FABER-KRAHN INEQUALITIES IN SHARP QUANTITATIVE FORM

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In this talk we present a sharp quantitative improvement of the celebrated *Faber-Krahn inequality*. The latter asserts that balls uniquely minimize the first eigenvalue of the Dirichlet-Laplacian, among sets with given volume. We prove that indeed more can be said: the difference between the first eigenvalue $\lambda(\Omega)$ of a set Ω and that of a ball of the same volume controls the deviation from spherical symmetry of Ω. Moreover, such a control is the sharpest possible. This settles a conjecture by Bhattacharya, Nadirashvili and Weitsman. The result is valid for more general geometric quantities, like

$$\lambda_{2,q}(\Omega) = \min_{u \in W_0^{1,2}(\Omega)} \left\{ \int_\Omega |\nabla u|^2 \,:\, \|u\|_{L^q(\Omega)} = 1 \right\}.$$

The proof is based on various reduction steps: among these, a central role is played by a Selection Principle for the torsional rigidity functional, which essentially permits to reduce the task to prove the desired result for small smooth deformations of a ball.

The result here presented is contained in a recent joint paper with Lorenzo Brasco (Marseille) and Bozhidar Velichkov (Pisa).