Yue *Advances in Difference Equations* (2016) 2016:1
DOI 10.1186/s13662-015-0739-5

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH Open Access

Extinction for a discrete competition system with the effect of toxic substances

Qin Yue*

*Correspondence:
xhper1978@sina.com
College of Finance and Mathematics, West Anhui University, Liuan, Anhui 237000, P.R. China

Abstract

A nonautonomous discrete competitive system with nonlinear inter-inhibition terms and one toxin producing species is studied in this paper. Sufficient conditions which guarantee the extinction of one of the components are obtained and the global attractivity of the other one is proved. Our results supplement some existing ones. Numerical simulations show the feasibility of our results.

MSC: 34C05; 34C25

Keywords: discrete; extinction; competitive; toxicology

1 Introduction

For any bounded sequence $\{f(n)\}, f^L = \inf_{n\in N}\{f(n)\}, f^M = \sup_{n\in N}\{f(n)\}$.

Recently, many authors considered the following discrete two species competitive system with nonlinear inter-inhibition terms (see [1–4]):

$$\begin{aligned} x_1(n+1) &= x_1(n)\exp\left\{r_1(n) - a_1(n)x_1(n) - \frac{c_2(n)x_2(n)}{1+x_2(n)}\right\}, \\ x_2(n+1) &= x_2(n)\exp\left\{r_2(n) - a_2(n)x_2(n) - \frac{c_1(n)x_1(n)}{1+x_1(n)}\right\}, \end{aligned} \tag{1.1}$$

where $r_i(n)$, $a_i(n)$, $c_i(n)$ $(i = 1, 2)$ are assumed to be bounded positive sequences and $x_1(n)$, $x_2(n)$ are population density of species x_1 and x_2 at the nth generation, respectively. For the ecological meaning of model (1.1), see [1]. Sufficient conditions which guarantee the permanence, existence, and global stability of positive periodic solutions of system (1.1) were established by Qin *et al.* [1]. By using the Lyapunov function, some analysis techniques, and preliminary lemmas, Wang and Liu [2] further established a criterion for the existence, uniqueness, and uniformly asymptotic stability of positive almost periodic solution of system (1.1) with almost periodic parameters. Noting that ecosystems in the real world are often distributed by unpredictable forces which can result in changes in biological parameters, Wang *et al.* [3] investigated the existence and uniformly asymptotic stability of the unique positive almost periodic solution of system (1.1) with almost periodic parameters and feedback controls. Yu [4] further showed that feedback control variables have no influence on the persistent property of the system. On the other hand, as we all know, the extinction property is also an important topic in the study of mathematical biology; however, until now there are still no scholar investigations of this property of system (1.1). One aim of this work is to obtain a set of sufficient conditions which guarantee the extinction of system (1.1).

In recent years, the competition system with toxic substance has been widely studied. Li and Chen [5] studied the extinction property and global attractivity of the following two species discrete competitive system:

$$\begin{aligned} x_1(n+1) &= x_1(n)\exp\bigl\{r_1(n) - a_{11}(n)x_1(n) - a_{12}(n)x_2(n) - b_1(n)x_1(n)x_2(n)\bigr\},\\ x_2(n+1) &= x_2(n)\exp\bigl\{r_2(n) - a_{21}(n)x_1(n) - a_{22}(n)x_2(n) - b_2(n)x_1(n)x_2(n)\bigr\}. \end{aligned} \tag{1.2}$$

Solé *et al.* [6] and Bandyopadhyay [7] considered the Lotka-Volterra system of two interacting phytoplankton species with one species that could be toxic, while the other one is non-toxic. The stability property of the equilibrium of the system is obtained. Motivated by the above ideas, Chen *et al.* [8] introduced the following system:

$$\begin{aligned} x_1(n+1) &= x_1(n)\exp\bigl\{r_1(n) - a_{11}(n)x_1(n) - a_{12}(n)x_2(n) - b_1(n)x_1(n)x_2(n)\bigr\},\\ x_2(n+1) &= x_2(n)\exp\bigl\{r_2(n) - a_{21}(n)x_1(n) - a_{22}x_2(n)\bigr\}, \end{aligned} \tag{1.3}$$

where $x_1(n)$ represents the density of non-toxic phytoplankton and $x_2(n)$ is the toxic liberating phytoplankton. The coefficients in system (1.3) have the same restriction as that in system (1.2) and system (1.3) is a special case of (1.2) with $b_2(n) \equiv 0$, *i.e.*, the first species could not be toxic. They obtain several sets of sufficient conditions which guarantee the extinction of the one species and the global stability of the other species. To the best of the author's knowledge, to this day, no work has been done previously on the discrete competitive system with nonlinear inter-inhibition terms and one toxin producing species. Hence, we consider the following system:

$$\begin{aligned} x_1(n+1) &= x_1(n)\exp\left\{r_1(n) - a_{11}(n)x_1(n) - \frac{a_{12}(n)x_2(n)}{1+x_2(n)} - b_1(n)x_1(n)x_2(n)\right\},\\ x_2(n+1) &= x_2(n)\exp\left\{r_2(n) - a_{22}(n)x_2(n) - \frac{a_{21}(n)x_1(n)}{1+x_1(n)}\right\}, \end{aligned} \tag{1.4}$$

where all the coefficients have the same meaning as that of systems (1.1)-(1.3), and for $i,j = 1,2$, $r_i(n)$, $a_{ij}(n)$, and $b_1(n)$ are bounded nonnegative sequences defined for $n \in N = \{0,1,2,\ldots\}$ such that

$$0 < r_i^L \le r_i(n) \le r_i^M, \qquad 0 < a_{ij}^L \le a_{ij}(n) \le a_{ij}^M, \qquad 0 < b_1^L \le b_1(n) \le b_1^M. \tag{1.5}$$

As regards the biological meaning, we assume (1.4) together with the initial conditions: $x_1(0) > 0$ and $x_2(0) > 0$. It is not difficult to see that the solutions of (1.4) are defined and remain positive for all $n \in N$. For more relevant work, one could refer to [5–19] and the references cited therein.

The remaining part of this paper is organized as follows. In Section 2, we study the extinction of some one species. The global stability of the other species when the previous species is eventually in extinction both for systems (1.4) and (1.1) is then studied in Section 3. Some examples together with their numerical simulations are presented in Section 4 to show the feasibility of our results. We give a brief discussion in the last section.

2 Extinction

In this section, we will establish sufficient conditions on the extinction of species x_2 or x_1. By a similar proof to Lemma 2.1 in Li and Chen [5], we can obtain the following result.

Lemma 2.1 *Any positive solution* $(x_1(n), x_2(n))^T$ *of system* (1.4) *satisfies*

$$\limsup_{n\to+\infty} x_i(n) \le B_i, \tag{2.1}$$

where $B_i = \frac{\exp(r_i^M - 1)}{a_{ii}^L}$, $i = 1, 2$.

Lemma 2.1 shows that the positive solutions of system (1.4) are bounded eventually. We now come to the study of the extinction of species x_2 of system (1.4).

Theorem 2.1 *Assume*

$$(\mathrm{H}_1)\quad \frac{r_2^M}{r_1^L} < \min\left\{\frac{a_{21}^L}{a_{11}^M(1+B_1)}, \frac{a_{22}^L}{a_{12}^M}\right\}$$

and

$$(\mathrm{H}_2)\quad b_1^M < \frac{1}{B_1 B_2}\min\left\{r_1^L - r_2^M\frac{a_{11}^M(1+B_1)}{a_{21}^L}, r_1^L - r_2^M\frac{a_{12}^M}{a_{22}^L}\right\}$$

hold, where B_i $(i = 1, 2)$ *is defined in Lemma* 2.1, *then the species* x_2 *will be driven to extinction, that is, for any positive solution* $(x_1(n), x_2(n))^T$ *of system* (1.4), $\lim_{n\to+\infty} x_2(n) = 0$.

Proof Conditions (H_1) and (H_2) can be rewritten as

$$\frac{r_2^M}{r_1^L - b_1^M B_1 B_2} < \min\left\{\frac{a_{21}^L}{a_{11}^M(1+B_1)}, \frac{a_{22}^L}{a_{12}^M}\right\}. \tag{2.2}$$

According to (2.2), one can choose a small enough positive constant ε_1 such that

$$\frac{r_2^M}{r_1^L - b_1^M(B_1+\varepsilon_1)(B_2+\varepsilon_1)} < \min\left\{\frac{a_{21}^L}{a_{11}^M(1+B_1+\varepsilon_1)}, \frac{a_{22}^L}{a_{12}^M}\right\}. \tag{2.3}$$

By (2.3), there exist positive constants α and β such that

$$\frac{r_2^M}{r_1^L - b_1^M(B_1+\varepsilon_1)(B_2+\varepsilon_1)} < \frac{\beta}{\alpha} < \min\left\{\frac{a_{21}^L}{a_{11}^M(1+B_1+\varepsilon_1)}, \frac{a_{22}^L}{a_{12}^M}\right\}. \tag{2.4}$$

Thus,

$$\beta a_{11}^M - \frac{\alpha a_{21}^L}{1+B_1} < 0, \qquad \beta a_{12}^M - \alpha a_{22}^L < 0 \tag{2.5}$$

and we can choose a constant $\delta_1 > 0$, such that

$$\alpha r_2^M - \beta r_1^L + \beta b_1^M(B_1+\varepsilon)(B_2+\varepsilon) < -\delta_1 < 0. \tag{2.6}$$

For the above ε_1, it follows from Lemma 2.1 that there exists a large enough N such that

$$x_i(n) < B_i + \varepsilon_1, \quad n > N. \tag{2.7}$$

For any $p > N$, according to the equations of system (1.4) and (2.7), we can get

$$\begin{aligned} \ln\frac{x_1(p+1)}{x_1(p)} &= r_1(p) - a_{11}(p)x_1(p) - \frac{a_{12}(p)x_2(p)}{1+x_2(p)} - b_1(p)x_1(p)x_2(p) \\ &\geq r_1^L - a_{11}^M x_1(p) - a_{12}^M x_2(p) - b_1^M x_1(p)x_2(p), \\ \ln\frac{x_2(p+1)}{x_2(p)} &= r_2(p) - a_{22}(p)x_2(p) - \frac{a_{21}(p)x_1(p)}{1+x_1(p)} \\ &\leq r_2^M - a_{22}^L x_2(p) - \frac{a_{21}^L x_1(p)}{1+B_1+\varepsilon_1}. \end{aligned} \tag{2.8}$$

Therefore, inequalities (2.5)-(2.8) show that

$$\begin{aligned} &\alpha\ln\frac{x_2(p+1)}{x_2(p)} - \beta\ln\frac{x_1(p+1)}{x_1(p)} \\ &\leq \left(\alpha r_2^M - \beta r_1^L\right) + \left(\beta a_{11}^M - \frac{\alpha a_{21}^L}{1+B_1+\varepsilon_1}\right)x_1(p) + \left(\beta a_{12}^M - \alpha a_{22}^L\right)x_2(p) \\ &\quad + \beta b_1^M x_1(p)x_2(p) \\ &< \alpha r_2^M - \beta r_1^L + \beta b_1^M (B_1+\varepsilon)(B_2+\varepsilon) \\ &< -\delta_1 < 0, \quad p > N. \end{aligned} \tag{2.9}$$

Summing both sides of the above inequalities from $N+1$ to $n-1$ leads to

$$\alpha\ln\frac{x_2(n)}{x_2(N+1)} - \beta\ln\frac{x_1(n)}{x_1(N+1)} < -\delta_1(n-N-1), \tag{2.10}$$

hence

$$x_2(n) < \left[\left(\frac{x_1(n)}{x_1(N+1)}\right)^{\beta}\left(x_2(N+1)\right)^{\alpha}\right]^{\frac{1}{\alpha}} \exp\left(-\frac{\delta_1}{\alpha}(n-N-1)\right). \tag{2.11}$$

The above inequality together with the ultimate boundedness of $x_1(n)$ shows that $\lim_{n\to+\infty} x_2(n) = 0$. The proof is completed. □

Theorem 2.2 *In addition to* (H$_1$), *further suppose that*

$$(\mathrm{H}_3) \quad b_1^M < \frac{r_1^L a_{21}^L - (1+B_1)a_{11}^M r_2^M}{(1+B_1)B_2 r_2^M}$$

holds, where B_i $(i = 1,2)$ *is defined in Lemma* 2.1, *then for any positive solution* $(x_1(n), x_2(n))^T$ *of system* (1.4), $\lim_{n\to+\infty} x_2(n) = 0$.

Proof It follows from conditions (H$_1$) and (H$_3$) that

$$\frac{r_2^M}{r_1^L} < \min\left\{\frac{a_{21}^L}{(a_{11}^M + b_1^M B_2)(1+B_1)}, \frac{a_{22}^L}{a_{12}^M}\right\}. \tag{2.12}$$

Thus, one can choose a small enough positive constant ε_2 such that

$$\frac{r_2^M}{r_1^L} < \min\left\{\frac{a_{21}^L}{(a_{11}^M + b_1^M(B_2+\varepsilon_2))(1+B_1+\varepsilon_2)}, \frac{a_{22}^L}{a_{12}^M}\right\}. \tag{2.13}$$

By (2.13), there exist positive constants α and β such that

$$\frac{r_2^M}{r_1^L} < \frac{\beta}{\alpha} < \min\left\{\frac{a_{21}^L}{(a_{11}^M + b_1^M(B_2+\varepsilon_2))(1+B_1+\varepsilon_2)}, \frac{a_{22}^L}{a_{12}^M}\right\}. \tag{2.14}$$

So

$$\beta a_{11}^M + \beta b_1^M(B_2+\varepsilon_2) - \frac{\alpha a_{21}^L}{1+B_1+\varepsilon_2} < 0, \qquad \beta a_{12}^M - \alpha a_{22}^L < 0 \tag{2.15}$$

and we can choose a constant $\delta_2 > 0$, such that

$$\alpha r_2^M - \beta r_1^L < -\delta_2 < 0. \tag{2.16}$$

Therefore, inequalities (2.15), (2.16), (2.7), and (2.8) show that

$$\begin{aligned}
&\alpha \ln\frac{x_2(p+1)}{x_2(p)} - \beta\ln\frac{x_1(p+1)}{x_1(p)} \\
&\quad\le \left(\alpha r_2^M - \beta r_1^L\right) + \left(\beta a_{11}^M - \frac{\alpha a_{21}^L}{1+B_1+\varepsilon_2}\right)x_1(p) + \left(\beta a_{12}^M - \alpha a_{22}^L\right)x_2(p) \\
&\qquad + \beta b_1^M x_1(p)x_2(p) \\
&\quad\le \left(\alpha r_2^M - \beta r_1^L\right) + \left(\beta a_{11}^M - \frac{\alpha a_{21}^L}{1+B_1+\varepsilon_2} + \beta b_1^M(B_2+\varepsilon_2)\right)x_1(p) \\
&\qquad + \left(\beta a_{12}^M - \alpha a_{22}^L\right)x_2(p) \\
&\quad< \alpha r_2^M - \beta r_1^L < -\delta_2 < 0, \quad p > N.
\end{aligned} \tag{2.17}$$

The rest of the proof is similar to that of the corresponding proof of Theorem 2.1, we omit the details here. This ends the proof of Theorem 2.2. □

Theorem 2.3 *Let* $(x_1(n), x_2(n))^T$ *be any positive solution of system* (1.4), *in addition to* (H_1), *further suppose that*

$$(H_4) \quad b_1^M < \frac{r_1^L a_{22}^L - r_2^M a_{12}^M}{B_1 r_2^M}$$

holds, where B_1 *is defined in Lemma* 2.1, *then* $\lim_{n\to+\infty} x_2(n) = 0$.

Proof It follows from conditions (H_1) and (H_4) that

$$\frac{r_2^M}{r_1^L} < \min\left\{\frac{a_{21}^L}{a_{11}^M(1+B_1)}, \frac{a_{22}^L}{a_{12}^M + b_1^M B_1}\right\}. \tag{2.18}$$

Thus, one can choose a small enough positive constant ε_3 such that

$$\frac{r_2^M}{r_1^L} < \min\left\{\frac{a_{21}^L}{a_{11}^M(1+B_1+\varepsilon_3)}, \frac{a_{22}^L}{a_{12}^M + b_1^M(B_1+\varepsilon_3)}\right\}. \tag{2.19}$$

By (2.19), there exist positive constants α and β such that

$$\frac{r_2^M}{r_1^L} < \frac{\beta}{\alpha} < \min\left\{\frac{a_{21}^L}{a_{11}^M(1+B_1+\varepsilon_3)}, \frac{a_{22}^L}{a_{12}^M + b_1^M(B_1+\varepsilon_3)}\right\}. \tag{2.20}$$

Thus,

$$\beta a_{11}^M - \frac{\alpha a_{21}^L}{1+B_1+\varepsilon_3} < 0, \qquad \beta a_{12}^M - \alpha a_{22}^L + \beta b_1^M(B_1+\varepsilon_3) < 0, \tag{2.21}$$

and we can choose a constant $\delta_3 > 0$, such that

$$\alpha r_2^M - \beta r_1^L < -\delta_3 < 0. \tag{2.22}$$

Therefore, inequalities (2.21), (2.22), (2.7), and (2.8) show that

$$\begin{aligned} &\alpha \ln\frac{x_2(p+1)}{x_2(p)} - \beta \ln\frac{x_1(p+1)}{x_1(p)} \\ &\quad \le \left(\alpha r_2^M - \beta r_1^L\right) + \left(\beta a_{11}^M - \frac{\alpha a_{21}^L}{1+B_1+\varepsilon_3}\right)x_1(p) \\ &\qquad + \left(\beta a_{12}^M - \alpha a_{22}^L + \beta b_1^M(B_1+\varepsilon_3)\right)x_2(p) \\ &\quad < \alpha r_2^M - \beta r_1^L < -\delta_3 < 0, \quad p > N. \end{aligned} \tag{2.23}$$

The rest of the proof is similar to that of the corresponding proof of Theorem 2.1, we omit the details here. This ends the proof of Theorem 2.3. □

Now, let us investigate the extinction property of species x_1 in system (1.4) which is also an interesting problem and we obtain the following result.

Theorem 2.4 *Let* $(x_1(n), x_2(n))^T$ *be any positive solution of system* (1.4). *Suppose*

$$(\mathrm{H}_5) \quad \frac{r_2^L}{r_1^M} > \max\left\{\frac{a_{21}^M}{a_{11}^L}, \frac{a_{22}^M(1+B_2)}{a_{12}^L}\right\}$$

holds, where B_2 *is defined in Lemma* 2.1, *then the species* x_1 *will be driven to extinction, that is,* $\lim_{n\to+\infty} x_1(n) = 0$.

Proof According to (H_5), one can choose a small enough positive constant ε_4 such that

$$\frac{r_2^L}{r_1^M} > \max\left\{\frac{a_{21}^M}{a_{11}^L}, \frac{a_{22}^M(1+B_2+\varepsilon_4)}{a_{12}^L}\right\}. \tag{2.24}$$

By (2.24), there exist positive constants α and β such that

$$\frac{r_2^L}{r_1^M} > \frac{\beta}{\alpha} > \max\left\{\frac{a_{21}^M}{a_{11}^L}, \frac{a_{22}^M(1+B_2+\varepsilon_4)}{a_{12}^L}\right\}. \tag{2.25}$$

Thus,

$$\alpha a_{21}^M - \beta a_{11}^L < 0, \qquad \alpha a_{22}^M - \beta \frac{a_{12}^L}{1 + B_2 + \varepsilon_4} < 0 \tag{2.26}$$

and we can choose a constant $\delta_4 > 0$, such that

$$\beta r_1^M - \alpha r_2^L < -\delta_4 < 0. \tag{2.27}$$

For any $p > N$, according to the equations of system (1.4) and (2.7), we can get

$$\begin{aligned} &\ln \frac{x_1(p+1)}{x_1(p)} \le r_1^M - a_{11}^L x_1(p) - \frac{a_{12}^L}{1 + B_2 + \varepsilon_4} x_2(p) - b_1^L x_1(p) x_2(p), \\ &\ln \frac{x_2(p+1)}{x_2(p)} \ge r_2^L - a_{22}^M x_2(p) - a_{21}^M x_1(p). \end{aligned} \tag{2.28}$$

Therefore, inequalities (2.26)-(2.28) show that

$$\begin{aligned} &\beta \ln \frac{x_1(p+1)}{x_1(p)} - \alpha \ln \frac{x_2(p+1)}{x_2(p)} \\ &\quad \le \left(\beta r_1^M - \alpha r_2^L\right) + \left(\alpha a_{21}^M - \beta a_{11}^L\right) x_1(p) + \left(\alpha a_{22}^M - \beta \frac{a_{12}^L}{1 + B_2 + \varepsilon_4}\right) x_2(p) \\ &\qquad - \beta b_1^L x_1(p) x_2(p) \\ &\quad < \beta r_1^M - \alpha r_2^L < -\delta_4 < 0, \quad p > N. \end{aligned} \tag{2.29}$$

The rest of the proof is similar to that of the corresponding proof of Theorem 2.1, we omit the details here. This ends the proof of Theorem 2.4. □

3 Global stability

In Section 2, we get sufficient conditions which guarantee the extinction of the first or second species in system which motives us to investigate the stability property of the rest species. Let us first state several lemmas which will be useful in the proof of the main result of this section.

Lemma 3.1 (see [20]) *Assume that* $\{x(n)\}$ *satisfies*

$$x(n+1) \ge x(n) \exp\bigl\{a(n) - b(n)x(n)\bigr\}, \quad n \ge N_0,$$

$\limsup_{n\to+\infty} x(n) \le x^*$, *and* $x(N_0) > 0$, *where* $a(n)$ *and* $b(n)$ *are nonnegative sequences bounded above and below by positive constants and* $N_0 \in N$. *Then*

$$\liminf_{n\to+\infty} x(n) \ge \min\left\{\frac{a^L}{b^U} \exp\bigl\{a^L - b^U x^*\bigr\}, \frac{a^L}{b^U}\right\}.$$

Lemma 3.2 *Suppose conditions in Theorem* 2.1 *or* 2.2, *or* 2.3 *hold, let* $(x_1(n), x_2(n))^T$ *be any positive solution of system* (1.4), *then*

$$A_1 \le \liminf_{n\to+\infty} x_1(n) \le \limsup_{n\to+\infty} x_1(n) \le B_1,$$

where $A_1 = \frac{r_1^L}{a_{11}^M} \exp\{r_1^L - a_{11}^M B_1\}$ *and* B_1 *is defined in Lemma* 2.1.

Proof It follows from Lemma 2.1 and Theorem 2.1 or 2.2, or 2.3 that

$$\lim_{n\to+\infty} x_2(n) = 0, \qquad \limsup_{n\to+\infty} x_1(n) \le B_1. \tag{3.1}$$

To end the proof of Lemma 3.2, it is enough to show that

$$\liminf_{n\to+\infty} x(n) \ge A_1. \tag{3.2}$$

Since $r_1^L > 0$, there exists a small enough $\varepsilon > 0$ such that

$$A_\varepsilon \triangleq r_1^L - a_{12}^M \varepsilon - b_1^M (B_1 + \varepsilon)\varepsilon > 0. \tag{3.3}$$

According to (3.1), for the above $\varepsilon > 0$, there exists a large enough $N_1 > 0$, such that, for $n \ge N_1$,

$$x_1(n) \le B_1 + \varepsilon, \qquad x_2(n) \le \varepsilon. \tag{3.4}$$

Thus, it follows from (3.4) and the first equation of system (1.4) that

$$x_1(n+1) \ge x_1(n) \exp\bigl\{r_1^L - a_{11}^M x_1(n) - a_{12}^M \varepsilon - b_1^M (B_1 + \varepsilon)\varepsilon\bigr\}. \tag{3.5}$$

Since $A_\varepsilon > 0$, by applying Lemma 3.1 to (3.5), it immediately follows that

$$\liminf_{n\to+\infty} x_1(n) \ge \min\left\{\frac{A_\varepsilon}{a_{11}^M} \exp\bigl\{A_\varepsilon - a_{11}^M B_1\bigr\}, \frac{A_\varepsilon}{a_{11}^M}\right\}.$$

Setting $\varepsilon \to 0$ in the above inequality, one can obtain

$$\liminf_{n\to+\infty} x_1(n) \ge \min\left\{\frac{r_1^L}{a_{11}^M} \exp\bigl\{r_1^L - a_{11}^M B_1\bigr\}, \frac{r_1^L}{a_{11}^M}\right\}. \tag{3.6}$$

By calculation, one can easily get

$$r_1^L - a_{11}^M B_1 = r_1^L - a_{11}^M \frac{\exp(r_1^M - 1)}{a_{11}^L} \le r_1^L - \exp\bigl(r_1^M - 1\bigr) \le r_1^L - r_1^M \le 0. \tag{3.7}$$

Inequality (3.6) together with (3.7) leads to

$$\liminf_{n\to+\infty} x_1(n) \ge \frac{r_1^L}{a_{11}^M} \exp\bigl\{r_1^L - a_{11}^M B_1\bigr\} \triangleq A_1, \tag{3.8}$$

that is to say, (3.2) holds. This ends the proof of Lemma 3.2. □

Lemma 3.3 *Suppose conditions in Theorem* 2.4 *hold, let* $(x_1(n), x_2(n))^T$ *be any positive solution of system* (1.4), *then*

$$A_2 \le \liminf_{n\to+\infty} x_2(n) \le \limsup_{n\to+\infty} x_2(n) \le B_2,$$

where $A_2 = \frac{r_2^L}{a_{22}^M} \exp\{r_2^L - a_{22}^M B_2\}$ *and* B_2 *is defined in Lemma* 2.1.

Proof The proof of Lemma 3.3 is similar to that of the proof of Lemma 3.2, we omit the details here. □

Consider the following discrete logistic equation:

$$x(n+1) = x(n)\exp\bigl(r_1(n) - a_{11}(n)x(n)\bigr), \quad n \in N, \tag{3.9}$$

where $r_1(n)$ and $a_{11}(n)$ are bounded nonnegative sequences.

Lemma 3.4 (see [8]) *For any positive solution $x(n)$ of* (3.9), *we have*

$$A_1 \le \liminf_{n\to+\infty} x(n) \le \limsup_{n\to+\infty} x(n) \le B_1,$$

where A_1, B_1 are defined by Lemma 3.2.

Consider the following discrete logistic equation:

$$x(n+1) = x(n)\exp\bigl(r_2(n) - a_{22}(n)x(n)\bigr), \quad n \in N, \tag{3.10}$$

where $r_2(n)$ and $a_{22}(n)$ are bounded nonnegative sequences.

Lemma 3.5 (see [8]) *For any positive solution $\tilde{x}(n)$ of* (3.10), *we have*

$$A_2 \le \liminf_{n\to+\infty} \tilde{x}(n) \le \limsup_{n\to+\infty} \tilde{x}(n) \le B_2,$$

where A_2, B_2 are defined by Lemma 3.3.

Now, we come to showing the main results of this section.

Theorem 3.1 *Suppose in addition the conditions of Theorem* 2.1 *or* 2.2, *or* 2.3 *hold, further suppose that*

$$(\mathrm{H}_6) \quad \frac{a_{11}^M}{a_{11}^L}\exp\bigl(r_1^M - 1\bigr) < 2.$$

Then for any positive solution $(x_1(n), x_2(n))^T$ of system (1.4), *we have*

$$\lim_{n\to+\infty}\bigl(x_1(n) - x(n)\bigr) = 0, \qquad \lim_{n\to+\infty} x_2(n) = 0,$$

where $x(n)$ is any positive solution of system (3.9).

Proof It follows from Theorem 2.1 or 2.2, or 2.3 that

$$\lim_{n\to+\infty} x_2(n) = 0. \tag{3.11}$$

Set $y(n) = \ln x_1(n) - \ln x(n)$, then it follows from the first equation of system (1.4) and (3.9) that

$$y(n+1) = y(n) - a_{11}(n)x(n)\bigl(\exp\bigl(y(n)\bigr) - 1\bigr) - \frac{a_{12}(n)x_2(n)}{1 + x_2(n)} - b_1(n)x_1(n)x_2(n). \tag{3.12}$$

Using the mean value theorem, we can obtain

$$\exp\bigl(y(n)\bigr) - 1 = \exp\bigl(\theta(n)y(n)\bigr)y(n), \quad \theta(n) \in (0,1). \tag{3.13}$$

Substituting (3.13) into the right side of equation (3.12), we can get

$$y(n+1) = \bigl(1 - a_{11}(n)x(n)\exp\bigl(\theta(n)y(n)\bigr)\bigr)y(n) - \left(\frac{a_{12}(n)}{1 + x_2(n)} + b_1(n)x_1(n)\right)x_2(n). \tag{3.14}$$

Considering (H_6) implies that $-1 < 1 - a_{11}^M B_1$, there exists a small enough $\varepsilon > 0$ such that

$$-1 < 1 - a_{11}^M(B_1 + \varepsilon). \tag{3.15}$$

According to Lemma 3.2, Lemma 3.4, and (3.11), for the above $\varepsilon > 0$, there exists large enough $N > 0$, such that, for $n \geq N$,

$$A_1 - \varepsilon \leq x_1(n) \leq B_1 + \varepsilon, \qquad x_2(n) \leq \varepsilon, \qquad A_1 - \varepsilon \leq x(n) \leq B_1 + \varepsilon. \tag{3.16}$$

Note that $\theta(n) \in (0,1)$ implies that $x(n)\exp(\theta(n)y(n))$ lies between $x(n)$ and $x_1(n)$. From (3.14) and (3.16), for $n \geq N$, one can get

$$\begin{aligned} \bigl|y(n+1)\bigr| &\leq \max\bigl\{\bigl|1 - a_{11}^M(B_1 + \varepsilon)\bigr|, \bigl|1 - a_{11}^L(A_1 - \varepsilon)\bigr|\bigr\}\bigl|y(n)\bigr| \\ &\quad + \bigl(a_{12}^M + b_1^M(B_1 + \varepsilon)\bigr)\varepsilon \\ &\triangleq \lambda_\varepsilon\bigl|y(n)\bigr| + M_\varepsilon\varepsilon, \end{aligned} \tag{3.17}$$

where $\lambda_\varepsilon = \max\{|1 - a_{11}^M(B_1 + \varepsilon)|, |1 - a_{11}^L(A_1 - \varepsilon)|\}$, $M_\varepsilon = a_{12}^M + b_1^M(B_1 + \varepsilon)$. This implies that

$$\bigl|y(n)\bigr| \leq \lambda_\varepsilon^{n-N}\bigl|y(N)\bigr| + \frac{1 - \lambda_\varepsilon^{n-N}}{1 - \lambda_\varepsilon}M_\varepsilon\varepsilon, \quad \text{for } n \geq N. \tag{3.18}$$

Note that $1 - a_{11}^M(B_1 + \varepsilon) \leq 1 - a_{11}^L(A_1 - \varepsilon) < 1$, hence $0 < \lambda_\varepsilon < 1$ according to (3.15). Thus, $\lim_{n\to+\infty} y(n) = 0$ can be immediately obtained by (3.18), and so $\lim_{n\to+\infty}(x_1(n) - x(n)) = 0$. This ends the proof of Theorem 3.1. □

Similarly, by using Lemmas 3.3 and 3.5, we have the following theorem.

Theorem 3.2 *In addition to the conditions of Theorem* 2.4, *further suppose that*

$$(\mathrm{H}_7) \quad \frac{a_{22}^M}{a_{22}^L}\exp\bigl(r_2^M - 1\bigr) < 2.$$

Then for any positive solution $(x_1(n), x_2(n))^T$ *of system* (1.4) *and any positive solution* $\tilde{x}(n)$ *of system* (3.10), *we have*

$$\lim_{n\to+\infty} x_1(n) = 0, \qquad \lim_{n\to+\infty}\bigl(x_2(n) - \tilde{x}(n)\bigr) = 0.$$

As a direct corollary of Theorem 3.1 and Theorem 3.2, we have the following corollary.

Corollary 3.1 *In addition to* (H_1), *further suppose that*

$$(\mathrm{H}_6)\quad \frac{a_{11}^M}{a_{11}^L}\exp\bigl(r_1^M-1\bigr)<2.$$

Then for any positive solution $(x_1(n),x_2(n))^T$ *of system* (1.1) *and any positive solution* $x(n)$ *of system* (3.9), *we have*

$$\lim_{n\to+\infty}\bigl(x_1(n)-x(n)\bigr)=0,\qquad \lim_{n\to+\infty}x_2(n)=0.$$

That is to say, the species x_2 *will be driven to extinction.*

Corollary 3.2 *Assume that the conditions of Theorem* 2.4 *hold, also*

$$(\mathrm{H}_7)\quad \frac{a_{22}^M}{a_{22}^L}\exp\bigl(r_2^M-1\bigr)<2.$$

Then for any positive solution $(x_1(n),x_2(n))^T$ *of system* (1.1) *and any positive solution* $\tilde{x}(n)$ *of system* (3.10), *we have*

$$\lim_{n\to+\infty}x_1(n)=0,\qquad \lim_{n\to+\infty}\bigl(x_2(n)-\tilde{x}(n)\bigr)=0.$$

That is to say, the species x_1 *will be driven to extinction.*

4 Examples and numeric simulation

In this section, we give the following two examples to verify the feasibilities of our results.

Example 4.1 Consider the following system:

$$\begin{aligned}
x_1(n+1)&=x_1(n)\exp\biggl\{1.2-1.5x_1(n)-\frac{(1+0.3\sin(\sqrt{7}n))x_2(n)}{1+x_2(n)}-b_1(n)x_1(n)x_2(n)\biggr\},\\
x_2(n+1)&=x_2(n)\exp\biggl\{0.6-1.3x_2(n)-\frac{(4+\cos(\sqrt{3}n))x_1(n)}{1+x_1(n)}\biggr\}.
\end{aligned}\tag{4.1}$$

Case 1. $b_1(n)=0.2$.

Take easy calculation, we have $\frac{r_2^M}{r_1^L}=0.5$, $B_1\approx 0.8143$, $B_2\approx 0.5156$, $\frac{a_{21}^L}{a_{11}^M(1+B_1)}\approx 1.1024$, $\frac{a_{22}^L}{a_{12}^M}=1$, $r_1^L-r_2^M\frac{a_{11}^M(1+B_1)}{a_{21}^L}\approx 0.6476$, $r_1^L-r_2^M\frac{a_{12}^M}{a_{22}^L}=0.6$, thus

$$\frac{r_2^M}{r_1^L}=0.5<\min\biggl\{\frac{a_{21}^L}{a_{11}^M(1+B_1)},\frac{a_{22}^L}{a_{12}^M}\biggr\}=1\tag{4.2}$$

and

$$\begin{aligned}
b_1^M&=0.2<\frac{1}{B_1B_2}\min\biggl\{r_1^L-r_2^M\frac{a_{11}^M(1+B_1)}{a_{21}^L},r_1^L-r_2^M\frac{a_{12}^M}{a_{22}^L}\biggr\}\approx 1.4291,\\
b_1^M&=0.2<\frac{r_1^La_{21}^L-(1+B_1)a_{11}^Mr_2^M}{(1+B_1)B_2r_2^M}\approx 3.5048,
\end{aligned}\tag{4.3}$$

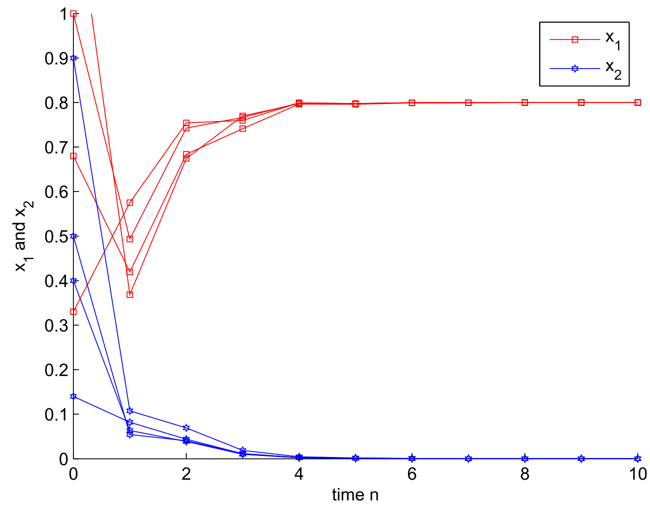

Figure 1 Dynamical behaviors of the system (4.1) with $b_1(n) = 0.2$ and the initial conditions $(x_1(0), x_2(0)) = (1, 0.4)^T$, $(0.33, 0.14)^T$, $(0.68, 0.9)^T$, and $(1.3, 0.5)^T$, respectively.

$$b_1^M = 0.2 < \frac{r_1^L a_{22}^L - r_2^M a_{12}^M}{B_1 r_2^M} \approx 1.5965,$$

(4.2)-(4.3) show that the coefficients of the system (4.1) satisfy the conditions of Theorems 2.1, 2.2, and 2.3. Moreover,

$$\frac{a_{11}^M}{a_{11}^L} \exp\left(r_1^M - 1\right) = \exp(1.2 - 1) \approx 1.2214 < 2.$$

Hence, condition (H_6) is also satisfied. It follows from Theorem 3.1 that, for any positive solution $(x_1(n), x_2(n))^T$ of system (4.1), we have $\lim_{n\to+\infty}(x_1(n) - x(n)) = 0$, $\lim_{n\to+\infty} x_2(n) = 0$, where $\{x(n)\}$ is any positive solution of the system

$$x(n+1) = x(n)\exp\left\{1.2 - 1.5x(n)\right\}.$$

Our numerical simulation supports our result (see Figure 1).

Case 2. $b_1(n) = 0$.

$b_1(n) = 0$ shows that the two species are all non-toxic. One can easily find that the conditions in Corollary 3.1 are satisfied, so for any positive solution $(x_1(n), x_2(n))^T$ of system (4.1) with $b_1(n) = 0$, $x_2(n)$ is in extinction while $x_1(n)$ will be globally attractive (see Figure 2).

Example 4.2 Consider the following system:

$$\begin{aligned} x_1(n+1) &= x_1(n)\exp\left\{0.3 - 1.5x_1(n) - \frac{(2.2 + 0.2\sin(\sqrt{5}n))x_2(n)}{1 + x_2(n)}\right. \\ &\qquad \left. - b_1(n)x_1(n)x_2(n)\right\}, \\ x_2(n+1) &= x_2(n)\exp\left\{1.2 - 1.3x_2(n) - \frac{(2 + \cos(\sqrt{7}n))x_1(n)}{1 + x_1(n)}\right\}. \end{aligned} \tag{4.4}$$

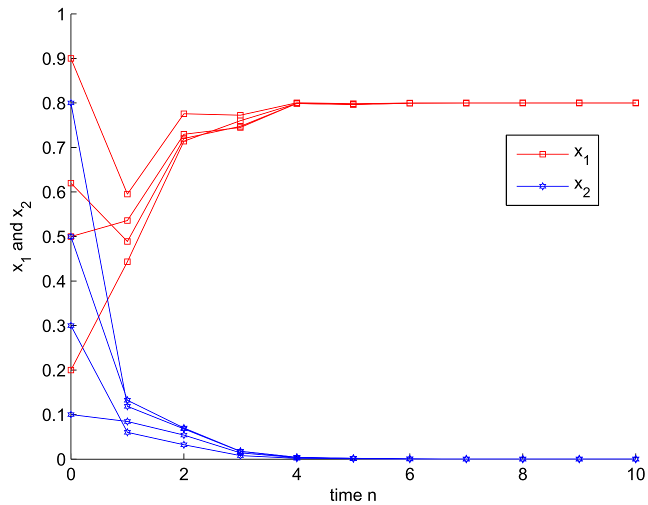

Figure 2 Dynamical behaviors of the system (4.1) with $b_1(n) = 0$ and the initial conditions $(x_1(0), x_2(0)) = (1, 0.4)^T$, $(0.33, 0.14)^T$, $(0.68, 0.9)^T$, and $(1.3, 0.5)^T$, respectively.

Case 1. $b_1(n) = 0.3$.

In this case, one could easily see that $\frac{r_2^L}{r_1^M} = 4$, $B_2 \approx 0.9395$, $\frac{a_{21}^M}{a_{11}^L} = 2$, $\frac{a_{22}^M(1+B_2)}{a_{12}^L} \approx 1.2607$, so

$$\frac{r_2^L}{r_1^M} = 4 > \max\left\{\frac{a_{21}^M}{a_{11}^L}, \frac{a_{22}^M(1 + B_2)}{a_{12}^L}\right\} = 2 \tag{4.5}$$

and

$$\frac{a_{22}^M}{a_{22}^L}\exp\bigl(r_2^M - 1\bigr) = \exp(1.2 - 1) \approx 1.2214 < 2, \tag{4.6}$$

(4.5) and (4.6) mean that all conditions of Theorem 3.2 are satisfied in system (4.4). Thus, for any positive solution $(x_1(n), x_2(n))^T$ of system (4.4) and any positive solution $\{\tilde{x}(n)\}$ of system (3.10), we have $\lim_{n\to+\infty} x_1(n) = 0$, $\lim_{n\to+\infty}(x_2(n) - \tilde{x}(n)) = 0$, where $\{\tilde{x}(n)\}$ is any positive solution of the system

$$x(n + 1) = x(n)\exp\bigl\{1.2 - 1.3x(n)\bigr\}.$$

Figure 3 shows the dynamical behavior of system (4.4) with $b_1(n) = 0.3$.

Case 2. $b_1(n) = 0$.

$b_1(n) = 0$ shows that the two species are all non-toxic. One can easily find that the conditions in Corollary 3.2 are satisfied, so for any positive solution $(x_1(n), x_2(n))^T$ of system (4.4) with $b_1(n) = 0$, $x_1(n)$ is in extinction while $x_2(n)$ will be globally attractive (see Figure 4).

5 Discussion

In this paper, we consider a two species nonautonomous discrete competitive system with nonlinear inter-inhibition terms and one toxin producing species, *i.e.*, (1.4). By developing the analysis technique of Chen *et al.* [8], sufficient conditions which guarantee the

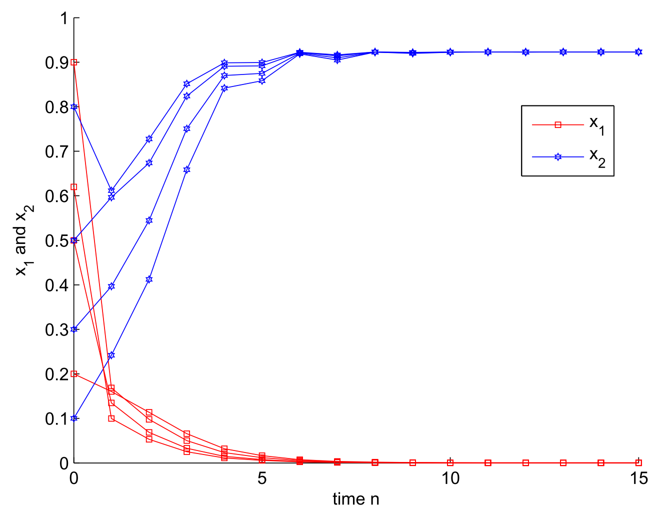

Figure 3 Dynamical behaviors of the system (4.2) with $b_1(n) = 0.3$ and the initial conditions $(x_1(0), x_2(0)) = (0.9, 0.3)^T$, $(0.2, 0.1)^T$, $(0.62, 0.8)^T$, and $(0.5, 0.5)^T$, respectively.

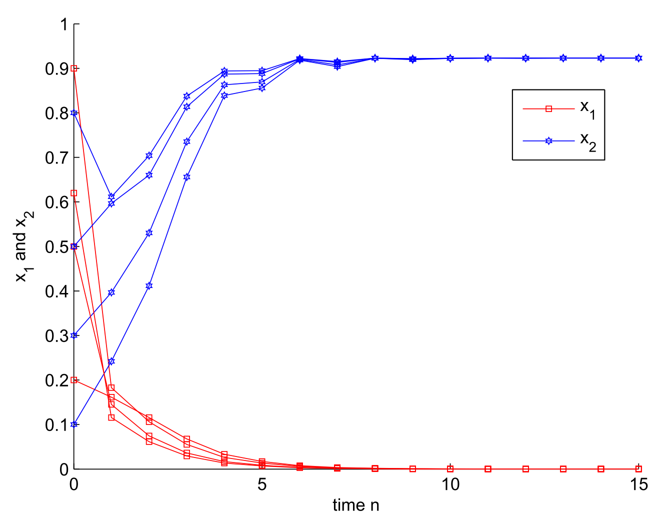

Figure 4 Dynamical behaviors of the system (4.2) with $b_1(n) = 0$ and the initial conditions $(x_1(0), x_2(0)) = (0.9, 0.3)^T$, $(0.2, 0.1)^T$, $(0.62, 0.8)^T$, and $(0.5, 0.5)^T$, respectively.

extinction of one of the two species are obtained and the stability property of the other species are proved. As direct results of Theorem 3.1 and Theorem 3.2, Corollaries 3.1 and 3.2 show the same conclusions for a non-toxic system, which supplements the results of [1, 2]. Moreover, by comparing Theorem 3.1 with Corollary 3.1, and Theorem 3.2 with Corollary 3.2, we also found that, for such a kind of system, a lower rate of toxic production has no influence on the extinction property of the system.

Competing interests
The author declares that they have no competing interests.

Author's contributions
The author wrote the manuscript carefully, and read and approved the final manuscript.

Acknowledgements
The author would like to thank the anonymous referees and the editor for their constructive suggestions on improving the presentation of the paper. Also, this research was supported by Anhui Province College Excellent Young Talents Support Plan Key Projects (No. gxyqZD2016240).

Received: 10 October 2015 Accepted: 21 December 2015 Published online: 04 January 2016

References

1. Qin, W, Liu, Z, Chen, Y: Permanence and global stability of positive periodic solutions of a discrete competitive system. Discrete Dyn. Nat. Soc. **2009**, Article ID 830537 (2009)
2. Wang, Q, Liu, Z: Uniformly asymptotic stability of positive almost periodic solutions for a discrete competitive system. J. Appl. Math. **2013**, Article ID 182158 (2013)
3. Wang, Q, Liu, Z, Li, Z: Positive almost periodic solutions for a discrete competitive system subject to feedback controls. J. Appl. Math. **2013**, Article ID 429163 (2013)
4. Yu, S: Permanence for a discrete competitive system with feedback controls. Commun. Math. Biol. Neurosci. **2015**, Article ID 16 (2015)
5. Li, Z, Chen, F: Extinction in two dimensional discrete Lotka-Volterra competitive system with the effect of toxic substances. Dyn. Contin. Discrete Impuls. Syst., Ser. B, Appl. Algorithms **15**, 165-178 (2008)
6. Solé, J, Garca-Ladona, E, Ruardij, P, Estrada, M: Modelling allelopathy among marine algae. Ecol. Model. **183**, 373-384 (2005)
7. Bandyopadhyay, M: Dynamical analysis of a allelopathic phytoplankton model. J. Biol. Syst. **14**, 205-217 (2006)
8. Chen, F, Gong, X, Chen, W: Extinction in two dimensional discrete Lotka-Volterra competitive system with the effect of toxic substances (II). Dyn. Contin. Discrete Impuls. Syst., Ser. B, Appl. Algorithms **20**, 449-461 (2013)
9. Chattopadhyay, J: Effect of toxic substances on a two-species competitive system. Ecol. Model. **84**, 287-289 (1996)
10. Mukhopadhyay, A, Chattopadhyay, J, Tapaswi, PK: A delay differential equations model of plankton allelopathy. Math. Biosci. **149**, 167-189 (1998)
11. Li, Z, Chen, F: Extinction in two dimensional nonautonomous Lotka-Volterra systems with the effect of toxic substances. Appl. Math. Comput. **182**, 684-690 (2006)
12. Chen, F, Li, Z, Chen, X, Laitochov, J: Dynamic behaviors of a delay differential equation model of plankton allelopathy. J. Comput. Appl. Math. **206**, 733-754 (2007)
13. Liu, Z, Chen, L: Positive periodic solution of a general discrete nonautonomous difference system of plankton allelopathy with delays. J. Comput. Appl. Math. **197**, 446-456 (2006)
14. Liu, Z, Chen, L: Periodic solution of a two-species competitive system with toxicant and birth pulse. Chaos Solitons Fractals **32**, 1703-1712 (2007)
15. Liu, Z, Wu, J, Chen, Y, Haque, M: Impulsive perturbations in a periodic delay differential equation model of plankton allelopathy. Nonlinear Anal., Real World Appl. **11**, 432-445 (2010)
16. Tian, C, Lin, Z: Asymptotic behavior of solutions of a periodic diffusion system of plankton allelopathy. Nonlinear Anal., Real World Appl. **11**, 1581-1588 (2010)
17. Li, Z, Chen, F, He, M: Asymptotic behavior of the reaction-diffusion model of plankton allelopathy with nonlocal delays. Nonlinear Anal., Real World Appl. **12**, 1748-1758 (2011)
18. Li, Z, Chen, F, He, M: Global stability of a delay differential equations model of plankton allelopathy. Appl. Math. Comput. **218**, 7155-7163 (2012)
19. Chen, L, Chen, F: Extinction in a discrete Lotka-Volterra competitive system with the effect of toxic substances and feedback controls. Int. J. Biomath. **8**, 1550012 (2015)
20. Chen, F: Permanence for the discrete mutualism model with time delays. Math. Comput. Model. **47**, 431-435 (2008)