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Extensions of Concept Languages for a Mechanical Engineering Application

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Extensions of Concept Languages for a Mechanical Engineering Application

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Abstract

We shall consider an application in mechanical engineering, and shall show that the adequate modeling of the terminology of this problem domain in a conventional concept language poses two main representation problems. The first requires access to concrete domains, such as real numbers, while the second asks for a construct which can be used to represent sequences of varying length. As shown in recent papers by the authors there exist extended concept languages—equipped with sound and complete reasoning algorithms—that satisfy the respective representation demands separately.

The main result presented in this paper is that the combination of both extensions leads to undecidable terminological inference problems. In particular, the important subsumption problem is undecidable. It should be noted that the need for these extensions is not particular to the considered problem domain; similar representation demands are likely to occur in other non-toy applications.

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1 Introduction

Concept languages based on KL-ONE [Brachman and Schmolze, 1985] are mostly used to represent the terminological knowledge of a particular problem domain on an abstract logical level. To describe this kind of knowledge, one starts with atomic concepts and roles, and defines new concepts using the operations provided by the language. Concepts can be considered as unary predicates which are interpreted as sets of individuals, and roles as binary predicates which are interpreted as binary relations between individuals. Examples for atomic concepts may be Human and Female, and for roles child. If the logical connective conjunction is present as language construct, one may describe the concept Woman as "humans who are female", and represent it by the expression Human \sqcap Female. Many languages provide quantification over role fillers which allows for example to describe the concept Mother by the expression Woman $\sqcap \exists child.Human$.

KL-ONE was first developed for the purpose of natural language processing [Brachman *et al.*, 1979], and some of the existing systems are still mostly used in this context (see e.g., SB-ONE [Kobsa, 1989]). However, its success in this area has also led to applications in other fields (see e.g., MESON [Edelmann and Owsnicki, 1986] which is used for computer configuration tasks, CLASSIC [Borgida *et al.*, 1989] which is e.g. used for retrieval in software information systems, or K-REP [Mays *et al.*, 1987; Mays *et al.*, 1988] which is used in a financial marketing domain).

In this paper we shall investigate how concept languages can be used in a mechanical engineering domain. More precisely, we shall consider the representation of concepts that are related to lathe workpieces. We shall describe this application in more detail in Section 2. There it will be pointed out that the adequate formalization of these concepts demands two substantial extensions of conventional concept languages. On the one hand, reference to concrete notions such as real numbers is mandatory to represent, for example, the geometric aspects of a lathe workpiece. On the other hand, the abstraction in this domain requires to describe classes of lathes which are sequences of geometric primitives. These sequences have a finite, but varying and not a priori bounded length.

The extensions needed to satisfy these demands have separately been considered in recent papers by the authors. Section 4 summarizes the schematic extension by concrete domains proposed in [Baader and Hanschke, 1991]. An instantiation of this scheme appropriate for representing the geometric aspects in our problem domain is obtained by taking the concrete domain "real numbers." In Section 5 we recall the extension of a conventional concept language by a transitive closure operator as proposed in [Baader, 1991]. This can be used to deal with the varying length aspect. Both papers not only introduce the extended formalisms, but also describe decision procedures for the common terminological inference problems such as subsumption.

In contrast to these positive results we shall show in the present paper (Section 6)

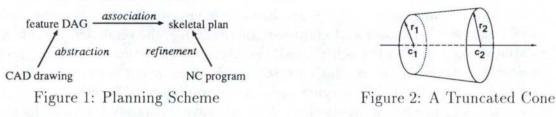
that the subsumption problem in a concept language combining both extensions is undecidable. The paper concludes with some remarks on how this algorithmic problem can be circumvented in a hybrid knowledge representation architecture. As the common base for both extensions the concept language \mathcal{ALCF} is introduced in Section 3. It provides abstract concept-forming operators as used in the introductory examples above.

2 Motivation and Problem Domain

As already mentioned, the domain we want to consider is production planning for CNC lathe machines. More precisely, our work has been motivated by the following application:

Given the geometry of a rotational-symmetric workpiece, generate abstract NC macros for turning the workpiece on a CNC lathe machine.

Reasoning in this application follows a scheme (Figure 1) that is inspired by William J. Clancey's *heuristic classification*: The input to the system is a CAD drawing describing the workpiece in terms of primitive surfaces and basic technological data. The *abstraction* phase generates a schematic description of the workpiece in terms of (CAD/CAM) features [Klauck *et al.*, 1991]. Such features are often associated with parts of the workpiece that are characteristic with respect to how these parts (or the whole lathe) may be manufactured. The second phase associates *skeletal (production) plans* to the features (i.e. to the nodes in the feature DAG). Finally, the third phase *refines* and merges the skeletal plans to a complete numerical control (NC) program.



This problem domain requires, among other things, the representation of geometric primitives used in the CAD drawing and of technological data of the workpiece. It also contains the features which characterize the workpiece. If this could be done with a concept language, the abstraction phase could be mapped naturally into a terminological framework:

- Arrange the features represented as concepts in a generalization hierarchy using the subsumption service of the terminological system.
- Represent a particular CAD drawing of the workpiece with its geometric and technological information as instances of appropriate concepts.

• Employ the so-called realization service [Nebel, 1990] to compute the most specific concepts that apply to the particular lathe. The features corresponding to these concepts are then the output of the abstraction phase to which the skeletal plans can be associated.

However it is easy to see that conventional concept languages cannot be used to adequately represent this problem domain. Consider for example the concept of a truncated cone (see Figure 2). Since we consider geometric objects as fixed to an axis, a truncated cone can be characterized by four real numbers, two for its radii and two for the corresponding centers. But of course not all quadruples of rational numbers represent a truncated cone. So we have to restrict the values such that the radii are positive. In addition, one has to exclude cases where the truncated cone degenerates to a line, a circle, or even a point. It seems to be impossible to represented this using only "abstract" concept terms without reference to predicates over, for example, real numbers, which shows the need for an integration of concrete domains.

Another problem is due to the fact that one has to describe classes of lathes which are sequences of geometric primitives. The problem is that these sequences have a finite, but varying and not a priori bounded length. It is quite simple to define concepts for features such as

'1 truncated cone', '2 truncated cones', ...

Assume, for example, that the following two concepts are already defined: Truncone, which stands for the class of truncated cones, and Neighboring, which characterizes when two truncated cones fit together. We shall see later how these concepts could be defined using our first extension. Then the mentioned sequence of concepts can be defined as follows:

> One = ∃head.Truncone □ ∀tail.Bottom Two = ∃head.Truncone □ ∃tail.One □ Neighboring

But it remains the problem to represent the most specific generalization (union) of these *infinitely* many features (concepts). The resulting concept could be termed a 'sequence of neighboring truncated cones'. It should be noted that its specialization 'ascending sequence of truncated cones' (see the definition in Section 6) is essential for characterizing the production classes of lathes. In Section 6 we shall give a formal representation of the kind of sequences exemplified in the figure.

3 The Concept Language ALCF

This section introduces the language \mathcal{ALCF} as a prototypical conventional concept language. It will be the starting point for the two extensions described in

the next two sections.

Definition 3.1 (syntax of ALCF) Concept terms are built from concept, role, and attribute¹ names using just the concept-forming operators

negation $(\neg C)$, disjunction $(C \sqcup D)$, conjunction $(C \sqcap D)$, exists-in restriction $(\exists R.C)$, and value restriction $(\forall R.C)$.

Here C and D are syntactic variables for concept terms and R is a role or attribute name.

Let A be a concept name and let D be a concept term. Then A = D is a terminological axiom. A terminology (T-box) is a finite set T of terminological axioms with the additional restrictions that (i) no concept name appears more than once as a left hand side of a definition, and (ii) T contains no cyclic definitions.²

Please note that the exists-in and the value restrictions are not only defined for roles but also for attributes. The next definition gives a model-theoretic semantics for the language introduced in Definition 3.1.

Definition 3.2 (semantics of \mathcal{ALCF}) An interpretation \mathcal{I} for \mathcal{ALCF} consists of a set dom(\mathcal{I}) and an interpretation function. The interpretation function associates with each concept name A a subset $A^{\mathcal{I}}$ of dom(\mathcal{I}), with each role name R a binary relation $\mathbb{R}^{\mathcal{I}}$ on dom(\mathcal{I}), i.e., a subset of dom(\mathcal{I}) × dom(\mathcal{I}), and with each attribute name f a partial function $f^{\mathcal{I}}$ from dom(\mathcal{I}) into dom(\mathcal{I}).

For such a partial function $f^{\mathcal{I}}$ the expression $f^{\mathcal{I}}(x) = y$ is sometimes written as $(x, y) \in f^{\mathcal{I}}$. The interpretation function—which gives an interpretation for atomic terms—can be extended to arbitrary concept terms as follows: Let C and D be concept terms and let R be a role or attribute name. Assume that $C^{\mathcal{I}}$ and $D^{\mathcal{I}}$ are already defined. Then

- 1. $(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}, \ (C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}, \ and \ (\neg C)^{\mathcal{I}} = dom(\mathcal{I}) \setminus C^{\mathcal{I}},$
- 2. $(\forall R.C)^{\mathcal{I}} = \{x \in dom(\mathcal{I}); \text{ for all } y \text{ such that } (x, y) \in R^{\mathcal{I}} \text{ we have } y \in C^{\mathcal{I}}\}$ and
 - $(\exists R.C)^{\mathcal{I}} = \{x \in dom(\mathcal{I}); \text{ there exists } y \text{ such that } (x, y) \in R^{\mathcal{I}} \text{ and } y \in C^{\mathcal{I}}\}$

An interpretation \mathcal{I} is a model of the T-box \mathcal{T} iff it satisfies $A^{\mathcal{I}} = D^{\mathcal{I}}$ for all terminological axioms A = D in \mathcal{T} .

An important service terminological representation systems provide is computing the subsumption hierarchy, i.e., computing the subconcept-superconcept relationships between the concepts of a T-box. This inferential service is usually called classification. The model-theoretic semantics introduced above allows the following formal definition of subsumption.

¹To avoid confusion with the 'CAD/CAM features' of the application domain we refer to the functional roles as attributes and not as features as, e.g., in [Baader and Hanschke, 1991].

²See [Nebel, 1989; Baader, 1990] for a treatment of cyclic definitions in concept languages.

Definition 3.3 (subsumption) Let \mathcal{T} be a T-box and let C, D be concept terms. Then D subsumes C with respect to \mathcal{T} iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ holds for all models \mathcal{I} of \mathcal{T} .

In addition to the formalism defined so far, terminological systems usually provide for an assertional component, and corresponding reasoning services [Nebel, 1990]. Because of the space limitations we shall not address this aspect in the present paper.

4 Integrating Concrete Domains

In this section we introduce a formalism that is capable to deal with the first representational problem mentioned in the introductory sections. Before we can define this extended language, we have to formalize the notion "concrete domain" which has until now only been used in an intuitive sense.

Definition 4.1 A concrete domain \mathcal{D} consists of a set dom (\mathcal{D}) , the domain of \mathcal{D} , and a set pred (\mathcal{D}) , the predicate names of \mathcal{D} . Each predicate name P is associated with an arity n, and an n-ary predicate $P^{\mathcal{D}} \subseteq \text{dom}(\mathcal{D})^n$.

An important example for our application is the concrete domain \mathcal{R} of real arithmetic. The domain of \mathcal{R} is the set of all real numbers, and the predicates of \mathcal{R} are given by formulae which are built by first order means (i.e., by using logical connectives and quantifiers) from equalities and inequalities between integer polynomials in several indeterminates.³ For example, $x + z^2 = y$ is an equality between the polynomials $p(x,z) = x + z^2$ and q(y) = y; and x > y is an inequality between very simple polynomials. From these equalities and inequalities one can e.g. build the formulae $\exists z(x + z^2 = y)$ and $\exists z(x + z^2 = y) \lor (x > y)$. The first formula yields a predicate name of arity 2 (since it has two free variables), and it is easy to see that the associated predicate is $\{(r,s); r \text{ and } s \text{ are real numbers and } r \leq s\}$. Consequently, the predicate associated to the second formula is $\{(r,s); r \text{ and } s \text{ are real numbers}\} = dom(\mathcal{R}) \times dom(\mathcal{R})$.

To get inference algorithms for the extended concept language which will be introduced below, the concrete domain has to satisfy some additional properties. For technical reasons we have to require that the set of predicate names of the concrete domain is *closed under negation*, e.g., if P is an n-ary predicate name in $pred(\mathcal{D})$ then there has to exist a predicate name Q in $pred(\mathcal{D})$ such that $Q^{\mathcal{D}} = dom(\mathcal{D})^n \setminus P^{\mathcal{D}}$. In addition, we need a unary predicate name which denotes the predicate $dom(\mathcal{D})$.

³For the sake of simplicity we assume here that the formula itself is the predicate name. In applications, the user will probably take his own intuitive names for these predicates.

The property which will be formulated now clarifies what kind of reasoning mechanisms are required in the concrete domain. Let $P_1, ..., P_k$ be k (not necessarily different) predicate names in $pred(\mathcal{D})$ of arities $n_1, ..., n_k$. We consider the conjunction

$$\bigwedge_{i=1}^{k} P_i(\underline{x}^{(i)}).$$

Here $\underline{x}^{(i)}$ stands for an n_i -tuple $(x_1^{(i)}, ..., x_{n_i}^{(i)})$ of variables. It is important to note that neither all variables in one tuple nor those in different tuples are assumed to be distinct. Such a conjunction is said to be *satisfiable* iff there exists an assignment of elements of $dom(\mathcal{D})$ to the variables such that the conjunction becomes true in \mathcal{D} .

For example, let $P_1(x, y)$ be the predicate $\exists z(x + z^2 = y)$ in $pred(\mathcal{R})$, and let $P_2(x, y)$ be the predicate x > y in $pred(\mathcal{R})$. Obviously, neither the conjunction $P_1(x, y) \wedge P_2(x, y)$ nor $P_2(x, x)$ is satisfiable.

Definition 4.2 A concrete domain \mathcal{D} is called admissible iff (i) the set of its predicate names is closed under negation and contains a name for dom(\mathcal{D}), and (ii) the satisfiability problem for finite conjunctions of the above mentioned form is decidable.

The concrete domain \mathcal{R} is admissible. This is a consequence of Tarski's decidability result for real arithmetic [Tarski, 1951; Collins, 1975]. However, for the linear case (where the polynomials in the equalities and inequalities have to be linear) there exist more efficient methods (see e.g. [Weispfenning, 1988; Loos and Weispfenning, 1990]). We are now ready to define the extension $\mathcal{ALCF}(\mathcal{D})$ of \mathcal{ALCF} which is parametrized by an admissible concrete domain \mathcal{D} .

Definition 4.3 $(\mathcal{ALCF}(\mathcal{D}))$ The concept formalism of \mathcal{ALCF} is extended by one new construct, called predicate restriction. Assume that f_1, \dots, f_m are m > 0attributes. Then $f_1 \dots f_m$ is called an attribute chaining. If u_1, \dots, u_n are attribute chainings and P is an n-ary concrete predicate then $P(u_1, \dots, u_n)$ (predicate restriction) is a concept term.

The only differences between interpretations \mathcal{I} of $\mathcal{ALCF}(\mathcal{D})$ and ALCF are:

- The abstract domain $dom(\mathcal{I})$ is required to be disjoint to $dom(\mathcal{D})$.
- Attributes f are interpreted as partial functions f^I: dom(I) → dom(I) ∪ dom(D). They establish the link between the abstract and the concrete domain.

The semantics of the predicate restriction is defined as

 $P(u_1, ..., u_n)^{\mathcal{I}} = \{ x \in dom(\mathcal{I}); \text{ there exist } r_1, ..., r_n \in dom(\mathcal{D}) \text{ such that} \\ u_1^{\mathcal{I}}(x) = r_1, ..., u_n^{\mathcal{I}}(x) = r_n \text{ and } (r_1, ..., r_n) \in P^{\mathcal{D}} \}.$

Here the application of the interpretation function to the attribute chainings u_i is the composition of the respective partial functions.

Under the assumption that the concrete domain is admissible [Baader and Hanschke, 1991] describes sound and complete algorithms for all reasoning services for this extended language that are usually provided for terminological systems. In particular, for two concept terms C and D and a terminology \mathcal{T} over $\mathcal{ALCF}(\mathcal{D})$ it is decidable whether C subsumes D w.r.t. \mathcal{T} . The basic algorithm relies on a form of tableaux calculus and is an extension of the procedures in [Schmidt-Schauß and Smolka, 1991; Hollunder *et al.*, 1990; Hollunder, 1990; Donini *et al.*, 1991] that have been designed for concept languages without a concrete domain. The following sample terminology shows how a part of the problem domain of lathes can be formalized using $\mathcal{ALCF}(\mathcal{R})$.

In this terminology r_1 , r_2 , c_1 , c_2 , head, and tail are attributes. The A is an arbitrary concept name used to define the empty concept Bottom. The concrete predicate truncone-condition restricts the attribute fillers for r_1 , r_2 , c_1 , and c_2 , according to the requirements mentioned in Section 2. For the concrete predicates = and \leq we have used infix notation to increase readability. The expression

(head $r_2 = tail$ head r_1) \sqcap (head $c_2 = tail$ head c_1)

in the definition of Neighboring ensure that the 'right' co-ordinates of the 'left' truncated cone coincide with the 'left' co-ordinates of the 'right' truncated cone.

5 Adding Transitive Closure

This section introduces an extension \mathcal{ALCF}^* of \mathcal{ALCF} that can satisfy the demands of the problem domain for representing concepts which contain objects that are sequences of finite but previously unknown length (varying length aspects). The basic idea of this extension is to allow role and attribute *terms* in value-restrictions and exists-in restrictions instead of just allowing role and attribute *names* as in \mathcal{ALCF} . **Definition 5.1 (syntax of** $ALCF^*$) The role/attribute terms are built from role and attribute names with

union $(R \sqcup S)$, composition $(R \circ S)$, and transitive closure (trans(R))

of roles and attributes. Concept terms in $ALCF^*$ are defined as in ALCF with the only difference that role/attribute terms can be used in value-restrictions and exists-in restrictions.

For example, a sequence of truncated cones can be defined as follows:

Sequence = \exists head.Truncone $\sqcap \forall$ trans(tail). \exists head.Truncone

Since \mathcal{ALCF}^* provides no concrete domains Truncone is a primitive (i.e., not further defined) concept in this terminology and we have not expressed that the truncated cones are neighboring.

Definition 5.2 (semantics of \mathcal{ALCF}^*) The interpretation can be easily extended to attribute/role terms in the obvious way: $R \sqcup S^T = R^T \cup S^T$, $R \circ S^T = \{(x, y); \exists z : (x, z) \in R^T \text{ and } (z, y) \in S^T\}$, and $\operatorname{trans}(R)^T := \bigcup_{n>1} (R^T)^n$.

In [Baader, 1991] it is shown that for \mathcal{ALC}^* (i.e., \mathcal{ALCF}^* without attributes) the subsumption problem is decidable. A close look at the algorithm for \mathcal{ALC}^* (which is much too complex to be sketched here) reveals that the result also holds for \mathcal{ALCF}^* , that means, for concept terms C, D and a terminology \mathcal{T} over \mathcal{ALCF}^* (with attributes and roles) it is decidable whether C subsumes D.

6 Combining the Extensions

Up to now we have considered two different extensions of our base language \mathcal{ALCF} : The extension by concrete domains and the extension by role/attribute terms involving transitive closure. Now we consider the language $\mathcal{ALCF}^*(\mathcal{D})$ which we obtain if we combine both extensions.

As mentioned in the introductory sections we should like to have both representational facilities available to solve our representation problems. With \mathcal{R} as the concrete domain this language is expressive enough to define concepts that are of great importance in our application domain, such as a 'sequence of neighboring truncated cones' (Seq-tc) and its specialization 'ascending sequence of truncated cones' (Aseq-tc):

Seq-tc=Sequence □ (Last □ Neighboring) □
∀trans(tail).(Last □ Neighboring)Aseq-tc=Seq-tc □ ∀head.Ascend □ ∀trans(tail) ∘ head.Ascend

where the other concepts are as in the sample terminology of Section 4.

The price we have to pay for this expressiveness is that we cannot decide in general whether a concept C subsumes a concept D in this language.

This will be shown by reducing the Post Correspondence Problem to the subsumption problem for this language. The reduction will use only very simple predicates from real arithmetic, namely equalities between linear polynomials in at most two variables.

First, we recall the definition of the Post Correspondence Problem. Let Σ be a finite alphabet. A Post Correspondence System (PCS) over Σ is a nonempty finite set $S = \{(l_i, r_i); i = 1, ..., m\}$ where the l_i, r_i are words over Σ . A nonempty sequence $1 \leq i_1, ..., i_n \leq m$ is called a solution of the system S iff $l_{i_1} \cdots l_{i_m} =$ $r_{i_1} \cdots r_{i_m}$. It is well-known that the Post Correspondence Problem, i.e., the question whether there exists a solution for a given system, is in general undecidable if the alphabet contains at least two symbols [Post, 1946].

A solution of a PCS is a sequence of pairs of words with a previously unknown size. The varying size is represented with the help of the transitive closure on the abstract level, whereas, the words and their concatenation is modeled by predicates of the concrete domain \mathcal{R} over real numbers.

The words are encoded into \mathcal{R} as follows. For $B := |\Sigma| + 1$ we can consider the elements of Σ as digits 1, 2, ..., B - 1 of numbers represented at base B. For a given nonempty word w over Σ we denote by \overline{w} the nonnegative integer (in ordinary representation at base 10) it represents at base B. We assume that the empty word ε represents the integer 0. Obviously, the mapping $w \mapsto \overline{w}$ is a 1-1mapping from Σ^* into the set of nonnegative integers. Concatenation of words is reflected on the corresponding numbers as follows. Let v, w be two words over Σ . Then we have $\overline{vw} = \overline{v} \cdot B^{|w|} + \overline{w}$, where |w| denotes the length of the word w. We are now ready to define names for the predicates of the concrete domain \mathcal{R} we shall use in our reduction. For i = 1, ..., m,

$$\begin{array}{lll} C_l^i(x,y,z) & \Longleftrightarrow & y = \overline{l_i} \wedge z = y + x \cdot B^{|l_i|}, \\ C_r^i(x,y,z) & \Longleftrightarrow & y = \overline{r_i} \wedge z = y + x \cdot B^{|r_i|}, \\ E(x,y) & \Longleftrightarrow & x = y, \quad \text{and} \quad L(x) \iff x = 0. \end{array}$$

Let l, r, w_l, w_r , and f be attribute names. The concept term C(S) corresponding to the Post Correspondence System S is now defined as follows:

$$C(S) = \bigsqcup_{i=1}^{m} \left(C_l^i(w_l, l, f w_l) \sqcap C_r^i(w_r, r, f w_r) \right) \sqcap$$
$$L(w_l) \sqcap L(w_r) \sqcap$$
$$\forall \mathsf{trans}(f). \left(\bigsqcup_{i=1}^{m} \left(C_l^i(w_l, l, f w_l) \sqcap C_r^i(w_r, r, f w_r) \right) \right) \sqcap$$
$$\exists \mathsf{trans}(f). E(w_l, w_r).$$

A concept term D is satisfiable iff there is a interpretation such that $D^{\mathcal{I}}$ is not empty. Obviously, a concept term D is satisfiable iff **Bottom** does not subsume D. Thus, undecidability of satisfiability implies undecidability of subsumption.

Theorem 6.1 The concept term C(S) is satisfiable if and only if the Post Correspondence System S has a solution. Consequently, satisfiability is in general undecidable for concept terms which may contain transitive closure of attributes and predicate restrictions of an admissible concrete domain.

Proof. Assume that S has a solution $i_1, ..., i_n$ of length n. We extend this sequence to an infinite sequence $i_1, ..., i_n, i_{n+1}, i_{n+2}, ...$ by choosing arbitrary indices $1 \leq i_{n+1}, i_{n+2}, ... \leq m$. This new sequence is used to define an interpretation \mathcal{I} as follows:

$$dom(\mathcal{I}) := \{k; k \ge 1\}, \text{ and for all } k \ge 1,$$

$$f^{\mathcal{I}}(k) := k + 1,$$

$$l^{\mathcal{I}}(k) := \overline{l_{i_k}} \text{ and } r^{\mathcal{I}}(k) := \overline{r_{i_k}},$$

$$w_l^{\mathcal{I}}(k) := \overline{l_{i_1} \cdots l_{i_{k-1}}} \text{ and } w_r^{\mathcal{I}}(k) := \overline{r_{i_1} \cdots r_{i_{k-1}}}$$

Please note that for k = 1, the word $l_{i_1} \cdots l_{i_{k-1}}$ is the empty word, and thus $\overline{l_{i_1} \cdots l_{i_{k-1}}} = 0$. It is now easy to show that $1 \in C(S)^{\mathcal{I}}$. Obviously, this implies that C(S) is satisfiable.

On the other hand, assume that C(S) is satisfiable, and let \mathcal{I} be an interpretation such that $C(S)^{\mathcal{I}} \neq \emptyset$. This interpretation can be used to find a solution of S. Consider an arbitrary element c of $C(S)^{\mathcal{I}}$. Obviously, $c \in (L(w_l) \sqcap L(w_r))^{\mathcal{I}}$ yields $w_l^{\mathcal{I}}(c) = 0 = w_r^{\mathcal{I}}(c)$. Since

$$c \in \left(\bigsqcup_{i=1}^{m} \left(C_l^i(w_l, l, f w_l) \sqcap C_r^i(w_r, r, f w_r) \right) \right)^{\mathcal{I}}$$

we know that there exists an index between 1 and m, say i_1 , such that

 $c \in \left(C_l^{i_1}(w_l, l, f w_l) \sqcap C_r^{i_1}(w_r, r, f w_r)\right)^{\mathcal{I}}.$

By the definition of the concrete predicates we get that $l^{\mathcal{I}}(c) = \overline{l_{i_1}}$ and $r^{\mathcal{I}}(c) = \overline{r_{i_1}}$, and $(f w_l)^{\mathcal{I}}(c) = \overline{l_{i_1}}$ and $(f w_r)^{\mathcal{I}}(c) = \overline{r_{i_1}}$.

Similarly, one can show by induction on k that for all $k\geq 0$ there exists an index i_{k+1} between 1 and m such that

$$(f^k l)^{\mathcal{I}}(c) = \overline{l_{i_{k+1}}}, \qquad (f^k r)^{\mathcal{I}}(c) = \overline{r_{i_{k+1}}}, (f^{k+1} w_l)^{\mathcal{I}}(c) = \overline{l_{i_1} \cdots l_{i_{k+1}}}, \qquad (f^{k+1} w_r)^{\mathcal{I}}(c) = \overline{r_{i_1} \cdots r_{i_{k+1}}},$$

From $c \in (\exists \mathsf{trans}(f).E(w_l,w_r))^{\mathcal{I}}$ we can now deduce that there exists a positive integer n such that $(f^n w_l)^{\mathcal{I}}(c) = (f^n w_r)^{\mathcal{I}}(c)$, and thus we have $\overline{l_{i_1} \cdots l_{i_n}} = \overline{r_{i_1} \cdots r_{i_n}}$. Consequently, $l_{i_1} \cdots l_{i_n} = r_{i_1} \cdots r_{i_n}$, which shows that the sequence i_1, \ldots, i_n is a solution of S.

7 How to Live with this Undecidability Result

Since terminological systems are only subsystems in larger representation architectures which usually have incomplete reasoners or provide only semidecision procedures, there are two possible ways to avoid undecidable inference problems in the concept language:

- Take $\mathcal{ALCF}(\mathcal{D})$ and deal with the varying length aspects in the surrounding formalism.
- Take \mathcal{ALCF}^* and provide for concrete domains in the surrounding system.

The first approach has been taken in the ARC-TEC project at DFKI, where the varying size aspects are shifted to a rule formalism [Hanschke and Baader, 1991; Hanschke and Hinkelmann, 1991].

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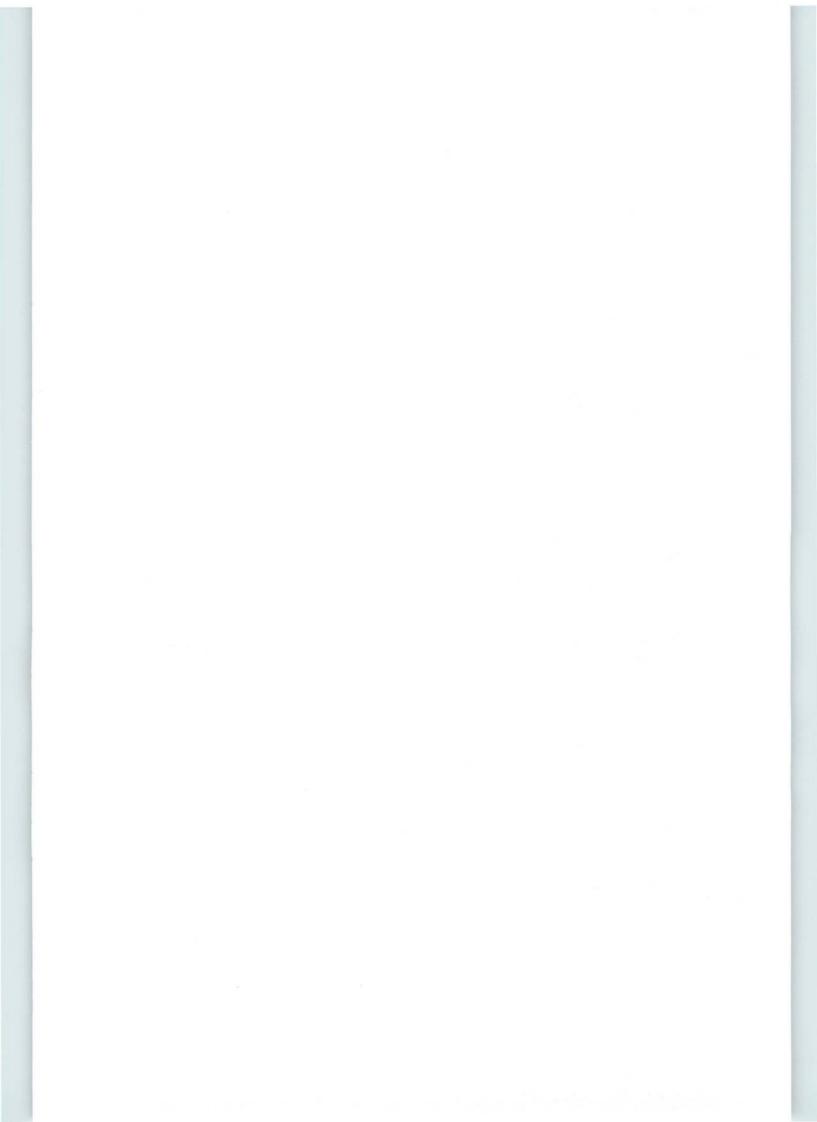
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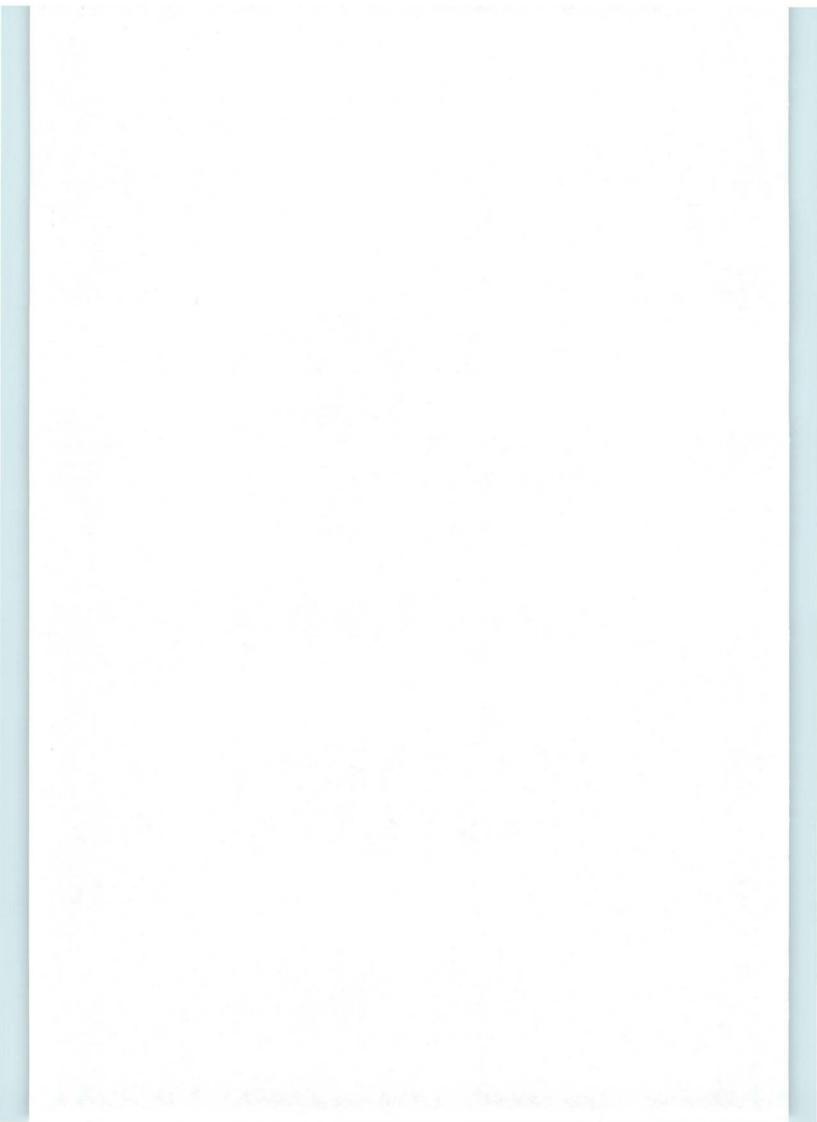
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