CHAPTER 11

EXTREME WAVE PREDICTION USING DIRECTIONAL DATA

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ABSTRACT

Potential inconsistencies in the predictions of long term wave heights can be experienced as a result of different methods of analysis possible when using directional wave data. This paper attempts to illustrate some of them. It involves analysis of two sets of directional wave data - one from a coastal location in the Irish Sea and another from an offshore location in the North Sea. An attempt is made to eliminate the discrepancies between the long term return-value wave height predictions based upon the conditional height distributions associated with different direction sectors and those derived from the omni-directional data set.

1. INTRODUCTION

The estimation of the long term wave heights is often made by fitting the wave data to a convenient probability distribution and extrapolating it upto the chosen 'return period' probabilities. A graphical technique using a probability paper is often employed.

The estimates made in this way normally do not account explicitly for the directions of the wave or those of winds generating them. It is implied by this procedure that all wave directions are equally likely to occur and that the conditions of wind speed and fetch generating the waves are directionally unbiased. If this is not the case, the method involves implicit extrapolation of sea states in direction sectors with restricted fetch beyond a physical upper limit.

Design engineers are increasingly asking for estimates of the probability of occurrence of extreme wave heights from different points of the compass. It is however observed that such estimates are not always consistent with the omni-directional estimates and it is quite

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possible that because of the sample size and fitting errors, the extreme wave height predictions from the worst direction may exceed the corresponding omni directional estimates.

Graham (1981) has suggested the simple factoring of the estimates from the worst direction to make them compatible with the omni-direction estimates to overcome this problem. An alternative approach is proposed here which is based on the convolution of the directional wave height distributions, that can subsequently be correlated against the omni-directional estimates to eliminate all inconsistencies.

2. DATA FOR THE STUDY

Two sets of directional wave data were available for the present studies. One **set** was the outcome of an earlier wave hindcasting study from seven years (1964-1970) of wind measurement at a nearshore site on the 'Mersey Bar' in the Irish Sea. The wave heights so derived have shown good correlation with observations at the site over a shorter period of one year (Burrows et al., 1985).

Figure 1 shows this site while Table 1 gives fetch length as well as corresponding estimates of significant wave height (Hs) generated from an extreme 'hurricane force' wind speed of 80 knots. It may be noted that the physical upper limit for the wave heights generated along different direction sectors varies from 4.12 to 10.06 m.

The other data set consisting of significant wave heights, average zero cross period (Tz) and wind direction, was from indtrumental recordings over the period 1975-1976 and 1980-1981 at a location in the North Sea (Figure 1). For such an open site, the fetch lengths corresponding to different directions were very large - the minimum value being 97 miles. Consequently, all of the corresponding Hs values at the 80 knots wind speed exceeded 11 m.

3. VARIABILITY IN DIRECTIONAL DISTRIBUTIONS

The data was categorized into different direction sectors and for each sector theoretical probability distributions of Gumbel, Weibull, extreme value Type III-U forms were fitted (expressions for these distributions are given in the Appendix). Estimates of the return value Hs were then abstracted.

Figure 2 shows the directional distributions of Hs values for the Irish Sea location along with selected 100-year return value probabilities. Table 1 gives the estimates of the '100 year' Hs values obtained by moments fitting to the Gumbel distribution in case of the Irish Sea data. The suitability of the Gumbel distribution fit for this

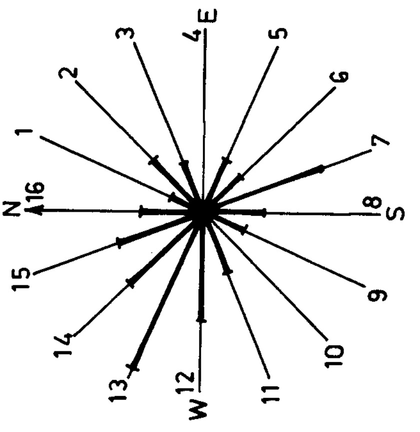
FIG.1 SITES OF THE DATA COLLECTION

Table 1 Estimation of '100 Year' Hs Values

(Gumbel Distribution (All Data)

(Site : Irish Sea)

Direction sector	Fetch (Nm)	Upper limit Hs(m)	Total number of sea states	'100 Year' Hs (m)	Percentage difference w.r t omni-direction estimate
1	33.49	7.16	503	4.65	− 41.36
2	25.16	6.66	1228	4.21	− 46.91
3	11.50	4.56	797	3.03	− 61.79
4	9.31	4.27	66	1.48	− 81.34
5	9.04	4.21	870	3.31	− 58.26
6	9.01	4.12	863	3.23	− 59.27
7	8.99	4.12	2190	3.70	− 53.34
8	9.98	4 42	972	3.33	− 58.01
9	9.84	4.42	752	3.61	− 54.48
10	11.80	4.57	135	1.56	− 80.33
11	16.06	5.18	1113	4.32	− 45.52
12	25.80	6.71	1851	5.66	− 28.63
13	56.97	8.84	2958	9.86	24.34
14	68.63	10.06	1781	11.10	39.97
15	51.28	8.54	1569	6.01	− 24.34
16	56.04	8.84	1003	8.59	8.32
Omni direction			18651	7.93	0.0



ROSE DIAGRAM INDICATING TOTAL NUMBER OF SEA STATES ALONG DIFFERENT DIRECTIONS

(SCALE - 1mm = 100 SEA STATES)

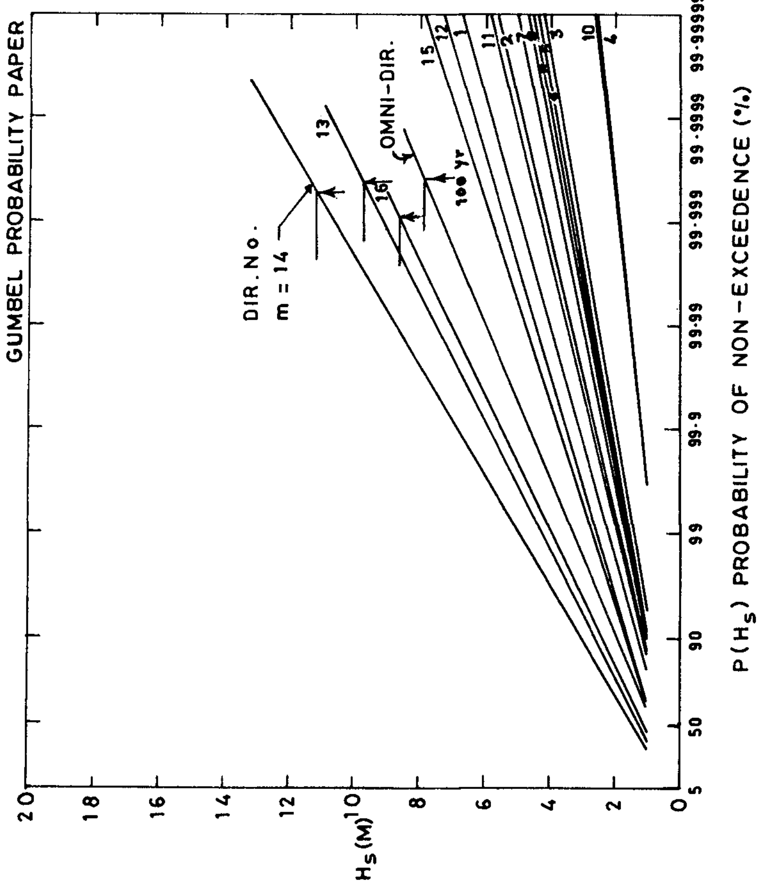


FIG. 2 DIRECTIONAL H_s DISTRIBUTIONS (Irish Sea Data)

location was checked (Deo and Burrows, 1985). It is immediately seen that the total number of sea states coming from different directions is not the same and hence the assumption to that effect, made implicitly in the normal extreme wave prediction procedure is not valid.

Figure 3 shows the similar distribution fittings to the North Sea data.

Both sets of data showed that predictions of extreme wave heights from certain worst direction sectors (e g. m = 13, 14, 16 in Table 1) exceed the equivalent prediction using the complete data set i.e. the normal omni-directional approach.

This outcome was found to be sensitive neither to the fitted distribution nor to the extrapolation techniques of moments or least squares. This behaviour departs from the outcome of statistical reasoning but can be explained by the different levels of uncertainty associated with the distribution fitting to data sets of different sizes.

One way to account for the above mentioned inconsistency between the directional and omni-directional estimates is to factor the former and make predictions from the worst directions compatible with the omni-directional distributions (Graham 1981) Whilst this satisfies the engineering requirements, it is not statistically correct but is nevertheless conservative. The error in this approach follows because the omni-directional predictions must exceed the worst direction values since there will generally be a finite chance of the extreme conditions (in 100 years, say) arising from other than the worst direction.

Herein, an alternative and statistically consistent technique is followed which is described in the following section.

4. CONVOLUTION OF CONDITIONAL (DIRECTIONAL) WAVE HEIGHT DISTRIBUTIONS

a) Long Term Distribution of Significant Wave Heights

The revised procedure to predict the return value estimates of Hs is as follows:

(i) Categorize the data into different direction sectors

(ii) Obtain the fitted conditional distribution of Hs for each direction

(iii) Introduce appropriate wave height ceilings to each direction sector

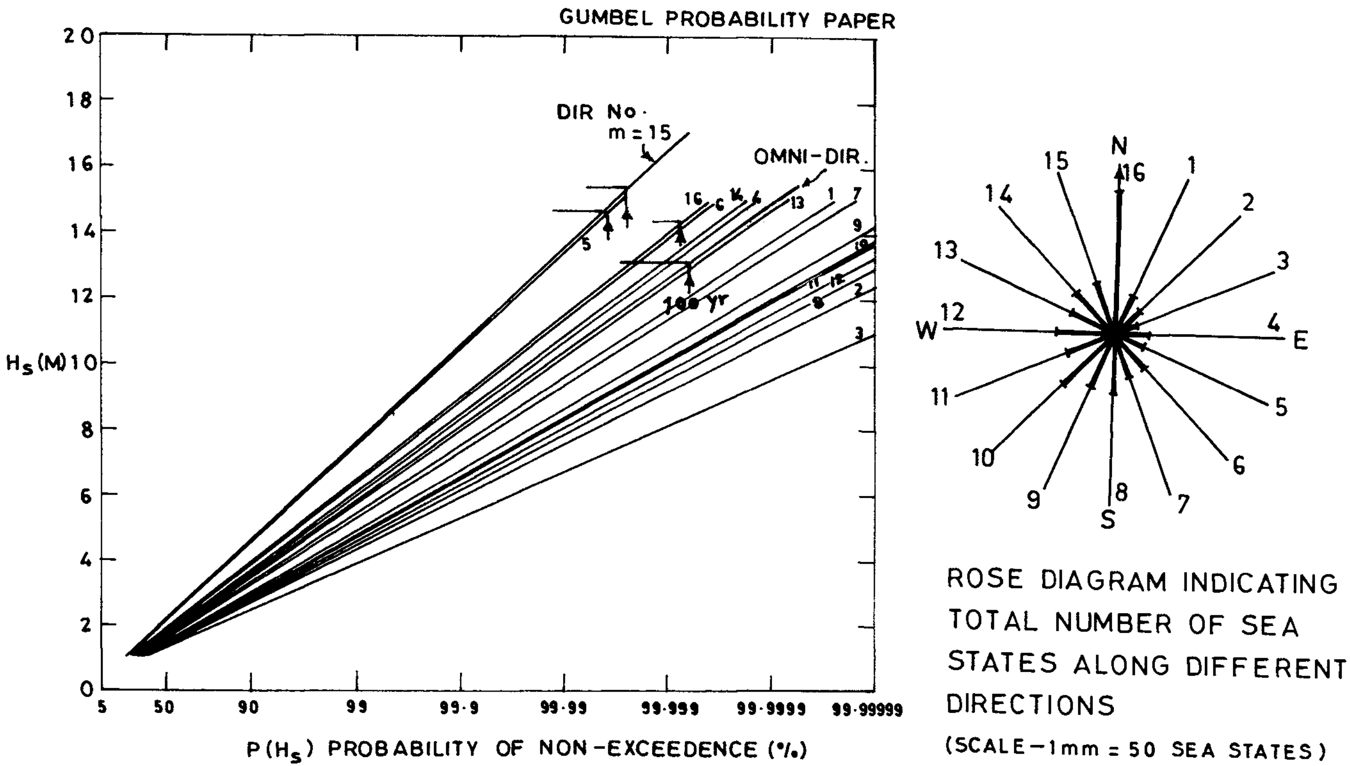


FIG.3 DIRECTIONAL H_S DISTRIBUTION (North Sea Data)

(iv) Convolute the different directional distributions and obtain an equivalent all direction distribution as follows:

$$P(Hs) = \sum_{\text{all } \theta} P(Hs/\theta)\, W(\theta) \qquad \ldots(1)$$

where

- P(Hs) = equivalent all direction distribution function of Hs
- θ = wave direction
- P(Hs/θ) = conditional distribution function of Hs for given θ
- W(θ) = weighting function representing the proportion of sea states along the direction θ in the entire population.

(v) Make predictions of long term Hs values on the basis of equation (1) <u>or</u> if there is no directional bias in fetch or wind field, divide the all-directional Hs value obtained as the outcome of equation (1) at the appropriate return period and use this as a factor for adjustment of the directional wave height predictions. This then makes them statistically consistent with the omni-directional values which in the situation may be viewed as the most reliable estimation since it is based on fitting and extrapolation of the entire data set.

b) Long Term Distribution of Individual Wave Heights (H)

This can be made by two alternative techniques:

Method 1 : (i) Use the equivalent all direction distribution of Hs values as per equation (1) to calculate the long term distribution P(H) in the normal manner (Battjes 1970; see Appendix) <u>or</u>

Method 2 : (i) Use the following convolution technique to get P(H) :

$$P(H) = \sum_{\text{all } \theta} P(H/\theta)\, W'(\theta) \qquad \ldots(2)$$

where

- P(H) = equivalent all-direction long term distribution
- P(H/θ) = conditional long term distribution of H along direction θ
- W'(θ) = weighting function representing proportion of individual waves in the entire population (Deo and Burrows 1985)

(ii) Predict return individual wave heights in a similar manner to step (v) in (a) above relating to significant wave height predictions.

For each data set the equivalent all-direction distribution of Hs was obtained according to equation (1) for each year separately as well as for the total data set. These distributions were then compared with the corresponding omni-directional distributions obtained by following the normal procedure. This was repeated by varying the underlying theoretical probability distribution as well as by varying the fitting technique.

Figure 4 shows a typical outcome of this exercise and it pertains to the case of Gumbel distribution fit to the Irish Sea data using the method of moments.

It was observed that the equivalent all-direction distribution does not follow the same theoretical form as the constituent conditional distributions (Gumbel, in the present case). This result is evident also from theoretical considerations. Further the introduction of appropriate wave height ceilings along different directions, in these cases, had a negligible effect on the resulting equivalent all-direction distribution. This is probably due to the fact that this imposition did not affect any of the worst directions that dictate the extreme tail of the distribution since the most frequent wind directions fall in the sectors of longest fetch.(Refer to limiting Hs values in Table 1). In other geographical circumstances this may not be the case. In the present case the effect of applying wave height limits for each direction is certainly overshadowed by other curve fitting and extrapolation uncertainties.

It was also noted from the above mentioned comparisons that the equivalent distribution resulting from convolution technique lies on the probability paper above the one obtained on the basis of the existing (omni-direction) procedure. This indicated that it would produce conservative estimates of the design wave heights. This outcome remained unchanged in case of the North Sea data as well (Figure 5). It was also not sensitive to the choice of the theoretical distributions and the fitting techniques.

Figure 6 pertains to the equivalent all-direction distribution of individual wave heights obtained by following the methods-1 and 2 as well as by using the normal procedure. This outcome is also in line with the one discussed above in case of Hs distributions.

The values of the 100-year return period wave heights were extracted from these plots and it was found that the equivalent all-direction distribution produced conservative estimates of Hs ranging from about 5 to 20 percent for all cases involved. The corresponding overestimation of individual wave heights by the use of methods-1 and 2 was 9 to 20 percent.

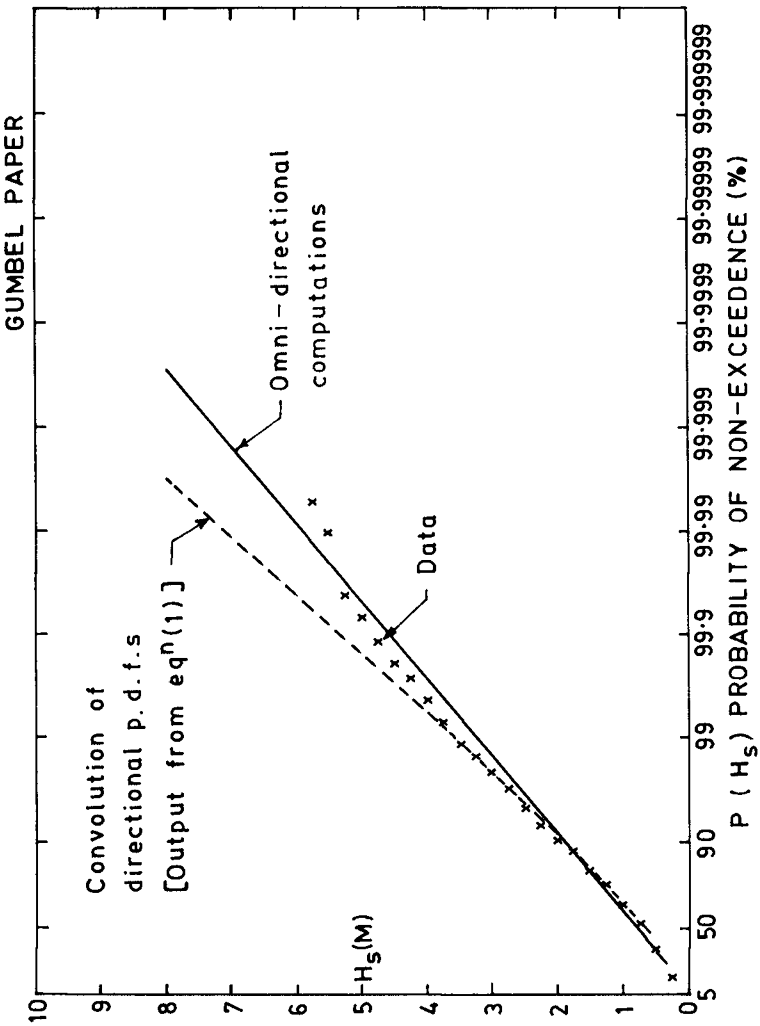


FIG. 4 COMPARISON OF H_s DISTRIBUTIONS (Irish Sea Data)

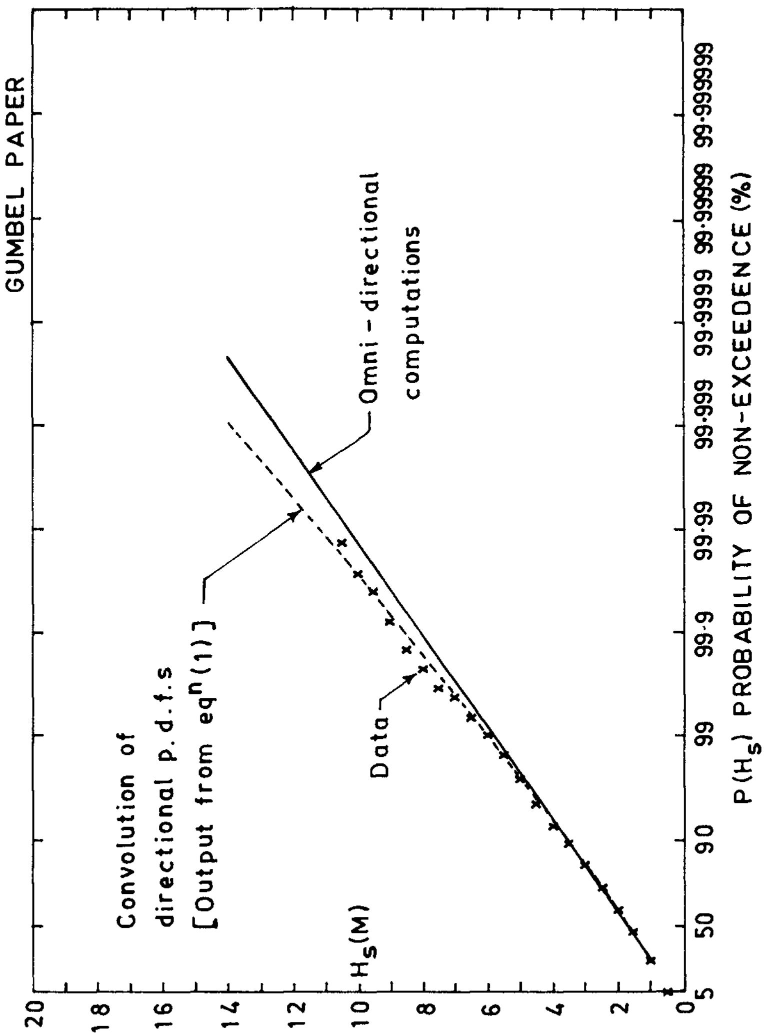


FIG. 5 COMPARISON OF H_s DISTRIBUTIONS (North Sea Data)

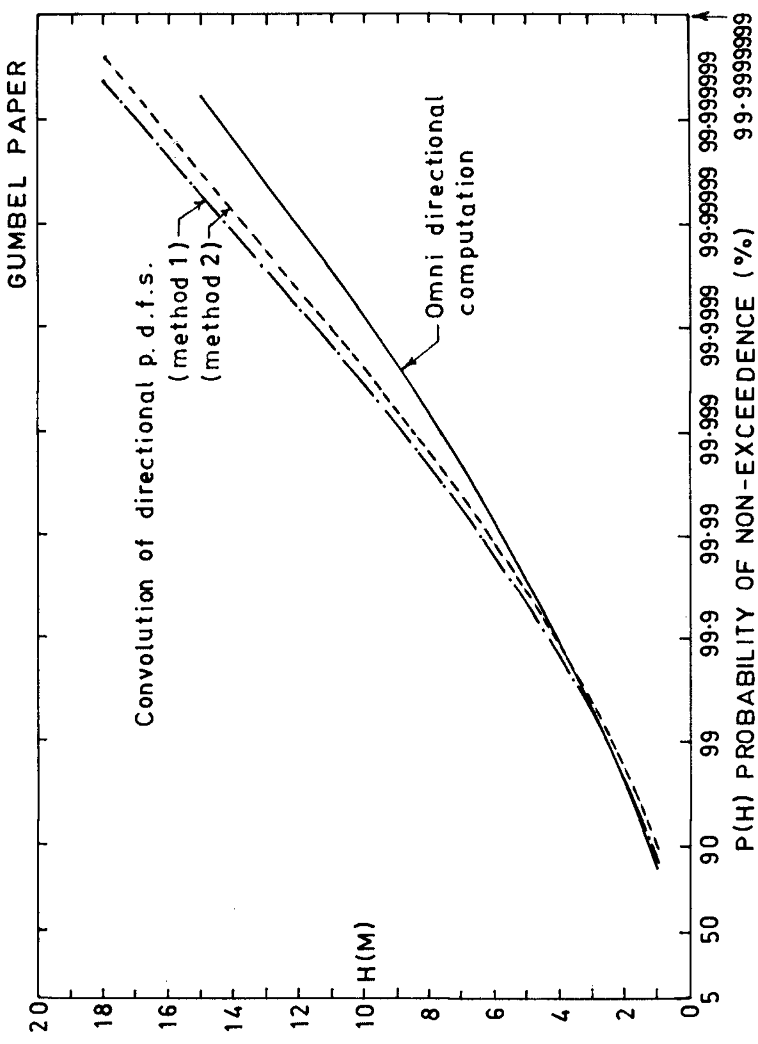


FIG. 6 COMPARISON OF 'H' DISTRIBUTIONS (Irish Sea Data)

5. CONCLUSIONS

(a) A discrepancy in the long term wave height predictions will generally arise if the directionality of the wave data is taken into account.

(b) The normal procedure to predict the long term values of the wave heights implicitly incorporates extrapolation of wave fields in certain direction sectors beyond potential physical limits. It is deficient, therefore, where severe directional bias in the fetch and wind field exists. In these circumstances, the methods presented here are more apporpriate, although they involve higher levels of uncertainties due to the smaller sample sizes considered in curve fitting and extrapolation procedures.

(c) When no such directional bias exists, the methodology enables the computation of a statistically consistent set of directional return-value wave heights which can be factored to bring them into line with the normal omni-direction calculations, having more statistical confidence due to the large sample sizes involved.

(d) Since all the procedures to predict the long term wave heights discussed here are essentially empirical in nature, their theoretical justifications are weak and hence the choice of a particular method should be left to the designer. A need for compatible sets of directional and omni-direction distributions, however, may call for the application of the methods presented herein.

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APPENDIX

Theoretical Probability Distributions Involved in the Analysis

1. Gumbel Distribution

(Or Extreme Value Type I Distribution)

$$P(Hs) = \exp\{-\exp[-\alpha(Hs - u)]\}$$

where P(Hs) = cumulative probability distribution of Hs

α and u = constants expressible in terms of the statistical moments of the data (References 3, 5)

2. Weibull Distribution

(Or Extreme Value Type III-L Distribution)

$$P(Hs) = 1 - \exp[\ (\frac{Hs - A}{B})^{c}\ ]$$

A, B, C = constants expressible in terms of the statistical moments of the data (References 3, 5)

3. Extreme Value Type III-u Distribution

$$P(Hs) = \exp[-(\frac{A - Hs}{B})^{c}]$$

A, B, C = constants expressible in terms of the statistical moments of the data (References 3, 5)

4 Long Term Distribution of Individual Wave Heights, H

$$P(H) \simeq 1 - \int_0^\infty\int_0^\infty \exp(-2H^2/Hs^2)\ \frac{Tz^{-1}}{\overline{Tz^{-1}}}\ P(Hs\ Tz)dHs\ dTz$$

where P(H) = long term distribution of H

$\overline{Tz^{-1}}$ = average number of waves per unit time in the long term

P(Hs Tz)dHsdTz = joint probability of occurrence of Hs and Tz