Extraction of tacho information from a vibration signal for improved synchronous averaging

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ABSTRACT

Time synchronous averaging (TSA) is a procedure that allows the extraction of a deterministic component from a vibration signal. TSA requires a constant frequency deterministic component. Practically, a vibration signal from rotating machinery contains small frequency variations, even when operating at nominally constant speed. To remove these variations, the signal is order-tracked with respect to the deterministic component. This is normally accomplished using a reference signal from a tachometer directly coupled to the rotating shaft of interest. When the tachometer cannot be directly coupled to the shaft of interest, alternative methods are required to generate a reference signal. A case presented here is for the high pressure shaft (HPS) of a gas turbine engine, where the tachometer is coupled to an auxiliary shaft via a gearbox, with unknown exact gear ratio. This paper proposes two approaches to generate the reference signal for order-tracking. The first is to accurately estimate the gear ratio between the auxiliary shaft and the HPS, which will then be combined with the tachometer to produce a suitable reference signal. The second approach extracts a reference signal directly from the vibration signal using phase demodulation. TSA results derived using both methods are compared to evaluate their effectiveness.

INTRODUCTION

Time synchronous averaging (TSA), or synchronous averaging, is a procedure that allows the extraction of a deterministic component from a vibration signal.

TSA consists of averaging together a series of signal segments each corresponding to one period of a synchronising signal (McFadden 1987). Thus:-

$$y_a(t) = \frac{1}{N} \sum_{n=0}^{N-1} y(t+nT) \qquad (1)$$

TSA is an important method in the analysis of many vibration signals, such as the vibration responses of complex gearboxes for Machine Condition Monitoring (MCM). In the complex gearbox case TSA allows the separation of deterministic gear responses from non-deterministic bearing responses, allowing the detailed analysis of both phenomena in isolation without mutual contamination affecting results. TSA is also used in both vibration and acoustic fields as a method to reduce noise in a signal. Other applications of TSA include machine balancing applications and operational modal analysis (Asayesh, Khodabandeloo & Siami 2009). Synchronous averaging methods are integrated into many commercially available signal analysers.

 Successful application of the synchronous averaging method requires a constant frequency deterministic component. Practically, a vibration signal from rotating machinery contains small frequency variations, even when operating at nominally constant speed. To remove these variations, the signal is order-tracked with respect to the deterministic component.

Order-tracking re-samples a signal from equal time increments to equal phase increments. This results in a signal containing constant angular frequency components which can be used with TSA. The equal phase spacing used to resample the signal is determined from a synchronously recorded reference signal.

Order-tracking is normally accomplished using a reference signal from a tachometer directly coupled to the rotating shaft of interest (Bossley, McKendrick, Harris & Mercer 1999). When the tachometer cannot be directly coupled to the shaft of interest, alternative methods are required to generate a reference signal.

In this paper, multiple methods to construct a suitable reference signal are investigated for situations where a directly coupled tachometer signal is unavailable.

The case investigated here was the application of synchronous averaging to the vibration signal from the high pressure shaft (HPS) of a gas turbine engine. Due to the casing design of the engine, a tachometer could not be directly fitted to the HPS. A tachometer could only be fitted to a shaft in the accessory gearbox which was geared to the HPS. As the shafts are geared together and hence both deterministic, the frequency of the tachometer signal from the accessory gearbox is directly proportional to the speed of the HPS of interest.

Knowing the exact rational gear ratio between the two shafts, a reference signal for the HPS could be easily constructed from the captured tachometer signal by multiplying the pulse timings of the accessory gearbox tachometer signal with the exact gear ratio.

Unfortunately, the gear ratio supplied by the manufacturer was a rounded decimal figure to 4 significant figures. Upon application of the synchronous averaging method, it was found that the deviation of the rounded decimal from the true exact ratio, although very small, resulted in error over each period of rotation, which compounded resulting in divergence of the constructed reference signal away from the HPS response along the record length. Since the reference signal was not a true representation of the HPS response, order-tracking with respect to the reference signal did not result in

correct order-tracking of the HPS. As a result the deterministic part, the HPS component, was not exactly synchronised. Hence synchronous averaging did not work correctly.

For this project, two main approaches were investigated to produce a suitable reference signal for the HPS.

The first approach was to more accurately determine the gear ratio between the accessory gearbox shaft and the HPS, which can be used to construct a more accurate reference signal for order-tracking. These approaches are investigated in section 4.

The second approach was to extract a reference signal directly from the vibration signal for use in order-tracking, based on previously developed demodulation based order-tracking methods (Bonnardot, El Badaoui, Randall, Daniére and Guillet 2005) (Coats 2006). An extracted reference signal bypasses the need for an accurate gear ratio, allowing synchronous averaging to be employed. These approaches are investigated in section 5.

Figure 1 summarises the letter designations which will be used for the different methods described in this paper.
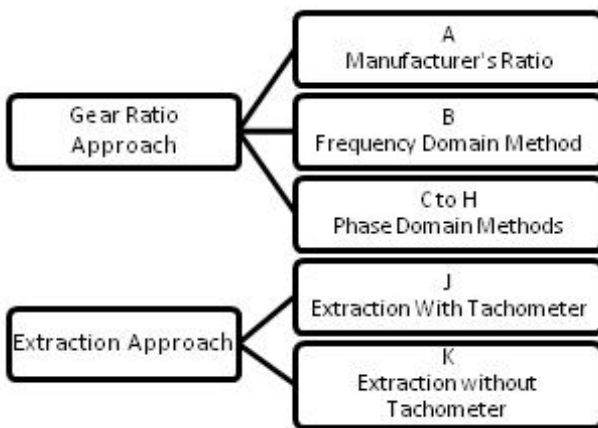

Figure 1. Letter Designations for Methods

EXPERIMENTAL DATA

The experimental data utilised in the case investigated here was captured from a Larzac gas turbine engine.

A vibration signal was captured from an accelerometer mounted on the casing of the high pressure stage compressor.

A one pulse per revolution tachometer was mounted on the low pressure stage main shaft (LPS), providing a reference signal for the low pressure shaft.

A sixty pulse per revolution tachometer was mounted on the accessory gearbox shaft, providing a reference signal in proportion to the HPS of interest.

The gear ratio supplied by the manufacturer between the accessory gearbox shaft and the HPS was 0.1861.

The number of gear stages between the accessory shaft and the HPS was unknown. The number of teeth on any of the gears was also unknown. As a result it was not possible to determine the exact rational ratio based on the supplied data.

TESTING PROCEDURE

In order to evaluate the effectiveness of each of the methods investigated to construct a suitable reference signal for order-tracking, each of the methods has been used to conduct synchronous averaging with respect to the HPS.

In each case evaluated, as much of the procedure has been left constant as possible to ensure that further accuracy differences from sources other than the reference signal construction methods are minimised.

Unless a specific different order-tracking method is being evaluated, standard order-tracking approaches have been used in all steps. Specifically a standard order-tracking toolbox used at UNSW written by Dominique Ho was used (Ho 1999). This is based on detection of reference signal pulse timings in the time domain, then using cubic interpolation to calculate new sample positions to order-track the signal.

Before any new synchronous averaging methods were tested, the vibration signals and the tachometer signal for the HPS were first order-tracked with respect to the Low Pressure shaft using the LPS tachometer signal. The vibration signals were then synchronously averaged, and the extracted TSA signal was subtracted from the original signal to produce the residual signal. This removed the discrete components of the Low Pressure Shaft from the signal. It was conducted to improve the clarity of the signal by removing components which would be smeared by order-tracking with respect to the HPS.

Synchronous averaging of the HPS with the developed methods was then implemented using the residual signal rather than the original vibration signals.

Two measures have been used for each reference signal construction method to evaluate the effectiveness of the synchronous averaging of the HPS.

The first measure was to visually compare the residual of the signal after synchronous averaging, to the signal immediately prior to synchronous averaging being conducted. The more successful the synchronous averaging, the more deterministic components will be removed from the residual. By comparing the effectiveness of the removal of various harmonics of the HPS, a rough subjective estimation can be made of the effectiveness of the synchronous averaging process after using the method being investigated to conduct order-tracking.

In order to visually compare the residual and the original signal, both are plotted using the Power Spectral Density function in MATLAB.

As both signals have been successfully order-tracked, the frequency values of the deterministic components of interest are present in the frequency domain at a very small number of samples, and appear to be a single discrete peak in the frequency domain. Although the frequency spectrum from a single FFT provides the true measure of how well the order-tracking has been implemented, it is extremely difficult to visually compare the difference between the residual and the original signal in the frequency domain. It is necessary to zoom to such an extent to localise a single frequency component that it is not possible to see the rest of the signal and make an estimation of the effectiveness of the synchronous averaging over the entire frequency spectrum. The windowing effect present in the PSD plot causes a smearing of the discrete frequency components over a number of samples. This smearing makes visual comparison of the effectiveness of the discrete component removal relatively easy, but should

not be taken as an indication that order-tracking has not been successfully implemented.

The second measure used to evaluate the effectiveness of the synchronous averaging was to divide the average power of the TSA signal by the average power of the residual signal. This measure has been named the separation index (SI), and can as seen as equation 1 below.

$$ SI = \frac{\dfrac{\sum_{i=0}^{n_y} y_i^2}{n_y}}{\dfrac{\sum_{i=0}^{n_x} x_i^2}{n_x}} \tag{2} $$

Where y is the frequency spectrum of the TSA signal, x is the frequency spectrum of the residual signal, and n is the number of samples in the signal.

The separation index can also be thought of as the ratio of the mean square values of the TSI and residual signals. The SI is equivalent to the ratio of the variance of the two signals, since the mean of both signals is zero as both are AC-coupled.

As the effectiveness of the synchronous averaging increases, the TSA signal will contain more of the discrete frequency component of interest and will increase in power. At the same time the residual will lower in power as the discrete components have been removed. Hence, as the synchronous averaging process improves, the SI will increase.

DETERMINING GEAR RATIO

The first approach investigated to construct a suitable reference signal for order-tracking with respect to the HPS was to accurately calculate the gear ratio between the accessory shaft and the HPS. The reference signal can then be constructed by multiplying the pulse timings of the accessory gearbox tachometer signal with the calculated gear ratio.

Firstly the supplied manufacturer's ratio was tested to provide a baseline for further testing.

Two basic methods were investigated to accurately calculate the gear ratio.

Method B was to determine the exact frequency of both the HPS of interest and the accessory gearbox shaft in the frequency domain. Once the frequencies of both are determined, the gear ratio is simply the ratio of both frequency values.

Method C was to extract the instantaneous phase of both the HPS of interest and the accessory gearbox shaft. Once the phase of both was extracted, the gear ratio was the ratio between both phase progressions.

Manufacturer's Gear Ratio – Method A

Although it was previously known that the supplied manufacturer's ratio could not be used to successfully implement synchronous averaging, the manufacturer's ratio was first tested to provide comparison to the other methods being investigated.

Figure 2 shows the difference between the PSD plot of the order-tracked signal and the residual signal when the manufacturer's ratio is used to construct a reference signal for order-tracking. The plot shows the 57th, 58th and 59th harmonics of the HPS.
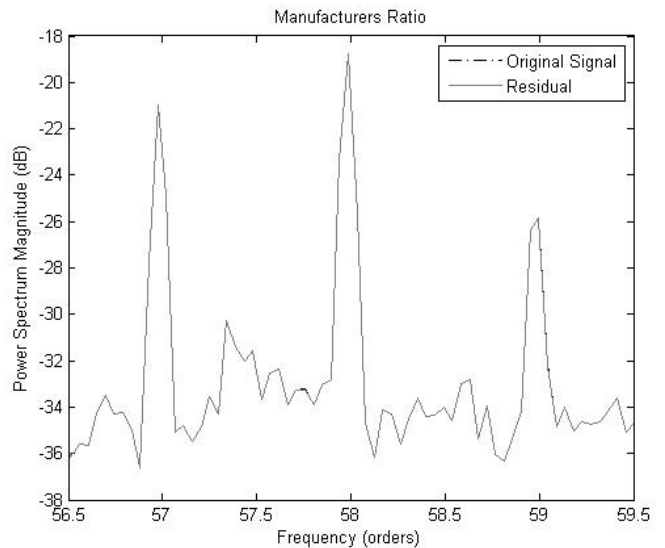

Figure 2. PSD plot of original and residual signal, using the manufacturer's gear ratio

Using the manufacturer's ratio to conduct order-tracking resulted in only part of the first few harmonics of the HPS being removed in the residual signal. As can be seen in figure 1, the higher harmonics have been completely unaffected.

The calculated SI using the manufacturer's ratio was:-

SI = 0.0649

Frequency Domain Method – Method B

In order to determine the exact frequencies of both the accessory gearbox shaft and the HPS in the time domain, the signals were first order-tracked with respect to the accessory gearbox shaft to remove any speed fluctuations.

The single record order tracking method (Coats 2006) was selected as the order-tracking method to order-track the signals.

The order tracking method involves using phase demodulation to extract the relationship between phase and time of the components of interest directly from a reference signal, which can be used to determine the new angular sampling for the order-tracked signal.

The order-tracking method involves 4 basic steps.

The first step of the order-tracking method was to select a suitable harmonic of the reference signal, in this case the accessory gearbox tacho signal. The demodulation process is blind, so it is necessary to select a harmonic which is clearly separable from the rest of the spectrum so contamination from other components is not included. A demodulation band is then selected encompassing this harmonic.
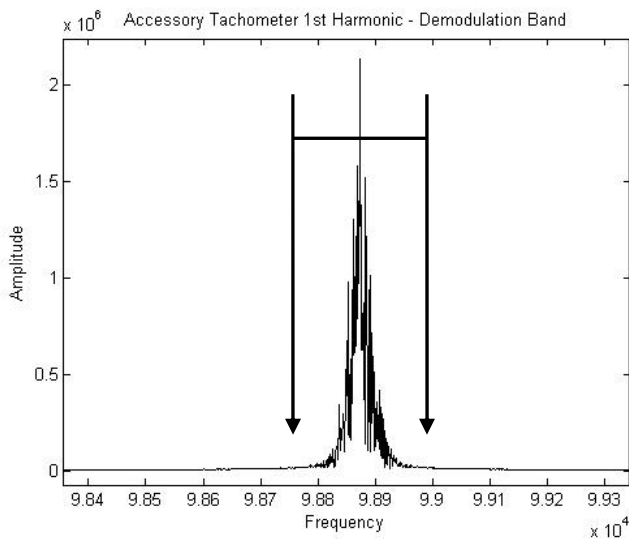
Figure 3. Frequency spectrum of accessory tachometer signal – First Harmonic and Demodulation Band

Figure 3 shows the first harmonic of the accessory tachometer signal which is clearly separable from the spectrum. Figure 3 also shows the selected demodulation band fully encompassing the harmonic.

In the second step, phase demodulation is then used to extract the phase of the selected harmonic. A carrier frequency is selected in the demodulation band, and then the frequency band is shifted so the carrier has zero frequency. Phase demodulation extracts the variation in phase of the frequency band around the carrier frequency. Finally the linear phase progression of the carrier frequency is then added to produce the true relationship between phase and time for the extracted harmonic.
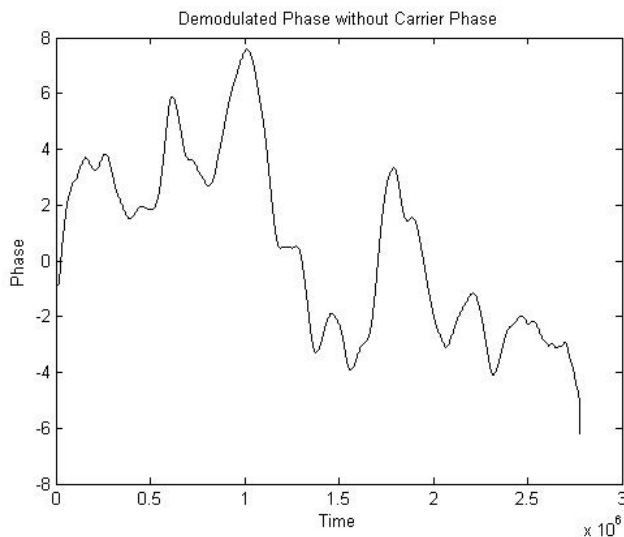

Figure 4. Demodulated Phase without Carrier Phase

Figure 4 shows the demodulated phase from the reference signal, without the linear phase progression of the median carrier frequency. This effectively shows the speed variation away from the median along the record length. As can be seen, minor speed variations are definitely present despite the engine being operated at constant speed.

In the third step, the phase-time relationship is used to calculate time values corresponding to equal increments of phase.

In the fourth step the signals are then resampled at these new time values, equally spaced in phase, to produce order-tracked signals.
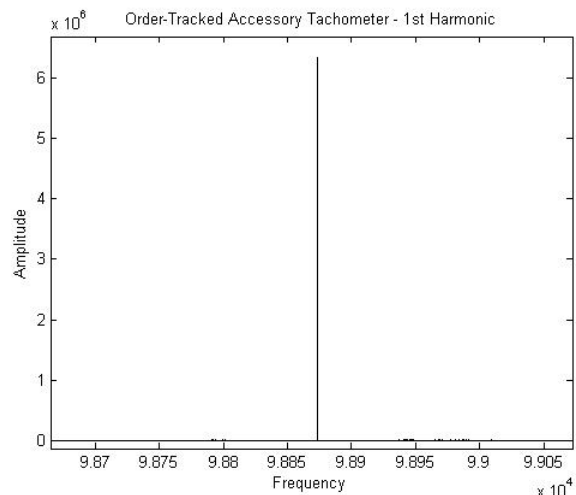

Figure 5. Accessory Tachometer Order Spectrum

Figure 5 shows the stem plot of the order-tracked accessory tachometer signal. As can be seen, the tachometer signal has been correctly order-tracked to one sample position.

From the order-tracked vibration signal, the frequency values of the accessory gearbox shaft and the HPS could be located. The 60th harmonic of each shaft was identified in the order spectrum. A higher harmonic was used to increase the accuracy of the readings.

The 60th harmonic of the accessory gearbox shaft was located at sample number 98,875.

The 60th harmonic of the HPS was located at sample number 531,101.

Taking the ratio between the two sample numbers produced the gear ratio of:-

$$\text{Gear ratio} = 0.18616986222959$$

Figure 6 shows the difference between the PSD plots when this gear ratio is used to order-track the vibration signal
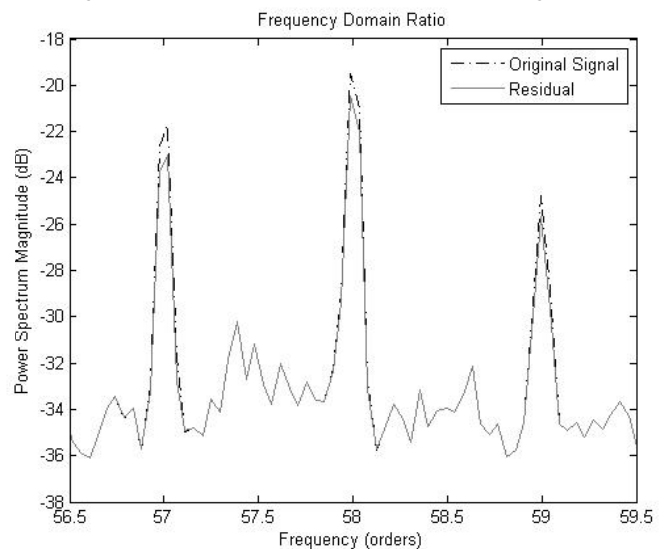

Figure 6. PSD plot of original and residual signal, using the frequency domain calculated gear ratio

The frequency domain calculated ratio provides acceptable HPS removal for low order harmonics, but for the higher harmonics shown in figure 2, the removal of the HPS components has been very minor.

The calculated SI for using the frequency domain calculated gear ratio was:-

$$SI = 0.2211$$

Phase Domain Method – Method C to H

The second method investigated to calculate the gear ratio more accurately was to determine the gear ratio by dividing the instantaneous phase relationship of one shaft of interest with the other at all points in time.

The phase of each shaft of interest was directly extracted from the vibration signal using the same phase demodulation process described in section 4.2 step 2. The main difference was that the shaft harmonics were extracted from the vibration signal, rather than from the accessory gearbox tacho signal.

From the vibration signal, the 60th harmonics of both the accessory gearbox shaft and the HPS were found to be clearly separable and suitable, and were selected. Locating a separable harmonic in the vibration signal is much more difficult than in the reference signal, as the spectrum contains significant noise as well as other significant components.

Figures 7 and 8 show the selected demodulation bands for the 60th harmonics of both the accessory gearbox shaft and the HPS.
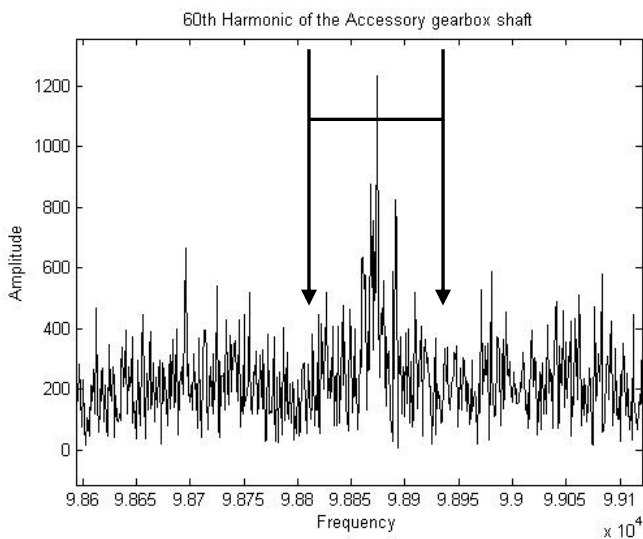

Figure 7. Demodulation band for 60th Harmonic of the Accessory shaft
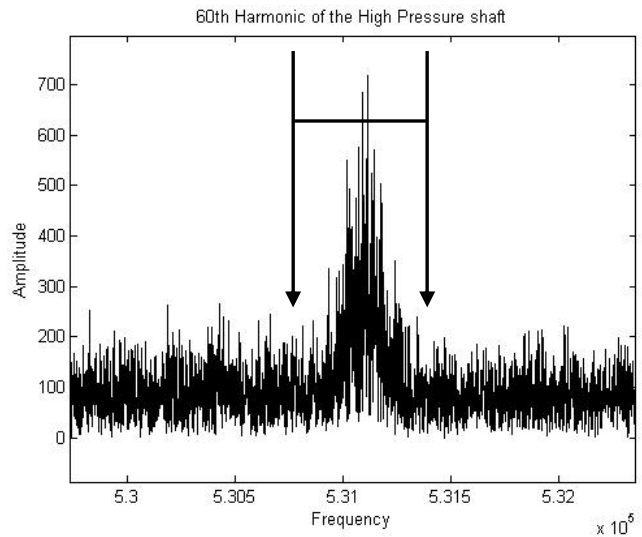

Figure 8. Demodulation band for 60th Harmonic of the High Pressure shaft

Both frequency harmonics of interest were phase demodulated to gain the phase progression of both shafts. The phase of one shaft was then divided by the phase of the other to produce an estimate of the instantaneous gear ratio along the signal length.
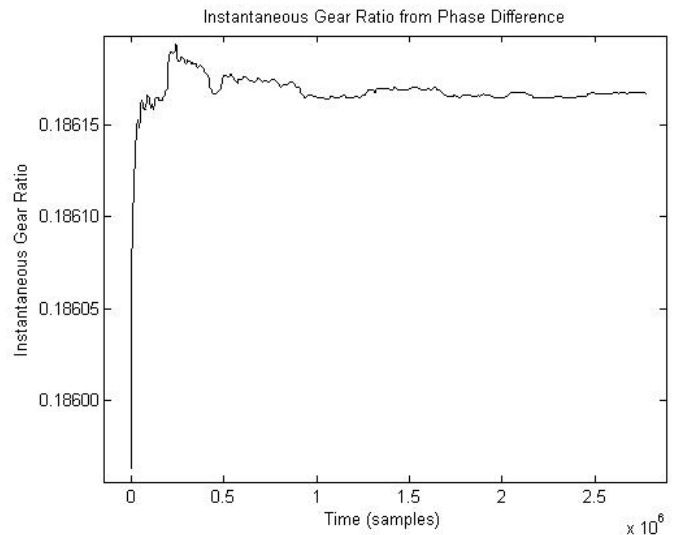

Figure 9. Instantaneous Gear ratio from the division of the shaft phase progressions

Figure 9 shows the calculated instantaneous gear ratio along the signal length. It can be seen that there is significant variation along the record length, particularly at the beginning of the record.

The ratio appears to be trending towards a steady value down the record length. This is to be expected, as the gear ratio is the ratio of two diverging phase progressions, which are equal at zero time. As the two phase progressions diverge, the ratio should become progressively more accurate.

To determine the best method of determining the gear ratio from the series of instantaneous ratios, multiple methods were used to calculate ratios for testing. The methods were used to extract a ratio were:-

C – The instantaneous ratio series was averaged

D – The first 1 million samples were cropped and the remainder was averaged, using a section of the signal which had roughly approached a steady state.

E – The first 2 million samples were cropped, and the remainder averaged.

F – The first 2.5 million samples were cropped, and the remainder averaged.

G – The first 1 million samples were cropped, and the midpoint between the remaining maximum and minimum ratios was calculated.

H - The first 2 million samples were cropped, and the midpoint between the remaining maximum and minimum ratios was calculated.

Figure 10 shows the difference between the PSD plots when gear ratio H is used to order-track the vibration signal. Of the 6 ratios, ratio H produced the best result.
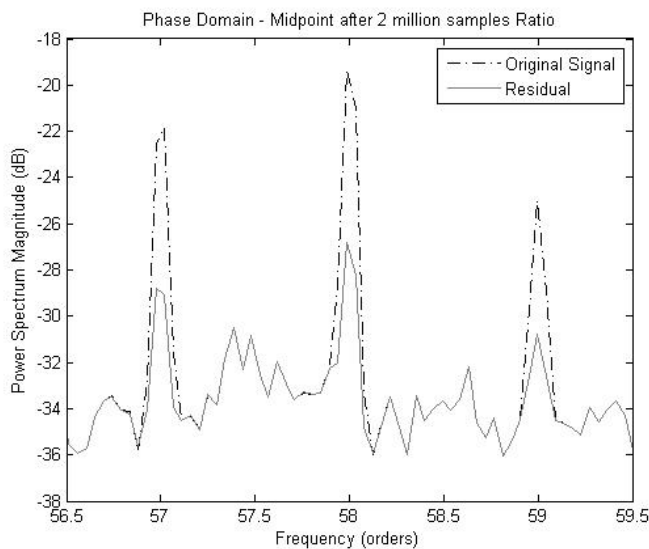

Figure 10. PSD plot of original and residual signal, using ratio H calculated from the Phase domain

Figure 10 shows that significant proportion of the HPS components have been removed at the higher frequencies shown. For the lower harmonics, most of the HPS has been separated from the residual signal.

The calculated accuracies for the six gear ratios calculated from the phase domain methods can be seen in table 1.

Table 1. Phase Domain Methods Ratios and SI

Ratio Letter	Gear Ratio	SI
C	0.18616835040315	0.2618
D	0.18616666323078	0.3800
E	0.186166234236261	0.4684
F	0.186167244952268	0.2999
G	0.186167063108743	0.3194
H	0.186166088918351	0.5025

From the results in Table 1, it can be concluded that taking ratios from the later section of the record length once the signal has trended towards a steady value produced better results. Taking a small section at the end of the record produced worse results however, so there is no indication that once the instantaneous ratio approaches a steady value the accuracy continues to improve down the record length.

EXTRACTING A REFERENCE SIGNAL

The second approach investigated to construct a suitable reference signal for order-tracking with respect to the HPS was to extract reference signal information directly from the vibration signal and to use this information to perform order-tracking.

When using a fixed ratio to multiply with the speed of the accessory gearbox shaft to produce a reference signal for the HPS, unless the exact ratio between the two shafts is used, any difference will result in the constructed reference signal gradually diverging from the true position of the HPS.

By extracting reference signal information directly from the HPS, rather than utilising a fixed non-exact ratio, the extracted reference signal will remain synchronous with the HPS. Although error due to jitter between the gears will still be present, there will be no error associated with progressive divergence of the reference signal from the true position of the HPS.

Two slightly different order-tracking methods were investigated which could extract a reference signal from the vibration signal.

Both methods utilise the basic single order-tracking method used previously in section 4.2 (Coats 2006). The only difference from the Single record Order tracking method is that rather than phase demodulating a single harmonic of a tachometer signal, a single harmonic of the HPS is phase demodulated from the vibration signal. This follows the basic order-tracking approach described by F. Bonnardot *et al* (2005).

The main difference between the two methods is that method J first order-tracks the vibration signal using the accessory gearbox shaft to remove as much speed fluctuation as possible beforehand. Method K does not employ a tachometer signal at all, and could be applied to situations where no separate synchronous reference signal is available.

Extracting a Reference Signal with a Tachometer signal used – Method J

As phase demodulating a band of the vibration signal will produce the phase of every component within the band, it is necessary to use a band which not only encompasses an entire harmonic of the HPS, but also to use a band which contains a minimum of extraneous components.

When using the vibration signal rather than the spectrum of a tachometer signal which is relatively clean, any band selected will unavoidably contain some degree of noise. It is necessary to select a band which contains a minimum of other components, as any extra information will be incorrectly accounted for in the order-tracking process and the resulting signal will not be correctly order-tracked with respect to the HPS.

In order to improve on the order-tracking process described by F. Bonnardot et al (2005) when using phase demodulation, the vibration signal was first order-tracked relative to the accessory gearbox shaft tachometer signal using standard order-tracking approaches. The order-tracking was conducted using the supplied manufacturer's gear ratio. Order-tracking with regards to a shaft directly coupled to the HPS removed a significant proportion of the speed variations of the HPS. This resulted in lower smearing of the harmonics of the HPS in the frequency spectrum, and allowed a narrower demodulation band to encompass an entire harmonic of the HPS. A narrower band should contain fewer contaminating compo-

nents, resulting in a more accurate order-tracking of the HPS and hence more accurate synchronous averaging.

From the vibration signal, the 141st harmonic of the HPS was selected for phase demodulation. This harmonic was clearly separable, and as it had the highest magnitude in the spectrum it should also have the highest signal to noise ratio, making it the most suitable harmonic to be used.

Figure 11 shows the difference between the PSD plots from synchronous averaging after the extracted reference signal has been used to order-track the vibration signal.
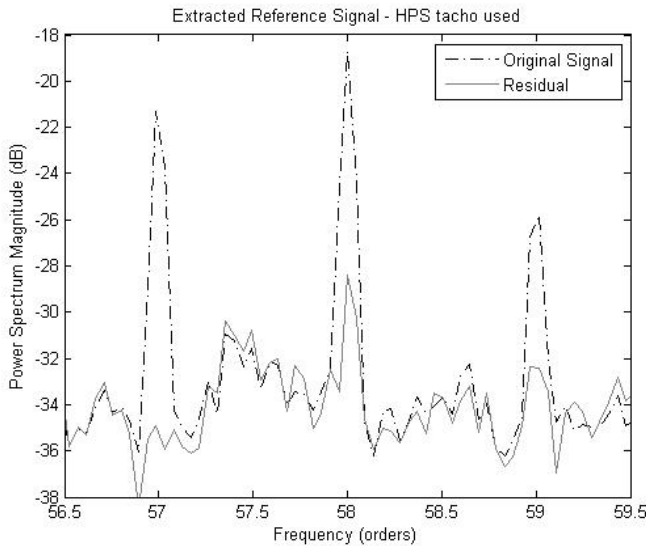

Figure 11. PSD plot of original and residual signal, using the extracted reference signal with the assistance of the accessory gearbox tachometer

From the PSD plot, it was clear that a large proportion of the HPS components had been extracted over the entire order spectrum. Figure 5 shows how most of the HPS components have been extracted at the shown frequencies.

The calculated SI for using an extracted reference signal with the assistance of the accessory gearbox tachometer signal was:-

$$SI = 0.6917$$

Extracting a Reference Signal without using a Tachometer signal – Method K

The first problem to overcome with extracting a reference signal for order-tracking without the use of a proportional reference signal is in locating a harmonic of the HPS.

In this case the position of the HPS components in the frequency spectrum was known from the accessory gearbox tachometer signal. In a case where a tachometer signal is not actually available, it is still necessary to have enough knowledge of the system to be able to locate a harmonic of the component of interest in the frequency spectrum. The frequency information for the component of interest does not need to be very accurate or synchronously recorded, but it needs to be accurate enough to identify the component of interest out of all the components present in a vibration signal. The difficulty in identifying the component of interest will be dependent on the complexity of the signal.

The second problem is that without the use of a proportional reference signal to remove the bulk of the speed variations of

the HPS, it is more difficult to locate a separable harmonic of the HPS for phase demodulation.

The higher the harmonic of the HPS that can be utilised, the more accurate the order-tracking should be. However, as the frequency smearing of a varying speed frequency component is proportional to the harmonic number, without initial order-tracking the higher harmonics will be much more smeared than the lower ones. The larger demodulation band necessary to encompass a higher harmonic will inadvertently introduce more contamination, and negate the increased accuracy of using a higher harmonic.

In order to utilise the increased accuracy of order-tracking with a higher harmonic, the order-tracking was conducted in multiple stages. The use of multiple stages to increase the allowable speed fluctuations differentiates method K from that described by F. Bonnardot et al (2005). Each stage utilised a higher harmonic, starting with the lowest harmonic of the HPS which is separable. Each stage reduced the smearing of subsequent harmonics, allowing a narrow band to be demodulated each time.

From the vibration spectrum, the first harmonic of the HPS was determined to be separable, and so was used for the first order-tracking stage.

From the resulting order-tracked spectrum, the highest harmonic located which was still clearly separable was the 43rd harmonic. This harmonic was used to conduct the second order-tracking stage.

Finally, in the resulting order-tracked spectrum, the 141st harmonic of the HPS was now clearly separable. As with the case using the accessory gearbox tachometer to assist in the extraction of a reference signal, the 141st harmonic was considered to be the best harmonic to be used for order-tracking, and so was used for the final order-tracking stage.

Figure 12 shows the difference between the PSD plots from synchronous averaging after the final extracted reference signal has been used to order-track the vibration signal.
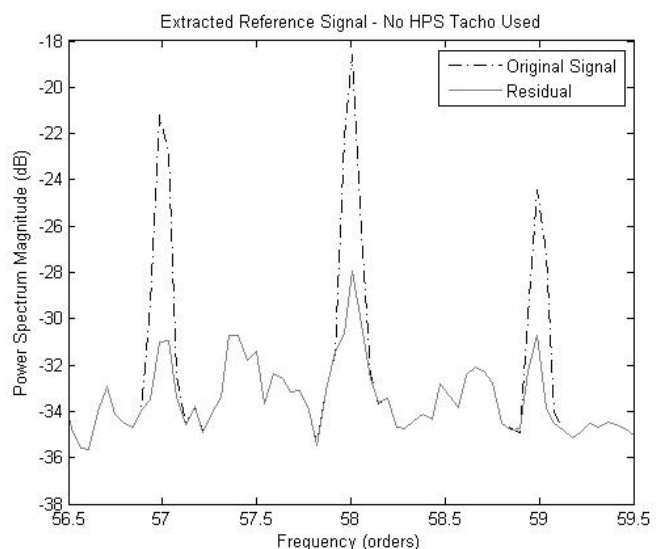

Figure 12. PSD plot of original and residual signal, using the extracted reference signal without the assistance of the accessory gearbox tachometer

From the PSD plot, the removal of the HPS components from the residual signal appeared to be equal to the previous case where the extraction was assisted with the use of the accessory gearbox tachometer signal. For the frequency compo-

nents shown in Figure 12, the extraction appears slightly worse in this case. However over the entire spectrum the differences between the methods J and K balance out.

The calculated SI for using an extracted reference signal without the assistance of the accessory gearbox tachometer signal was:-

$$SI = 0.6785$$

CONCLUSIONS

In this paper, multiple approaches were investigated to successfully produce a suitable reference signal for order-tracking to be used in conjunction with synchronous averaging when a directly coupled reference signal is unavailable.

The methods were classified into two main approaches. The first approach calculated more accurate gear ratios than were supplied by the manufacturer, which could be combined with the tachometer signal coupled to the shaft of interest to conduct order-tracking. The second approach investigated extracting a reference signal directly from the vibration signals.

Table 2 shows the summary of the accuracy values calculated for all the tested methods.

Table 2. Summary of SI results for all Methods

Method		SI
Gear Ratios		
A - Manufacturer's Ratio		0.0649
B - Frequency domain Approach		0.2211
Phase Domain Approach	Ratio no	
	C	0.2618
	D	0.3800
	E	0.4684
	F	0.2999
	G	0.3194
	H	0.5025
Extraction Approach		
J - Using tachometer signal		0.6917
K - Without using tachometer		0.6785

From the results summarised in table 2, it can firstly be concluded that all the methods investigated to order-track the signal resulted in better synchronous averaging results than those gained using the supplied manufacturer's gear ratio.

Secondly, it can be concluded that using an extracted reference signal to conduct the order-tracking resulted in more successful synchronous averaging than when any of the gear-ratio-based methods were employed.

Finally, it can be concluded that when a reference signal is extracted directly from the vibration signal for order-tracking, equivalent results are produced regardless of whether a coupled tachometer signal is first used to order-track the signal. This is conditional on both methods being possible however as without initial order-tracking with a coupled tachometer signal, there is less chance that a separable harmonic will be available and that phase information can be extracted.

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