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Extraction of m_s and $|V_{us}|$ from Hadronic Tau Decays [1]

Elvira Gámiz[a)], **Matthias Jamin**[b)], **Antonio Pich**[c)], **Joaquim Prades**[d)] **and Felix Schwab**[b,e)]

[a)] Department of Physics & Astronomy, University of Glasgow
Glasgow G12 8QQ, United Kingdom.

[b)] Physik Department, Technische Universität München
D-85747 Garching, Germany.

[c)] Departament de Física Teòrica, IFIC, Universitat de València–CSIC
Apt. de Correus 22085, E-46071 València, Spain.

[d)] Centro Andaluz de Física de las Partículas Elementales (CAFPE) and
Departamento de Física Teórica y del Cosmos, Universidad de Granada
Campus de Fuente Nueva, E-18002 Granada, Spain.

[e)] Max-Planck-Institut für Physik – Werner-Heisenberg-Institut
D-80805 München, Germany.

Abstract

We review recent work to determine the strange quark mass m_s as well as the proposal to determine $|V_{us}|$ using hadronic τ decay data. The recent update of the strange spectral function by OPAL and their moments of the invariant mass distribution are employed. Our results are $|V_{us}| = 0.2208 \pm 0.0034$ and $m_s(2\,\mathrm{GeV}) = 81 \pm 22$ MeV. Our result is already competitive to the standard extraction of $|V_{us}|$ from K_{e3} decays and to the new proposals to determine it. The error on $|V_{us}|$ is dominated by experiment and will be eventually much improved by the B-factories hadronic τ data. Ultimately, a simultaneous fit of both m_s and $|V_{us}|$ to a set of moments of the hadronic τ decays invariant mass distribution will provide one of the most accurate determinations of these Standard Model parameters.

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1 Introduction: Theoretical Framework

The high precision status reached by

$$R_\tau \equiv \frac{\Gamma\left[\tau^- \to \text{hadrons}(\gamma)\right]}{\Gamma\left[\tau^- \to e^- \overline{\nu}_e \nu_\tau(\gamma)\right]} \tag{1}$$

and related observables using the data of the LEP experiments ALEPH [1] and OPAL [2] at CERN and the CESR experiment CLEO [3] at Cornell can be used to determine fundamental QCD parameters [4]. Indeed, the analysis of the non-strange inclusive data has led to accurate measurements of $a_\tau \equiv \alpha_s(M_\tau)/\pi$ which complement and compete with the current world average [1, 2].

The SU(3) breaking induce sizable corrections in the semi-inclusive τ-decay width into Cabibbo-suppressed modes, which can be analyzed with precision determinations of the strange spectral function [6, 7, 8], providing accurate measurements of the strange quark mass $m_s(M_\tau)$ [9, 10, 11, 12, 13, 14, 15, 16].

More recently, it has been pointed out that precision determinations of the strange spectral function can be used to obtain an accurate value for the Cabibbo–Kobayashi–Maskawa matrix element module $|V_{us}|$ [15, 16, 17]. The advantage of this method is that the experimental uncertainty is expected to be reduced drastically at the present B-factories: BABAR and BELLE.

The basic objects one needs to perform the QCD analysis of (1) and related observables are Green's two-point functions for vector $V^\mu_{ij} \equiv \overline{q}_i \gamma^\mu q_j$ and axial-vector $A^\mu_{ij} \equiv \overline{q}_i \gamma^\mu \gamma_5 q_j$ color singlets,

$$\begin{aligned}\Pi^{\mu\nu}_{V,ij}(q) &\equiv i \int \mathrm{d}^4x\, e^{iq\cdot x} \langle 0|T\left([V^\mu_{ij}]^\dagger(x) V^\nu_{ij}(0)\right)|0\rangle\,, \\ \Pi^{\mu\nu}_{A,ij}(q) &\equiv i \int \mathrm{d}^4x\, e^{iq\cdot x} \langle 0|T\left([A^\mu_{ij}]^\dagger(x) A^\nu_{ij}(0)\right)|0\rangle\,. \end{aligned} \tag{2}$$

The subscripts i, j denote light quark flavors (up, down and strange). These correlators admit the Lorentz decompositions

$$\begin{aligned}\Pi^{\mu\nu}_{ij,V/A}(q) &= \left(-g_{\mu\nu} q^2 + q^\mu q^\nu\right) \Pi^T_{ij,V/A}(q^2) \\ &+ q^\mu q^\nu\, \Pi^L_{ij,V/A}(q^2)\end{aligned} \tag{3}$$

where the superscripts in the transverse and longitudinal components denote the spin $J = 1\,(T)$ and $J = 0\,(L)$ in the hadronic rest frame.

Using the analytic properties of $\Pi^J(s)$, one can express R_τ as a contour integral in the complex s-plane running counter-clockwise around the circle $|s| = M_\tau^2$

$$\begin{aligned}R_\tau &\equiv -i\pi \oint_{|s|=M_\tau^2} \frac{\mathrm{d}s}{s} \left[1 - \frac{s}{M_\tau^2}\right]^3 \\ &\times \left\{3\left[1 + \frac{s}{M_\tau^2}\right] D^{L+T}(s) + 4\, D^L(s)\right\}\,.\end{aligned} \tag{4}$$

We have used integration by parts to rewrite R_τ in terms of the logarithmic derivatives of the relevant correlators

$$\begin{aligned} D^{L+T}(s) &\equiv -s\frac{\mathrm{d}}{\mathrm{d}s}[\Pi^{L+T}(s)]\,; \\ D^{L}(s) &\equiv \frac{s}{M_\tau^2}\frac{\mathrm{d}}{\mathrm{d}s}[s\Pi^{L}(s)]\,, \end{aligned} \tag{5}$$

which satisfy homogeneous renormalization group equations, eliminate unwanted renormalization scheme dependent subtraction constants in $\Pi^L(s)$ and produce a triple zero in the real axis. For large enough Euclidean Q^2, the correlators $\Pi^{L+T}(Q^2)$ and $\Pi^L(Q^2)$ can be organized in series of dimensional operators using the operator product expansion (OPE) in QCD.

Moreover, we can decompose R_τ into

$$R_\tau \equiv R_{\tau,V} + R_{\tau,A} + R_{\tau,S} \tag{6}$$

according to the quark content

$$\begin{aligned} \Pi^J(s) &\equiv |V_{ud}|^2\left\{\Pi^J_{V,ud}(s) + \Pi^J_{A,ud}(s)\right\} \\ &+ |V_{us}|^2\left\{\Pi^J_{V,us}(s) + \Pi^J_{A,us}(s)\right\}\,. \end{aligned} \tag{7}$$

Additional information can be obtained from the measured invariant mass distribution of the final state hadrons. The corresponding moments

$$R_\tau^{(k,l)} \equiv \int_0^{M_\tau^2} \mathrm{d}s \left(1-\frac{s}{M_\tau^2}\right)^k \left(\frac{s}{M_\tau^2}\right)^l \frac{\mathrm{d}R_\tau}{\mathrm{d}s} \tag{8}$$

can be calculated analogously to $R_\tau = R_\tau^{(0,0)}$ with the QCD OPE. One gets,

$$\begin{aligned} R_\tau^{(k,l)} \equiv N_c S_{\mathrm{EW}}\Big\{&(|V_{ud}|^2+|V_{us}|^2)\left[1+\delta^{(k,l)(0)}\right] \\ &+\sum_{D\geq 2}\left[|V_{ud}|^2\delta_{ud}^{(k,l)(D)} + |V_{us}|^2\delta_{us}^{(k,l)(D)}\right]\Big\}\,. \end{aligned} \tag{9}$$

The electroweak radiative correction $S_{\mathrm{EW}} = 1.0201 \pm 0.0003$ [18] has been pulled out explicitly and $\delta^{(k,l)(0)}$ denotes the purely perturbative dimension-zero contribution. The symbols $\delta_{ij}^{(k,l)(D)}$ stand for higher dimensional corrections in the OPE from dimension $D \geq 2$ operators which contain implicit $1/M_\tau^D$ suppression factors [4, 10, 12, 19].

The dominant SU(3) breaking corrections in (8) are the dimension $D = 2$ quark mass squared terms and the dimension $D = 4$ terms proportional to $m_s\langle\overline{q}q\rangle$. The separate measurement of the Cabibbo-allowed and Cabibbo-suppressed τ decay widths allows to quantify this SU(3) breaking through the difference

$$\begin{aligned} \delta R_\tau^{(k,l)} &\equiv \frac{R_{\tau,V+A}^{(k,l)}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{(k,l)}}{|V_{us}|^2} \\ &= N_c\, S_{EW} \sum_{D\geq 2}\left[\delta_{ud}^{(k,l)(D)} - \delta_{us}^{(k,l)(D)}\right]\,. \end{aligned} \tag{10}$$

These observables vanish in the SU(3) limit which, apart from enhancing the sensitivity to the strange quark mass, also help to reduce many theoretical uncertainties which vanish in that limit.

The dimension two corrections $\delta_{ij}^{(k,l)(2)}$ are known to $\mathcal{O}(a^3)$ for both $J = L$ and $J = L + T$ components –see [10, 12, 20] for references. The $\mathcal{O}(a^3)$ correction for the $J = L + T$ component has been presented at this workshop [20]. The results obtained in [16] and discussed here do not contain these $\mathcal{O}(a^3)$ corrections and a full analysis taking them into account will be presented elsewhere [21].

An extensive analysis of the perturbative series for the dimension two corrections was done in [10]. The conclusions there were that while the perturbative series for $J = L + T$ converges very well the one for the $J = L$ behaves very badly.

In the following applications, the dimension four corrections $\delta_{ij}^{(k,l)(4)}$ are fully included while the dimension six corrections $\delta_{ij}^{(k,l)(6)}$ were estimated to be of the order or smaller than the error of the dimension four [12] and therefore will not be included.

2 Fixed m_s: Determination of $|V_{us}|$

Taking advantage of the large sensitivity of the SU(3) quantities $\delta R_\tau^{(k,l)}$ to $|V_{us}|$, one can obtain a determination of this Cabibbo-Kobayashi-Maskawa (CKM) matrix element using as input a fixed value for m_s. We use as input value $m_s(2\,\mathrm{GeV}) = (95 \pm 20)$ MeV which includes the most recent determinations of m_s from QCD Sum Rules [22, 23, 24], lattice QCD [25] and τ hadronic data [9, 10, 11, 12, 13, 14, 15]. With this strange quark mass input, one can calculate $\delta R_\tau^{(k,l)}$ in (10) from theory.

In order to circumvent the problem of the bad QCD behavior of the $J = L$ component in $\delta R_\tau^{(k,l)}$, we replace the QCD expression for scalar and pseudo-scalar correlators by the corresponding phenomenological hadronic parameterizations [15]. In particular, the pseudo-scalar spectral functions are dominated by far by the well known kaon pole to which we add suppressed contributions from the pion pole as well as higher excited pseudo-scalar states whose parameters have been estimated in [22].

For the strange scalar spectral function we take the one obtained from a study of S-wave $K\pi$ scattering [26] in the framework of resonance chiral perturbation theory [27] and used in [23] in a scalar QCD sum rule determination of the strange quark mass. For comparison, we show in Table 1 the results obtained for the different components either using the OPE or the phenomenological hadronic parameterizations. Being both compatible, the uncertainties are much smaller for the phenomenological results which we therefore take to replace the corresponding OPE contributions to $\delta R_\tau^{(k,l)L}$ while we take $\delta R_\tau^{(k,l)L+T}$ from the QCD OPE as mentioned above [12]. As a result the final theoretical uncertainty for $\delta R_\tau^{(k,l)}$ is much reduced.

Table 1: Comparison between the OPE and the phenomenological hadronic parameterizations explained in the text for the longitudinal component of $R^{(0,0)}_{\tau,V/A}$.

	$R^{(0,0)L}_{us,A}$	$R^{(0,0)L}_{us,V}$	$R^{(0,0)L}_{ud,A} \times 10^3$
OPE	-0.144 ± 0.024	-0.028 ± 0.021	-7.79 ± 0.14
Pheno.	-0.135 ± 0.003	-0.028 ± 0.004	-7.77 ± 0.08

The smallest theoretical uncertainty arises for the moment $(k,l) = (0,0)$, for which we get

$$\delta R^{(0,0)}_{\tau,\rm th} = (0.162 \pm 0.013) + (6.1 \pm 0.6)\, m_s^2 - (7.8 \pm 0.8)\, m_s^4 = 0.218 \pm 0.026 \qquad (11)$$

where m_s denotes the strange quark mass in MeV units, defined in the $\overline{MS}$ scheme at 2 GeV. Here, one observes explicitly the relatively small sensitivity of $\delta R^{(0,0)}_{\tau,\rm th}$ to the actual value of the strange quark mass once the longitudinal component has been substituted by its phenomenological parameterization. We have also replaced the leading dimension four correction $\langle m_s \overline{s}s - m_d \overline{d}d\rangle$ by its chiral perturbation theory (CHPT) expression [12].

OPAL has recently updated the strange spectral function in [8]. In particular, they measure a larger branching fraction $B(\tau^- \to K^-\pi^+\pi^-\nu)$ which agrees with the previously one measured by CLEO [7]. From the OPAL data and using

$$|V_{us}|^2 = \frac{R^{(0,0)}_{\tau,S}}{\frac{R^{(0,0)}_{\tau,V+A}}{|V_{ud}|^2} - \delta R^{(0,0)}_{\tau,\rm th}}\,; \qquad (12)$$

we get the result

$$|V_{us}| = 0.2208 \pm 0.0033_{\rm exp} \pm 0.0009_{\rm th} = 0.2208 \pm 0.0034\,; \qquad (13)$$

where we have used as input strange quark mass the one discussed at the beginning of this section and the PDG value for $|V_{ud}| = 0.9738 \pm 0.0005$. Clearly, the uncertainty for the $|V_{us}|$ determination with hadronic τ decays becomes an experimental issue which certainly will be much reduced at the BABAR and BELLE B-factories.

One remark is in order here, the branching fraction $B(\tau \to K\nu(\gamma))$ can be predicted using its relation to the $K_{\mu 2}$ branching fraction. This relation is under rather good theoretical control [28]. Updating the numerics used there, one gets

$B(\tau \to K\nu(\gamma)) = (0.715 \pm 0.004)\%$ which is more precise than the present world average $B(\tau \to K\nu(\gamma)) = (0.686 \pm 0.023)\%$. Using this theoretical prediction one gets $|V_{us}| = 0.2219 \pm 0.0034$. A new more precise determination of this branching fraction would be very welcomed and certainly attainable at the current B-factories.

3 Fixed $|V_{us}|$: Determination of m_s

One can now use the value obtained for $|V_{us}|$ in the previous section in a determination of m_s using the updated OPAL spectral function. This procedure is meaningful since the moment $(0,0)$ is much more sensitive to $|V_{us}|$ than to m_s. We again replace the phenomenological result for $\delta R^{(k,l),L}_{\tau,\mathrm{phen}}$ by its OPE counterpart. Thus we use [12]

$$m_s^2(M_\tau) \simeq \frac{M_\tau^2}{1-\varepsilon_d^2} \frac{1}{\Delta^{L+T(2)}_{(k,l)}(a_\tau)} \times \left[\frac{\delta R_\tau^{(k,l)L+T}}{18 S_{EW}} + \frac{8}{3}\pi^2 \frac{\delta O_4(M_\tau)}{M_\tau^4} Q^{L+T}_{(k,l)}(a_\tau) \right] ; \tag{14}$$

with $\delta O_4(M_\tau) \equiv \langle m_s \bar{s}s - m_d \bar{d}d \rangle$, $\varepsilon_d \equiv m_d/m_s$ and as input $\delta R_\tau^{(k,l)L+T} = \delta R_\tau^{(k,l)} - \delta R^{(k,l)L}_{\tau,\mathrm{phen}}$.

The term $\Delta^{L+T}_{(k,l)}(a_\tau)$ is known in perturbative QCD and the convergence of the series is very good to $\mathcal{O}(a^2)$ [10]. The term $Q^{L+T}_{(k,l)}(a_\tau)$ is also known in perturbative QCD to $\mathcal{O}(a^2)$ –see [12] for references. The effect of the new $\mathcal{O}(a^3)$ corrections to $\Delta^{L+T}_{(k,l)}(a_\tau)$ presented at this workshop [20] will be investigated in [21].

The results we get for the different moments are shown in Table 2 –for the sources of the individual errors see [16]. The moments $(0,0)$ and $(1,0)$ produce results with uncertainties larger than 100% due to their small sensitivity to m_s and therefore do not contribute to the final weighted average.

Table 2: Results for $m_s(M_\tau)$ extracted from the different moments.

Moment	(2,0)	(3,0)	(4,0)
$m_s(M_\tau)$ MeV	93.2^{+34}_{-44}	86.3^{+25}_{-30}	79.2^{+21}_{-23}

The weighted average of the strange mass values obtained for the different moments give

$$m_s(M_\tau) = 84 \pm 23\,\mathrm{MeV}\,, \quad \Rightarrow m_s(2\,\mathrm{GeV}) = 81 \pm 22\,\mathrm{MeV}\ . \tag{15}$$

The final uncertainty corresponds to that of the $(4,0)$ moment. The dominant theoretical uncertainties originate from higher order perturbative corrections as well as the SU(3)-breaking ratio of the quark condensates $\langle \bar{s}s \rangle / \langle \bar{d}d \rangle = 0.8 \pm 0.2$ [29].

There are two clear features of the result with respect to the analysis using the ALEPH spectral function [15]: first, the strong $(k,0)$-moment dependence is reduced and second, a somewhat reduced but fully compatible value for the strange quark mass. They are both related to the larger OPAL and CLEO branching fraction $B(\tau^- \to K^- \pi^+ \pi^- \nu)$. This manifests the very important task of the B-factories reducing the uncertainties in the strange spectral function.

4 Combined Fit to $|V_{us}|$ and m_s

The ultimate procedure to determine $|V_{us}|$ and the strange quark mass from τ hadronic data will be a simultaneous fit of both to a set of moments. A detailed study including theoretical and experimental correlations will be presented elsewhere [21].

The first step toward this goal was already presented in [16]. There, we neglected all correlations and use the five OPAL moments [8] from $R_\tau^{(0,0)}$ to $R_\tau^{(4,0)}$. The result of this exercise is

$$|V_{us}| = 0.2196 \quad \text{and} \quad m_s(2\,\text{GeV}) = 76\,\text{MeV}\,. \tag{16}$$

These values are in very good agreement with the results (13) and (15) obtained in the previous sections. The expected final uncertainties are expected to be somewhat smaller than the individual errors for those but only slightly since the correlations between different moments are rather strong.

5 Results and Conclusions

The high precision hadronic τ Cabibbo-suppressed decay data from ALEPH and OPAL at LEP and CLEO at CESR provide already competitive results for $|V_{us}|$ and m_s. Using the strange spectral function updated by OPAL [8], we get

$$|V_{us}| = 0.2208 \pm 0.0033_{\text{exp}} \pm 0.0009_{\text{th}}\,, \tag{17}$$

and

$$m_s(2\,\text{GeV}) = 81 \pm 22\,\text{MeV}\,; \tag{18}$$

as discussed in Sections 2 and 3 respectively. The combined fit to determine both is under way and will be ready soon.

The actual status of the $|V_{us}|$ determinations has been nicely reviewed recently in [30, 31]. Though the CKM unitarity discrepancy has certainly decreased with the new theoretical and experimental advances, the situation is not yet as good as one could wish.

The classical way of determining $|V_{us}|$ has been through K_{e3} decays. The uncertainty of this determination is dominated by the theoretical prediction of the form factor $f_+(0)$, which is known model independently just to $\mathcal{O}(p^4)$ in CHPT. Recently, there have appeared several works updating the old model calculation of the $\mathcal{O}(p^6)$ corrections to $f_+(0)$ by Leutwyler and Roos [32]. The present status of this form factor can be inferred from the calculation in [33] using resonance chiral perturbation theory and from the calculation in [34] which presented a quenched lattice calculation of the order p^6 corrections. $|V_{us}|$ can be also determined from hyperon decays [35, 36]. Another recent proposal has been to use the lattice QCD determination of f_K/f_π [37, 38]. The accuracy of all these determinations is still in the $\geq 1\%$ range for the uncertainty, which will be difficult to decrease at short or even medium term. One long term possibility to reduce the error in predicting $f_+(0)$ is the proposal in [39].

Thus, there is room for hadronic τ decays to make an accurate determination for $|V_{us}|$ with the eventual accurate measurement of the strange spectral function at BABAR and BELLE.

Our result for the strange quark mass is on the low side of previous determinations, but certainly compatible with them. It is also borderline of being compatible with lower bounds on m_s from sum rules [22, 40, 41, 42, 43]. It is also compatible with the sum rules determinations of $2\hat{m} \equiv m_u + m_d$ which can be combined with the ratio $m_s/\hat{m}$ from CHPT determinations. If one uses for instance the result for $m_u + m_s$ from [44] and the ratio from [45] one gets $m_s(2\,\mathrm{GeV}) = 114 \pm 22$ MeV which is just one σ from (18).

There are however several open questions that will have also to be addressed. The $(k, 0)$-moment dependence of the m_s prediction has been reduced after the recent OPAL and CLEO analyses finding larger branching fractions for $\tau^- \to K^-\pi^+\pi^-\nu$. That this moment dependence could be due to missing contributions from the higher energy region of the spectrum was speculated in [17]. What is the origin of the remaining moment dependence is still open.

In [14], it was checked that the m_s determination fulfills quark-hadron duality between the QCD OPE and the ALEPH data. What happens with $|V_{us}|$ is another open question. A first look at this question has been presented at this workshop [46]. We will eventually come back to all these questions in [21].

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