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EXTENSIONS OF DERIVATIONS

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ABSTRACT. We show that for a class of algebras including separable algebras one can extend derivations of the center to derivations of the algebra.

The following theorem was proved in the special cases that C is a field by Hochschild [**Ho**] and C is a semilocal ring by Roy and Sridharan [**R, S**] [and for any C in [**Kn**]]. It is also a trivial consequence of a more general result proved by a short cohomological argument.

THEOREM 1. *Let A be an algebra separable over its center C and M be an $A\otimes_C A^{\mathrm{op}}$-module. Then any derivation $d: C\to M^A$ extends to a derivation $\bar{d}: A\to M$.*

Since an algebra separable over its center C is C-projective [**A, G**, p. 379], Theorem 1 follows from

THEOREM 2. *Let A be a C-algebra, projective over C, of Hochschild dimension one and let M be an $A\otimes_C A^{\mathrm{op}}$-module. Then any derivation $d: C\to M^A$ extends to a derivation $\bar{d}: A\to M$.*

PROOF. Let B be the split extension of A by M. That is, B is the additive group $A\oplus M$ with $(a, m)(a', m')=(aa', am'+ma')$. If we let C operate on B by $c(a, m)=(ca, cm+dc\cdot a)=(a, m)c$, then B is a C-algebra and the projection of B to A is a C-algebra homomorphism. It is also C-linearly split since A is C-projective. Thus the extension is an element of $H_C^2(A, M)$ which is zero by hypothesis. This means that there is a C-algebra splitting of $B\to A$, the second coordinate of which is easily seen to be a derivation extending d.

COROLLARY 1. *Let A, C, M be as above and A_0 be a separable C-subalgebra of A. Then any derivation $d: A_0\to M$ which takes C to M^A can be extended to a derivation $\bar{d}: A\to M$.*

PROOF. First restrict to C, then extend to A. The difference, on A_0, is C-linear and hence inner.

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COROLLARY 2. *Let A be a R-algebra separable over its center C and let M be a $A \otimes_C A^{\mathrm{op}}$-module. Then* $H_R^1(A, M) \cong \mathrm{Der}_R(C, M^A)$.

PROOF. It is evident that any derivation of A to M restricts to a derivation $C \rightarrow M^A$.

REFERENCES

[**Ho**]. G. Hochschild, *Restricted Lie algebras and simple associative algebras of characteristic p*, Trans. Amer. Math. Soc. **80** (1955), 135–147. MR **17**, 282.

[**R, S**]. A. Roy and R. Sridharan, *Derivations in Azumaya algebras*, J. Math. Kyoto Univ. **7** (1967), 161–167. MR **36** #5125.

[**Kn**]. M. A. Knus, *Sur le théorème de Skolem-Noether et sur les dérivations des algèbres d'Azumaya*, C. R. Acad. Sci. Paris **270** (1970), 637–639.

[**A, G**]. M. Auslander and O. Goldman, *The Brauer group of a commutative ring*, Trans. Amer. Math. Soc. **97** (1960), 367–409. MR **22** #12130.

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