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Extraordinary Surface Voltage Effect in the Invisibility Cloak with an Active Device Inside

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The electromagnetic field solution for a spherical invisibility cloak with an active device inside is established. Extraordinary electric and magnetic surface voltages are induced at the inner boundary of a spherical cloak, which prevent electromagnetic waves from going out. The phase and handness of polarized waves obliquely incident on such boundaries are kept in the reflected waves. The surface voltages due to an electric dipole inside the concealed region are found equal to the auxiliary scalar potentials at the inner boundary, which consequently gain physical counterparts in this case.

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Pendry *et al.* [1] first proposed the invisibility cloak based on coordinate transforms where a “hole” is created in the transformed coordinate system and an object in the hole can be concealed from detection. Another method was reported to produce similar effects in the geometric limit [2]. Designs of invisibility cloaks based on other methods have also been published [3–6]. From the viewpoint of the transformation method [1], the hole creation does not result in an electromagnetic vacuum but rather a complete separation of electromagnetic domains into a cloaked region and those outside [1,7]. More precisely, a “true” cloak should not only cloak passive objects from incoming waves, but also cloak active devices by preventing waves from going out and being detected. The effectiveness of a transformation based cloak design for hiding a passive object was first confirmed computationally in the geometric optics limit [1,8], in full-wave finite-element simulations [9,10], and in full-wave analytic scattering models with deeper physical interpretations [11,12]. It has also been demonstrated experimentally with a simplified practical model [13]. On the other hand, however, the electromagnetic wave behavior in this concealed region with an active device inside remains unknown, due to the fact that the concealed region, or the hole, created by the transformation method does not exist before transformation and has no counterpart in the original Euclidian space. A rigorous mathematical treatment of cloaking shows that “finite energy solutions” to Maxwell’s equations do not exist in the presence of active sources inside the concealed region [14], which strongly implies that some special physical phenomenon must exist but has not been revealed. Also, the exact electromagnetic field solution to the Maxwell equations in this case, as well as whether waves can go out of the concealed region, still have not been established or confirmed.

In this Letter, the exact field solution to the Maxwell equations with an active source inside the concealed region of a spherical cloak is established. We show that electric and magnetic surface voltages are induced due to an infinite polarization of the material at the inner boundary,

which prevents the electromagnetic waves from going out. The special property of the material at the inner boundary of the cloak is able to keep not only the handness of polarization, but also the phase information, in the waves reflected from the inner boundary, whose behavior is different from a perfect electric or magnetic conductor (PEC/PMC), or the artificial soft and hard surface (SHS) in electrical engineering [15]. The induced surface electric and magnetic voltages are shown to be exactly equal to the auxiliary scalar electric and magnetic potentials at the inner boundary, respectively. Therefore, these auxiliary potentials, which are commonly introduced accompanied with vector potentials as strictly mathematical tools for most engineers [16], have physical counterparts at the inner boundary in this case.

Figure 1 shows the configuration of a spherical cloak with outer radius R_2 and inner radius R_1 . The cloak layer within $R_1 < r < R_2$ is a specified anisotropic and inhomogeneous medium with permittivity tensor $\bar{\epsilon} = \epsilon_r \hat{r} \hat{r} + \epsilon_t \hat{\theta} \hat{\theta} + \epsilon_t \hat{\phi} \hat{\phi}$ and permeability tensor $\bar{\mu} = \mu_r \hat{r} \hat{r} + \mu_t \hat{\theta} \hat{\theta} + \mu_t \hat{\phi} \hat{\phi}$. According to Ref. [1], it is chosen such that $\epsilon_t/\epsilon_0 = \mu_t/\mu_0 = R_2/(R_2 - R_1)$ and $\epsilon_r/\epsilon_t = \mu_r/\mu_t = (r - R_1)^2/r^2$. Without loss of generality, we assume that the background material in the region $r < R_1$ has permittivity ϵ_1 and permeability μ_1 . A time-harmonic

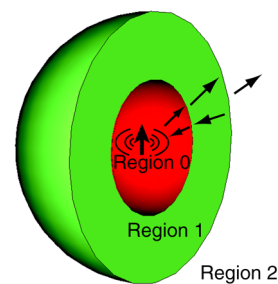


FIG. 1 (color online). Reflection and transmission of electromagnetic waves with an electric dipole inside the concealed region of a spherical cloak. Region 0: concealed region; Region 1: cloak layer; Region 2: free space.

electric dipole is put inside as an active device. The electromagnetic waves from the dipole as well as the response of the surrounding environment can be decomposed into TE and TM modes with respect to \hat{r} , corresponding to scalar potentials Ψ_{TE} and Ψ_{TM} , whose expressions in the current case have been derived in Ref. [11].

Since TE and TM modes in such a radially inhomogeneous medium can be shown to be decoupled [17], the derivations for these two kinds of modes are identical to each other. We start with the case that an outgoing TM wave is excited in the concealed region. This outgoing wave will induce a standing wave in region $r < R_1$, an outgoing wave and a standing wave in region $R_1 < r < R_2$, and an outgoing wave in region $r > R_2$ [17], as shown in Fig. 1. Thus the scalar potentials in these three regions can be written as

$$\Psi_{\text{TM}}^{\text{int}} = [\zeta_n(k_1 r) + R^{\text{TM}} \psi_n(k_1 r)] P_n^m(\cos\theta) e^{i m \phi}, \quad (1)$$

$$1/\sqrt{\mu_t \epsilon_1} [d_n^M \psi'_n(k_t(R_2 - R_1)) + f_n^M \chi'_n(k_t(R_2 - R_1))] = 1/\sqrt{\mu_0 \epsilon_0} T^{\text{TM}} \zeta'_n(k_0 R_2), \quad (4)$$

$$1/\mu_t [d_n^M \psi_n(k_t(R_2 - R_1)) + f_n^M \chi_n(k_t(R_2 - R_1))] = 1/\mu_0 T^{\text{TM}} \zeta_n(k_0 R_2), \quad (5)$$

$$1/\sqrt{\mu_1 \epsilon_1} [\zeta'_n(k_1(R_1 + \delta)) + R^{\text{TM}} \psi'_n(k_1(R_1 + \delta))] = 1/\sqrt{\mu_t \epsilon_t} [d_n^M \psi'_n(k_t \delta) + f_n^M \chi'_n(k_t \delta)], \quad (6)$$

$$1/\mu_1 [\zeta_n(k_1(R_1 + \delta)) + R^{\text{TM}} \psi_n(k_1(R_1 + \delta))] = 1/\mu_t [d_n^M \psi_n(k_t \delta) + f_n^M \chi_n(k_t \delta)]. \quad (7)$$

After solving all the equations, it can be obtained that $d_n^M = f_n^M = T^{\text{TM}} = 0$, indicating that no field exists in the cloak layer as well as the outside space. Meanwhile we can get that $R^{\text{TM}} = -\zeta_n(k_1 R_1)/\psi_n(k_1 R_1)$, which is important for later use. In addition, it follows that in the limit $\delta \rightarrow 0$, $f_n^M \chi'_n(k_t \delta)$ is nonzero. Obviously, the value of this product in Eq. (6) is nonzero only at the inner boundary, bringing out the discontinuity of the tangential E field across the inner boundary. This result is very interesting. Since both μ and ϵ are finite everywhere and no conductive media exist, it is unreasonable to include surface current to support this discontinuity, as the common boundary conditions do. It should be noted that due to the same reason the displacement surface currents introduced in the cylindrical cloak [12,18] are not applicable in the present case.

In order to understand this discontinuity, let us first consider a similar case in Cartesian coordinates as shown in Fig. 2(a), where a TM wave $\vec{H}_i = \hat{y} e^{ik_{tx}x + ik_z z}$ is obliquely incident ($k_z \neq 0$) from free space onto an uniaxial medium with permittivity tensor $\epsilon_x \hat{x} \hat{x} + \epsilon_t \hat{y} \hat{y} + \epsilon_t \hat{z} \hat{z}$ and permeability μ_t . The dispersion relation in this medium is $k_{tx}^2/(\omega^2 \epsilon_t \mu_t) + k_z^2/(\omega^2 \epsilon_x \mu_t) = 1$. Thus, when ϵ_x is very small, k_{tx} becomes imaginary and the transmitted wave becomes evanescent. In the limit $\epsilon_x \rightarrow 0$, it can be found that the transmitted wave is so strongly evanescent that no fields exist in the region $x > 0$. More interestingly,

$$\Psi_{\text{TM}}^c = [d_n^M \psi_n(k_t(r - R_1)) + f_n^M \chi_n(k_t(r - R_1))] \cdot P_n^m(\cos\theta) e^{i m \phi} \quad (2)$$

$$\Psi_{\text{TM}}^{\text{out}} = T^{\text{TM}} \zeta_n(k_0 r) P_n^m(\cos\theta) e^{i m \phi}, \quad (3)$$

where ψ_n , χ_n , and ζ_n are Riccati-Bessel functions of the first, the second, and the third kind, respectively; R^{TM} , d_n^M , f_n^M , and T^{TM} are the unknown expansion coefficients. Especially, R^{TM} and T^{TM} are called the general reflection coefficient and general transmission coefficient, respectively [17].

For the sake of illustration, the inner boundary of the cloak is set at $r = R_1 + \delta$ instead of $r = R_1$ and then the limit $\delta \rightarrow 0$ is taken [12]. Consequently, four boundary equations can be listed utilizing the continuities of tangential E and tangential H at the outer boundary [Eqs. (4) and (5)] and at the inner boundary [Eqs. (6) and (7)]:

in the limit $\epsilon_x \rightarrow 0$, the integration $\int_0^\infty E_{tx} dx$ has a finite value $-2i\eta_0 \cos\theta e^{ik_z z}/k_z$. In other words, E_{tx} is compressed on the interface like a delta function. This special finite and nonzero value can be named as an electric surface voltage V_E . When a free charge q moves to the interface, this voltage will push it to the other side and transfer energy qV_E to it. This voltage is not caused by conductive charges but an infinite polarization of the material on the interface; i.e., it corresponds to a distribution of polarized dipole moments on the interface. In addition, the tangential electric field at the left side of the interface is

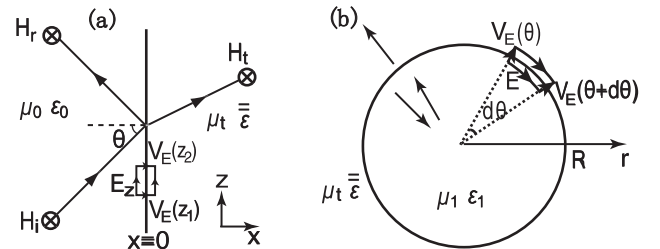


FIG. 2. Reflection and transmission of (a) a TM wave in Cartesian coordinate system incident from free space onto an uniaxial medium whose ϵ_x goes to zero, and (b) an outgoing TM wave in spherical coordinate system from the homogeneous and isotropic hole through a homogeneous uniaxial background medium whose radial permittivity goes to zero.

$E_{iz} + E_{rz} = -2\eta_0 \cos\theta e^{ik_z z}$ while that at the right side is zero, meaning the tangential E field is discontinuous across the interface. However, since $(E_{iz} + E_{rz}) \Delta z + V_E(z_2) - V_E(z_1) = 0$, as shown in Fig. 2(a), Faraday's law still holds on this interface. Clearly, using this uniaxial material, which is the same with the inner boundary of the cloak, E_x becomes a delta function and forms the electric surface voltage which supports the discontinuity of the tangential E field. Meanwhile, the reflection coefficient becomes -1 , meaning that this medium behaves like a PMC by means of controlling the medium's electric response.

Similarly, the reflection coefficient for a TE wave is also -1 if μ_x goes to zero. Thus, the special uniaxial medium whose ϵ_x and μ_x go to zero simultaneously behaves like a PMC for TM waves due to electric surface voltages, and a PEC for TE waves due to magnetic surface voltages. This leads to another interesting aspect of the reflection. First, in the sense that there is a complete reflection, the interface behaves like a mirror. Second, this special mirror not only keeps the polarization but also the phase information of the reflected waves. For example, for a right-handed circularly polarized wave incident onto this boundary, the reflected wave retains its handedness, but for a mere PEC or PMC boundary, the reflected wave becomes left-handed, as shown in Fig. 3. This property is similar to the SHS boundary used in radar and microwave engineering [15]. But for a SHS with its conducting vector fixed, if the incident plane changes, the phase of the reflected wave also changes. However, the phase of the reflected wave in Fig. 3(b) is independent on the incident plane, meaning it only depends on the optical path the wave travels. So, this mirror behaves the same in any plane of incidence, and the information of a source including the polarization and phase is entirely retained in the reflected wave.

From the above discussion in Cartesian coordinate system, we see that the surface voltages are introduced by the zero permittivity and permeability in the normal direction of the interface, which contribute to the discontinuity of the tangential electromagnetic fields across the boundary. This is also true for a spherical interface in spherical coordinate system. For example, as shown in Fig. 2(b), a sphere with permittivity ϵ_1 and permeability μ_1 is embedded in the

homogeneous background medium with permittivity tensor $\bar{\epsilon} = \epsilon_r \hat{r} \hat{r} + \epsilon_t \hat{\theta} \hat{\theta} + \epsilon_t \hat{\phi} \hat{\phi}$ and permeability μ_t . Similar to the case in Fig. 2(a), for TM waves, in the limit $\epsilon_r \rightarrow 0$, no fields exist in the region $r > R_1$, but the electric surface voltage V_E is induced at the boundary. Since $E_\theta^{\text{int}} R_1 d\theta + V_E(\theta + d\theta) - V_E(\theta) = 0$, Faraday's law still holds across the boundary. The similar result can be obtained for E_ϕ component. Outgoing TE waves have similar derivation when μ_r goes to zero. Therefore the condition where the material at the inner boundary of a spherical cloak has radial permittivity and permeability of zero is sufficient for total reflection of all waves back by inducing surface voltages, no matter whether the outside medium satisfies the relation of constitutive parameters proposed in Ref. [1] or not. Mathematical treatment in time domain in Ref. [19] has also gotten the similar result of complete reflection.

Based on the above discussion, the electric and magnetic surface voltages at the inner boundary as well as the field distribution inside the concealed region due to an electric dipole \bar{p} located at an arbitrary position (r', θ', ϕ') , where $r' < R_1$, can be derived. By expanding the wave from the dipole into spherical waves, the corresponding scalar potentials Ψ_{TM}^i and Ψ_{TE}^i for the incident waves can be obtained. Since it is known that the reflection coefficient for both TE and TM waves is $-\zeta_n(k_1 R_1)/\psi_n(k_1 R_1)$, the scalar potentials of reflected waves, Ψ_{TE}^r and Ψ_{TM}^r , can be easily obtained. Consequently, the induced electric and magnetic surface voltages at the inner boundary of the spherical cloak can be calculated as follows:

$$V_E = \int_{R_1^-}^{R_1^+} E_r dr = \frac{-i}{\omega \mu_1 \epsilon_1} \frac{\partial}{\partial r} (\Psi_{\text{TM}}^i + \Psi_{\text{TM}}^r) |_{r=R_1^-}, \quad (8)$$

$$V_H = \int_{R_1^-}^{R_1^+} H_r dr = \frac{-i}{\omega \mu_1 \epsilon_1} \frac{\partial}{\partial r} (\Psi_{\text{TE}}^i + \Psi_{\text{TE}}^r) |_{r=R_1^-}. \quad (9)$$

Figure 4 plots the amplitude of V_E at the inner boundary of a spherical cloak and the field H_y inside the concealed region in the x - z plane, due to an electric dipole pointing in z direction and located at $(R_1/2, \pi/4, \pi)$. First, it is seen that surface voltages distributions are not uniform on the surface. But for an outside observer, the dipole is invisible since no wave propagates outside. Second, the field inside exists in the form of standing waves. Figure 4(b) shows the field at the moment that the magnetic field reaches maximum. After a quarter of cycle, the magnetic field becomes zero while the electric field reaches maximum. Since E and H are always out of phase, the time-averaged Poynting power is zero anywhere, meaning no time-averaged power flowing inside. In other words, the energy radiated from the dipole at this moment will be returned to the dipole the next moment. Thus the total energy inside will not blow up.

It can be calculated from Eqs. (8) and (9) that, in the presence of an electric dipole inside, when ω decreases to zero, V_H becomes zero while V_E survives. Similarly, if a static magnetic dipole is inside instead of a static electric

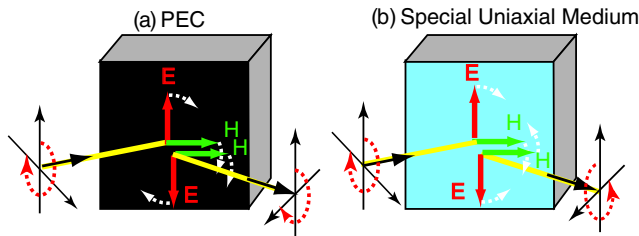


FIG. 3 (color online). Reflection of a right-handed circularly polarized wave from (a) a PEC and (b) a medium with normal permittivity and permeability go to zero simultaneously. The dotted arrows represent the direction of rotation.

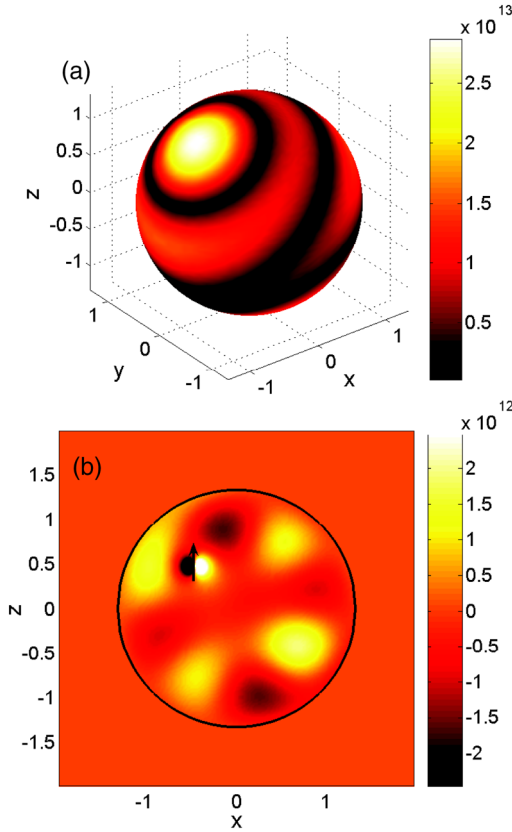


FIG. 4 (color online). Distributions of (a) the amplitude of V_E at the inner boundary and (b) H_y in the concealed region $r < R_1$ due to a unit electric dipole oriented in z direction and located at $(R_1/2, \pi/4, \pi)$ indicated by a small arrow in (b). $R_1 = 1.33\lambda_1$.

dipole, V_E vanishes while V_H survives. The cloak for the static magnetic field can be realized artificially [20]. Since there is no magnetic charge in nature, this magnetic surface voltage induced by a static magnetic dipole must exist in the form of its equivalent electric surface current. The inner boundary in Ref. [20] is made of superconductor which makes this surface current realizable.

Furthermore, the value of surface voltages can relate to another parameter directly. In derivation of the scalar potential in Ref. [11], the condition $\frac{\partial}{\partial r}\Psi_{\text{TM}} = i\omega\epsilon\mu\varphi_e$ [16], where φ_e represents the auxiliary electric scalar potential, is applied. It is interesting to note that $V_E = \varphi_e|_{r=R_1^-}$. Similarly, $V_H = \varphi_m|_{r=R_1^-}$. Thus these auxiliary scalar potentials, φ_e and φ_m , which were introduced originally as mathematical tools, have direct physical counterparts at the inner boundary of the cloak, i.e., surface voltages in this case.

In conclusion, the exact electromagnetic field solution to the Maxwell equations for a spherical cloak with an active source inside the concealed region is established. It is

shown that, an infinite polarization of the material at the inner boundary of the cloak will induce the electric and magnetic surface voltages, which prevent all waves from going out. These peculiar surface voltages are rare in nature, but they do not violate the Maxwell equations. The handedness of the polarization and phase information of the waves reflected from the inner boundary of the cloak are unchanged. Finally, these surface voltages due to an electric dipole inside the concealed region are found to be exactly equal to the auxiliary scalar potentials at the inner boundary of the cloak which gain physical counterparts in this case.

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