  |  |  | AAH |
| 544 RCC | 11.70889 | -52.00000 | -21.85000 | -. 00098 | 16.50000 | -. 00000 | AAI |
| 544 | 2.50000 |  |  |  |  |  | AAI |
| 545 RCC | 18.70800 | -37.00000 | -21.85000 | . 00482 | . 00000 | -15.00000 | AAJ |
| 545 | 1.50000 |  |  |  |  |  | AAJ |
| 546 RCC | 11.70800 | -37.00000 | $-21.85000$ | . 00482 | . 00000 | -15.00000 | AAK |
| 546 | 1.50000 |  |  |  |  |  | AAK |
| 547 ELL | 0.00000 | -52.00000 | 3.65000 | 0.00000 | -52.00000 | 7.15000 | AAA |
| 547 | 9.00000 |  |  |  |  |  | AAA |
| 548REC | 5.75000 | -52.00000 | -13.27500 | -11.50000 | . 00000 | . 00000 | AAC |
| 548 | 0.00000 | -. 00000 | 11.07500 | 0.00000 | 4.00000 | -. 00000 | AAC |
| 549 RCC | 0.00000 | -52.00000 | -3.45000 | 0.00000 | -. 00000 | 5.00000 | AAB |
| 549 | 2.00000 |  |  |  |  |  | AAB |
| 550RCC | 7.00000 | -52.00000 | -16.60000 | 0.00000 | -. 00000 | 14.40000 | AAD |
| 550 | 1.25000 |  |  |  |  |  | AAD |
| 551RCC | -7.00000 | -52.00000 | -16.60000 | 0.00000 | -. 00000 | 14.40000 | AAE |
| 551 | 1.25000 |  |  |  |  |  | AAE |

FIGURE ll: Output of sample problem - title card, parameter card and body table.


FIGURE 11, continued: remainder of body table and region table from output of sample problem.

|  | 536 | 10 | -0 | HEAD |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 537 | 11 | -0 | NECK |  |
|  | 538 | 12 | -0 | BODY |  |
|  | 539 | 13 | -0 | UPPER | ARM RIGHT |
|  | 540 | 14 | -0 | UPPER | ARM LEFT |
|  | 541 | 15 | -0 | LOWER | ARM RIGHT |
|  | 542 | 16 | -0 | LOWER | ARM LEFT |
|  | 543 | 17 | -0 | THIGH | RIGHT |
|  | 544 | 18 | -0 | THIG | LEFT |
|  | 545 | 19 | -0 | LOWER | LEG RIGHT |
|  | 546 | 20 | -0 | LOWER | LEG LEFT |
|  | 547 | 10 | -0 | HEAD |  |
|  | 548 | 11 | -0 | NECK |  |
|  | 549 | 12 | -0 | BODY |  |
|  | 550 | 13 | -0 | UPPER | ARM RIGHT |
|  | 551 | 14 | -0 | UPPER | ARM LEFT |
|  | 552 | 15 | -0 | LOWER | ARM RIGHT |
|  | 553 | 16 | -0 | LOWER | ARM LEFT |
|  | 554 | 17 | -0 | THIG | RIGHT |
|  | 555 | 18 | -0 | THIG | LEFT |
| $\stackrel{\omega}{\omega}$ | 556 | 19 | -0 | LOWER | LEG RIGHT |
|  | 557 | 20 | -0 | LOWER | LEG LEFT |

FIGURE 1l, continued: region identification table from output of sample problem.

NAMR (1000)

NI (30)
$\operatorname{LBOD}(30,30)$
NM (30)

JTYPE (2000) array of numbers giving the body type and data form versus body number
JBOD (30,30) work storage area for Boolean operations. OR'd regions are stored by row. The columns of a row contain bodies which intersect to form the OR'd region.
array of region names ( 3 characters) versus region numbers
number of intersecting bodies in an OR'd region indexed on row number of JBOD.
similar to JBOD
similar to NI
2. The following list briefly describes the subroutines comprising the program BRITL.

BRITL - is the main program, responsible for sequencing the entire calculation. Most of the input is read by BRITL, the exceptions being the thickness data for thick bodies (read by THICKN), the region transformation data (read by GENR), and the pipe data (read by PIPE). As the input body and region tables are processed, BRITL stores the output bodies and output regions on TAPE8 and TAPE9, respectively. After these files are complete, TAPE8 and TAPE9 are copied to TAPE7 in order. The input region identification table is read and processed and the results added to TAPE7. BRITL then calls the subroutine STRIP to remove unwanted regions (as indicated on the region identification card), to remove unused bodies and to resequence bodies and regions consistently, starting from the initial body number and initial region number given on the last input card. The final output is on TAPE7.

PIPE(L1,L2) - is described in detail in III D. Reads all but the leading body card for the body type PIP, produces all bodies for either a thin or a thick PIP, storing the data in locations $L 1$ to $L 2$ of the ASTER array.

THICKN (Ll,L2,K) - is called by BRITL when an input body (whose data is stored in MASTER starting at $\mathrm{K}+1$ ) is determined to be thick. It reads the thickness card and produces the new body, stores the data starting at $L$ l in the MASTER array and returns the final location L2 to BRITL. BRITL then calls WRITEB(Ll,L2) to print and punch the body cards.

ARBT (MASTER ( $\mathrm{K}+\mathrm{l}$ ), IV,THICK) - is called by THICKN to produce the second body for a thick ARB. The face descriptions, contained in IV $(4,6)$, are used to identify the three planes whose intersection is the vertex under consideration. New planes are obtained by moving each plane along its normal without rotation by a distance equal to the thickness for the corresponding face. The new vertex is the intersection of the three new planes.

AQST (THICK,Ll,K,L2) - is called by THICKN to take the input AQS data (stored in MASTER starting at $K+1$ ), produce a paraboloidal surface and, using the local normal, displace each of the original points along the normal by the input thickness to produce a second AQS to be stored in MASTER starting at Ll and extending to L 2 , the latter address being returned to THICKN.

NOTR - takes a region located in the MASTER array, negates this region, leaves the results, reduced to the conjunctive form, in the array JBOD. NOTR sets the arrays (NI (I), $I=1, N 1),((J B O D(J, I), J=1, N I(I)), I=1, N 1)$.

RANDR - intersects two regions, each in the conjunctive form, to produce a final region in the conjunctive form. One of the original regions resides in the MASTER array and the other is stored in LBOD. The results are ultimately stored in LBOD, NM and N2.

DEROW - is called by NOTR and RANDR to complete the simplification of a region description. In essence, DEROW removes non-geometric redundancies by removing OR'd regions which are explicitly contained in other OR'd regions.

ORSTOR (LBOD,N2,NM,MASTER (Ll), L2) - takes the contents of the array ( $(\operatorname{LBOD}(\mathrm{L}, \mathrm{M}), \mathrm{L}=1, \mathrm{NM}(\mathrm{M}), \mathrm{M}=1, \mathrm{~N} 2)$ and arranges it in MASTER along with the previousily stored data for the current region. At this point the data in LBOD is the result of processing a single "OR" of an input region description. THIS "OR" has been reduced (by substitution and Boolean manipulation) to N2 "OR"s, the Mth of which has NM (M) intersecting bodies. The NF OR's already in the MASTER array are stored as follows:

$$
\begin{aligned}
& \operatorname{MASTER}(L l)=N F \\
& \operatorname{MASTER}(L 1+M)=N M(M), M=1, \ldots \ldots, N F \\
& \operatorname{MASTER}\left(L 1+N 2+\sum_{M=1}^{K-1} N M(M)+I\right)=
\end{aligned}
$$

body number (with sign) for the Ith body of the Kth "OR".

ORSTOR returns a value of L 2 , the last location occupied in the MASTER array.

GENR (Ll,L2) - is called by BRITL to produce a new region series from a model region series by transformation. GENR reads the card giving the number of regions in the series and the cards giving the 4-point transformation, deduces a displacement vector and a rotation matrix (or a reflection or distortion matrix), and then applies these to each body in the model regions in turn, thus producing a new sequence of consecutively numbered bodies. Concurrently, the new regions are constructed by substituting new body numbers for old in the description of the model regions. GENR calls WRITEB(L1,L2) for each new body. The region write is invoked by GENR with a call to WRITEF (NR,MASTER (L1),L2) for each region NR in the new set.

WRITEB (Ll,L2) - prints and punches body data from the storage area MASTER(L1) to MASTER(L2). The punched (or stored) cards are in the format acceptable to the GIFT program.

WRITEF (NR,MASTER(Ll),L2) - takes the completed region description for region NR from the storage area MASTER (II) to MASTER(L2) and constructs the region cards in the format acceptable to the GIFT program. The data are both printed and punched (or stored). L2 is calculated internally and returned to the calling routine.

STRIP - is the last subprogram called by BRITL and produces the final file on TAPE7. STRIP examines the region identification cards to determine which regions are to be retained. Bodies not mentioned in these regions are deleted, the remaining bodies are resequenced and the references in the region descriptions are corrected. The region cards and region identification cards are then resequenced. Initial body number and initial region number are obtained from the last input card, which is read by STRIP.

CROSS (A,B,C) - computes the cross product of $B$ and $C$ and stores the answer in $A$.

CUBIC ( $C, R, N$ ) - computes the roots $R$ of the cubic equation whose coefficients are input as $C$ (unit coefficient assumed for cubic term). Also returns the number of real roots N .

DOT (A,B) - computes the dot product of $A$ and $B$.

SOLVE ( $A, B, N, K S$ ) - is an IBM routine that solves a set of linear equations, order $N$, matrix of coefficients $A$, constant terms in vector $B$. The solution vector is returned in $B$. $K S$ is a signal: 0 for successful matrix inversion, 1 for failure.

UNIT $(A, B)$ - normalizes the vector $A$ and puts the results in $B$ without destroying $A$.

UNPACK - is a utility routine used in connection with ARB data. It returns an index related to the number of vertices.
III. A COMGEOM MODEL OF THE MIO9A1 HOWITZER

A COMGEOM model of the M109Al Howitzer has been prepared for use with the BRI-GIFT program. The model requires 939 bodies and 753 regions.

Some of the model transformation capabilities of BRITL (see Section II of this report) were used to generate copies of repeated structures in the proper position and orientation, e.g. wheels and crew members.

A number of small programs were written for the checking and correcting of body parameters, e.g. mutual orthogonality of the BOX vectors and planar faces for the ARB.

A major checking device, namely the GIFT program itself, was made available toward the end of the contract period, and has been used mainly for region checking, $i_{0} e$. to discover overlaps.

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