Extraction of Knowledge with Population-Based Metaheuristics Fuzzy Rules Applied to Credit Risk

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Abstract. One of the goals of financial institutions is to reduce credit risk. Consequently they must properly select customers. There are a variety of methodologies for credit scoring, which analyzes a wide variety of personal and financial variables of the potential client. These variables are heterogeneous making that their analysis is long and tedious. This paper presents an alternative method that, based on the subject information, offers a set of classification rules with three main characteristics: adequate precision, low cardinality and easy interpretation. This is because the antecedent consists of a small number of attributes that can be modeled as fuzzy variables. This feature, together with a reduced set of rules allows obtaining useful patterns to understand the relationships between data, and make the right decisions for the financial institutions. The smaller the number of analyzed variables of the potential customer, the simpler the model will be. In this way, credit officers may give an answer to the loan application in the shorter time, achieving a competitive advantage for the financial institution. The proposed method has been applied to two databases from the UCI repository, and a database from a credit unions cooperative in Ecuador. The results are satisfactory, as highlighted in the conclusions. Some future lines of research are suggested.

Keywords: VarPSO (Variable Particle Swarm Optimization)
FR (Fuzzy Rules) credit risk

1 Introduction

Currently, people apply for a wide variety of loans in financial institutions: commercial loans, consumer loans, mortgages, and microcredits. This leads to financial institutions to analyze a large number of micro-economic variables that allow assess the customer,

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and thus give an answer about the access to financial resources. This assessment should advice financial institutions on the amount of the loan and the repayment period. Thanks to technological progress, operations are recorded automatically, giving rise to large repositories of historical information. This records contain not only financial information from customers, but also the result of decisions made, which motivates the interest in learning from past situations, seeking to identify the selection criteria. The aim of this paper is modeling credit risk through classification rules. Proper identification of the most important features will help the credit officer in the decision making process, conducting analysis of the subject of credit in less time. This paper presents a methodology for credit risk which allows obtaining classification rules. The antecedent of the rule is formed by fuzzy variables and nominal variables that contain the knowledge of the credit expert in the database, basically in the membership function that is assigned to each of the fuzzy variables. The use of fuzzy variables will allow credit officer to interpret them more easily and can make decisions properly. To measure the performance of the proposed method, different solutions are analyzed; especially considering the simplicity of the model regarding to:

Cardinality of rules: the lower the number of rules, the better to analyze the generated model,

Average length of the antecedent of the rules and type of variables: the fewer conditions used to form the antecedent of each rule, using fuzzy variables, more easily will be the interpretation of the model.

An association rule is an expression of the form *IF condition1 and condition2 THEN condition3* where condition1 and condition2 may contain fuzzy variables, to allow for a conjunction of propositions of the form (attribute IN fuzzy set X), and whose sole restriction is that the attributes involved in the antecedent of the rule are not part of the consequent. Attributes may be nominal and fuzzy. When the set of association rules shows in the consequent the same attribute, is said that it is a set of classification rules [2, 14].

This article presents a method for obtaining classification rules that combines a neural network with an optimization technique. Emphasis is put on achieving good coverage using a small number of rules, whose antecedent includes fuzzy variables. In this sense it is an extension of previous works [18, 19, 20], aimed at the identification of better classification methods for credit scoring.

The organization is the following: Sect. 2 briefly describes some related work; Sect. 3 develops the proposed method; Sect. 4 presents the results. Finally, Sect. 5 summarizes the findings and describes some future lines of work.

2 Related Work

In the 1960's, the development of the capital markets in United States, showed the need for more scientific models to evaluate the corporate economic and financial 'health'. As a result, Altman [3] developed the first model, known as z score. A survey of techniques applied in the financial area published towards the end of the 1990s [4], does not provide explicit reports of the application of hazard rate models [16] or partial likelihood [17]. However, the survey gives evidence of the use of statistical techniques such as probit and logit, together with techniques of state transition, and other so-called "derivation of actuarial-like probability of default" associated with the bond default. In the following decade, there were developments of application of survival analysis to the measurement of the credit risk [5, 11, 12]. In Latin America, savings and credit cooperatives are considered as a growing industry. It is usual the association between a financial institution with a household appliances store, in order to offer customers quick credit a line. The existence of such financial instrument helps to increase sales. This partnership creates a conflict of interest. On the one hand, the appliance store wants to sell products to all customers; so it is interested in promoting an attractive credit policy. On the other hand, the financial institution wants to maximize revenue from loans, leading to a strict surveillance of the losses on loans. The ideal situation is the existence of transparent policies between appliances shops and financial institutions. There is also the case of financial institutions that grant credits for consumption or production, and also whose goal is the minimization of credit risk. One way of developing such a policy is the construction of objective rules in order to decide to grant or deny a credit application.

Using intelligent computational techniques could produce better results. These techniques, without being exhaustive, include artificial neural networks, theory of fuzzy sets, decision trees, vector support machines, genetic algorithms, among others. In regard to neural networks, there are different architectures, depending on the type of problem to solve. These architectures include popular models, such as back propagation networks, self-organizing maps (SOM) and learning vector quantization (LVQ). Fuzzy sets theory, developed from the seminal work by Zadeh [15] is very useful in cases such as the classification of credit, where the boundaries are not well defined. The data can also be structured in the form of trees, with their respective branches, where the objective is to test the attributes of each branch of the tree. It can also be used support vector machines that, according to the type of discriminant function, enable to build extremely powerful linear and non-linear models. Genetic algorithms as well as particles swarm optimization of particles, are population-based optimization techniques inspired by various biological processes.

If the goal is to obtain association rules, the a priori method [1] or any of its variants can be used. This method is responsible for identifying the sets of attributes that are more common in different nominal, numerical and fuzzy representations. Then it combines them to obtain a set of rules. There are variants of the a priori method that are responsible for reducing computing time. When working with classification rules, the literature identifies different tree-based methods such as the C4.5 [10] or pruned trees as PART [6]. In either case, the fundamental thing is to obtain a set of rules covering the examples, and fulfilling a preset error bound. Tree-based construction methods, which splits the set of samples into subsets, are based on different metrics of the attributes in order to estimate their coverage ability.

3 Methodology

This article presents a hybrid methodology based on the combination of fuzzy rules, optimization by particles swarms of variable population (varPSO), along with LVQ competitive neural networks, which are used to begin the search in promising sectors of the search space. While there are methods for obtaining of rules using the PSO [9], in the first part of this methodology, numeric attributes are fuzzified. In doing so, membership functions are set for each of them. The limits will be defined by credit expert. Nominal attributes are not subject to fuzzification. In this work, we compared the performance of various methods using fuzzy and nominal attributes that combine fixed and variable population. PSO begins with two competitive neural networks, LVQ and SOM. The optimization technique is used to identify the numerical fuzzy and nominal attributes that are more representative. They will form the antecedent of the rules. In other words, the optimization technique is responsible for generating the rules that will be incorporated into the system based on fuzzy rules, with the aim of obtaining good accuracy, interpretability and cardinality.

3.1 Learning Vector Quantization (LVQ)

Learning Vector Quantization (LVQ) is a supervised classification algorithm based on centroids or prototypes [Kohonen, 1990]. This algorithm can be interpreted as a competitive neural network composed of three layers. The first layer is only input. The second is where the competition takes place. The third layer is the output, responsible for the classification. Each neuron in the competitive layer carries a numeric vector of equal dimension than the examples of input and a label which indicates the class which is going to represent. These vectors are the ones that, at the end of the adaptive process contain the information of the centroids or prototypes of the classification. There are different versions of the training algorithm. In the following paragraph, we describe the one used in this article.

At the start of the algorithm, the quantity of K centroids to be used, should be indicated. This allows to define the network's architecture where the number of inputs and outputs are defined by the problem to be solved.

Centroids are initialized by selecting K random examples. Then each of the examples is entered and adapt the position of the centroids. The closest centroid is identified, using a preset distance metric. As it is a supervised process, it is possible to determine if the example and the centroid correspond to the same class. If the centroid and the example belong to the same class, the centroid "approaches" to the example with the objective of strengthening the representation. On contrary, if the classes are different, the centroid "moves away" of the example. These movements are performed using a factor or adaptation speed, which allows to consider the step that is to be performed.

This process is repeated until the change lies below a preset threshold, or until the examples are identified with the same centroids in two consecutive iterations, whichever comes first.

As a variant on the implementation in this article, it is also considered to the second closest centroid, provided that the class to which they belong is different from the example analyzed, and is located at a distance less than 1.2 times the distance of the first centroid, due to the factor of inertia that was established previously and the applied "detachment". Variations of LVQ can be found in [8].

3.2 Fuzzy Rules (FR)

Fuzzy logic is derived from the theory of fuzzy sets. It takes as a basis the human reasoning, which is approximate, considering that it can be taken as an alternative to classical logic. Fuzzy logic enables to handle human reasoning, interpreting better the inaccurate real world. For example, we can be considered the use of vague data in the analysis of credit management. For example, the variable income "USD. 4000", can be considered as "High income with a membership of 0.3", and as "Median with a membership of 0.6". To provide the membership level of the fuzzy set, we should work with experts, since they know the system. When the antecedent of the rule consists of variables that use the conjunction operator for various conditions, the min or product operator between degrees of membership of the variables can be used.

3.3 Particle Swarm Optimization (PSO)

Particle Swarm Optimization (PSO) is a population metaheuristic proposed by Kennedy and Eberhart [7]. Each individual of the population, called particle, represents a possible solution to the problem, and fits following three factors: its knowledge about the environment (its fitness value), its historical knowledge or previous experiences (memory) and the historical knowledge or past experience of the individuals located in its neighborhood (its social knowledge).

Obtaining classification rules using the PSO, able to operate on numerical, nominal and fuzzy attributes requires a combination of some methods mentioned above, because it is necessary to determine the attributes that will be part of the antecedent. In the case of fuzzy variables, it is necessary to know the membership degree of them.

Taking into account that it is a population-based technique, it is necessary to analyze the required information in each individual of the population. Additionally, we must decide between represent a single rule or the complete set of rules by individual. Finally, we have to choose the scheme of representation of each rule. According to the goals of this work, we follow the Iterative Rule Learning (IRL) [13], in which each individual represents a single rule, and the solution of the problem is built from the best individuals obtained in a sequence of executions. Using this approach implies that population technique is applied iteratively until reaching the required coverage, obtaining a single rule at each iteration: the best individual of the population. It has also decided to use a fixed-length representation, where the antecedent of the rule will only be encoded. Given this approach, we will follow an iterative process involving all individuals of the population with a class by default, which do not requires the encoding of the consequent. PSO uses fuzzified variables, reducing the amount of attributes to choose, which form the antecedent of the rule. Additionally, it uses a criterion of "voting": whenever the fitness function is evaluated, the average degree of membership of the examples that abide by the rule is computed. This information is also used in the movement of the individual.

3.4 Proposed Method for Obtaining Rules: Fuzzy Variables + LVQ + PSO

The sets are determined according to the knowledge of the experts. Fuzzy sets can be represented by triangular or trapezoidal functions, depending on the variable. For example, "age" is represented by a triangular function, since it was defined as young, middle and old. The variable "number of children" is represented by a trapezoidal function. When the variable is equal to 0 or 1, the membership degree to the "low set" is equal to 1. When the variable is equal to 3 the membership is 0.5 to the low set and 0.5 high set. Finally when the variable is equal to or greater than 4, the membership degree is 1 to the high set.

To obtain the rules we use fuzzy variables. Such rules are obtained through an iterative process that analyzes examples that have not been covered by each of the classes starting with those that have higher number of elements. Then an average degree of membership of the examples that satisfy the rule is computed. When a rule has been obtained, the set of examples covered by the generated rule is removed from the input database. This process is performed iteratively, until it reaches to the maximum number of iterations, or until all examples are covered or until the number of examples of each of the resulting classes are considered too few. When the examples are covered by the generated rule, they are removed from the input data set. In order to classify a new example, rules must be applied in the order they were obtained, and the example will be classed with the class corresponding to the consequent of the first rule whose antecedent is verified for the example under examination. Even though, the original data are numerical and nominal, neural networks use numerical attributes. Therefore, the nominal variables are encoded in such a way that each of them has as many binary digits as different values have. Numeric attributes are scaled between 0 and 1. The membership degree of the fuzzy variables defined above, can be treated as nominal or numeric. The similarity measure used is the Euclidean distance. Once the training is completed, each centroid will contain roughly the average of the examples that it represents. For obtaining each of the rules, we need to determine, first of all, the class corresponding to the consequent. In this way it is obtained the rules with high support. The minimum support of each of the classes decreases in the iteration process, as long as the examples of the corresponding class are covered. Consequently, the first generated rules have greater supports. Figure 1 shows the pseudocode of the proposed method.

4 Results

This section benchmarks the performance of the proposed method, with PART and C4.5. This empirical validation is done on two public databases of credit application from the UCI repository, and a database from a savings and credit cooperative from Ecuador. This cooperative is classified as segment 2 by the Superintendency of Popular and Solidary Economy (regulatory authority), given that its assets are between 20,000,000.00 and 80,000,000.00 USD. Regarding the last database, the following variables of the applicants were considered: year and month of credit application,

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Determine the fuzzy set of each of the variables
Determine the membership function for each of the variables in
each of the defined fuzzy sets.
Train the LVQ network using the training examples
Determine the minimum support for each of the classes
Iteration = 0
While stop criterion is not achieve do
     Choose the class that has the largest uncovered examples
     Construct a population considering LVQ centroids
     Evolve the population using variable population PSO, con-
     sidering the average of the membership degree of the exam-
     ples tan comply with the rule
     Obtain the best population rule
     If the rule complies with the support and confidence then
          Add the rule to the rule set
          Consider as covered the examples that are covered by
          the rule
          Recalculate the minimum support of the considered
          class
      Iteration = Iteration + 1
      End If
End While
```

Fig. 1. Pseudocode of the proposed method

province, loan's purpose, cash savings, total income, total assets, total expenses and total debt. It is also known if the requests for credit was denied or approved.

In the case of the UCI repository databases, triangular fuzzy sets were defined for continuous numerical variables, and trapezoidal fuzzy sets for discrete variables. We used three sets for each continuous numeric variable and two fuzzy sets for discrete numeric variables. To define them, the range of each variable was divided in an equitable manner. Regarding the Ecuadorian database, an expert in the area of credit risk was asked to define fuzzy sets for each of the variables, according to the economy of Ecuador. The following attributes were fuzzified: amount requested, cash savings, total income, total expenses, total assets, and total debts.

We processed the data described above, and compared the performance of several methods that combine two types of PSO, one of fixed population and other variable population, initialized with two different competitive neural networks: LVQ and SOM. These solutions are compared with the C4.5 and PART methods. The way of finding classification rules in the proposed and control methods are different. C4.5 is a pruned tree method, whose branches are mutually exclusive, and allow to classify examples. PART gives a list of rules equivalent to those generated by the proposed method of classification, but in a deterministic way. PART performance is based on the construction of partial trees. Each tree is created in a similar manner to C4.5, but during the process construction errors of each branch are calculated. These errors determine tree pruning.

The proposed method uses random values that makes the movement of the particle not overly deterministic, as is the case with PART. The most important feature of the results obtained, is the combination of an attribute search algorithm (which may be diffuse, numerical or qualitative), with a competitive neural network. As a consequence, we obtain a set of rules with fuzzy variables in the antecedent with a significantly low cardinality (fewer rules). The proposed solution provides greater accuracy, with a reduced set of rules, which makes it easier to understand. The accuracy of the classification obtained in PSO is good. Thus the proposed method meets the objective: that the credit officer can respond fast and accurately, verifying the fewest rules. We believe that this method is an excellent alternative to be used in financial institutions. Results are displayed in Tables 1, 2 and 3

Table 1. Results of fuzzy rules with the Australian database – UCI Repository

Method	Type of prediction	Denied	Accepted	Precision	#rules	Antecedent length
SOM + fuzzy PSO	Denied	0,4472 0.0081	0.1154 0.0211	0.8550	3.0083	1.3076
	Accepted	0,0295 0.0050	0.4079 0.0211	0.0131	0.0009	0.1433
SOM + fuzzy varPSO	Denied	0.4526 0.0101	0.0787 0.0112	0.8957	3.0000	1.3333
	Accepted	0.0255 0.0066	0.4430 0.0120	0.0098	0.0000	0.0896
LVQ + fuzzy PSO	Denied	0.4504 0.0113	0.1066 0.0095	0.8578	3.0000	1.2897
	Accepted	0.0356 0.0060	0.4074 0.0161	0.0109	0.0000	0.2254
LVQ + fuzzy varPSO	Denied	0.4547 0.0123	0.1022 0,0092	0.8689	3.0000	1.4511
	Accepted	0.0288 0.0088	0.4142 0.0178	0.0122	0.0000	0.1493
C4.5	Denied	0.4618 0.0063	0,0847 0.0066	0.8528	18.2200	4.8394
	Accepted	0.0625 0.0120	0.3910 0.0121	0.0124	2.0825	0.2810
PART	Denied	0.3906 0.0288	0.1562 0.0289	0.7469	33.343	2.4926
	Accepted	0.0969 0.0134	0.3564 0.0136	0.0292	1.5793	0.0934

Table 2. Results of fuzzy rules using German database – UCI Repository

Method	Type of prediction	Denied	Accepted	Precision	#rules	Antecedent length
SOM + fuzzy PSO	Denied	0,6031 0.0160	0.0896 0.0201	0.7636	7.7612	2.7926
	Accepted	0,1459 0.0133	0.1605 0.0107	0.0101	0.6540	0.1449
SOM + fuzzy varPSO	Denied	0.5920 0.0131	0.0915 0.0100	0.7697	8.1848	2.8433
	Accepted	0.1385 0.0190	0.1777 0.0111	0.0081	0.5141	0.4493
LVQ + fuzzy PSO	Denied	0.6009 0.0128	0.0961 0.0109	0.7578	8.3595	2.9163
	Accepted	0.1461 0.0078	0.1569 0.0068	0.0091	0.6087	0.2996
LVQ + fuzzy varPSO	Denied	0.5992 0.0132	0.0985 0,0324	0.7592	8.4120	2.6937
	Accepted	0.1418 0.0133	0.1601 0.0124	0.0058	0.5641	0.1921
C4.5	Denied	0.5894 0.0070	0,1106 0.0070	0.7113	86.4600	5.6267
	Accepted	0.1781 0.0069	0.1219 0.0069	0.0079	4.0788	0.1382
PART	Denied	0.5185 0.0091	0.1687 0.0135	0.6940	70.913	3.0138
	Accepted	0.1372 0.0170	0.1754 0.0120	0.0139	2.1575	0.0561

Table 3. Result of fuzzy rules with data from a savings and credit cooperative from Ecuador, belonging to segment 2 of Superintendencia de Economía Popular y Solidaria (assets between 20′ 000.000,00 and 80′000.000,00 USD)

Method	Type of prediction	Denied	Accepted	Precision	#rules	Antecedent length
SOM + fuzzy PSO	Denied	0,6288 0.0098	0.1067 0.0065	0.8142	3.5925	3.0164
	Accepted	0,0785 0.0078	0.1854 0.0065	0.0062	0.2147	0.2459
SOM + fuzzy varPSO	Denied	0.6238 0.0102	0.0825 0.0128	0.8332	3.9957	2.4328
	Accepted	0.0829 0.0057	0.2094 0.0092	0.0026	0.2968	0.2367
LVQ +fuzzy PSO	Denied	0.6129 0.0143	0.1043 0.0153	0.7448	3.1214	2.1427
	Accepted	0.0778 0.0118	0.1819 0.0043	0.0070	0.2272	0.1971

(*continued*)

Table 3. (*continued*)

Method	Type of prediction	Denied	Accepted	Precision	#rules	Antecedent length
LVQ + fuzzy varPSO	Denied	0.6298 0.0056	0.0780 0,0127	0.8444	4.1498	2.3770
	Accepted	0.0775 0.0094	0.2146 0.0091	0.0089	0.2787	0.1145
C4.5	Denied	0.6320 0.0014	0,1075 0.0013	0.8106	114.2600	9.6752
	Accepted	0.0819 0.0013	0.1786 0.0013	0.0011	6.0543	0.1144
PART	Denied	0.6229 0.0065	0.1036 0.0064	0.8054	42.3567	4.6956
	Accepted	0.0910 0.0065	0.1825 0.0064	0.0023	2.1661	0.0880

5 Conclusions

In this paper, we present a new method of classification rules, whose antecedent is formed by fuzzy variables. We apply this method to the analysis of credit risk, combining competitive neural networks (SOM and LVQ) and population-based optimization techniques (PSO and varPSO). To verify the performance of this method, we used two credit databases. One database is in the UC Irvine Machine Learning Repository. The other database is from a savings and credit cooperative from Ecuador. The results have been satisfactory. The measurements reached by the proposed method has a reduced rule set, which could be used by the credit officer with very good accuracy.

This technique can be considered an optimal model for the credit officer in determining the credit scoring as numerical, nominal and fuzzy attributes from credit applications are being used. A limited number of rules are obtained, whose antecedent is formed by fuzzy variables, facilitating the understanding of the model. Credit officers can assess credit applications in a shorter time frame, with more accuracy, leading to a decrease of credit risk. In future lines of research, we would consider adding to the model the defuzzification of the output variable, indicating the percentage of risk involved in granting the credit. Additionally, we would like to combine in the antecedent of the rule macroeconomic and microeconomic variables, which allow a simpler model while maintaining an adequate accuracy.

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